# Order and Disorder in Open Systems

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In isolated physical systems differences in pressure, temperature, and chemical potential tend to even out. This principle of decay is called the second law of thermodynamics. It can be derived from the assumed randomness of molecular motion. The second law was postulated by the French physicist Sadi Carnot in his 1824 paper *Reflections on the Motive Power of Fire* [1], which presented the view that the work done by heat engines, including car engines are due to the flow of heat from a hot to cold body. Entropy is a measure of how much this evening-out process has progressed. Entropy is a measure of disorder. The entropy change $dS$ of a system undergoing any infinitesimal reversible process is given by $dq / T$, where $dq$ is the heat supplied to the system and $T$ is the absolute temperature of the system. The second law of thermodynamics states that the entropy in an isolated system increases. This means that the limiting state of an isolated system is a state of maximum disorder.

In open systems the situation is different. For instance, when a heat flow is applied to an object, temperature differences within the object will not even out. In the limiting state a pattern of temperature differences within the object accommodates the flow of the heat through the object. Due to the pattern of temperature differences, the entropy of the system is not at its maximum and the system is in an ordered state. If the system is initially in a patternless state, its entropy decreases over time. However, if the initial state has more structure than the limiting state, its entropy will increase over time. Therefore, in open systems the entropy can increase or decrease over time. This phenomenon is illustrated in Fig. 1. In this experiment several hundred conducting particles are placed in a horizontal dish with a circular grounded electrode [2]. When charges are sprayed from above, the particles agglomerate into fractal patterns. Figure 1 shows pictures of an experiment where the particles are initially randomly distributed. In this case, the entropy decreases, because the limiting pattern is not random [2]. By way of comparison, Figure 2 shows pictures of an experiment where the particles are initially in the center. In this case, the entropy increases, because the limiting state has a much more complicated structure.

Since there is no clear trend for the entropy in open systems, scientists have been looking for other organizing principles. Onsager proposed in 1931 reciprocal relations between flows and forces in open systems [3] and received the Nobel Prize in Chemistry for this work in 1968. For example in a fluid, temperature differences induce heat flow and pressure differences cause matter to flow. What is less obvious is that if the pressure is constant, temperature differences can cause matter flow (convection) and pressure differences can cause heat flow even if the temperature is constant. Onsager discovered that the heat flow per unit of pressure difference and the matter flow per unit of temperature difference are equal. Onsager's theory applies if the system is locally in equilibrium, and if inertial terms, such as Coriolis forces and magnetic forces, are small.

In 1957 Ziegler, and others after, proposed a principle of Maximum rate of Entropy Production (MEP): The steady macrostate observed corresponds to microdynamics that maximize thermodynamic entropy production [4]. MEP applies to systems with a gradient dynamics and states that the system chooses the path that creates disorder at the largest rate possible. For instance, if an object is submerged in oil and slowly slides down a two-dimensional uneven slope due to gravity, it will choose the path of steepest decent and heat up the oil at the fastest rate possible. This increases the entropy at the largest rate possible. Again, the theory applies if the system is locally in equilibrium, and if inertial terms are small.

While MEP describes the path leading towards the limiting state, Ilya Prigogine's principle of minimum entropy production describes the rate at which disorder is produced as the system approaches very near it limiting state [5]. Prigogine received the Nobel Prize in Chemistry in 1977 on his work on dissipative structures, complex systems, and irreversibility. Prigogine's principle has the same limitations as those mentioned earlier. In addition, the system must be very close to equilibrium [6], the flow through the system boundary must be kept constant, and the temperature must be kept constant throughout the system. For example, Prigogine's law and Onsager's theory predict a linear temperature profile for heat flow through a rod; whereas, convection in a fluid, a far-from-equilibrium steady state, is not covered [7]. A principle, related to Prigogine's, but with a larger range of applicability, is the Minimum Resistance Principle (MRP): In an open system the limiting resistance to the flow is minimal.

Figure 3 illustrates the MRP. There are two fixed electrodes submerged in viscous oil with high resistivity. The electrodes are attached to a battery. The temperature is kept constant. Between the two electrodes there is a conducting particle. Due to the induced charges the particle moves to the center. The resistance $R$ between the electrodes depends on the distance of the particle from the electrodes. If the particle is in the center, the resistance is smallest and it increases monotonically if the particle if further outward. If the battery provides a constant voltage $V$, then the Ohmic heating of the oil P is proportional to $V^2 / R$ and reaches a maximum at the limiting state, because the resistance is minimal. In contrast, if the battery provides a constant current $I$ then the Ohmic heating of the oil is proportional to $I^2 R$ and reaches a minimum at the limiting state, because the resistance is minimal. In both cases, the specific entropy production of the limiting state $p = P / I^2$ is minimal. This means the rate at which disorder is created is minimal.

Since the availability of energy is one of the prerequisites of life, MRP is broadly important: It states that there is a tendency in systems with a constant flow to minimize energy consumption. Nonetheless, there are several important lessons to bear in mind when applying thermodynamic principles to natural organization. First, as we illustrated, one must keep the range of applicability clearly in sight. Different boundary conditions can change a maximum into a minimum, for example, or, worse, simply render the "Principle'" incorrect. Second, one must always keep the fundamental dynamics of the system in mind. This is what determines which structures and behaviors appear. And those, in turn, dictate which, if any, principles describe their emergence. In a sense, this is why statistical mechanics came into being during the 20th century. It is the attempt to derive macroscopic properties from microscopic dynamics. Finally, given the astounding complexity of biological systems and the sheer difficulty of directly applying statistical mechanics, the search for thermodynamic principles of organization will continue. The promise is to distill Nature's multifarious complication into a few easily understood and easily applied principles.

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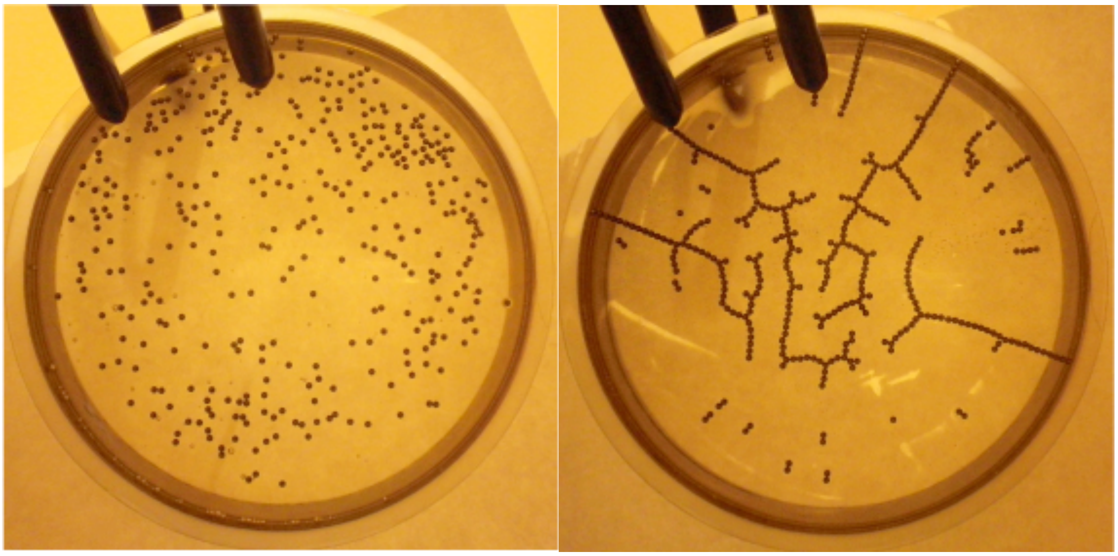

*Figure 1: The initial state (left) and the limiting state (right) of a particle agglomeration process driven by a high-voltage current. The order is increasing, because the initial state is random.*

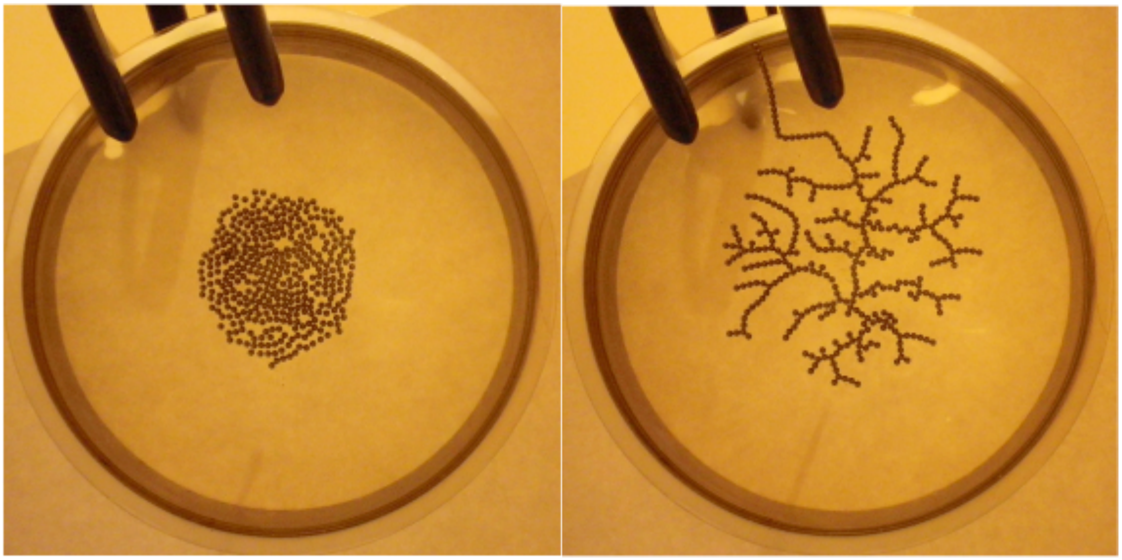

*Figure 2: The initial state (left) and the limiting state (right) of a particle agglomeration process driven by a high-voltage current. The order is decreasing, because the initial state is highly ordered.*

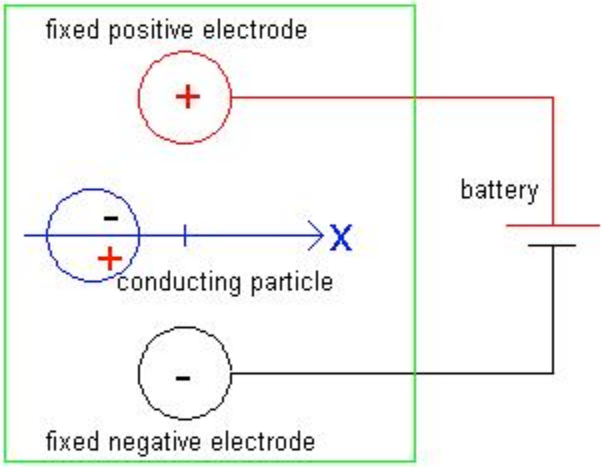

*Figure 3: Illustration of the minimum resistance principle: A conducting particle between two electrodes moves to the state of minimum entropy production.*