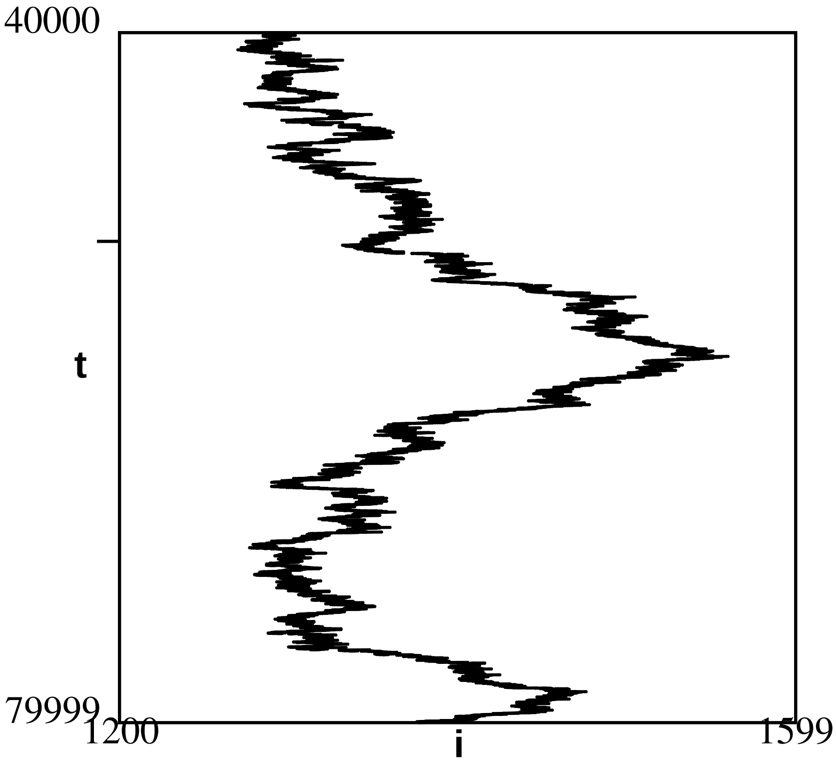

Fig. 10.24 Expanded view of a part of fig. 10.23. Every iteration is displayed. The tick mark at $t = 52000$ shows the location of fig. 10.25. The apparent break in the trajectory just below that point shows the recognized dislocation making a large lateral jump to the right in a single time step.

of the action of the CA rule. In this subsection, we will show how the domain-wall analysis of a CA helps with the process of *experimentally* determining the motion of its dislocations—so that the methods of this section can be applied equally well to other rules.

As a first step, we show that the two-domain language $\Lambda_{18}^{0,0}$, consisting of all states with a single dislocation, is temporally invariant. This is less straightforward than the proof of domain invariance because $\Lambda_{18}^{0,0}$ is not spatially homogeneous. The homogeneity is broken by the presence of the dislocation: a given cell is either to the left or to the right of it, or directly under it. For reasons that will become clear later, we also include in $\Lambda_{18}^{0,0}$ the null language 0^* consisting of the null states for all lattice sizes. The minimal DFA for $\Lambda_{18}^{0,0}$ is shown in fig. 10.27. Note that apart from the synchronizing start state, it consists of two copies of the machine for Λ_{18}^{0} connected by a dislocation.

Proposition. $\Lambda_{18}^{0,0}$ is a time-invariant (fixed-point) regular language, although it is not a domain.

Proof: We begin by calculating the image of $\mathrm{M}\left(\Lambda_{18}^{0,0}\right)$ under ECA 18 by means of the FME algorithm. Fig. 10.28 shows the machine obtained part of the way through the calculation, immediately after composing $\mathrm{M}\left(\Lambda_{18}^{0,0}\right)$ with the ECA 18 update transducer

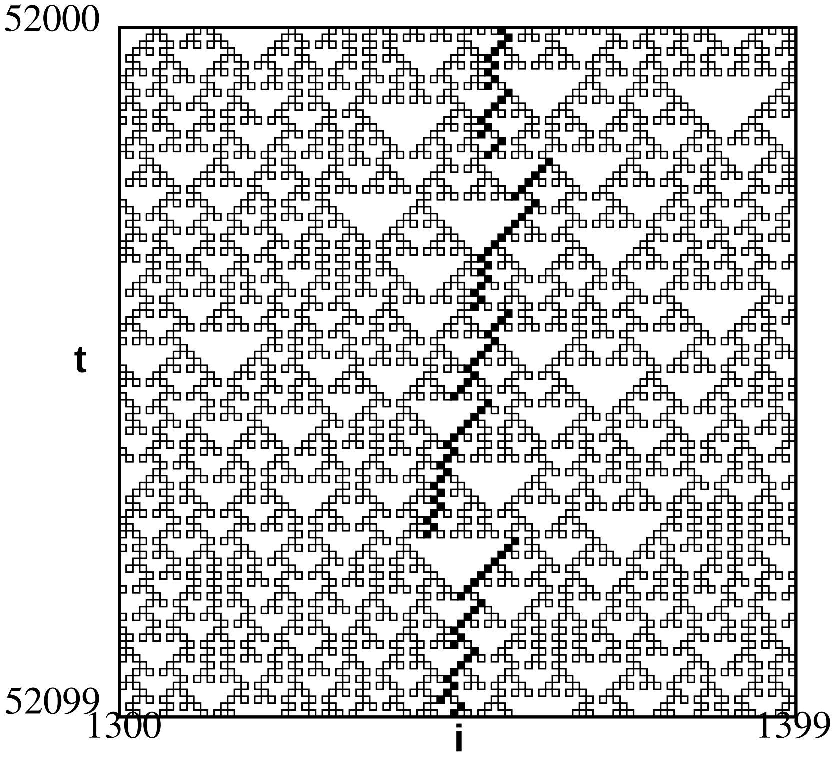

Fig. 10.25 Trajectory of a single dislocation recognized by $T_{\Lambda^0_{18}}$. This is an expanded view of a part of fig. 10.24, located at the tick mark on the vertical axis of that figure. The region shown is in fact smaller than the width of the tick.

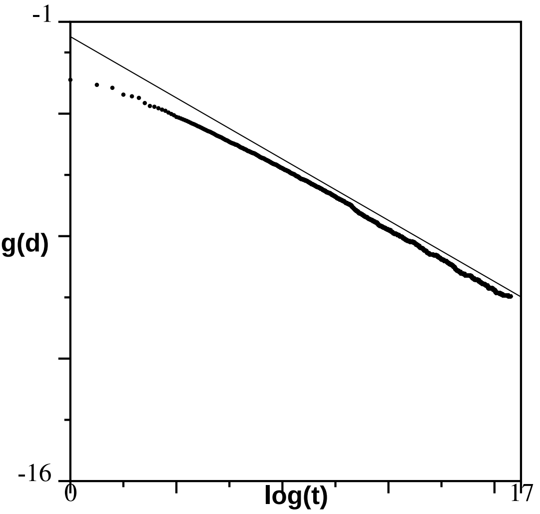

Fig. 10.26 Dislocation number density vs. time on a periodic lattice of 10^5 sites. The initial pattern was random. The density is plotted at each of $100,000$ time steps. Again, logarithms are base 2. The straight line has slope $\frac{1}{2}$.

T_{18}. After finishing the FME algorithm, we find that the resulting machine is slightly different from $M\left(\Lambda^{0,0}_{18}\right)$: there is a set of input strings accepted by $M\left(\Lambda^{0,0}_{18}\right)$ whose images under ECA 18 are accepted by $M\left(\Lambda^{0}_{18}\right)$. This is a spurious effect due to the