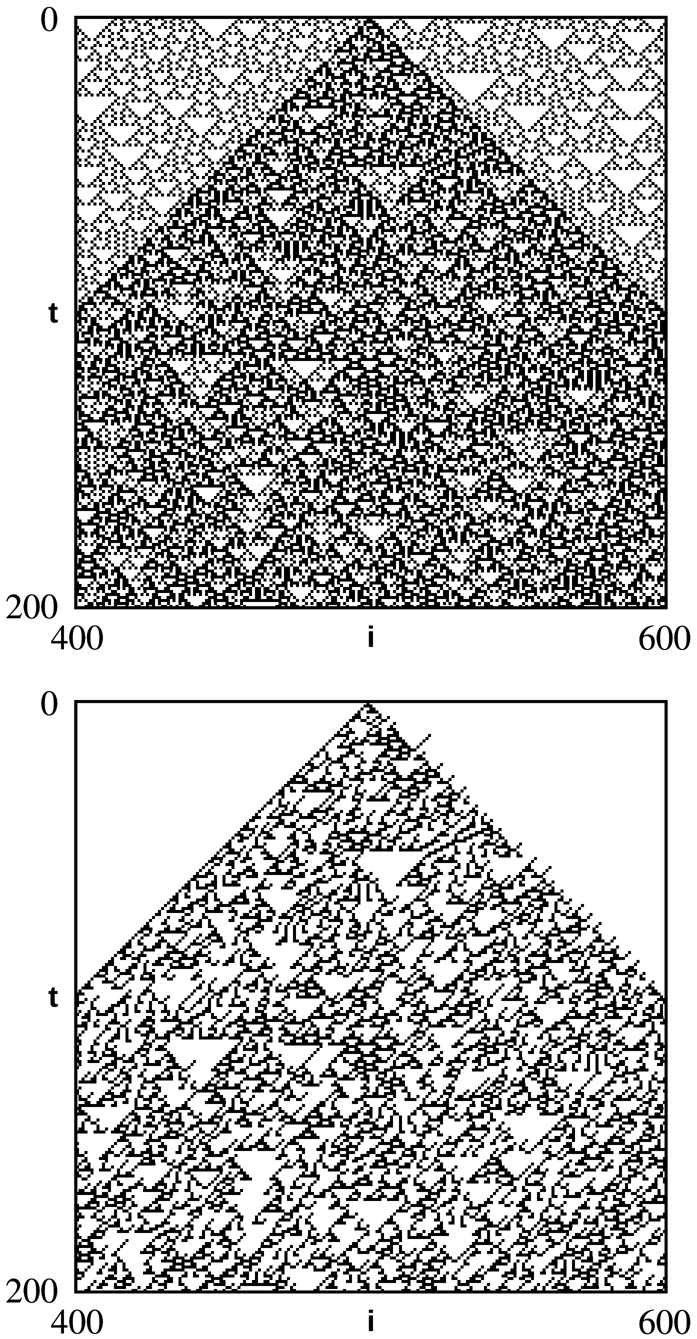

Fig. 10.22 Raw and filtered space-time plots of ECA 90 with an initial condition consisting of two domains in Λ^0_{18} with a single dislocation between. The filter is ECA 18's domain transducer. Note the rapid multiplication of dislocations, unlike the similar picture for ECA 18.

question is to decide what happens to these dislocations as the state evolves in time. One fundamental property of dislocations in ECA 18 is shown in the following lemma.

Lemma.[65] The number of dislocations in an arbitrary CA state evolving under ECA 18 decreases or remains constant in time. Furthermore, for all states except those that evolve immediately to the null state (i.e., all s such that $\Phi_{18}(\mathrm{s}) \notin 0^*$), dislocations always annihilate in pairs.

Proof: (Outline) From the CA rule table. There are three cases, each concerning a spatially local configuration: (i) no dislocations, (ii) an isolated dislocation, and (iii) two or more nearby dislocations that may or may not collide during the next time step. The first case has been addressed by showing that Λ^0_{18} is an invariant language: no dislocations are created in time. The single-dislocation case is proven by enumerating all local neighborhoods that contain one dislocation. The result is that the dislocation moves to the left or right, but does not disappear or multiply. Thus, when the initial ensemble $\mathcal{L}_0$ consists of configurations in Λ^0_{18} or $\Lambda^{0,0}_{18}$, no dislocations are created in time. Case (iii) is proven as follows. The local action of the update rule ensures that unless the dislocations' closest boundaries are separated by less than $2r+1=5$ cells, the dislocations do not interact, and can be treated under case (ii) above. The sole exception to this is when two dislocations and the cells between them are mapped to a region of consecutive 0s, in which case they annihilate and the number of dislocations decreases by two. Enumeration of all 5-cell neighborhoods shows that the number of dislocations either remains the same or decreases by an even number. ▌

In addition, the domain walls appear to execute a random walk on the lattice. This suggests a stochastic model of dislocation motion, in which they perform independent random walks on the lattice until they collide, at which time they annihilate. The stochastic model is further motivated by a proof that the motion of a single dislocation in ECA 18 performs a random walk.[35] The classical model of diffusing annihilating particles was studied by Griffeath,[43] who proved that with probability 1 the density of dislocations asymptotically falls to zero. In essence, diffusive motion ensures that almost every dislocation eventually will collide with another and the pair will annihilate. On finite lattices, the dislocations annihilate as far as possible. For infinite lattices the random walk implies that the limiting set of states with a positive dislocation density has vanishing size relative to the full state space.

Dislocation decay statistics

For ECA 18 running the domain transducer $\mathrm{T}_{\Lambda^0_{18}}$ over the space-time data gives an encoded picture that shows only the dislocation trajectories. Figs. 10.23, 10.24, and 10.25 are typical of simulations starting with a random initial condition. Each of these pictures shows a different space-time region of the same simulation. Note the progressively reduced scale in figs. 10.23 and 10.24 used to make the diffusive behavior more apparent. The thickness of the lines is due to the figure's finite resolution. In fig. 10.25, blackened squares correspond to dislocations and hollow squares show the nonzero cells of the original pattern.

To complete the statistical analysis of ECA 18 diffusion dynamics facilitated by the dislocation detection transducer, we re-measure the temporal decay rate of the dislocation number density. Grassberger[41] estimated it using a rarefied dislocation