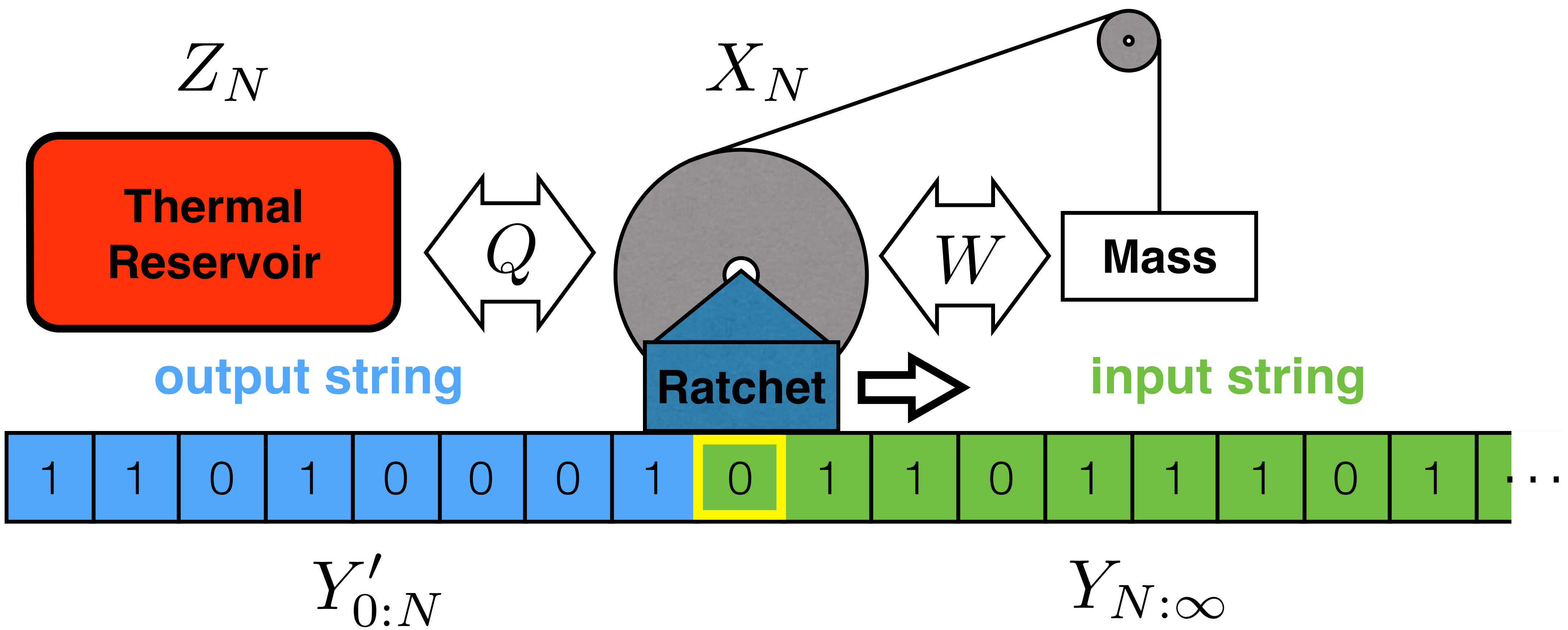
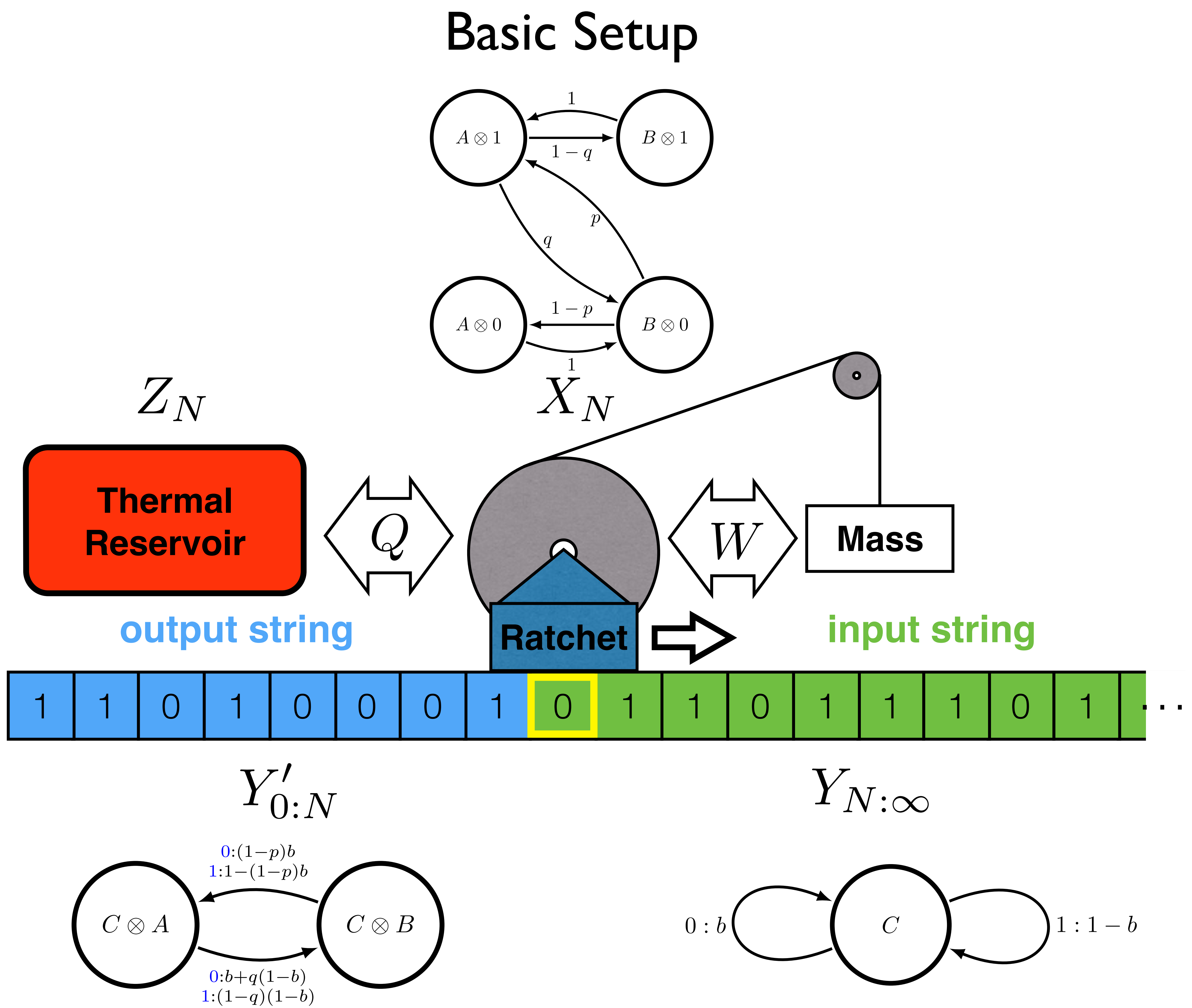
Memory and correlation in autonomous Maxwellian Demons

Alec Boyd, Dibyendu Mandal, James Crutchfield

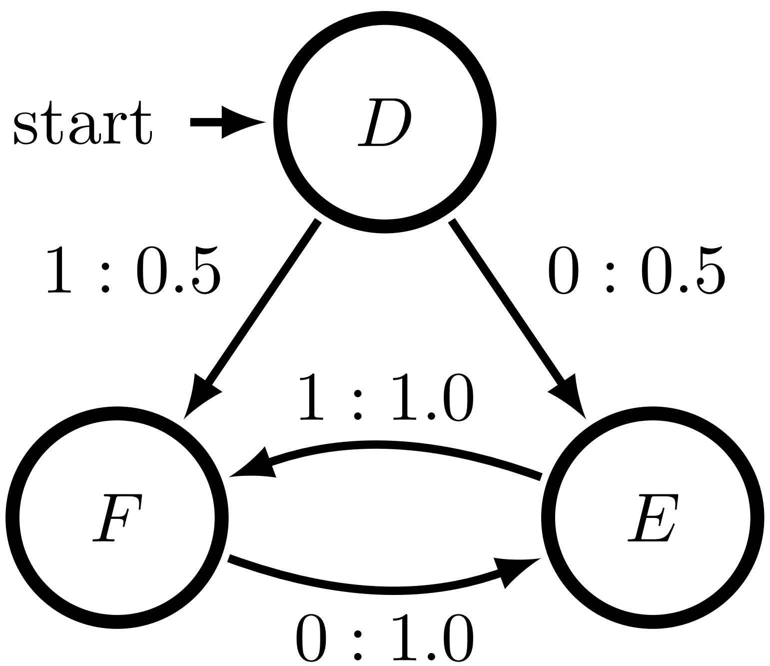




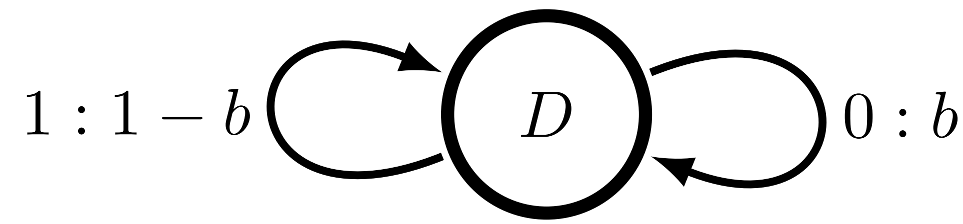
Basic Setup
$A \otimes 1$
$B \otimes 1$
$A \otimes 0$
$B \otimes 0$
1
$1-q$
p
q
$1-p$
1
Z_N
X_N
Thermal Reservoir
Q
W
Mass
Ratchet
output string
input string
1 1 0 1 0 0 0 1 0 1 1 0 1 1 1 0 1 ...
$Y'_{0:N}$
$Y_{N:\infty}$
$C \otimes A$
$C \otimes B$
0:$(1-p)b$
1:$1-(1-p)b$
0:$b+q(1-b)$
1:$(1-q)(1-b)$
C
$0:b$
$1:1-b$

Memory in Bit Strings

Memoryful:



Memoryless:



Bit strings generated by HMMs

Memory in Bit Strings

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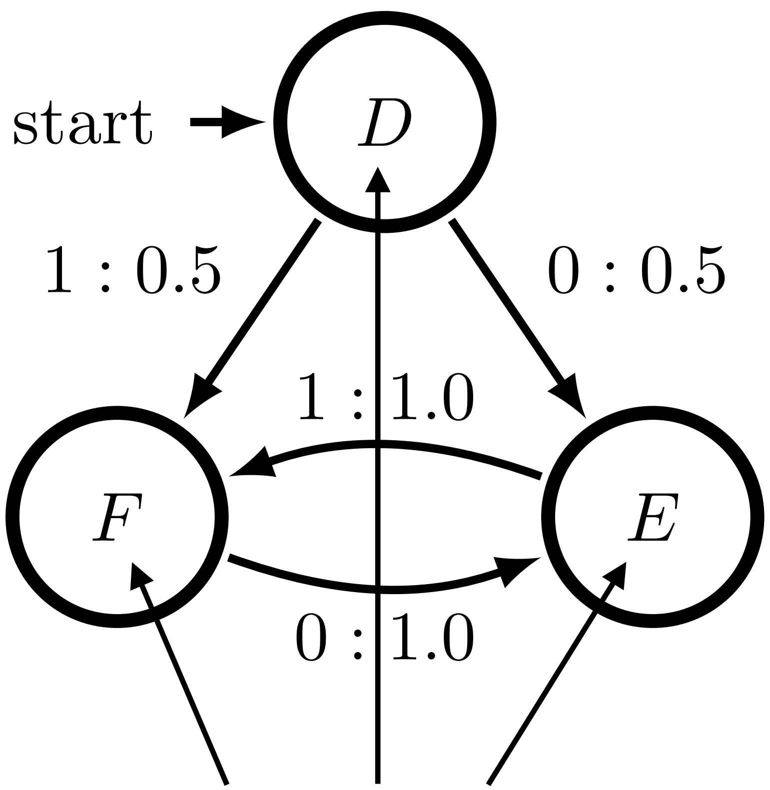
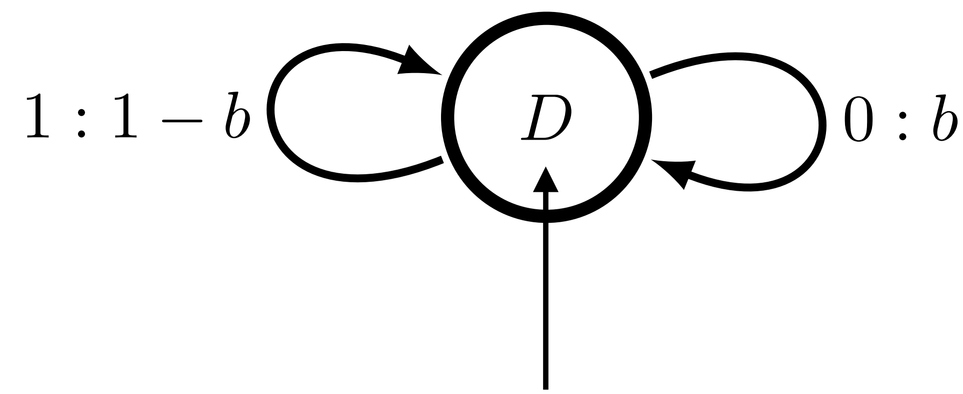


multiple hidden states

$$T^{(y_i)}_{s_i \to s_{i+1}} = \Pr(Y_i = y_i, S_{i+1} = s_{i+1} | S_i = s_i)$$

$$\Pr(Y_{0:\infty} = y_{0:\infty}) = \sum_{s_{0:\infty} \in \mathcal{S}^\infty} \Pr(S_0 = s_0) \prod_{i=0}^{\infty} T^{(y_i)}_{s_i \to s_{i+1}}$$

Memoryless:



single hidden state

$$\Pr(Y_{0:\infty} = y_{0:\infty}) = \prod_{i=0}^{\infty} \Pr(Y_i = y_i)$$

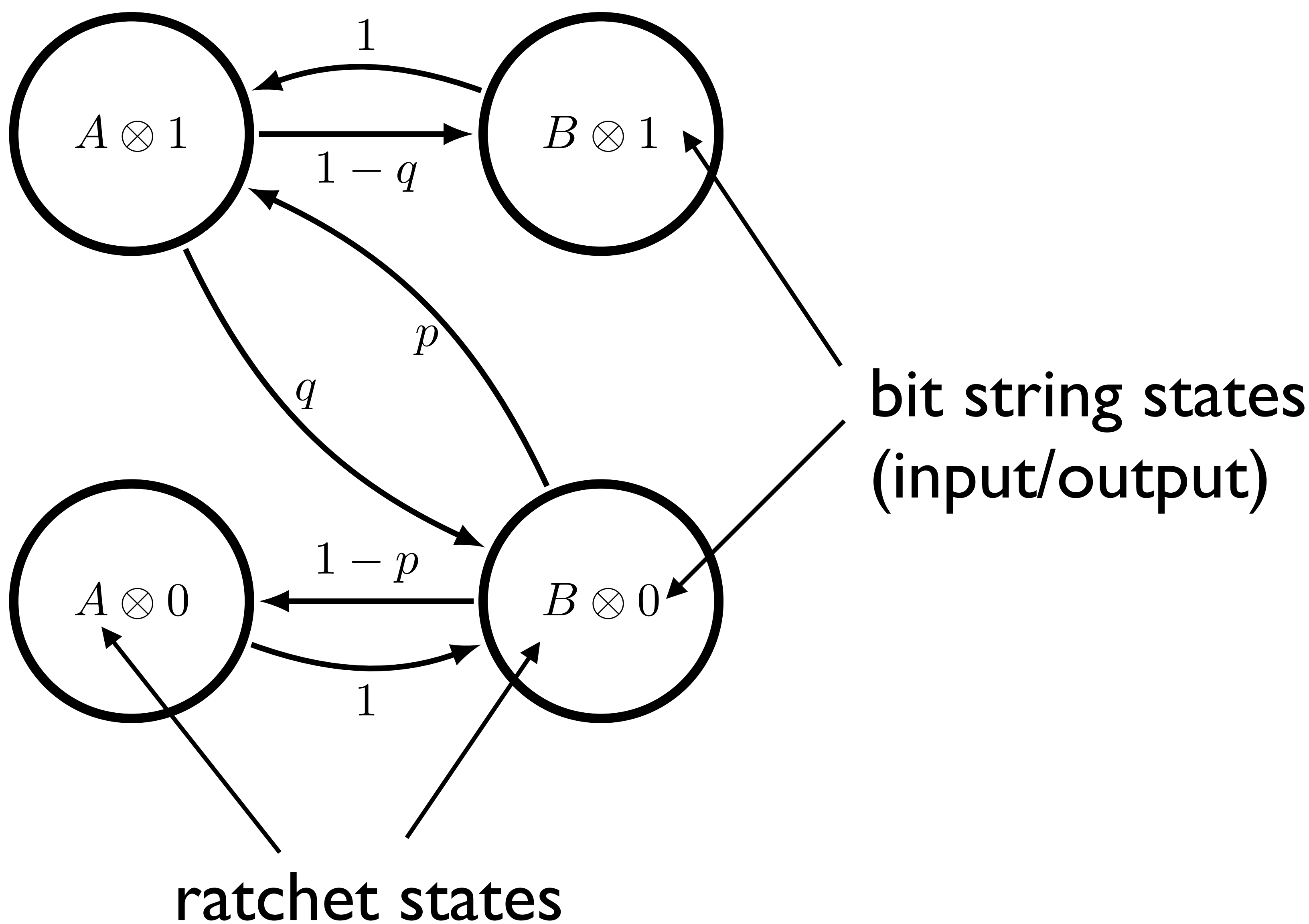
$$\Pr(Y_i = y) = \Pr(Y_j = y) \, \forall \, i, j$$

Bit strings generated by HMMs

Ratchet Bit Interaction

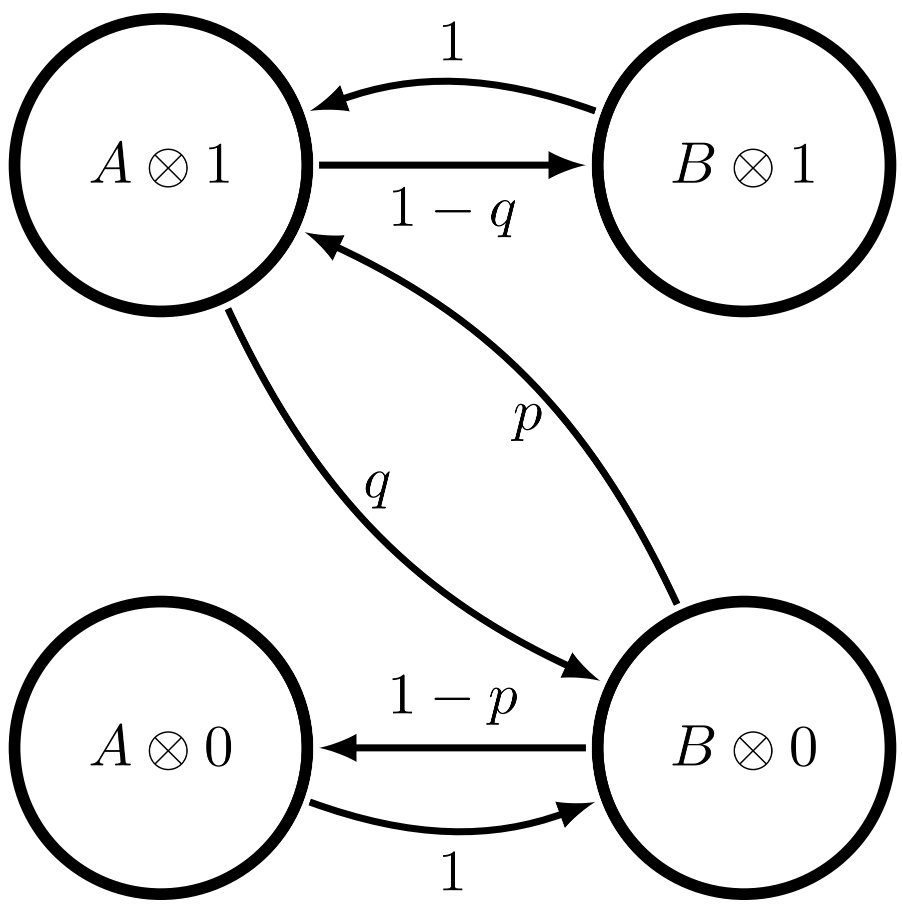
Ratchet and bit interact by evolving according to Markovian chain

$$M_{x_i \otimes y_i \to x_{i+1} \otimes y_i} = \Pr(X_{i+1} = x_{i+1}, Y'_i = y'_i | X_i = x_i, Y_i = y_i)$$

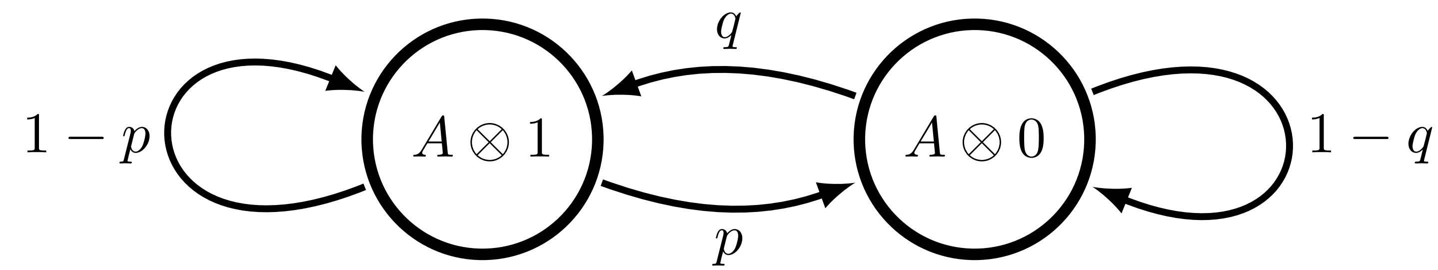


Memory in Ratchets

Memoryful:



Memoryless:



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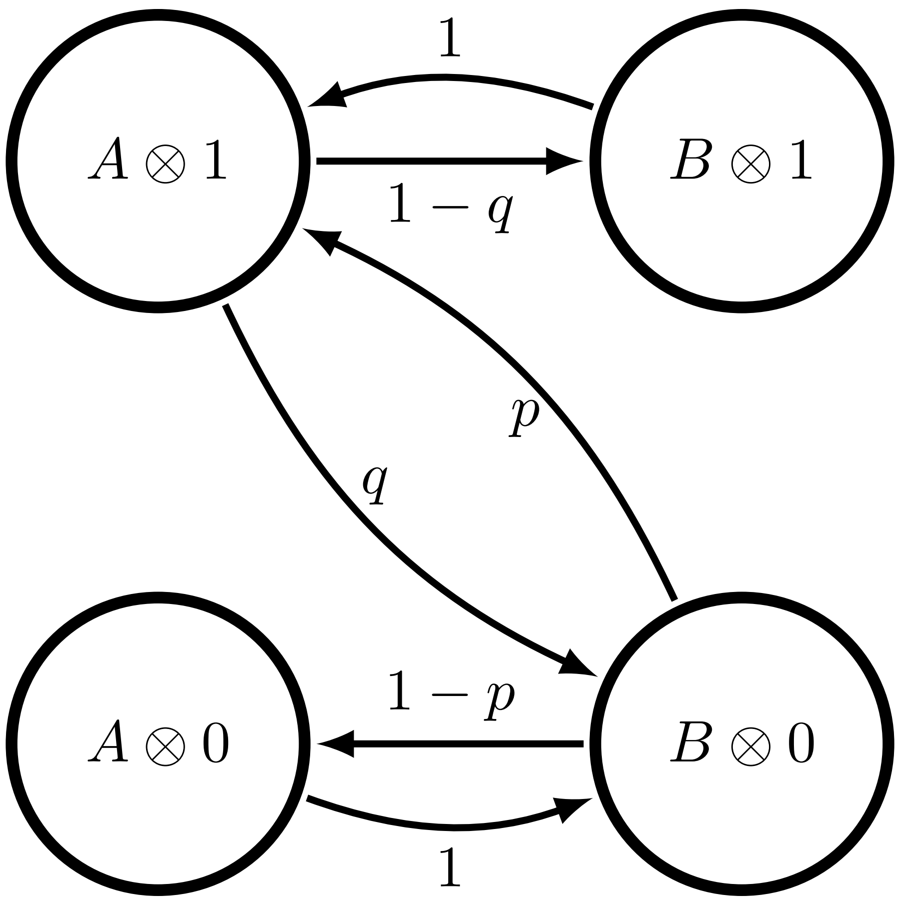
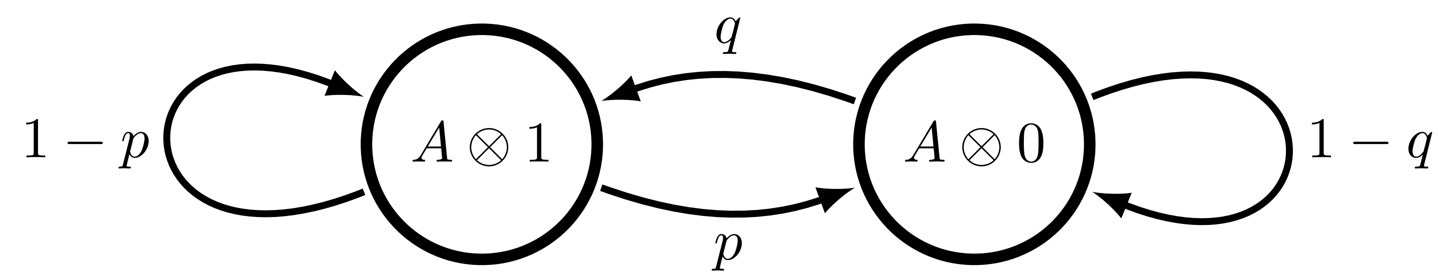


Memoryless:



Both have two bit states:

$$\mathcal{Y} = \{0, 1\}$$

multiple ratchet states:

$$\mathcal{X} = \{A, B\}$$

one ratchet state:

$$\mathcal{X} = \{A\}$$

Memory and Information Bounds

Two candidate bounds on work production:

$$W \leq k_B T \ln 2 \, \Delta h_\mu$$

$$W \leq k_B T \ln 2 \, \Delta H(1)$$

Ratchet

		memoryless	memoryful
Input	memoryless	$W \leq \Delta H(1) = \Delta h_\mu$	$W \leq \Delta h_\mu \leq \Delta H(1)$
	memoryful	$W \leq \Delta H(1) \leq \Delta h_\mu$	$W \leq \Delta h_\mu$ ~~$W \leq \Delta H(1)$~~

$(k_B T \ln 2 = 1)$

Work Production

$$\langle W \rangle = \sum_{x,x' \in \mathcal{X}\; y,y' \in \mathcal{Y}} \pi_{x,y} M_{x \otimes y \to x' \otimes y'} \Delta E_{x \otimes y \to x' \otimes y'}$$

The stationary distribution is highly dependent on the input

$$\pi_{x,y} = \lim_{N \to \infty} \Pr(X_N = x, Y_N = y)$$

Remaining factors are dependent on the ratchet transitions

$$\Delta E_{x \otimes y \to x' \otimes y'} = k_B T \ln \frac{M_{x' \otimes y' \to x \otimes y}}{M_{x \otimes y \to x' \otimes y'}}$$

Memoryless Ratchets

Work done is only dependent on input symbol bias:

$$\langle W \rangle = \sum_{x,x' \in \mathcal{X}\, y,y' \in \mathcal{Y}} \boxed{\pi_{x,y}} M_{x \otimes y \rightarrow x' \otimes y'} \Delta E_{x \otimes y \rightarrow x' \otimes y'}$$

$$\uparrow$$

$$\lim_{N \rightarrow \infty} \Pr(Y_N = y)$$

Work done is insensitive to order beyond single symbols. (Need memory to leverage correlations)

Memoryless Ratchets

ANY INPUT: Change in single symbol entropy is a stricter and accurate bound (assumes no "feedback", meaning no memory of the past outputs) N. Merhav, J. Stat. Mech.: Theor. Exp. (2015) P06037

$$W \leq k_B T \ln 2 \, \Delta H(1) \leq k_B T \ln 2 \, \Delta h_\mu$$

MEMORYLESS INPUT: The outputs are also memoryless, so

$$\Delta h_\mu = h'_\mu - h_\mu = H[Y'_i] - H[Y_i] = \Delta H(1)$$

$$\langle W \rangle \leq k_B T \ln 2 \, \Delta H(1) = k_B T \ln 2 \, \Delta h_\mu$$

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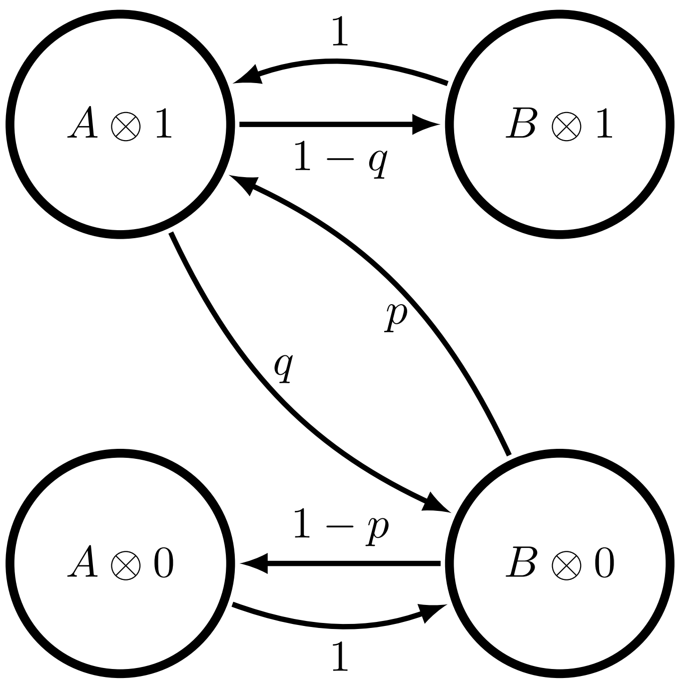
Memoryful Ratchets Memoryless Inputs

This is the case we've already demonstrated

A. B. Boyd, D. Mandal, and J. P. Crutchfield, arXiv: 1507.01537

Ratchet:



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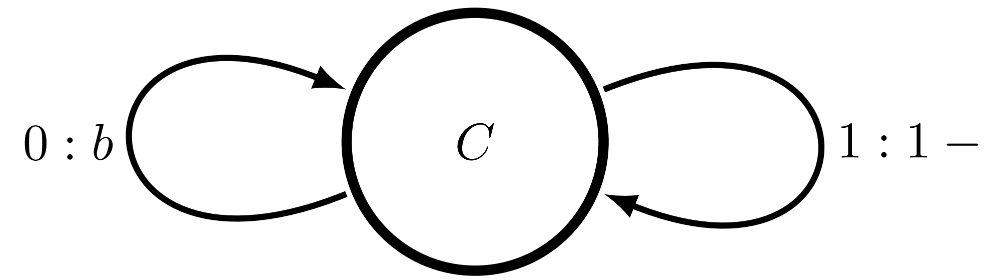

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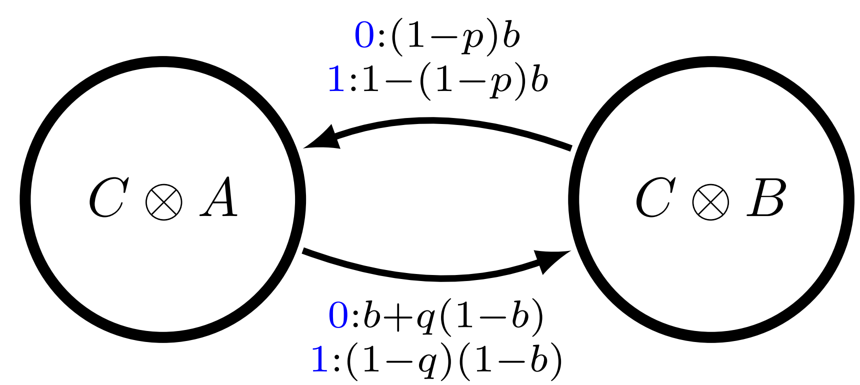

Input is IID, but output isn't necessarily.

$$\Delta h_\mu = h'_\mu - h_\mu = h'_\mu - H[Y_i] \leq H[Y'_i] - H[Y_i] = \Delta H(1)$$

$$\langle W \rangle \leq k_B T \ln 2 \, \Delta h_\mu \leq k_B T \ln 2 \, \Delta H(1)$$

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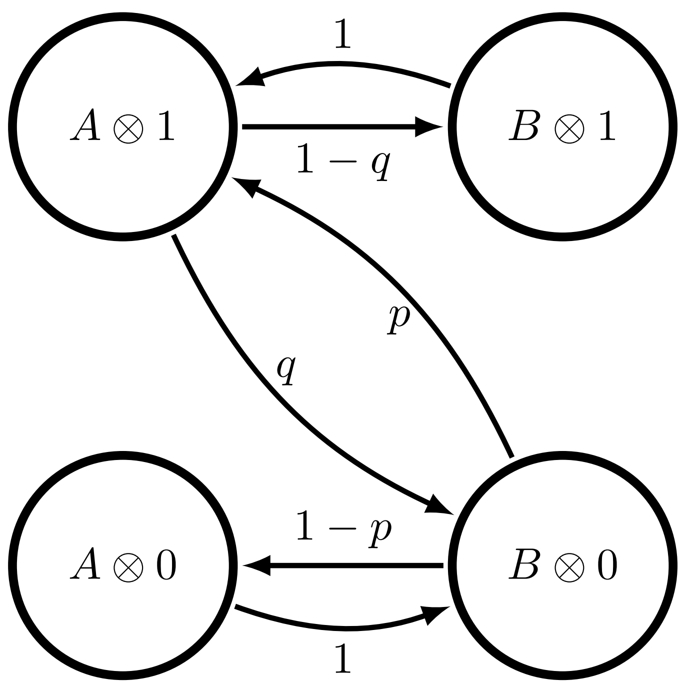

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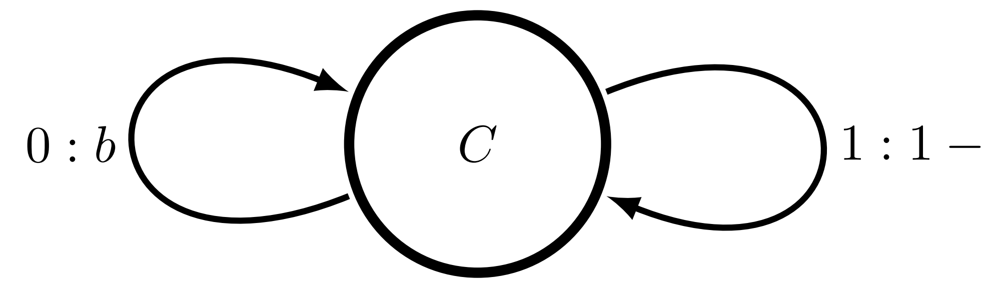
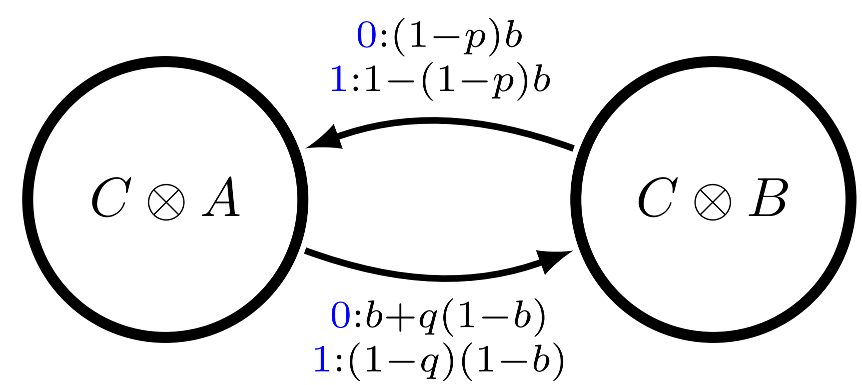


Out:



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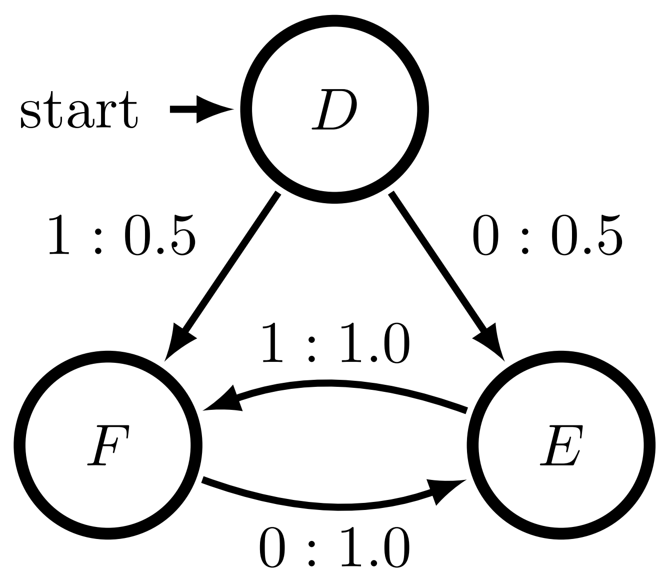
$$\langle W \rangle \leq k_B T \ln 2 \, \Delta h_\mu \leq k_B T \ln 2 \, \Delta H(1) \quad \checkmark$$

Memoryful Ratchets Memoryful Inputs

Theorem: $\langle W \rangle \leq k_B T \ln 2 \, \Delta h_\mu$ proven for all finite ratchets.

-Unknown if single symbol bounds hold.

-Consider correlated inputs: period-2



$$\Pr(Y_{0:\infty} = 010101...) = \Pr(Y_{0:\infty} = 101010...) = 1/2$$

$h_\mu = 0$ $\quad H[Y_i] = 1$

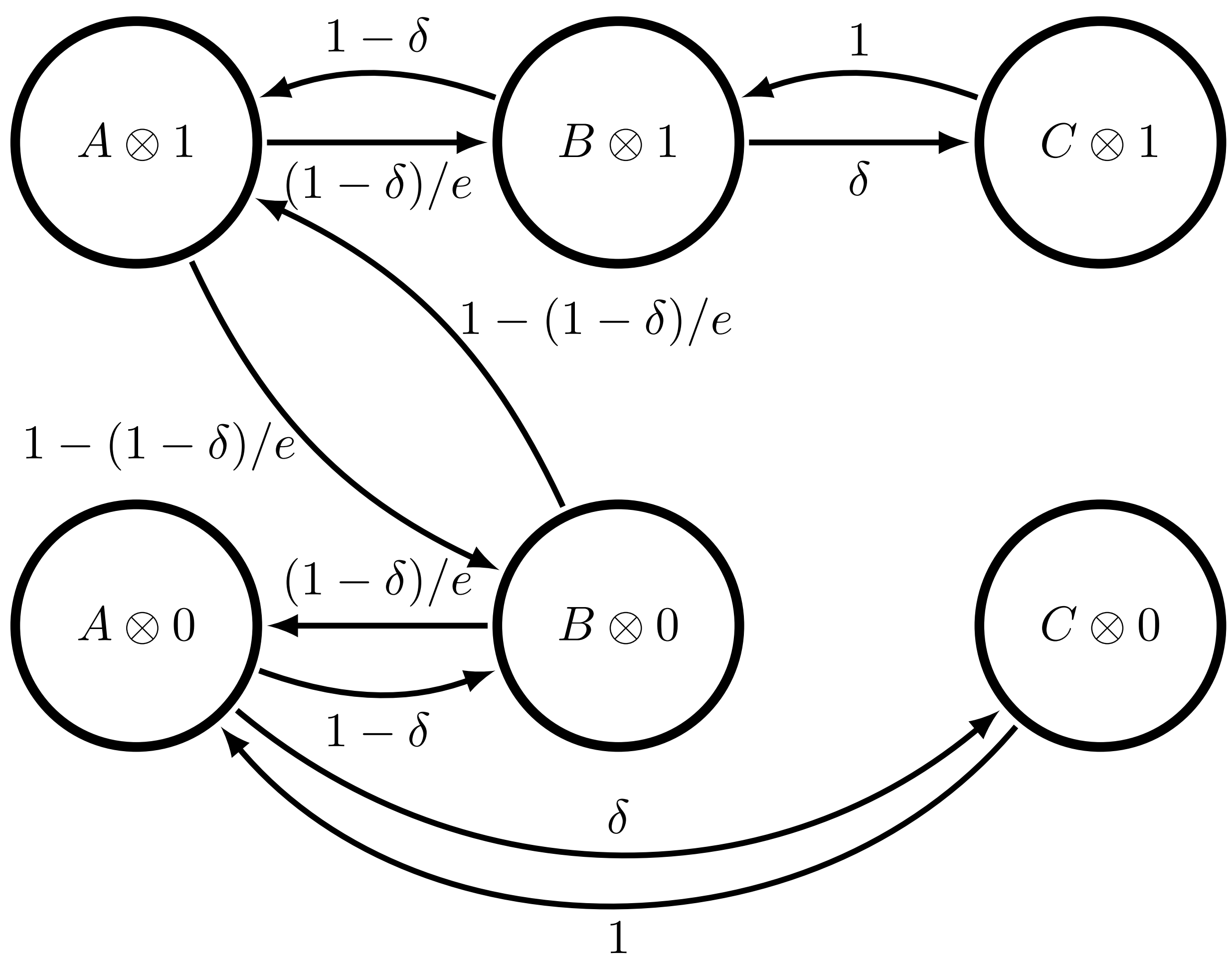
Memoryful Ratchets Memoryful Inputs

Single symbol bound: $\Delta H(1) \leq 0$
suggests no work can be done.

Entropy rate bound: $\Delta h_\mu \geq 0$
suggests work can be done.

Memoryful Ratchets Memoryful Inputs

Period-2 Ratchet:



$$\langle W \rangle = \frac{1 - \delta}{e} > 0$$

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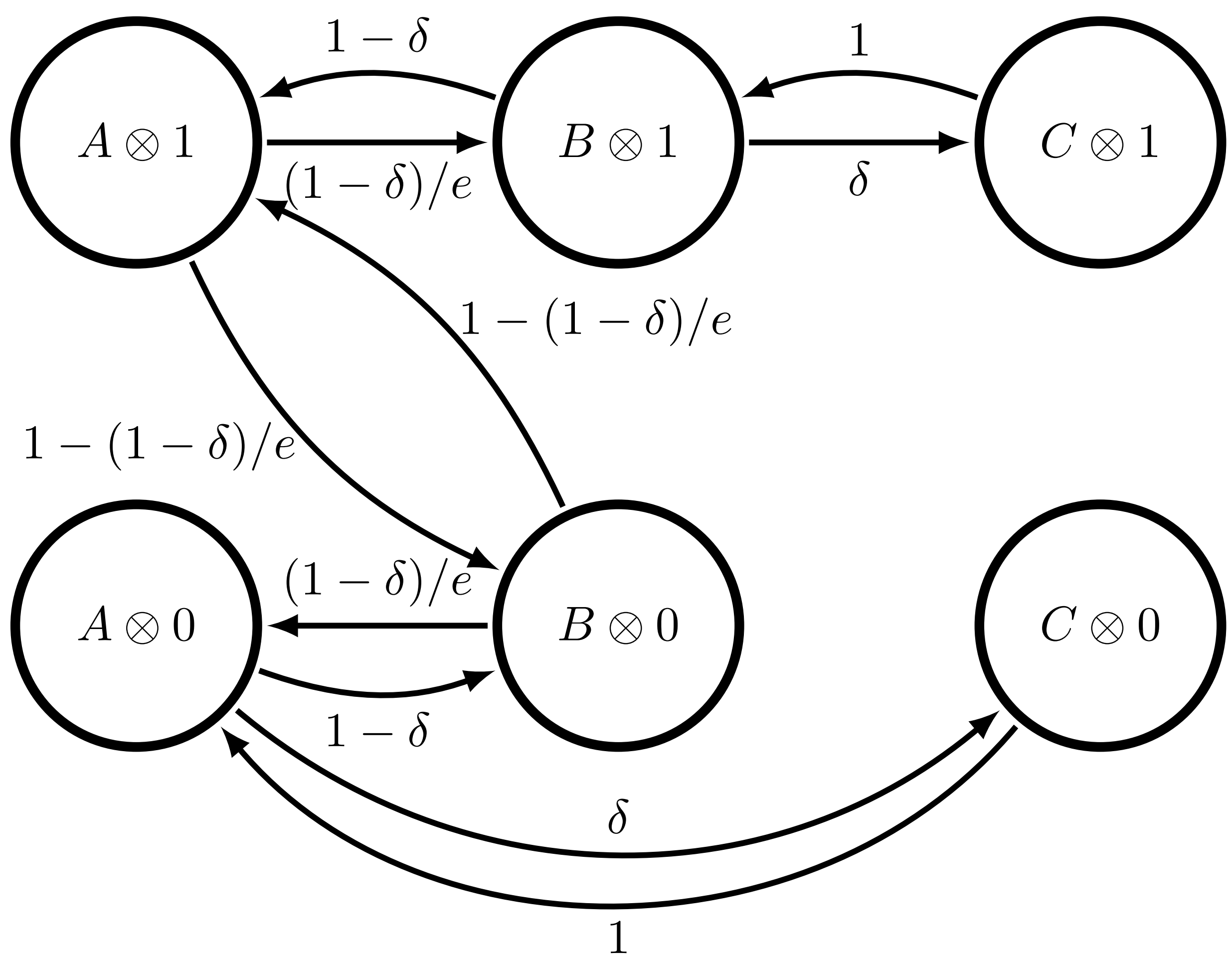
Memoryful Ratchets Memoryful Inputs

Period-2 Ratchet:



$$\langle W \rangle = \frac{1-\delta}{e} > 0 \quad \checkmark$$

$$\langle W \rangle \leq \Delta h_\mu = H((1-\delta)/e) \quad \checkmark$$

$$\langle W \rangle \leq k_B T \ln 2 \Delta H(1) \quad \checkmark$$

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$(k_B T \ln 2 = 1)$

Acknowledgements

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-Dibyendu Mandal (UC Berkeley)

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