Reacting Flows and Turnstiles

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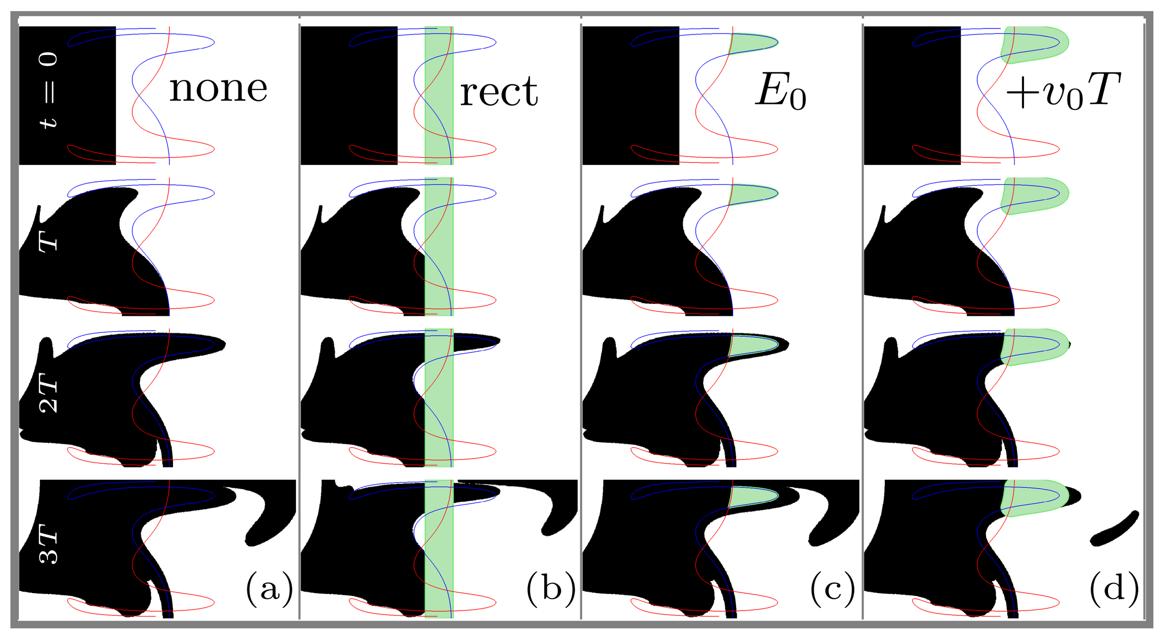
Can we prevent the algae invasion?

We begin with a time-periodic 2D fluid flow in which something is actively spreading. We imagine this is an algae bloom growing on top of the moving ocean surface. Our goal is to prevent the spread of algae to the right. To this end, we demonstrate three algae-cide treatment protocols.

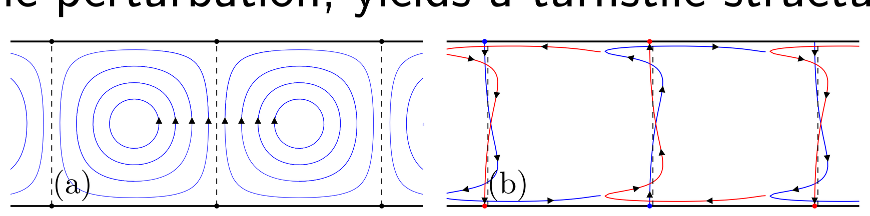


Time increases down. Algae (black), advective invariant manifolds (red, blue). Columns (a) No treatment, algae invades to the right. (b) Rectangular region (green) treated at each forcing cycle, almost no long-range effect. (c) Turnstile lobe E_0 treated, no effect. (d) "Fat" turnstile lobe (v_0 is front speed) treated, mitigation only temporary.

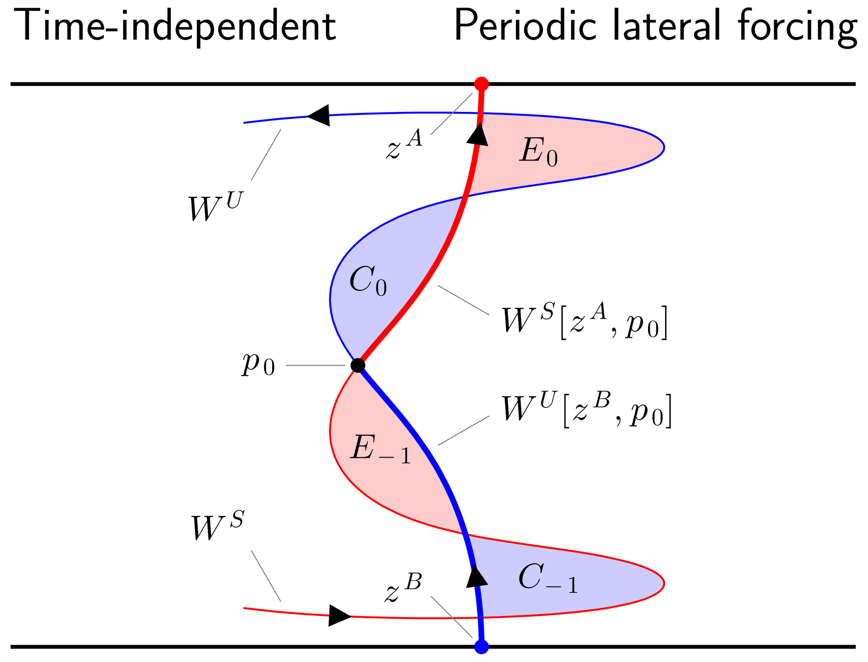
We observe that the advective turnstile structure is not respected by the algae.

Turnstile review

Our model fluid flow is a vortex chain, which under periodic perturbation, yields a turnstile structure.

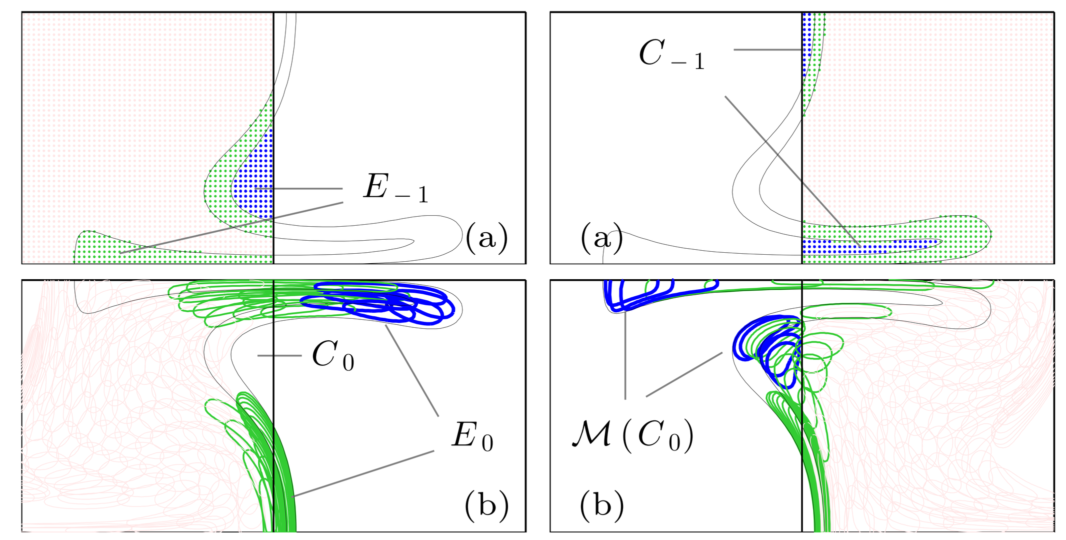


Time-independent Periodic lateral forcing



$$E_{-1} = \{p \in L : M(p) \in R\}$$
$$E_0 = \{p \in R : \exists q \in L, M(q) = p\}$$
$$C_{-1} = \{p \in R : M(p) \in L\}$$
$$C_0 = \{p \in L : \exists q \in R, M(q) = p\}$$

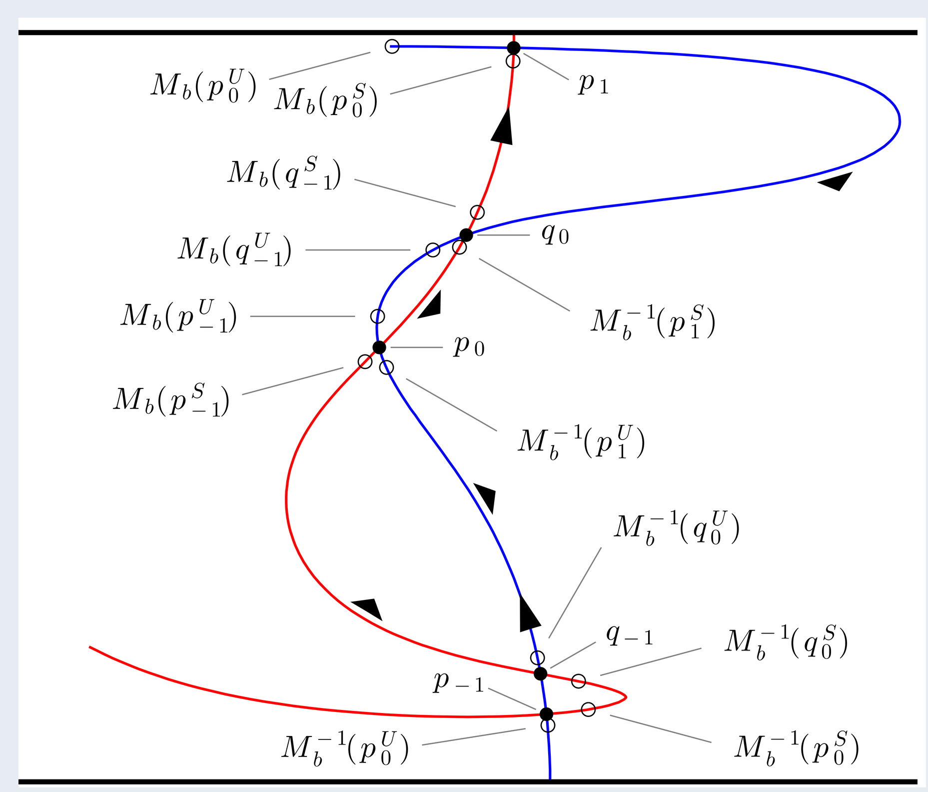
Set-definition of burning lobes



Point-stimulations on LHS/RHS map either fully across (blue), partially across (green), or not across (red) the midline.

$$E_{-1} = \{p \in L : \mathcal{M}(p) \cap R \neq \emptyset\}$$
$$E_0 = \{p \in R : \exists q \in L, p \in \mathcal{M}(q)\}$$
$$C_{-1} = \{p \in R : \mathcal{M}(p) \cap R = \emptyset\}$$
$$C_0 = \{p \in L : \exists q \in L, p \notin \mathcal{M}(q)\}$$

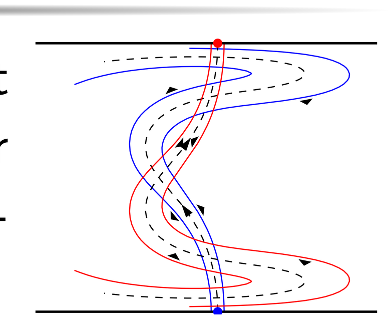
Burnstile



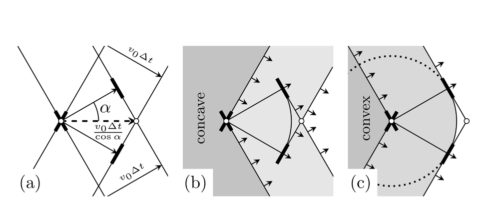
Generalization of turnstile for reacting flows.

Burning invariant manifolds

BIMs are *oriented* barriers to front propagation in flows. They appear on either side of the advective manifolds.

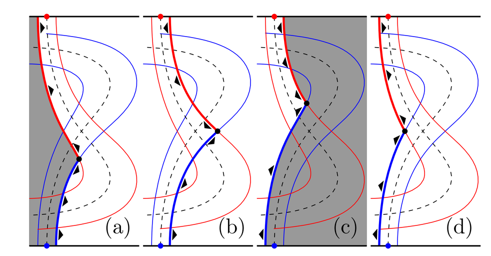


pip splitting



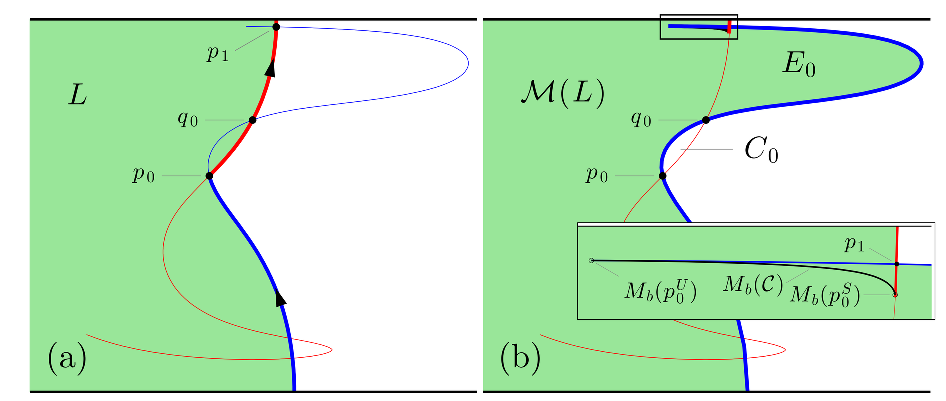
Intersections (pips) of fronts (BIMs) do not map to future intersections (pips). An intersection on the boundary of a reacted region may remain on the boundary, or run ahead.

Coorientation

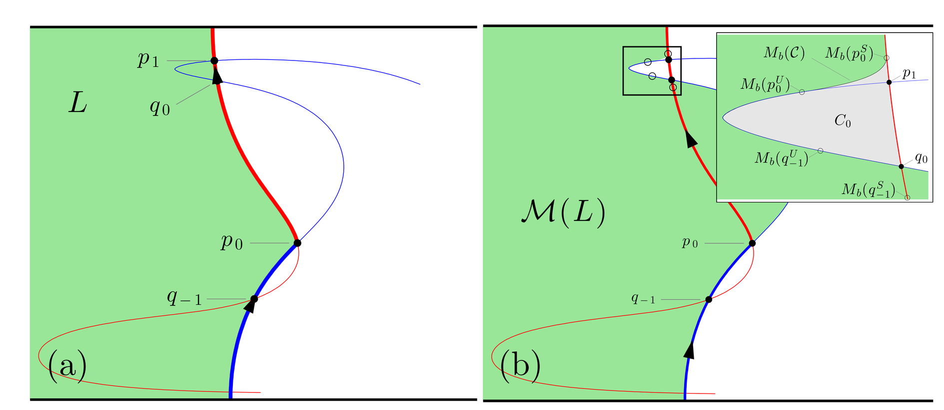


We must choose how to form the transport barrier. Only the two configurations with cooriented BIMs are physically meaningful.

Map forward



Choose concave pip: the iterate of the boundary lies within the burned region. Therefore BIMs are still boundaries of the lobes.



Choose convex pip: the iterate of the boundary is exposed - BIMs are not complete boundaries.

Map backward

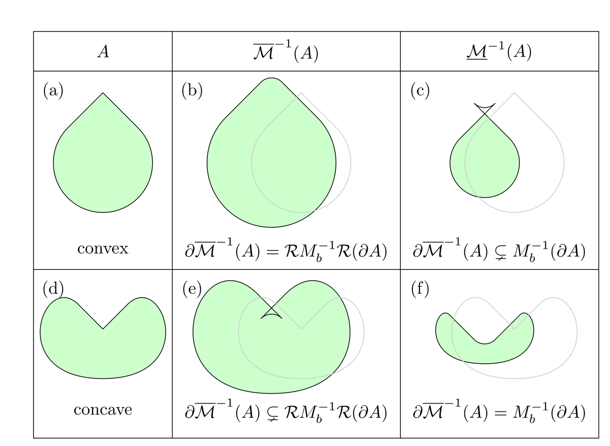
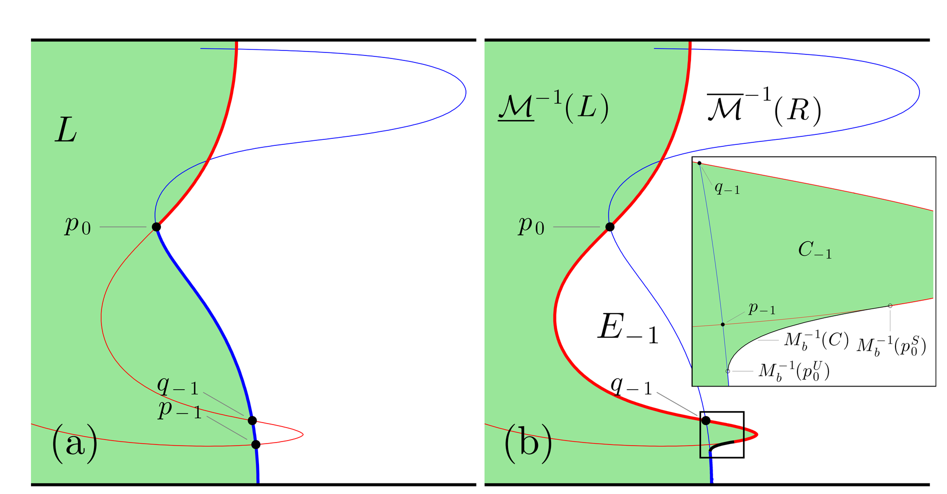


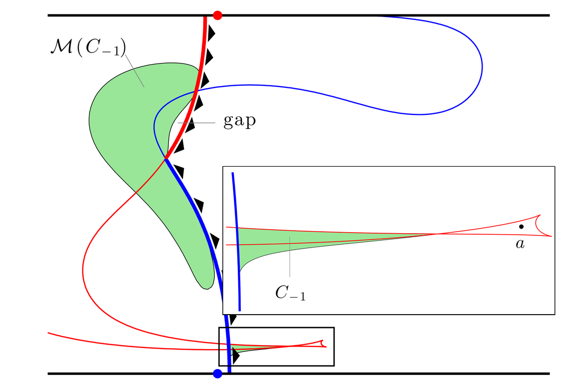
Illustration of “loose” and “tight” preimages of burned regions.

$$\overline{\mathcal{M}}^{-1}(A) = \{p : M(p) \cap A \neq \emptyset\}$$
$$\underline{\mathcal{M}}^{-1}(A) = \{p : M(p) \subset A\}$$

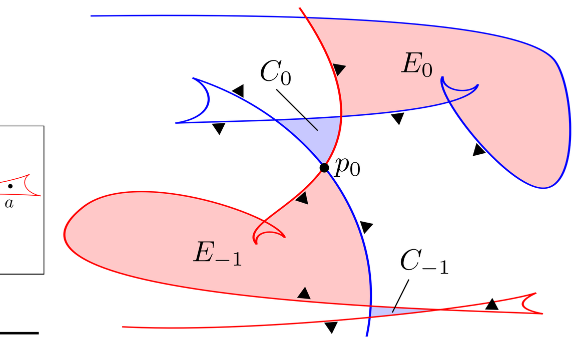


Choice of a concave pip leads to a non-BIM bounding component of the C_{-1} lobe.

Swallowtails



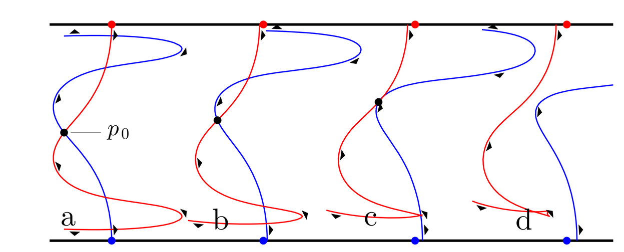
A swallowtail in the BIM leads to a reduction in the lobe size, and also to a gap between the lobe iterate and the BIMs.



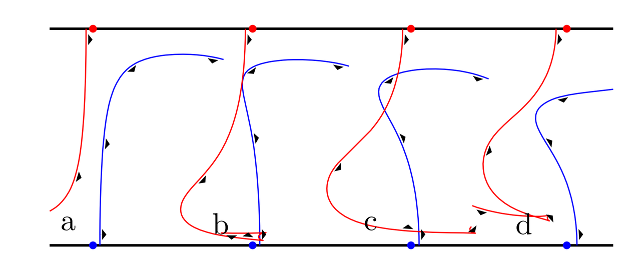
Swallowtails may occur in any of the lobes, illustrated here as a cartoon.

Non-trivial pip behavior

Reacting flow transport may be lobe-mediated or not. The transition between these two regimes is still not well understood.

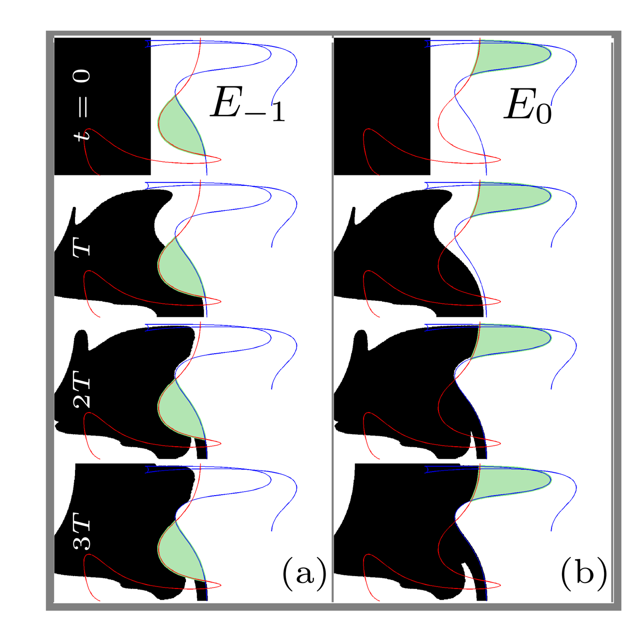


Increased burning speed leads to a loss of burning pips.



Increased forcing amplitude has a nontrivial effect on the burning pips.

Invasion prevented!



These two protocols target the burnstile lobes. Treatment of either the E_{-1} lobe (a), or the E_0 lobe (b) completely prevents the progress of algae to the right.

A turnstile mechanism for fronts propagating in fluid flows, J. Mahoney & K. Mitchell, Chaos 2013.