Noise phenomena in Josephson junctions

B. A. Huberman
Xerox Palo Alto Research Center, Palo Alto, California 94304
J. P. Crutchfield and N. H. Packard
Physics Department, University of California, Santa Cruz, California 95064

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We suggest that the reported noise-rise phenomenon observed in Josephson oscillators can be understood in terms of the full nonlinear and deterministic junction dynamics. We show that the drive damped pendulum equation describing the junction behavior exhibits chaotic solutions associated with the appearance of strange attractors in phase space. These results are relevant to the general problem of turbulent behavior of anharmonic systems.

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Several experiments on Josephson junction oscillators have revealed a striking noise-rise phenomena which cannot be accounted for in terms of thermal fluctuations. Chiao and co-workers[1] have reported that when superconducting junctions are used as unbiased parameteric amplifiers (SUPARAMPS), an increase in the amplitude of the oscillatory driving signal can lead to broad-band voltage fluctuations with equivalent noise temperatures of 5×10^4 K. This behavior, which has been observed in many other experiments,[2,3] defies explanations based on either amplification of thermal noise[4] or stability analysis of the equations governing the phase oscillations.[5]

In this letter, we suggest that this broad-band noise-rise phenomenon can be understood in terms of the existence of chaotic solutions to the full nonlinear junction dynamics. This turbulent behavior, which gives rise to broad-band power spectra, is associated with the appearance of a strange attractor[6] in phase space. Besides providing an explanation for the observed voltage fluctuations in some Josephson devices, our theory points to these junctions as attractive experimental tools for the study of solid-state turbulence and nonlinear dynamics. Also, since the equations that we study appear in a number of different systems, our results are relevant to problems that range from soliton dynamics[7] to solid-state turbulence.[8,9]

Consider a Josephson oscillator in the presence of microwave radiation and described by a current-driven shunted-junction model.[10] If C is the junction capacitance, R the normal-state resistance, and V the potential difference across the junction, the superconducting phase φ is determined by the following equations:

$$C\frac{dV}{dt} + \frac{V}{R} + I_c \sin\varphi = I_{rf} \cos\omega_d t, \tag{1}$$

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V, \tag{2}$$

where I_c is the critical supercurrent and I_{rf} the amplitude of the microwave field at the driving frequency ω_d. Replacing the potential in Eq. (1) by its expression in terms of the phase [Eq. (2)], we obtain the nonlinear differential equation for φ

$$\frac{d^2\varphi}{dt^2} + \frac{1}{\tau}\frac{d\varphi}{dt} + \omega_0^2 \sin\varphi = \frac{2e}{\hbar C} I_{rf} \cos\omega_d t, \tag{3}$$

where $\tau \equiv (RC)$ is the damping time and $\omega_0 = (2eI_c/\hbar C)^{1/2}$ the plasma frequency of the junction. This description of the driven damped motion of a particle in a spatially periodic potential forms the basis of extensive work on a number of devices utilizing either point junctions or microbridges.[2,10–13] It also appears as a generalization of the anharmonicity problem in solid-state systems driven by periodic fields.[8,9]

For small enough values of the amplitude of the phase oscillations, it is possible to study the stability of Eq. (3) against fluctuations by converting it into a Mathieu-type equation.[12] As the value of φ increases, however, the first two terms of a Taylor-series expansion for the $\sin\varphi$ term lead to a cascade of bifurcations into a chaotic regime which cannot be obtained via perturbation theory.[8]

In order to study the full nonlinear solutions of Eq. (3) we express it in terms of dimensionless variables. Introducing a new time scale $t' = t/\alpha$ and writing $\Gamma \equiv (2e^2\alpha/\hbar C)I_{rf}$ and $\Omega_0 \equiv \alpha\omega_0$, we obtain

$$\frac{d^2\varphi}{dt^2} + \frac{\alpha}{\tau}\frac{d\varphi}{dt} + \Omega_0^2 \sin\varphi = \Gamma \cos\alpha\omega_0 t. \tag{4}$$

This equation was solved by using a hybrid digital-analog computer system. Starting with typical junctions parameters such as $R = 4\,\Omega$, $C = 5$ pF, $I_c = 100\,\mu$A, and choosing $\alpha = 10^{-11}$, we find $\alpha/\tau = 0.5$ and $\Omega_0^2 = 6.4$.

Although a detailed description of the possible solutions to Eq. (4) will be published elsewhere, our main result can be summarized in the bifurcation diagram of Fig. 1, where we show the types of behaviors to be expected for different values of the driving amplitude and frequency. As can be seen, for frequencies that are either much smaller or much larger than ω_0 one encounters periodic solutions which can, in some cases, become fairly complicated (i.e., subharmonic and harmonic content, hysteresis loops, etc.). A noteworthy periodic regime, which occurs at fairly high values of Γ, is the one in which φ undergoes successive 2π rotations in phase with the driving frequency, corresponding to the periodic motion of the particle from one potential well to another (region A). This kind of behavior represents the running periodic solutions described by Levi *et al.*[12]

In region B, the phase amplitude is confined to one potential well, i.e., $0 \leqslant \varphi \leqslant 2\pi$. In this regime the solutions exhib-

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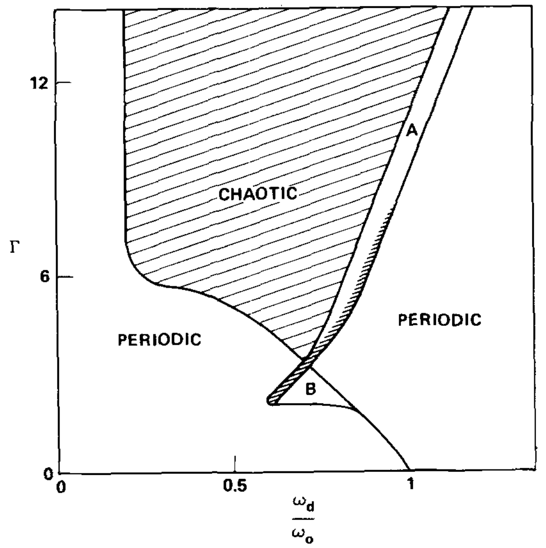

FIG. 1. Bifurcation diagram for Eq. (4) with $a/\tau = 0.5$, $\Omega_0^2 = 6.4$, and $\alpha = 10^{-11}$, obtained by varying ω_d at constant Γ. The chaotic regime contains small regions of periodic solutions. Region A: periodic running solutions. Region B: amplitude hysteresis and cascading bifurcations of Ref. 8. Narrow shaded region: full extent of period-doubling chaos.

it the amplitude hysteresis and the set of period-doubling cascading bifurcations into a chaotic state which was found for the anharmonic potential.[8] Beyond this regime, φ is no longer bounded and a complicated turbulent behavior ensues, characterized by strange attractors in phase space[14] whose Poincaré sections[15] display an infinite lattice structure with the spatial periodicity of the potential (Fig. 2). The structure of these strange attractors can be understood in terms of two distinct time scales which the motion exhibits: The shorter time scale corresponds to fast oscillations between a small number of wells; the longer time scale is associated with a slower diffusion throughout the lattice. This turbulent behavior is best characterized by the power spectral density shown in Fig. 3, which was obtained for the parameter values $\Gamma = 3.8$, $\omega_d/\omega_0 = 0.64$. $S(\omega)$ denotes the Fourier transform of the autocorrelation function for the time derivative of the phase which, by Eq. (2), is proportional to the voltage fluctuations across the junction. This broad-band spectrum, generated by the deterministic Eq. (4), is quite similar to some of those observed in Josephson oscillators

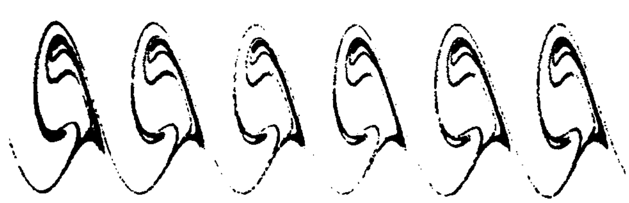

FIG. 2. Poincaré section of the strange attractor at parameter values $\Gamma = 3.8$ and $\omega_d/\omega_0 = 0.64$. Points comprising the section are taken at positive-going zero crossings of the driving force (zero phase). The section represents a six-well segment of the strange attractor lattice. The vertical and horizontal coordinates denote $\dot{\varphi}$ and φ, respectively.

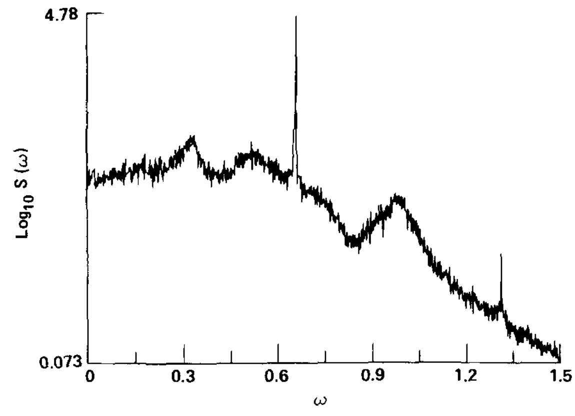

FIG. 3. Power spectrum of the strange attractor for same parameter values of Fig. 2. $S(\omega)$ is the Fourier transform of the voltage autocorrelation function, computed from 12 averages of a 4096-point fast Fourier transform. The frequency has been normalized to ω_0.

operated as parametric amplifiers.[1-3] We should also add that even though the sharp peak observed at the driving frequency appears narrow, it broadens at larger driving amplitudes.

Our results suggest an explanation for the noise-rise phenomenon based on the intrinsic nonlinearity of the junction dynamics. Whether this theory accurately describes the observed behavior depends on the extent to which the driven damped pendulum models the actual junction dynamics. If that were the case, the phase diagram of Fig. 1 could also provide some guidelines for operating superconducting parametric amplifiers in noise-free regions. Moreover, if broad-band noise in Josephson oscillators is due to the presence of strange attractors in phase space, they could become likely candidates for the study of solid-state turbulence and nonlinear dynamics, a subject which is just beginning to be studied experimentally.

Finally, we should point out that these results are of relevance to the wide variety of problems that can be modeled by the driven damped pendulum of Eq. (4). In particular, they show that the range of parameter values for which chaotic solutions can occur is much larger than that found for the single-well anharmonic problem.[8] This is of importance to experiments dealing with turbulent properties of solids.

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[6]That is, a region of phase space characterized by the fact that (i) all trajectories in its neighborhood must enter it and (ii) almost all trajectories diverge within the attractor. For a more rigorous definition, see D. Ruelle and F. Takens, Commun. Math. Phys. **20**, 167 (1971).
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