ing because they have the same velocity (zero). All of these domains are unstable to perturbations introducing other types of particle with different velocities.

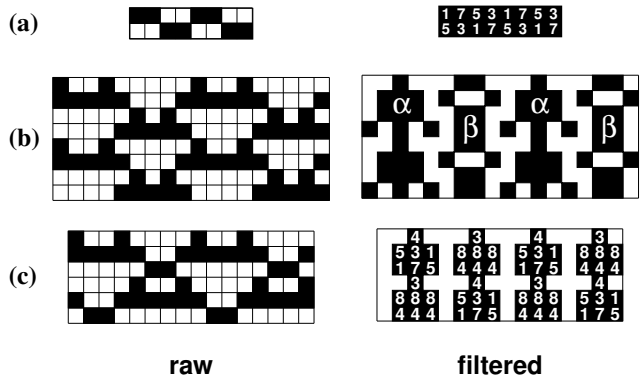

Figure 14. Unfiltered and filtered space-time diagrams of a few of ECA 54's other domains. . (a) An unstable domain. (b) A domain consisting of alternating αs and βs. (c) A domain of "pseudo-αs" in which every block of nonzero filtered cells is either part of an α or is a string occurring in an α-γ interaction. In the filtered plots, domain cells are white, particles are black and non-particle defects are black with T^0_{54} output symbol.

Figure 14(a) shows a space-time plot of a domain with configurations having spatial period 4 and temporal period 2. The domain itself is period 1. This domain is unstable: its boundaries always grow inward with speed $c = 1$ site per iteration. Initial conditions having spatial regions in this domain are responsible for the initial nonzero fractions of unrecognized defect types $\{1, 3, 5, 7\}$ shown in fig. 11. The fast decay in those fractions is due to this domain's instability.

A more complex, and perhaps also more interesting, domain consisting of alternating αs and βs is shown in fig. 14(b). Domain configurations have spatial period 9 and temporal period 4. This example again points out the inherent duality between domains and particles. In certain cases, particles may coalesce to form domains. The raw configurations can be interpreted in two ways, as the context of analysis requires. This is similar to the case of the β particle, which could be interpreted as a single entity or as the bound state of two γs.

Figure 14(c) shows another unstable domain, in this case consisting of strings occurring in the α particle and in α-γ interactions. The spatial period is 8. The temporal period of configurations is 6, but that of the pattern is 3.

The "gamma gun" (called a "radiating particle" in ref. [14]), shown in fig. 15, is a periodic particle formed by the decay of the domain of fig. 14(c). When that domain is bounded on both sides by regions of Λ_{54}, the two boundaries gradually approach each other while emitting γs that propagate out into the Λ_{54} regions. If the number of repetitions of the basic domain string is even (e.g., filtered string $...0(53108880)^n 0...$ with n even), then the ultimate collision of the two approaching boundaries results in a configuration in the domain Λ_{54}—i.e., the secondary domain disappears completely. If the number of repetitions of the basic string is odd, the ultimate result is the gamma gun. Vestiges of the vanished domain can still be seen in the gamma gun's space-time signature. For example, in the eighth row of fig. 15, the gamma gun's filtered spatial string is $...053108880...$, identical to one period of the basic domain string noted above.

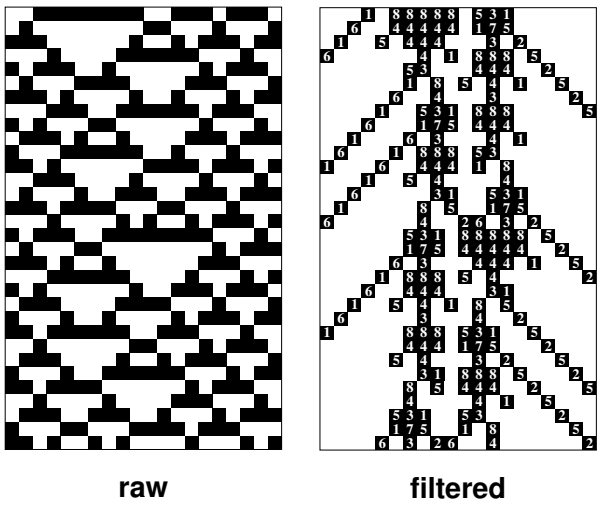

Figure 15. Raw (a) and filtered (b) space-time diagrams of the gamma gun. The gamma gun arises from the decay of the unstable domain shown in fig. 14(b).

Finally, we return to our consideration of previously unrecognized defect types 4 and 8. By construction, T^1_{54} classifies short blocks of 4s and 8s as follows: blocks of length 1 are α-γ interactions, blocks of length 2 are β particles, and blocks of length 3 are α

particles. This means that unrecognized 4s and 8s can occur in only two cases: (i) isolated blocks of length 4 or greater; or (ii) adjacent to other defect types (i.e., not isolated). The particle frequency statistics discussed in the last section show that all other unrecognized defect types vanish in the first few iterations. Therefore, only case (i) is possible at intermediate to long times. Application of the inverse domain filter Q^0_{54} to long blocks of 4s and 8s shows that they respectively correspond to long blocks of 0s and 1s in the unfiltered behavior. These can arise in a number of ways, the most common of which is when a pair of incoming parallel γs collides with an α particle—i.e., $\gamma^+ + \gamma^+ + \alpha$ or $\alpha + \gamma^- + \gamma^-$. Before the outcome of the first collision has precipitated into isolated particles, the second γ hits the interaction region, resulting in a block of 8s. Another way in which blocks of 8s arise is when two or more adjacent αs with the same temporal phase are pushed into each other by incoming γs.

From the mapping $\widehat{\Phi}_{54}$ for filtered configurations, or alternatively from the CA rule Φ_{54}, we see that a block of n 8s bounded by one or more 0s at each end is mapped onto a block of n 4s. Blocks of 4s shrink over time, though at alternate time steps they also emit short lived γs in the form of isolated 1s (γ^-) and 5s (γ^+). The general pattern is shown in fig. 16. There are four possible outcomes, based on the initial length n of the block of 8s. If $n \ mod \ 4 = 0$, then $n/4$ iterations later the filtered string will be ...050010..., which maps to a β particle by the γ^+-γ^- collision shown in fig. 9(e). If $n \ mod \ 4 = 1$, then $n/4$ iterations later the filtered string will be ...0504010..., which corresponds to an α particle with temporal phase α_1, as indicated in fig 8(b). If $n \ mod \ 4 = 2$, then $n/4$ iterations later the filtered string will be ...05044010..., corresponding to the three-particle collision $\gamma^+ + \beta + \gamma^-$ shown in fig. 9(g), which results in the mutual annihilation of all three particles. Finally, if $n \ mod \ 4 = 3$, then $n/4$ iterations later the filtered string will be ...050444010..., which is identical to the three-particle collision $\gamma^+ + \alpha + \gamma^-$ shown in fig. 9(f), and results in the three particles $\gamma^- + \alpha + \gamma^+$. Differentiating among these four possible outcomes for classification and filtering is done by calculating $n \ mod \ 4$ and remembering the location of the block's beginning. The latter requires adding a queue of unlimited depth to the particle filter. This means that a complete description of the pattern dynamics of ECA 54, whatever else it may entail, requires at least a computational model including a queue—i.e., a more sophisticated model class than finite state machines.

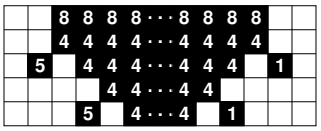

Figure 16. Evolution of long blocks of 4s and 8s in the domain filter output.

7. The particle physics of ECA 54

This article has described a number of properties of elementary CA rule 54—some new, some already known—using the unified, general-purpose framework of computational mechanics. As such, it presents a case for the utility of that framework, by showing how otherwise disparate calculations and results can be brought together under a single set of techniques, and by presenting new results that would be otherwise inaccessible, or at best ad hoc.

The analysis of ECA 54's patterns and pattern dynamics began with the identification of its most important computationally homogeneous pattern, the regular domain Λ_{54}. From this basic building block or "pattern basis", a domain filter was constructed to factor out spatio-temporal regions conforming to the domain pattern, locating and highlighting exactly those space-time cells at which defects in the pattern occur. The resulting filtered space-time data was found to be described by the iteration of a deterministic mapping. A second-level particle filter was constructed to automatically classify the important particles or coherent structures existing at the boundaries of domain regions. Straightforward modifications to the particle filter enabled it to recognize and classify patterns of all fundamental interactions, and ultimately of all but an exponentially small fraction of asymptotic spatio-temporal behavior.

In describing Λ_{54}, we were also able to introduce the space-time machine, an important extension to the techniques of computational mechanics, and present as an example the space-time machine corresponding to ECA 54's fundamental domain. Clearly the ideas underlying the space-time machine and its relatives deserve extensive treatment; this must of necessity be presented elsewhere.

We conclude by elaborating on a point made in the introduction: computational mechanics, in contrast with much work concerning CA and computation, takes a distinctly *naturalistic* stance. Each cellular automaton rule may be regarded as a specification of the microscopic equations of motion of a model universe. That is, the CA lookup table directly determines only the local space-time rules of cell interaction and cell modification. Confronted with the behavior that appears within such a universe over long temporal and spatial scales, computational mechanics attempts to answer the questions, What are the emergent structures? And what are the emergent physical laws governing their interactions?

The result here of our computational mechanics analysis was a near-complete particle-level description of ECA54's space-time behavior. The emergent structures were the regular domain—a sort of vacuum state—and the fundamental particles—excitations above that ground state. The laws were expressed in terms of the particle interactions and their properties, including the statistical features of the short and long term behavior. The collection of these behavioral components constitutes the "physics" of ECA 54's universe. More concretely, we showed that the space-time behavior condenses relatively quickly into these structures by the emergence time τ. Every site in configurations generated beyond that time participates in one of the identified structures. There may be, however, compound structures built up out of the fundamental ones. As was seen in the particle decay statistics, for example, at around $16,000$ iterations a scaling regime sets in, described by the power law decay in α particles. Presumably, this new level of organization is associated with the appearance of some new mechanism that removes these particles systematically.

8. Acknowledgments

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Appendix A. Deterministic Equation of Motion of the Particle-Level Model

In this appendix, we give the FST for the mapping $\widehat{\Phi}_{54}$ defined in eq. 4, that takes the filtered string $\mathbf{z}_t = \mathrm{T}^0_{54}(\mathbf{s}_t)$ and maps it to the filtered string $\mathbf{z}_{t+1} = \mathrm{T}^0_{54}(\mathbf{s}_{t+1}) = \mathrm{T}^0_{54}(\Phi_{54}(\mathbf{s}_t))$. The alphabet of $\widehat{\Phi}_{54}$ is $\mathcal{A}' = \{0-8\}$, though the mapping is defined only over the set $\left[\mathrm{T}^0_{54}\right]_{out}$, i.e., the set of possible outputs of the domain filter. For this reason, most states in $\mathrm{M}\left(\widehat{\Phi}_{54}\right)$ are lacking edges for many of the input symbols in $\mathcal{A}'$.

The machine will be specified by its *transition table*, which lists each state and its transitions in the following format:

$$> *state \quad [s_{in}|s_{out}] \rightarrow dest \quad [s_{in}|s_{out}] \rightarrow dest \ \ ...$$

where s_{in} is the input symbol, s_{out} the output, and $dest$ the destination state, of the given transition. Optional prefixes $>$ and $*$ indicate that the state is a start state and/or accept state, respectively. Transitions are listed in order of input symbol. Transitions with no outputs are indicated by $s_{out} = \lambda$.

The start state of $\mathrm{M}\left(\widehat{\Phi}_{54}\right)$ (state 0) has a self-loop on input 0, corresponding to a domain input symbol, and eight other edges for the eight different defect types. Once a nonzero input is read, the machine uses the next six input symbols to synchronize, finally passing down into its recurrent part, consisting of accept states $\{45, \ldots, 64\}$, where it begins writing output symbols. On reaching the end of the CA lattice, the machine remains in its current state and wraps around to the beginning of the lattice, where it produces outputs for the input symbols read during synchronization. If the machine is still in state 0 at the end of the lattice, then an output string $\mathbf{z}_{t+1} = 0^N$ is written.

> 0	[0|λ]→0	[1|λ]→1
	[2|λ]→2	[3|λ]→3
	[4|λ]→4	[5|λ]→5
	[6|λ]→6	[7|λ]→7
	[8|λ]→8	
1	[0|λ]→9	[7|λ]→10
2	[0|λ]→11	[6|λ]→12
3	[0|λ]→12	[1|λ]→11
4	[0|λ]→13	[4|λ]→14
5	[0|λ]→14	[3|λ]→13
6	[0|λ]→15	[2|λ]→16
7	[0|λ]→16	[5|λ]→15
8	[0|λ]→10	[8|λ]→9
9	[0|λ]→17	[8|λ]→18
10	[0|λ]→19	[5|λ]→20
11	[0|λ]→18	[7|λ]→17
12	[0|λ]→20	[2|λ]→19
13	[0|λ]→21	[1|λ]→22
14	[0|λ]→23	[4|λ]→24
15	[0|λ]→24	[3|λ]→23
16	[0|λ]→22	[6|λ]→21
17	[0|λ]→25	[5|λ]→26
18	[0|λ]→27	[8|λ]→28
19	[0|λ]→29	[6|λ]→30
20	[0|λ]→31	[3|λ]→32
21	[0|λ]→26	[2|λ]→25
22	[0|λ]→28	[7|λ]→27
23	[0|λ]→30	[1|λ]→29
24	[0|λ]→32	[4|λ]→31
25	[0|λ]→33	[6|λ]→34
26	[0|λ]→35	[3|λ]→36
27	[0|λ]→37	[5|λ]→38
28	[0|λ]→39	[8|λ]→40
29	[0|λ]→40	[7|λ]→39
30	[0|λ]→41	[2|λ]→42
31	[0|λ]→43	[4|λ]→44
32	[0|λ]→34	[1|λ]→33
33	[0|λ]→45	[7|λ]→46
34	[0|λ]→47	[2|λ]→48
35	[0|λ]→49	[4|λ]→50
36	[0|λ]→51	[1|λ]→52
37	[0|λ]→53	[6|λ]→54
38	[0|λ]→55	[3|λ]→56
39	[0|λ]→57	[5|λ]→58
40	[0|λ]→59	[8|λ]→60
41	[0|λ]→61	[3|λ]→62
42	[0|λ]→52	[6|λ]→51
43	[0|λ]→54	[1|λ]→53
44	[0|λ]→63	[4|λ]→64
*45	[0|0]→59	[8|0]→60
*46	[0|3]→57	[5|3]→58
*47	[0|0]→61	[3|8]→62
*48	[0|0]→52	[6|8]→51
*49	[0|0]→54	[1|6]→53
*50	[0|3]→63	[4|0]→64
*51	[0|8]→47	[2|8]→48
*52	[0|5]→45	[7|5]→46
*53	[0|0]→45	[7|0]→46
*54	[0|0]→47	[2|0]→48
*55	[0|2]→49	[4|0]→50

*56	[0|0]→51	[1|7]→52
*57	[0|0]→53	[6|1]→54
*58	[0|0]→55	[3|1]→56
*59	[0|0]→57	[5|0]→58
*60	[0|4]→59	[8|4]→60
*61	[0|0]→49	[4|5]→50
*62	[0|8]→51	[1|0]→52
*63	[0|1]→54	[1|0]→53
*64	[0|0]→63	[4|4]→64

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Refs. [2–5, 17, 18] are available via the World Wide Web from the Computational Mechanics Archive, http://www.santafe.edu/projects/CompMech. Refs. [13, 19] are available from the Evolving Cellular Automata archive, http://www.santafe.edu/projects/evca.

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