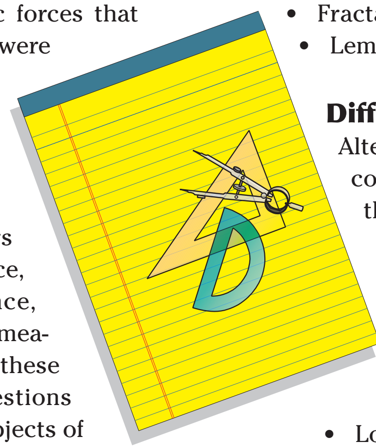
Measures of Complexity: A Nonexhaustive List

By Seth Lloyd

The world has grown more complex recently, and the number of ways of measuring complexity has grown even faster. This multiplication of measures has been taken by some to indicate confusion in the field of complex systems. In fact, the many measures of complexity represent variations on a few underlying themes. This column presents an (incomplete) categorization and tabulation of measures of complexity.

An historical analog to the problem of measuring complexity is the problem of describing electromagnetism before Maxwell's equations. In the case of electromagnetism, quantities such as electric and magnetic forces that arose in different experimental contexts were originally regarded as fundamentally different. Eventually it became clear that electricity and magnetism were in fact closely related aspects of the same fundamental quantity, the electromagnetic field. Similarly, contemporary researchers in architecture, biology, computer science, dynamical systems, engineering, finance, game theory, etc., have defined different measures of complexity for each field. Because these researchers were asking the same questions about the complexity of their different subjects of research, however, their answers have much in common.



Three questions that are frequently posed when attempting to quantify the complexity of the thing (house, bacterium, problem, process, investment scheme) under study are

1) How hard is it to describe?
2) How hard is it to create?
3) What is its degree of organization?

I use these questions to group measures of complexity. Measures within a group are typically closely related quantities.

Difficulty of Description

The complexity of a system is often associated with the degree of difficulty involved in completely describing the system. Prominent examples of these measures (typically quantified in bits) include the following:

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- Information;
- Entropy;
- Algorithmic complexity or algorithmic information content;
- Minimum description length;
- Fisher information;
- Renyi entropy;
- Code length (prefix-free, Huffman, Shannon-Fano, error-correcting, Hamming);
- Chernoff information;
- Dimension;
- Fractal dimension;
- Lempel-Ziv complexity.

Difficulty of Creation

Alternatively, the degree of difficulty involved in constructing or duplicating a system can form the basis of a complexity measure. Units in this case are typically those of time, energy, money, etc. Examples include:

- Computational complexity;
- Time computational complexity;
- Space computational complexity;
- Information-based complexity;
- Logical depth;
- Thermodynamic depth;
- Cost;
- Crypticity.

Degree of Organization

Complexity is also often related to organizational aspects. Measures in this category can be further subdivided into two subclasses.

a) Difficulty of describing organizational structure, whether corporate, chemical, cellular, etc.:

- Effective complexity;
- Metric entropy;
- Fractal dimension;
- Excess entropy;
- Stochastic complexity;
- Sophistication;
- Effective measure complexity;
- True measure complexity;
- Topological epsilon-machine size;
- Conditional information;
- Conditional algorithmic information content;
- Schema length;
- Ideal complexity;
- Hierarchical complexity;

- Tree subgraph diversity;
- Homogeneous complexity;
- Grammatical complexity.

b) Amount of information shared between the parts of a system as the result of its organizational structure:

- Mutual information;
- Algorithmic mutual information;
- Channel capacity;
- Correlation;
- Stored information;
- Organization.

Related Concepts

In addition to the above measures, there are a number of related concepts that are not quantitative measures of complexity per se, but that are closely related. Such concepts include:

- Long-range order;
- Self-organization;
- Complex adaptive systems;
- Edge of chaos.

I welcome additions to this list, whether or not they fall in the classification scheme adopted here.