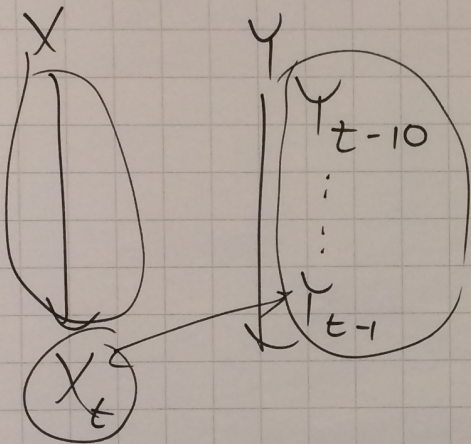


$$|A - B| \leq |A| \Rightarrow H(X|Y) \leq H(X)$$

$$|(A \cap B) - C| \leq |A \cap B|$$

$$\hookrightarrow I(X:Y|Z) \leq I(X:Y)$$

$I(X:Y|Z)$  = "The info shared by X and Y removing the effects of Z"



X	Y	Z	
0	0	0	1/4
0	1	1	1/4
1	0	1	1/4
1	1	0	1/4

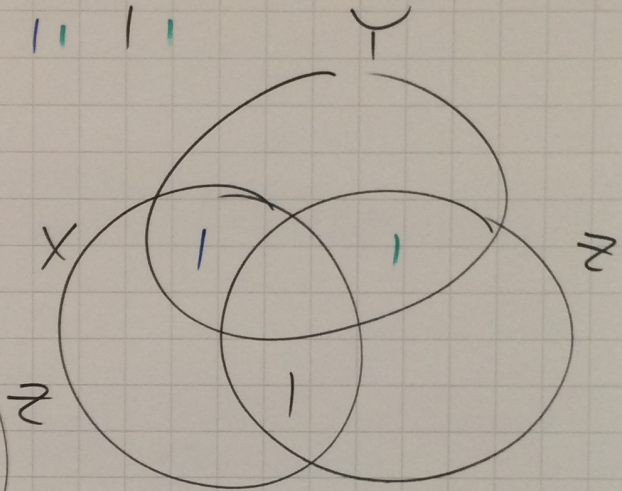
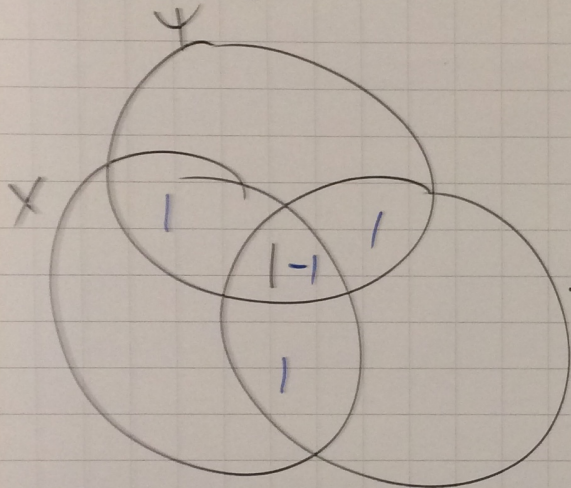
$$P(X,Y) = P(X)P(Y)$$

X	Y	Z
0	0	0
0	0	1
1	1	0
1	1	1

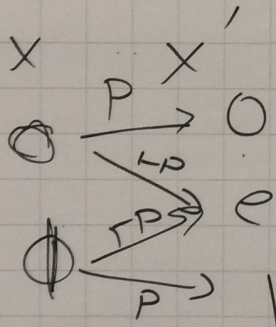
STAPLES

X	Y	Z
00	00	00
00	01	01
01	00	01
01	01	00
10	10	10
10	11	11
11	10	11
11	11	10

X	Y	Z
00	00	00
00	01	01
01	00	10
01	01	11
10	10	00
10	11	01
11	10	10
11	11	11

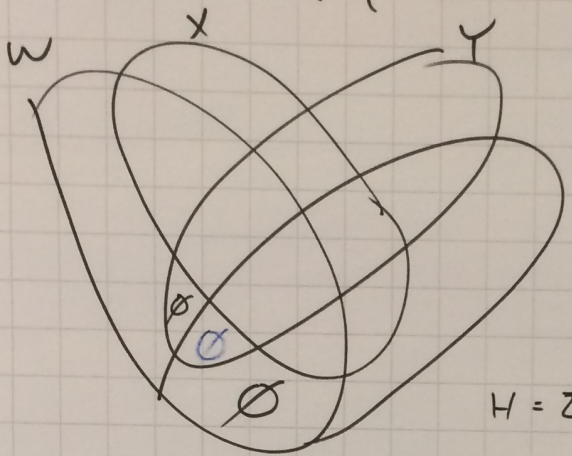


X	Y	Z
00	00	00
11	11	11



if  $P(x, y, z, w) > 0$ :

$$\left. \begin{matrix} W \perp Z | XY \\ W \perp Y | XZ \end{matrix} \right\} \Rightarrow W \perp YZ | X$$



$X_0$	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1

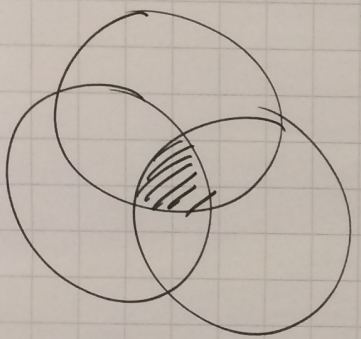
$$P(X_0, X_1) = \begin{matrix} 00 & 1/2 \\ 11 & 1/2 \end{matrix}$$

$$P(X_2, X_3)$$

$$P(X_4, X_5)$$

$$\vdots$$

$$H = \sum_j \sigma_j + \sum_{j,k} \sigma_j \sigma_k + \sum_{j,k,l} \sigma_j \sigma_k \sigma_l \dots$$



$$D_{KL}(P(xyz) \parallel P^z(xyz))$$

$$P^z(xyz) = \text{arg max}_q H_q(xyz)$$

$$\begin{matrix} P(xy) = q(xy) \\ P(xz) = q(xz) \\ P(yz) = q(yz) \end{matrix}$$

AND

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

$$I(X:Y:Z) = -0.188$$

$$\begin{matrix} P(xy) = 00 & 1/4 & 01 & 1/4 & \dots \\ P(xz) = 00 & 1/2 & 10 & 1/4 & 11 & 1/4 \\ P(yz) = 00 & 1/4 & 10 & 1/4 & 11 & 1/4 \end{matrix}$$

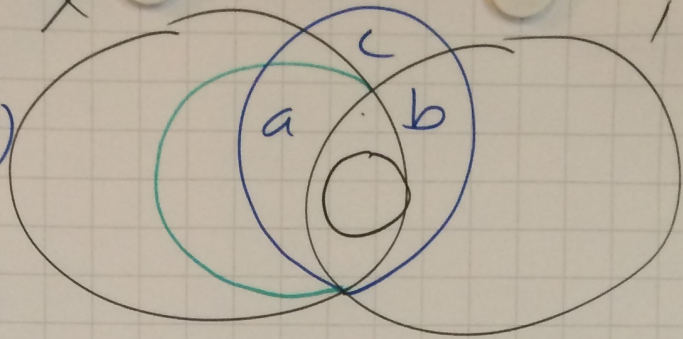
000  $P_0 =$

000
001
010
011
100
101
110
111

$$C(X:Y) = \min_{X-V-Y} I(V:XY)$$

$$G(X:Y) = \min_{X-V-Y} H(V)$$

$$K \leq I \leq C \leq G$$



$$\left. \begin{aligned} I(X:Y|V) &= 0 \\ H(V) - I(X:Y) &= 0 \end{aligned} \right\} \text{not possible}$$

$$K(X:Y) = \max_{\substack{V=f(X) \\ =g(Y)}} \{H(V)\}$$

$$X = f(Y) \Leftrightarrow H(X|Y) = 0$$

X	Y	P
0	0	1/8
0	1	1/8
1	0	1/8
1	1	1/8
2	2	1/2

	X	Y	V
A	0	0	1/2
	1	1	1/2
B	2	2	1/2

$$\begin{aligned} X &= (X', V) \\ Y &= (Y', V) \end{aligned}$$

$$I(X:Y|Z)$$

$$= \min_{P(\bar{Z}|Z)} I(X:Y|\bar{Z})$$

$$I(X_0:Y|\bar{X})$$

$$I(X:Y|Z)$$

X	Y	Z	U
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	0
2	2	0	1
3	3	1	1

$$= \min H(U) + I(X:Y|ZU)$$

$$d(P, Q) = \sum_{\ell=1}^{\infty} 2^{-\ell} \sqrt{D_{JS}(P^{(\ell)} \| Q^{(\ell)})}$$

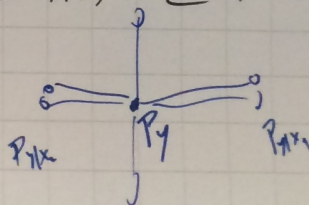
$$D_{JS}(P_1, P_2) = \frac{1}{2} D_{KL}(P_1 \| \frac{P_1+P_2}{2}) + \frac{1}{2} D_{KL}(P_2 \| \frac{P_1+P_2}{2})$$

$$= H(\frac{P_1+P_2}{2}) - \frac{1}{2} H(P_1) - \frac{1}{2} H(P_2)$$

$$= \sum_{\ell} H(\sum \pi_i P_i) - \sum \pi_i H(P_i)$$

$$= I(M, Z)$$

$$M = \sum \pi_i P_i$$



STAPLES