# A Brief History of Partial Information Decomposition

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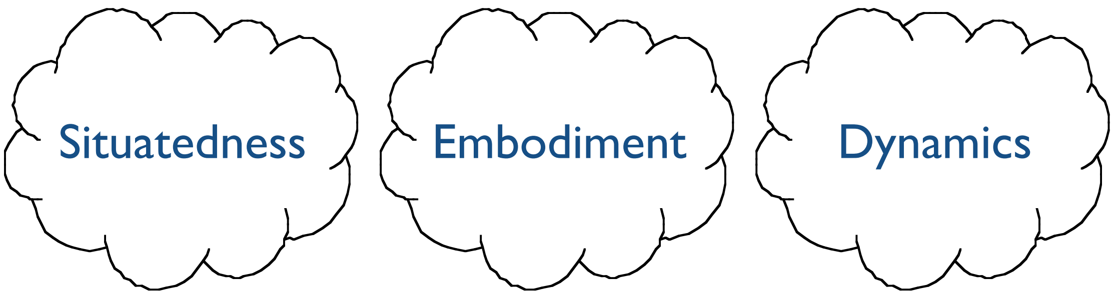
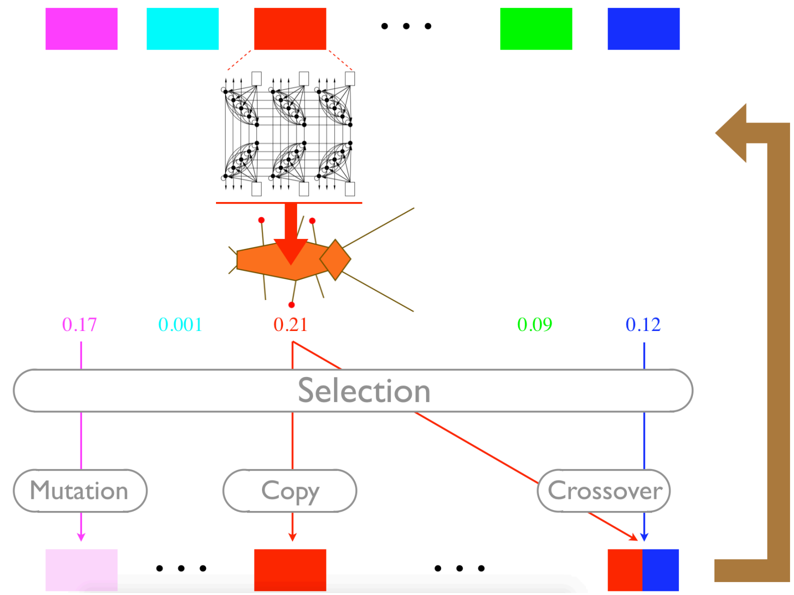
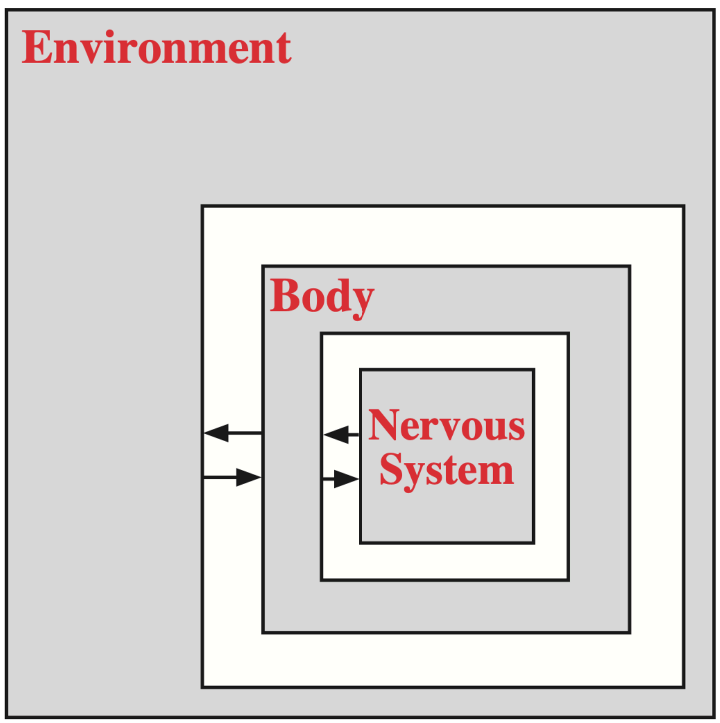
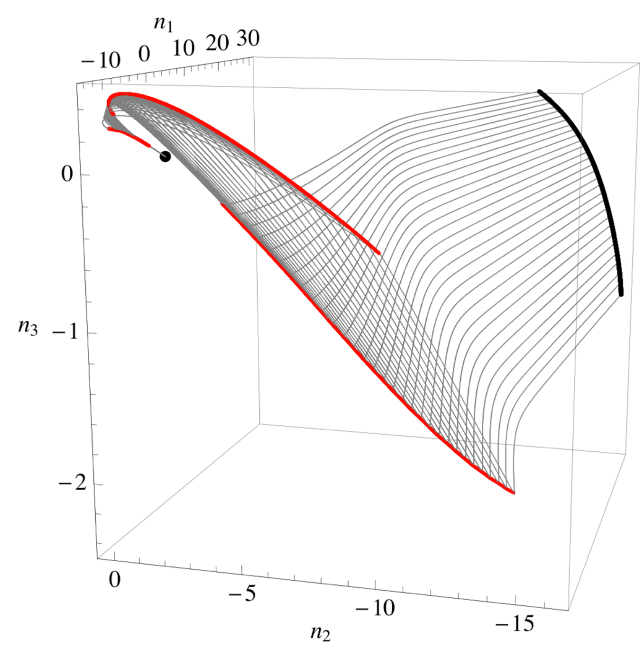
# The Dynamics of Brain-Body-Environment Systems



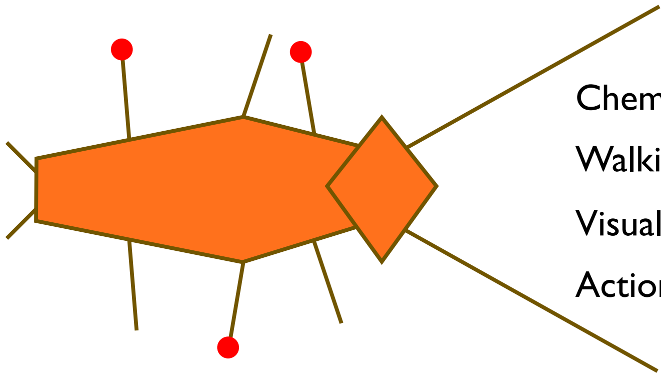




Beer, R.D. (1992/1995). A dynamical systems perspective on agent-environment interaction. *Artificial Intelligence* **72**:173-215.

# Examples

## Sensorimotor Behavior



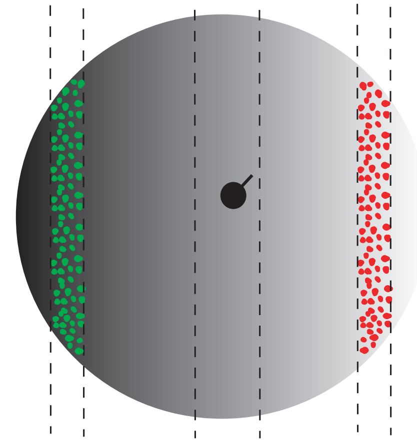
Chemotaxis

Walking

Visually-Guided Walking

Action Switching

## Learning Behavior

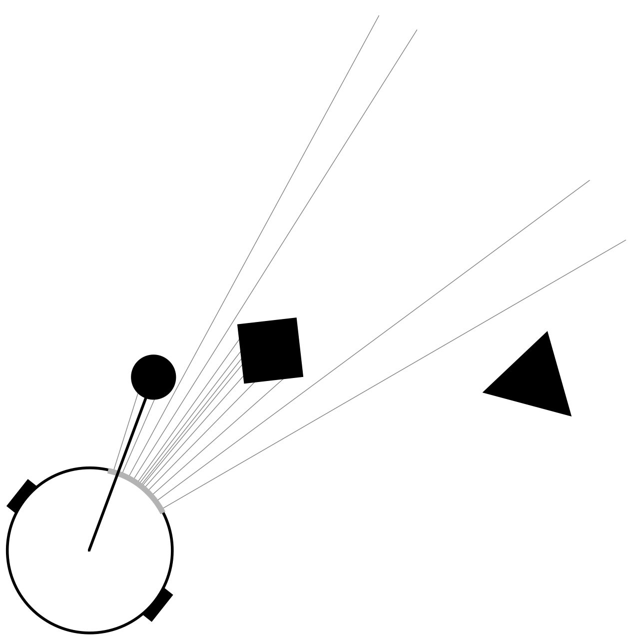


Sequential Decision Learning

Food Edibility Learning

Temperature Learning

## Minimally Cognitive Behavior



Visually-Guided Catching

Perception of Body-Scaled Affordances

Object Categorization

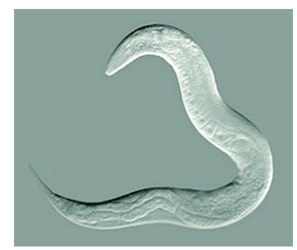
Short-Term Memory

Selective Attention

Relational Categorization

Referential Communication

## Empirically-Grounded Applications

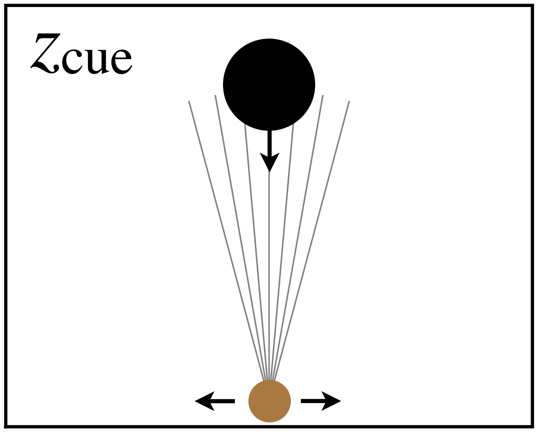


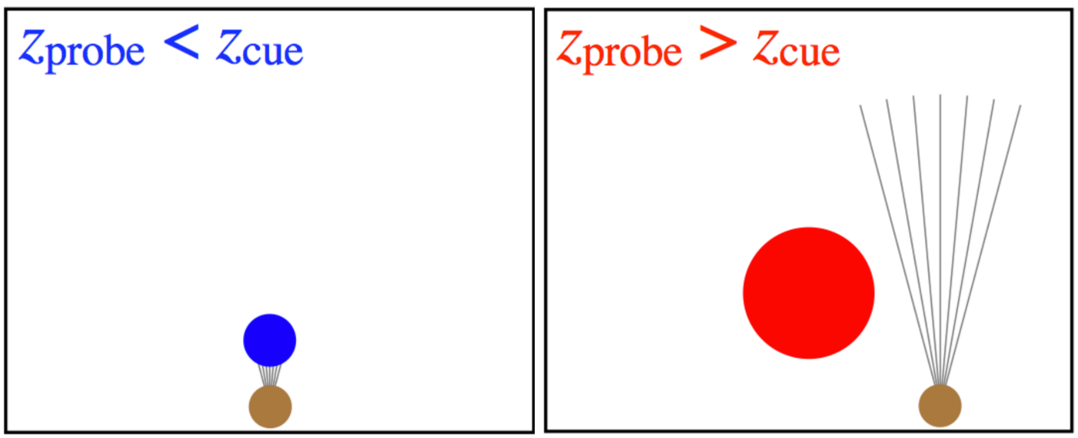
*C. elegans*

Visually-Guided Braking

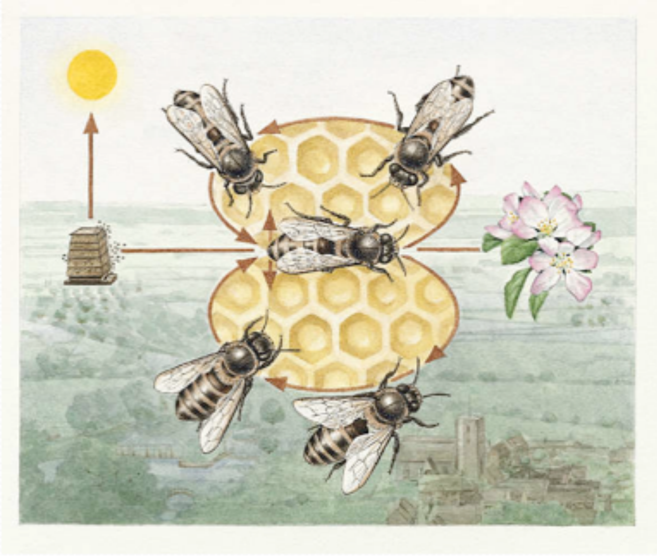
# Two Motivating Examples

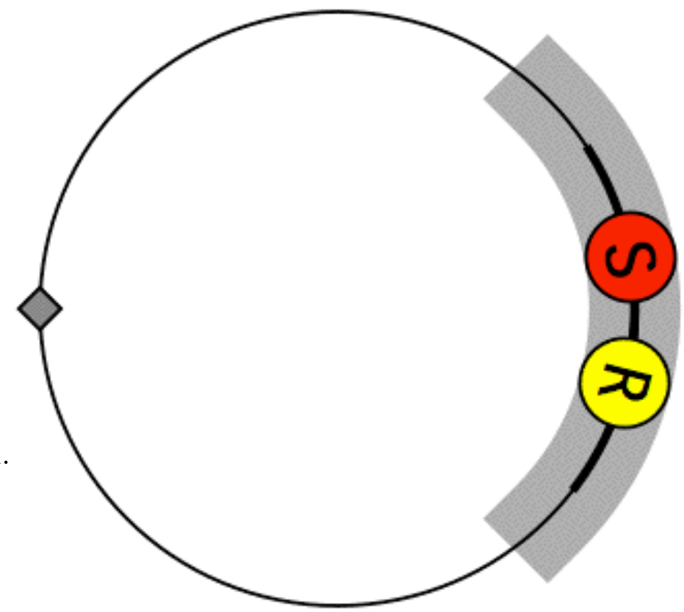
## Relational Categorization




## Referential Communication



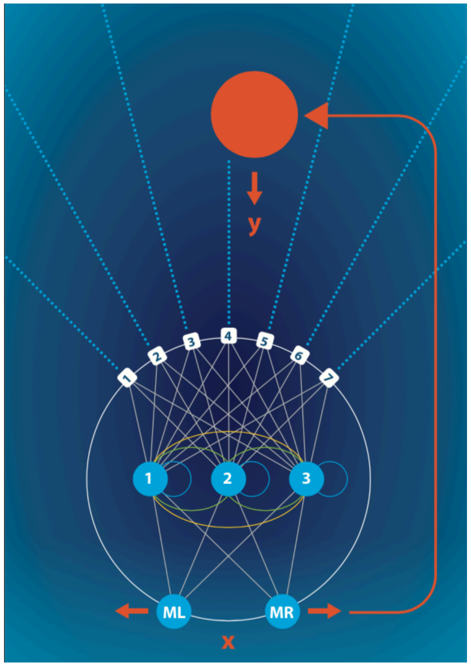




Williams, P.L., Beer, R.D., and Gasser, M. (2008). An embodied dynamical approach to relational categorization. In B.C. Love, K. McRae and V.M. Sloutsky (Eds.), *Proceedings of the 30th Annual Conference of the Cognitive Science Society* (pp. 223-228).

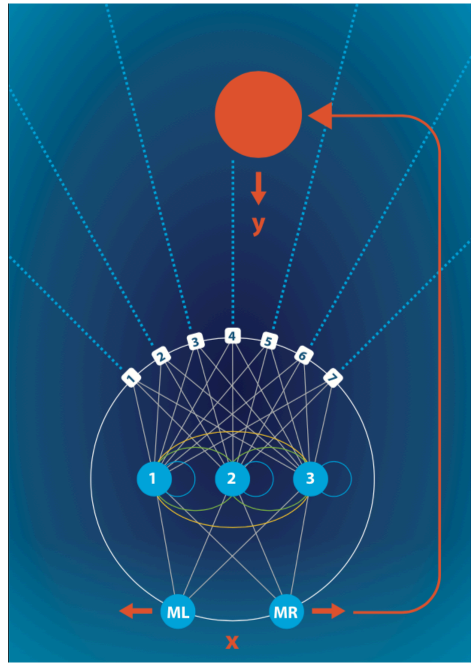
Williams, P.L., Beer, R.D., and Gasser, M. (2008). Evolving referential communication in embodied dynamical agents. In S. Bullock et al. (Eds.), *Artificial Life XI: Proceedings of the Eleventh International Conference on the Simulation and Synthesis of Living Systems* (pp. 702-709). MIT Press.

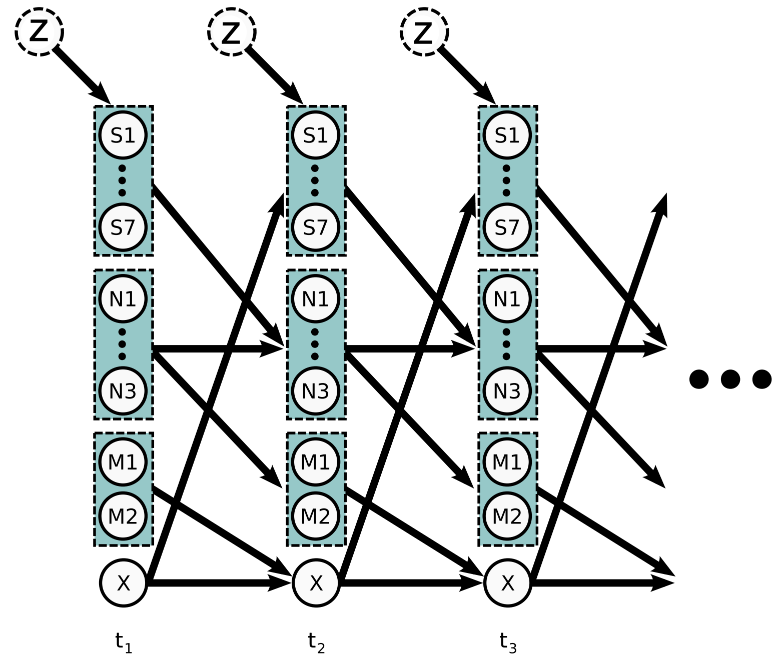
# Some Questions




- How is absolute object size extracted?
- Where is information about absolute object size stored?
- How is information about relative object size extracted?
- Where is information about relative object size stored?
- How does this information move through the system over time?
- How is this information combined into a classification decision?

# Some Special Features




- Deterministic system
- Causal diagram is available
- Interested in information about an external stimulus feature
- Temporal aspect is crucial

# The Dynamics of Information I

## Stimulus Information in a Stochastic Process

$$I(S; X_t)$$

## Information Gain

$$I_G(S; X_t) = I(S; X_t \mid X_{t-1})$$

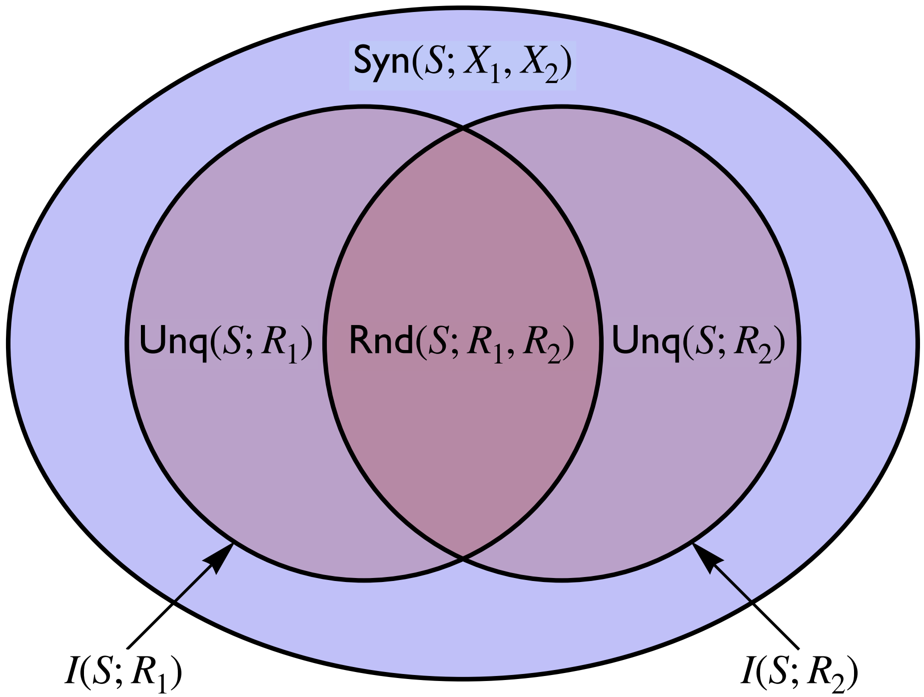
## Specific Information/Surprisal

$$I(S = s; X) = \sum_{x \in X} p(x \mid s) \ \log \frac{p(x \mid s)}{p(s)}$$

Williams, P.L. and Beer, R.D. (2010). Information dynamics of evolved agents. In S. Doncieux et al. (Eds), *From Animals to Animals 11: Proceedings of the International Conference on Simulation of Adaptive Behavior* (pp. 38-49). Springer-Verlag.

# The Structure of Multivariate Information

$I(S;R_1,R_2)$




$$I(S;R_1,R_2) = \text{Unq}(S;R_1) + \text{Unq}(S;R_2) + \text{Rnd}(S;R_1,R_2) + \text{Syn}(S;R_1,R_2)$$


Williams, P.L. and Beer, R.D. (2010). Nonnegative decomposition of multivariate information. arXiv:1004.2515

Williams, P.L. (2011). *Information Dynamics: Its Theory and Application to Embodied Cognitive Systems*. Ph.D. Dissertation, Cognitive Science Program, Indiana University.

# Measuring Shared Information

$$I_{\min}(S; \mathbf{A}_1, \ldots, \mathbf{A}_n) = \sum_{s \in S} p(s) \min_{\mathbf{A}_i} I(S = s; \mathbf{A}_i)$$

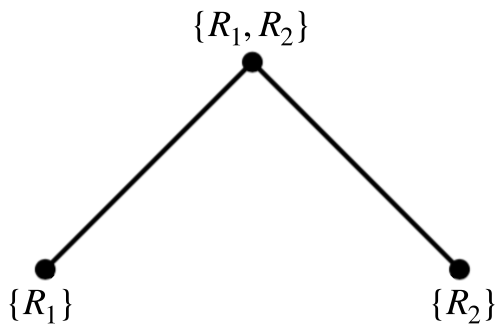
$$I_{\cap}(S; \mathbf{A}_1, \ldots, \mathbf{A}_k)$$

**Axiom 1** (symmetry): $I_{\cap}$ is symmetric in the $\mathbf{A}_i$‘s

**Axiom 2** (self-redundancy): $I_{\cap}(S; \mathbf{A}) = I(S; \mathbf{A})$

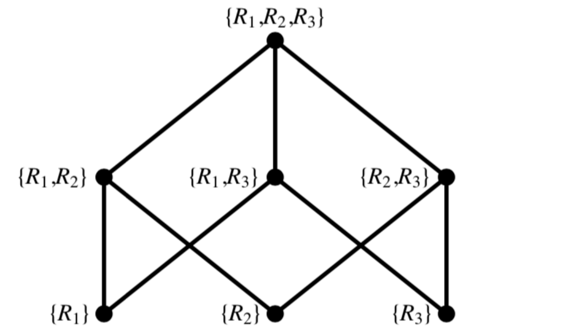
**Axiom 3** (monotonicity): $I_{\cap}(S; \mathbf{A}_1, \ldots, \mathbf{A}_k) \leq I_{\cap}(S; \mathbf{A}_1, \ldots, \mathbf{A}_{k-1})$
with equality when $\mathbf{A}_{k-1} \subseteq \mathbf{A}_k$

# The Partial Information Lattice




$$\langle \mathscr{P}^{+}(\mathbf{R}), \subseteq \rangle$$

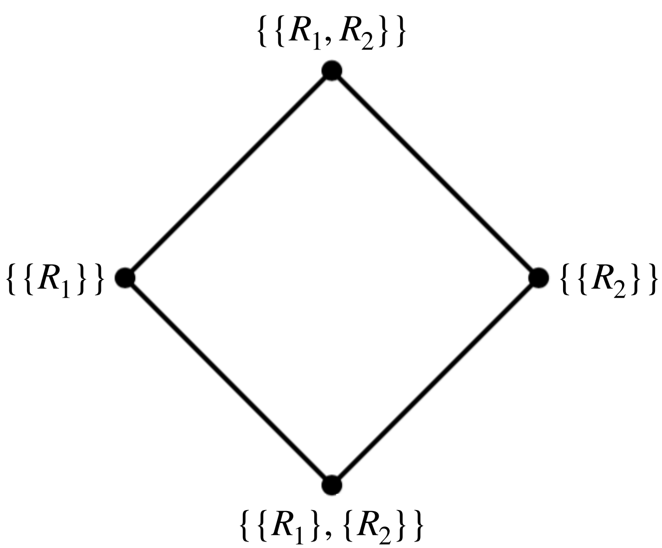
Source inclusion lattice




$\{\{\{R_1\}\}, \{\{R_2\}\}, \{\{R_1\}, \{R_2\}\}, \{\{R_1, R_2\}\}\}$

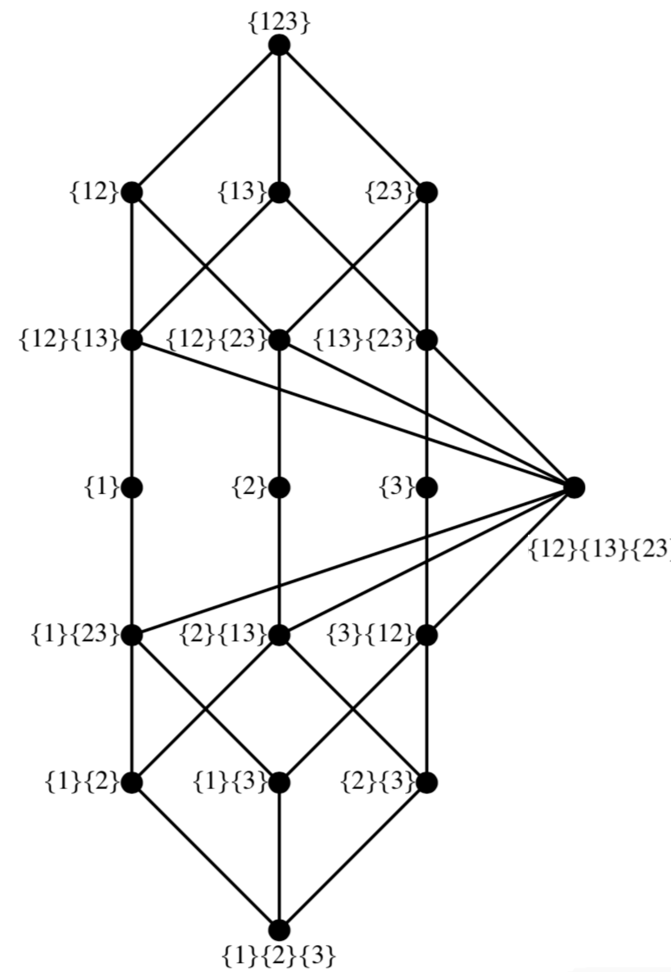
$$\mathscr{A}(\mathbf{R})$$

Set of antichains




$$\langle \mathscr{A}(\mathbf{R}), \preceq \rangle$$

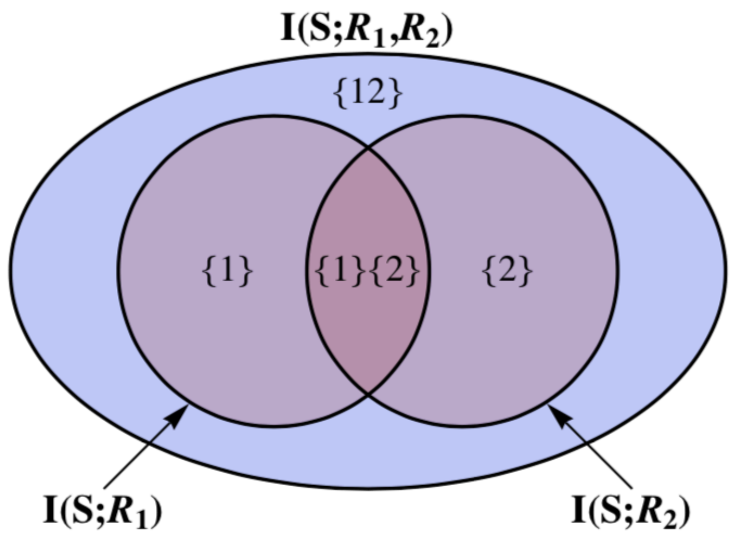
Partial information lattice



# Partial Information Decomposition

$$I_{\cap}(S;\alpha) = \sum_{\beta \preceq \alpha} I_{\partial}(S;\beta)$$

$$I(S;R_1,R_2) = \underbrace{I_{\partial}(S;\{1\})}_{\text{Unique}} + \underbrace{I_{\partial}(S;\{2\})}_{\text{Unique}} + \underbrace{I_{\partial}(S;\{1\}\{2\})}_{\text{Redundant}} + \underbrace{I_{\partial}(S;\{12\})}_{\text{Synergistic}}$$



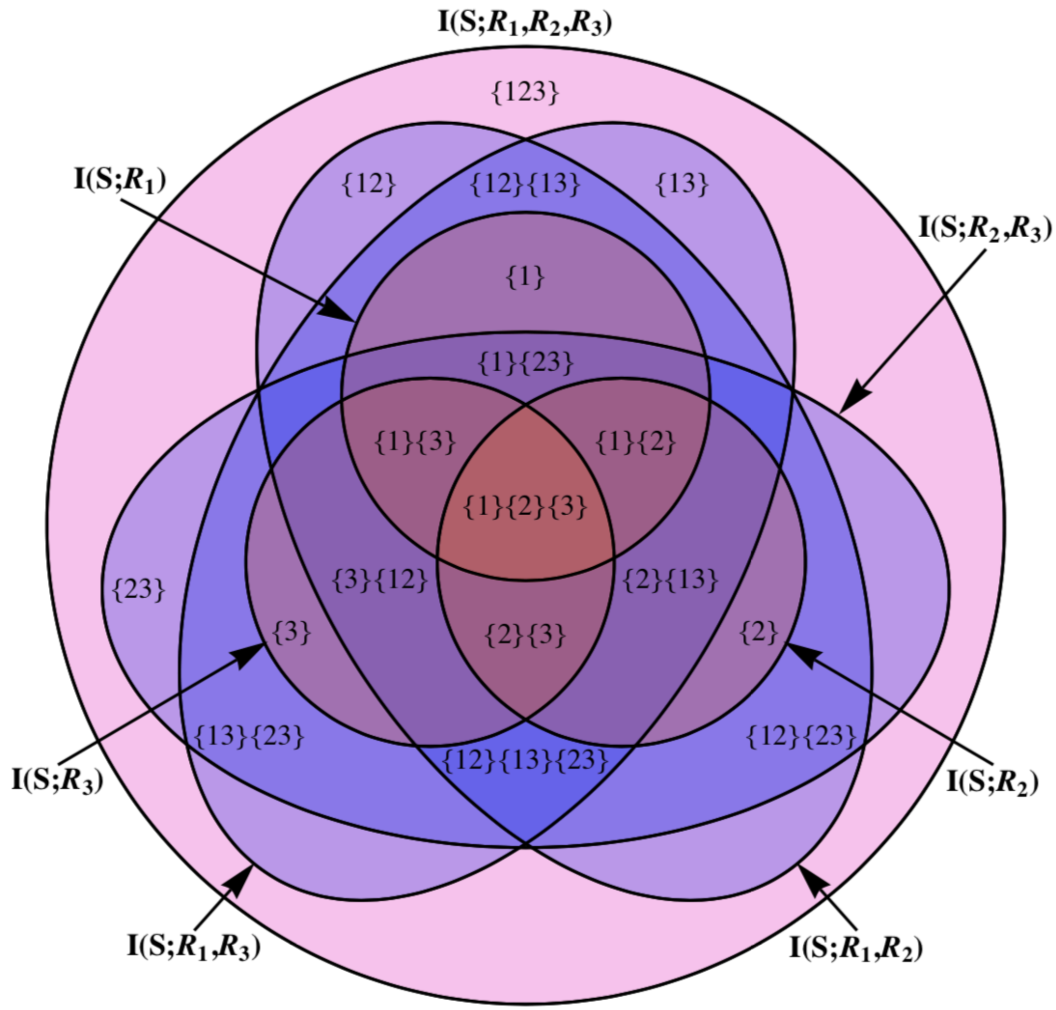

$$I_{\partial}(S;\{1\}\{2\}) = I_{\cap}(S;R_1,R_2)$$

$$I_{\partial}(S;\{1\}) = I(S;R_1) - I_{\cap}(S;R_1,R_2)$$

$$I_{\partial}(S;\{2\}) = I(S;R_2) - I_{\cap}(S;R_1,R_2)$$

$$I_{\partial}(S;\{12\}) = I(S;R_1,R_2) - I(S;R_1) - I(S;R_2) + I_{\cap}(S;R_1,R_2)$$

# Partial Information Decomposition



# Some Consequences

## Interaction Information

$$I(S;R_1;R_2) = I_\partial(S;\{12\}) - I_\partial(S;\{1\}\{2\})$$

Synergistic — Redundant

## Conditional Information

$$I(S;R_1 \mid R_2) = I_\partial(S;\{1\}) + I_\partial(S;\{12\})$$

Unique — Synergistic

## Transfer Entropy

$$T_{X \to Y} = I(Y_t;X_{t-1} \mid Y_{t-1}) = I_\partial(Y_t;\{X_{t-1}\}) + I_\partial(Y_t;\{X_{t-1},Y_{t-1}\})$$

Unique (state-independent) — Synergistic (state-dependent)

Williams, P.L. and Beer, R.D. (2011). Generalized measures of information transfer. arXiv:1102.1507.

# The Dynamics of Information II

## Stimulus Information in a Stochastic Process

$$I(S; X_t)$$

## Specific Information/Surprisal

$$I(S = s; X) = \sum_{x \in X} p(x \mid s) \ \log \frac{p(x \mid s)}{p(s)}$$

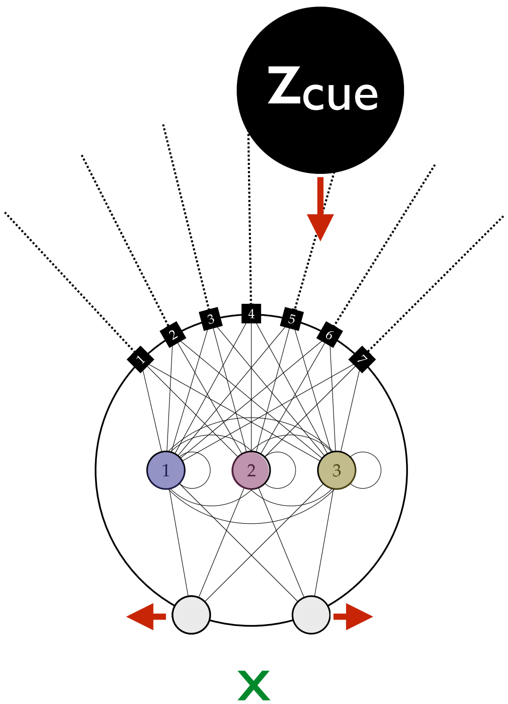
## Information Gain

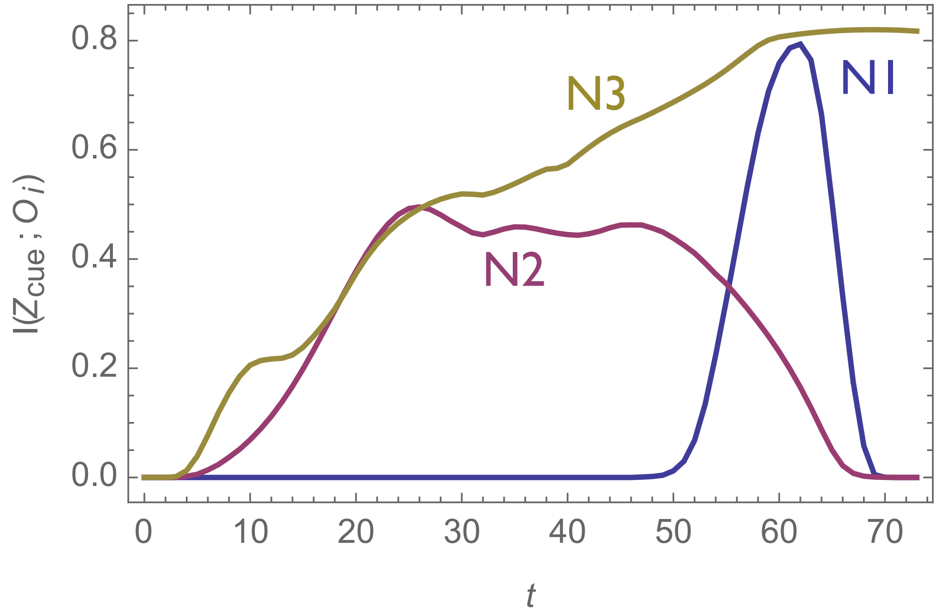
$$I_G(S; X_t) = I(S; X_t) - I_{\min}(S; X_{t-1}, X_t)$$

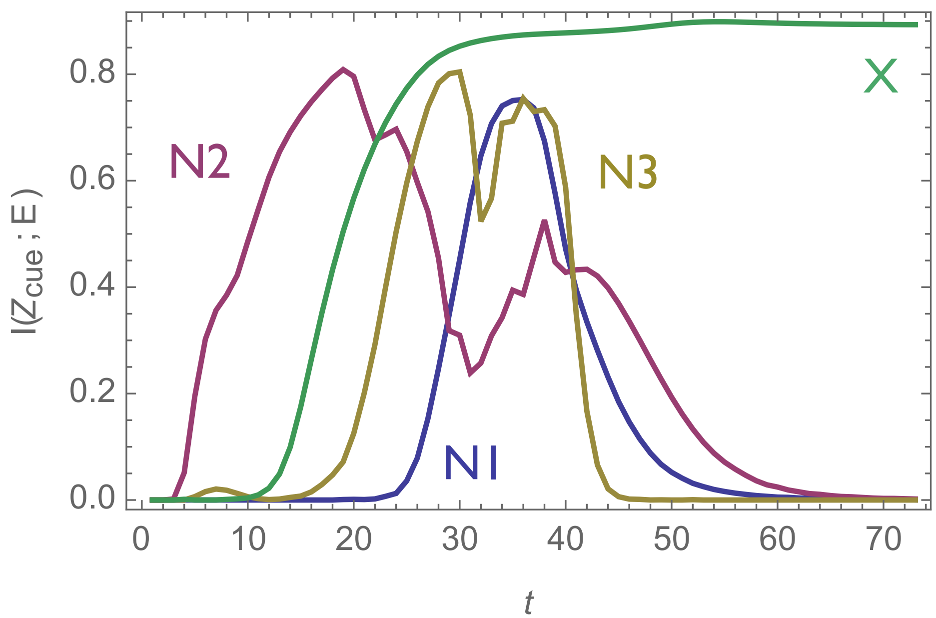
## Information Transfer

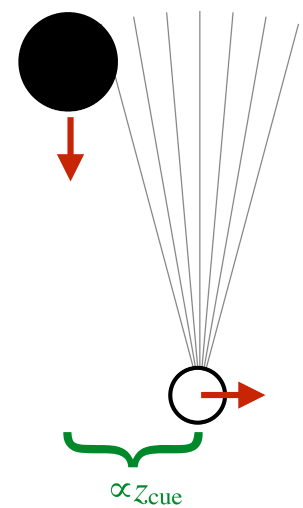
$$I_T(S; X_{t-1} \rightarrow Y_t) = I_{\min}(S; Y_t, \{X_{t-1}, Y_{t-1}\}) - I_{\min}(S; Y_{t-1}, Y_t)$$

# Where is Cue Object Size Stored?



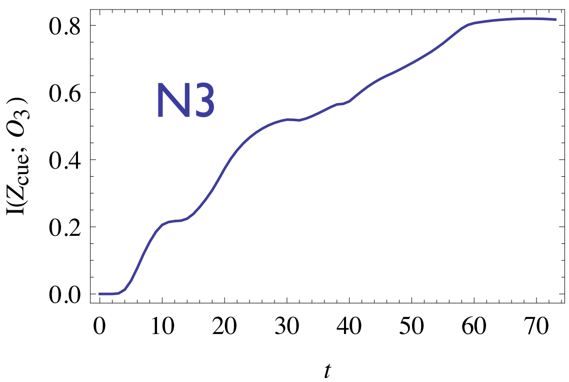



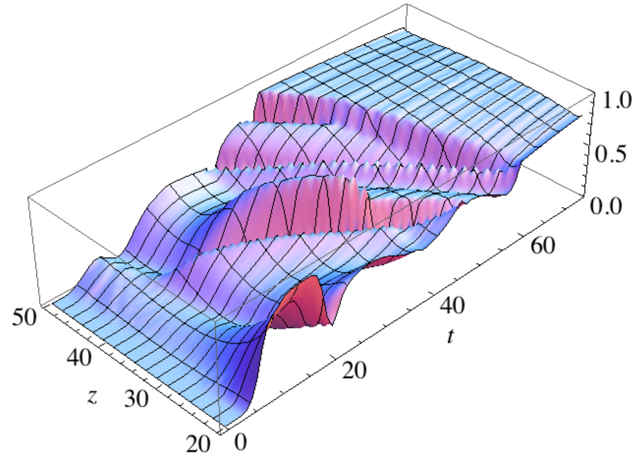



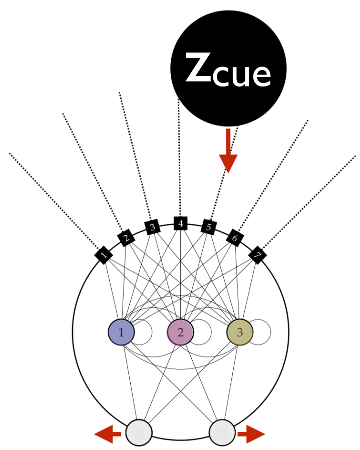

Beer, R.D. and Williams, P.L. (2015). Information processing and dynamics in minimally cognitive agents. *Cognitive Science* **39**:1-38.

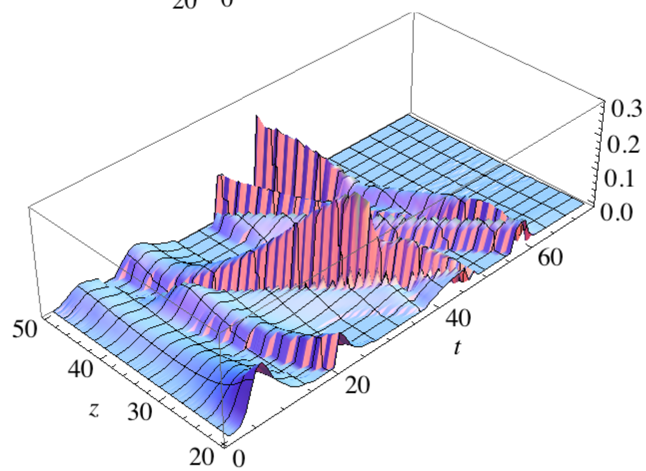
# Cue Stage Information Flow



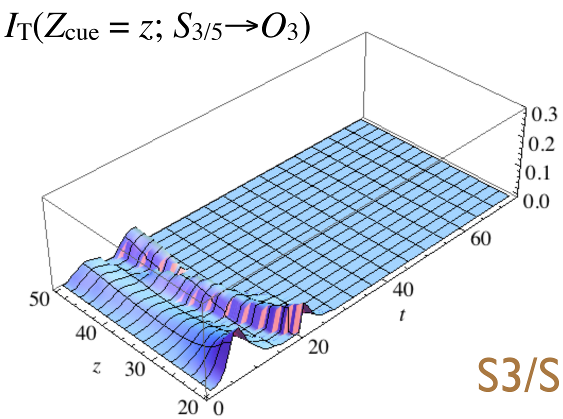

Specific Information



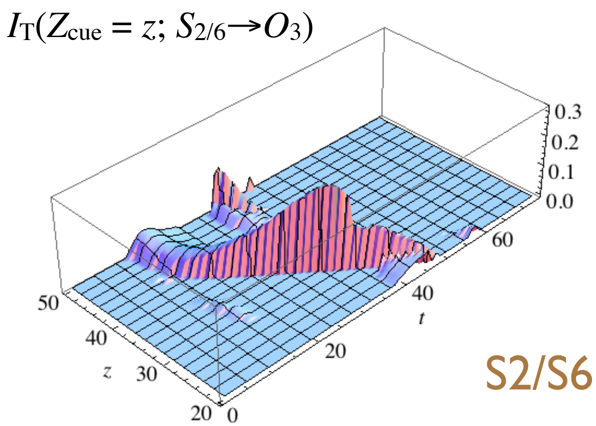




Information Gain

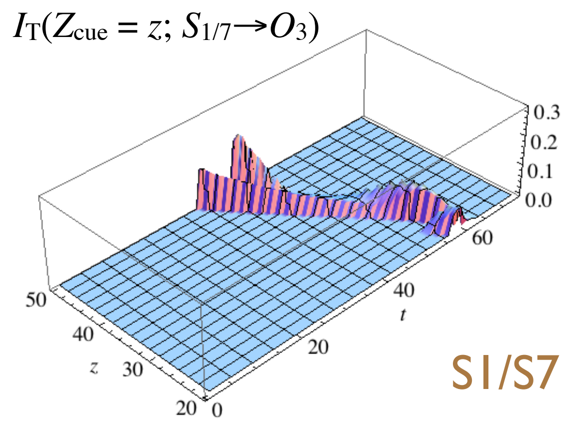
Information Transfer




S3/S5



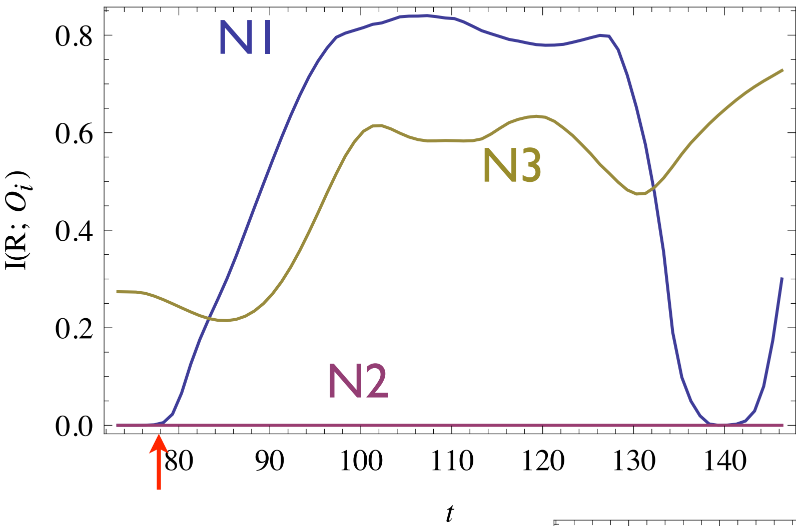

S2/S6




S1/S7

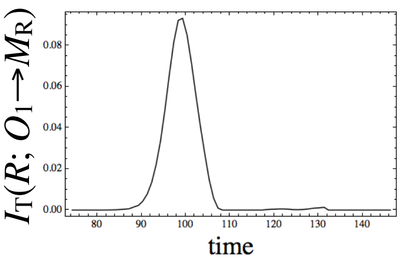
Beer

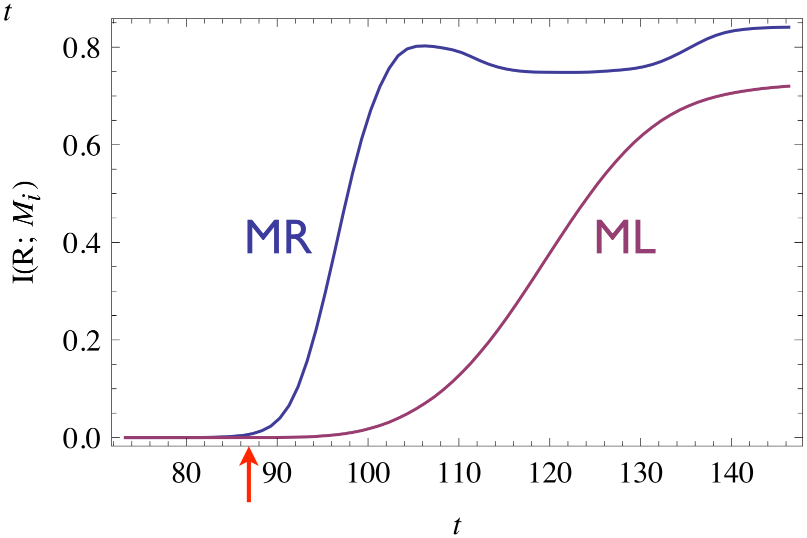
# Probe Stage Information Flow

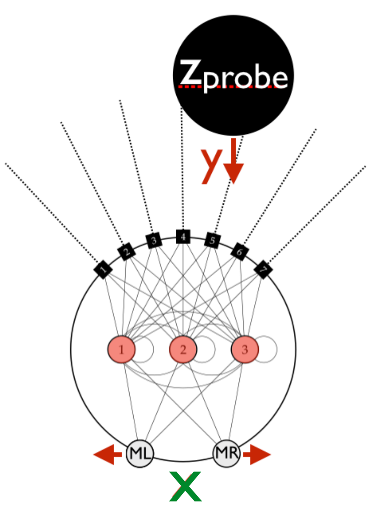


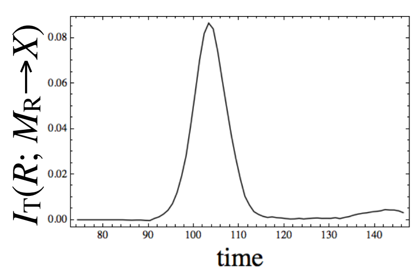

## Relative Object Size

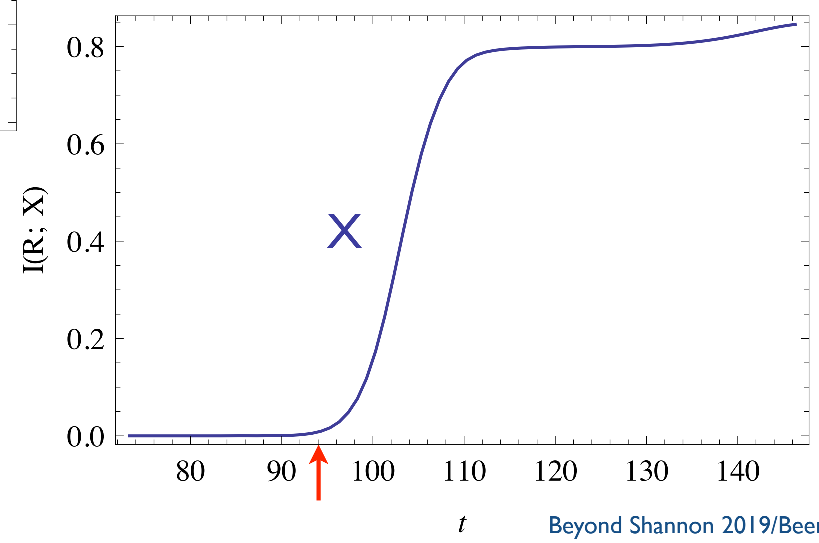
$$r = \begin{cases} 1 & z_{probe} \leq z_{cue} \\ 0 & z_{probe} > z_{cue} \end{cases}$$



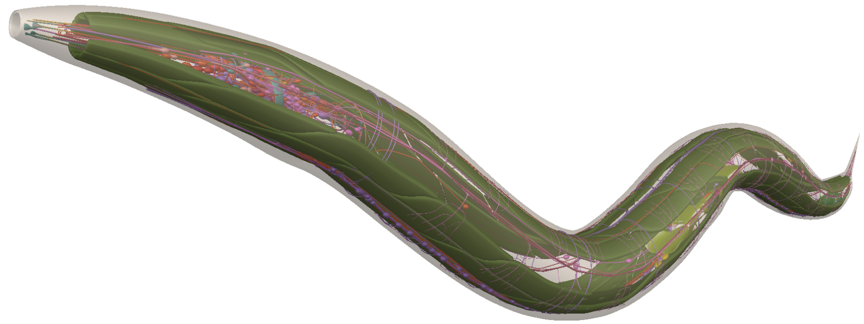



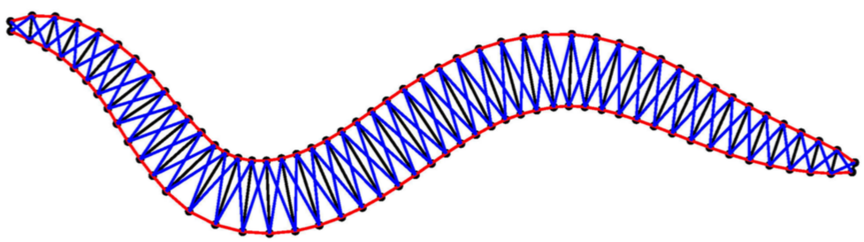



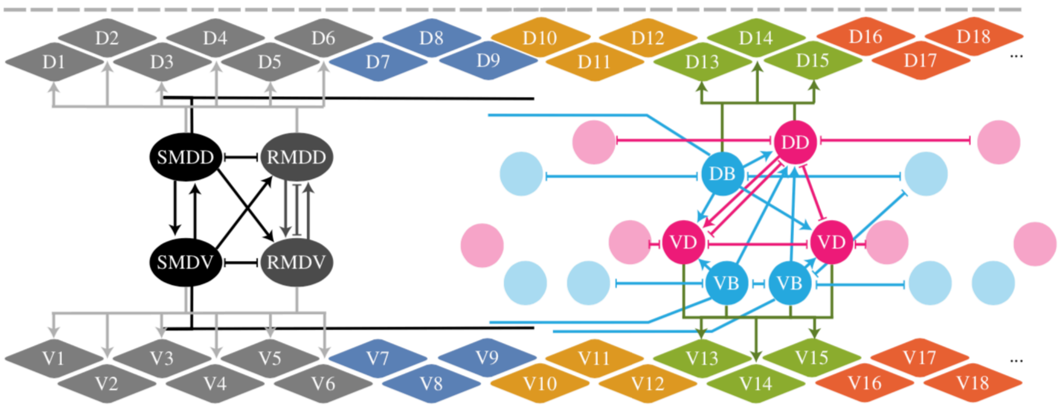



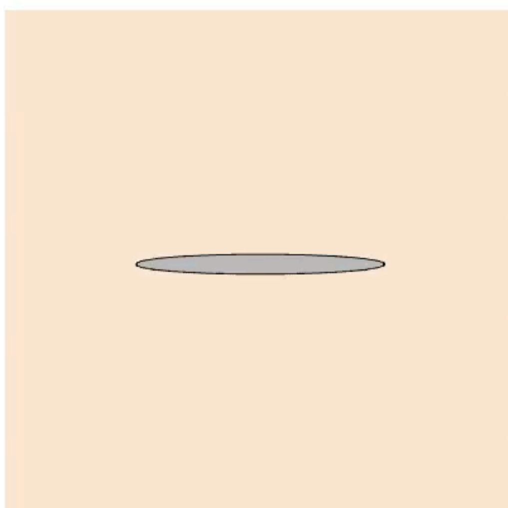



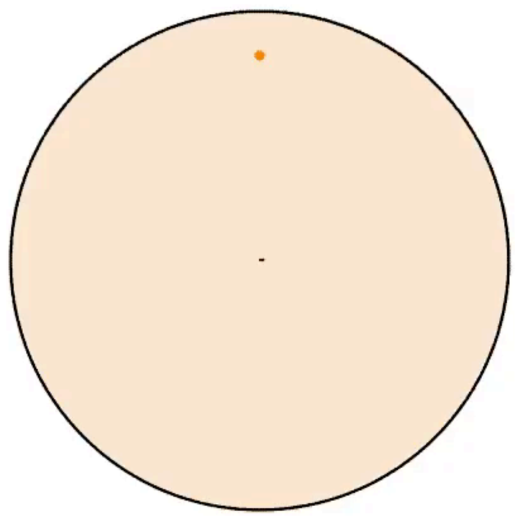
# A Biological Application: *Caenorhabditis elegans*



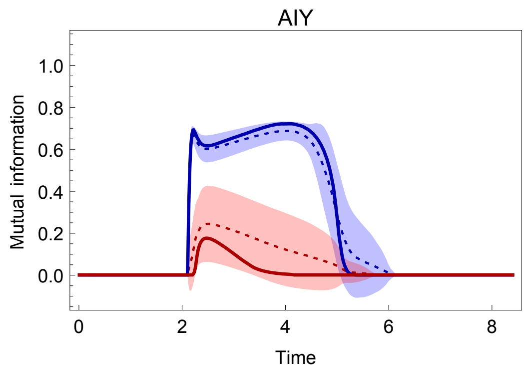


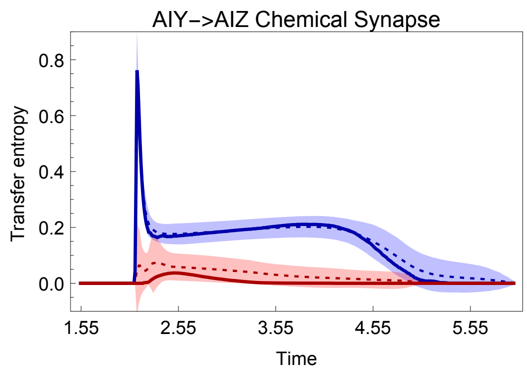


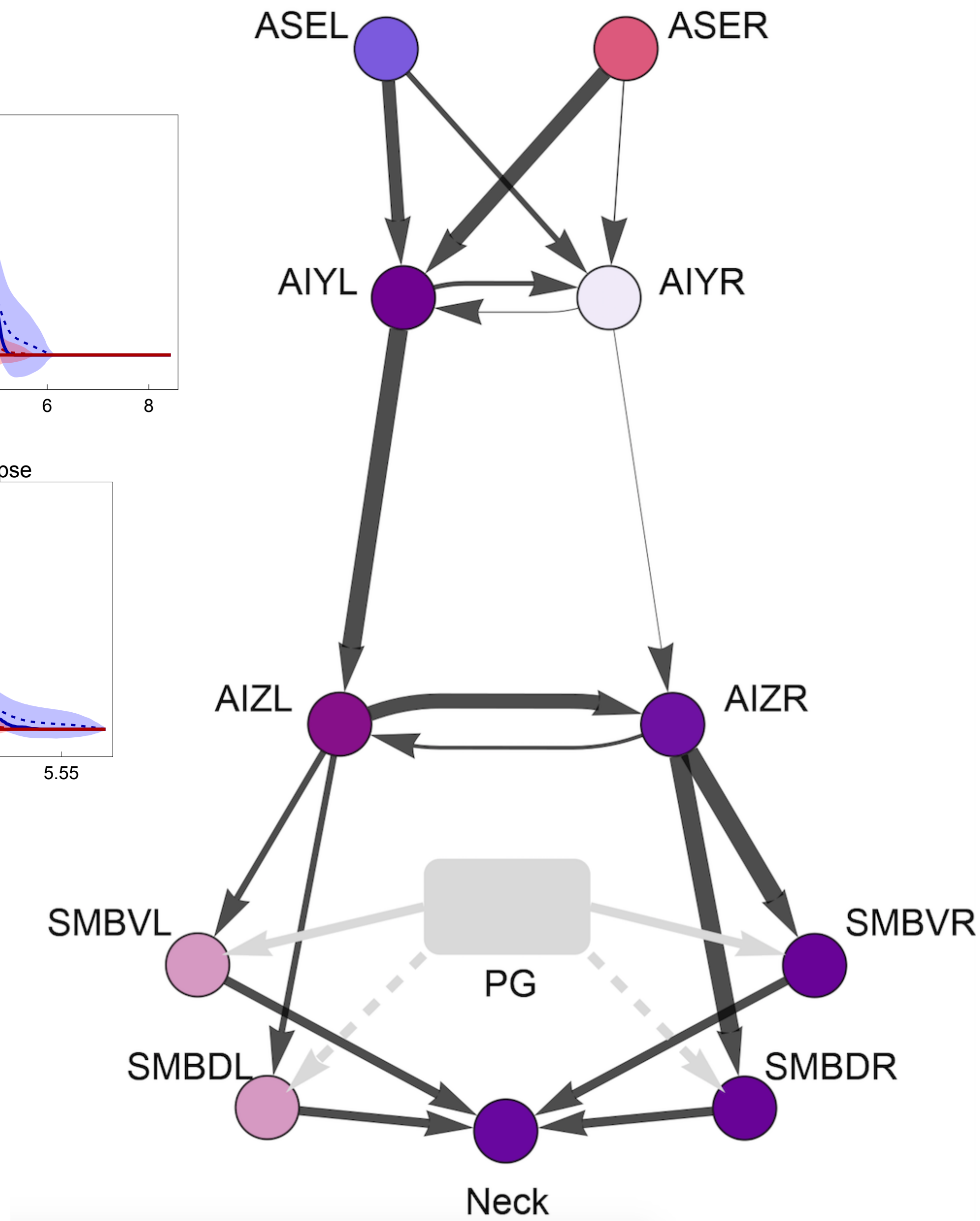





# Information Architecture of *C. elegans* Klinotaxis




Izquierdo, E.J., Williams, P.L. and Beer, R.D. (2015). Information flow through a model of the *C. elegans* klinotaxis circuit. *PLoS ONE* 10(10):e0140397. doi:10.1371/journal.pone.0140397.

# Many Other Applications

## Computation is concentrated in rich clubs of local cortical networks

Samantha P. Faber[1], Nicholas M. Timme[2], John M. Beggs[3], and Ehren L. Newman[1]

## High-Degree Neurons Feed Cortical Computations

Nicholas M. Timme[1]*, Shinya Ito[2], Maxym Myroshnychenko[3], Sunny Nigam[1], Masanori Shimono[4], Fang-Chin Yeh[5], Pawel Hottowy[6], Alan M. Litke[2], John M. Beggs[1]

## Apical Function in Neocortical Pyramidal Cells: A Common Pathway by Which General Anesthetics Can Affect Mental State

*William A. Phillips[1], Talis Bachmann[2]* and Johan F. Storm[3]*

## Maternal deprivation induces alterations in cognitive and cortical function in adulthood

Sarine S. Janetsian-Fritz[1], Nicholas M. Timme[1], Maureen M. Timm[1], Aqilah M. McCane[1], Anthony J. Baucum II[2], Brian F. O'Donnell[3] and Christopher C. Lapish[1,4,5]

## The Partial Information Decomposition of Generative Neural Network Models

Tycho M.S. Tax [1,*,†], Pedro A.M. Mediano [2,*,†] and Murray Shanahan [2]

## Multivariate Mutual Information of Interferometric Radar Altimeter

For an Application to Terrain-Referenced Navigation

Youngjoo Kim Hyochoong Bang

## Synergistic Information Processing Encrypts Strategic Reasoning in Poker

Seth Frey,[a,b,c] Dominic K. Albino,[d,†] Paul L. Williams[c]

## Information Theoretical Study of Cross-Talk Mediated Signal Transduction in MAPK Pathways

Alok Kumar Maity [1,†], Pinaki Chaudhury [1] and Suman K. Banik [2,*]

## Temporal information partitioning: Characterizing synergy, uniqueness, and redundancy in interacting environmental variables

Allison E. Goodwell[1] and Praveen Kumar[1,2]

## Dynamic process connectivity explains ecohydrologic responses to rainfall pulses and drought

Allison E. Goodwell[a,b], Praveen Kumar[a,1], Aaron W. Fellows[c], and Gerald N. Flerchinger[c]

## Nonlinear higher order abiotic interactions explain riverine biodiversity

Masahiro Ryo[1,2,3] | Eric Harvey[1,4] | Christopher T. Robinson[1,5] | Florian Altermatt[1,4]

## Stem Cell Differentiation as a Non-Markov Stochastic Process

Patrick S. Stumpf,[1,2] Rosanna C.G. Smith,[1,2] Michael Lenz,[3,4,5] Andreas Schuppert,[3,4] Franz-Josef Müller,[6,7] Ann Babtie,[8] Thalia E. Chan,[8] Michael P.H. Stumpf,[8] Colin P. Please,[9] Sam D. Howison,[9] Fumio Arai,[10] and Ben D. MacArthur[1,2,11,12,*]
[1]Centre for Human Development, Stem Cells and Regeneration, Faculty of Medicine, University of Southampton, Southampton SO17 1BJ, UK
[2]Institute for Life Sciences, University of Southampton, Southampton SO17 1BJ, UK
[3]Joint Research Center for Computational Biomedicine, RWTH Aachen University, 52056 Aachen, Germany
[4]Aachen Institute for Advanced Study in Computational Engineering Science, RWTH Aachen University, 52062 Aachen, Germany
[5]Maastricht Centre for Systems Biology, Maastricht University, 6229 ER Maastricht, the Netherlands
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