# Spontaneous Mutations in Population Dynamics and Phase Transitions

**Physics of Computation**
**(Physics 256B, Spring 2022)**

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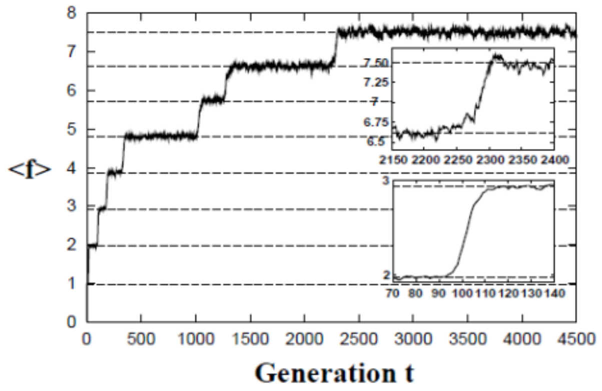
# Introduction and Previous models

- History of mutation-selection models
- Notion of exploration of the space of fitnesses through random mutations



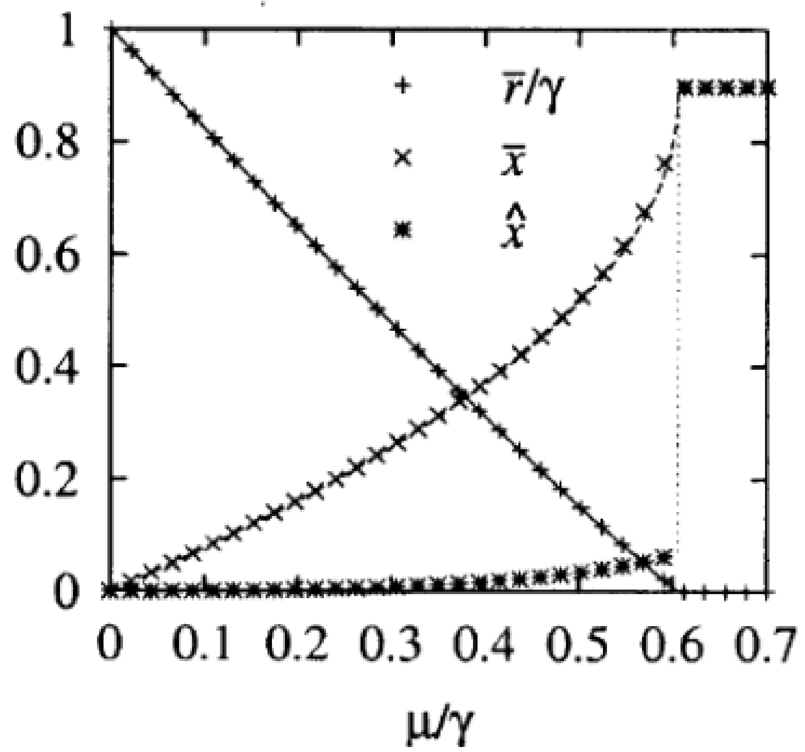

Erik van Nimwegen, James P. Crutchfield, and Melanie Mitchell, "Finite Populations Induce Metastability in Evolutionary Search", Physics Letters A, ***229***:3 (1997) 144-150.

- Mean fitness increases in a jump pattern separated by stagnating periods
- Increasing the population size smoothens the increase in fitness, the stagnating periods are really metastable.

# Transition between order and disorder

- Influence of the mutation probability
- Notion of mutation threshold

- Above the threshold, 0 mean fitness and large fluctuations
- High mutation rate leads to disorder



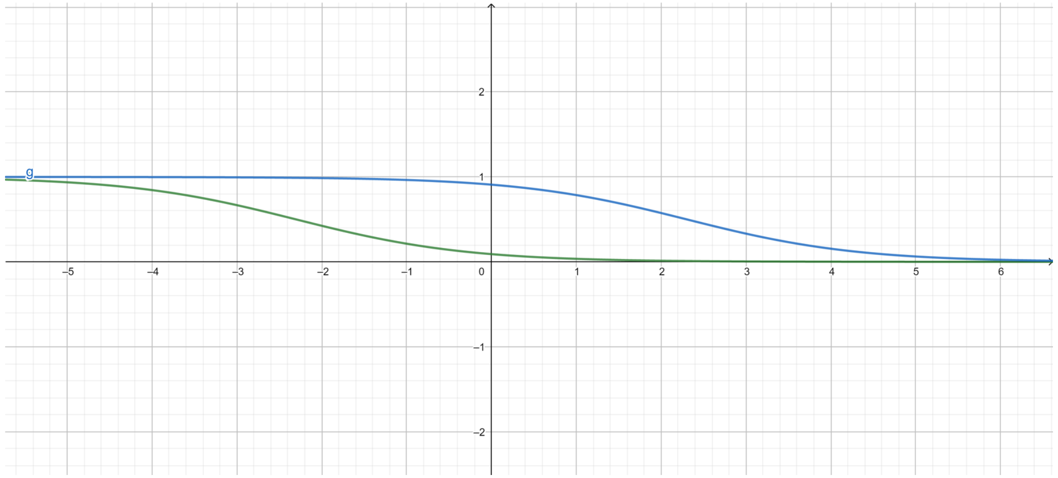

Hermisson, J., Redner, O., Wagner, H., & Baake, E. (2002). Mutation–selection balance: ancestry, load, and maximum principle. *Theoretical population biology*, *62*(1), 9-46.

# Motivations and Goals

- Study population growth through mutation-selection
- Build a suited model with time-varying population size
- Influence of the mutation rate on the growth (or decay) of populations
- Can we relate disorder (entropy) and growth ?

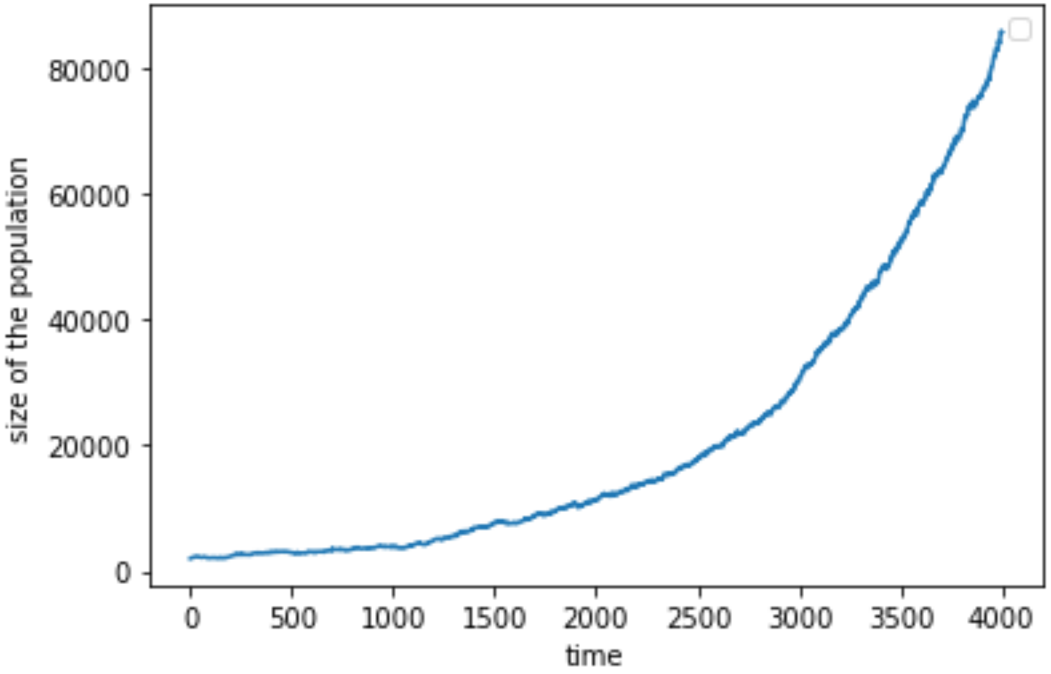
# The model

- Iterative mutation selection model
- Mutation step : Every individual has a fixed mutation rate. The new fitness is taken from a given distribution (2 choices). The parameters of the model quantify this step.
- Selection step : 3 outcomes possible for each individual : death, reproduction (actually cloning), or neither of the two.

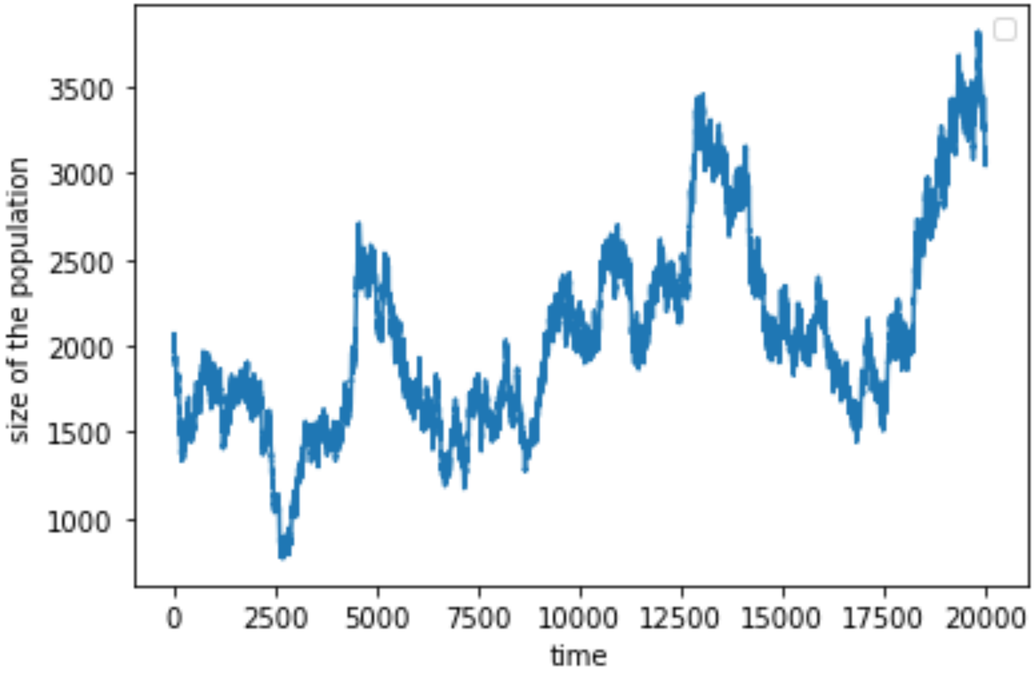



The probabilities for the selection step depend on the gap between the fitness of the individual and the mean fitness

# First results with correlation, some possible behaviours



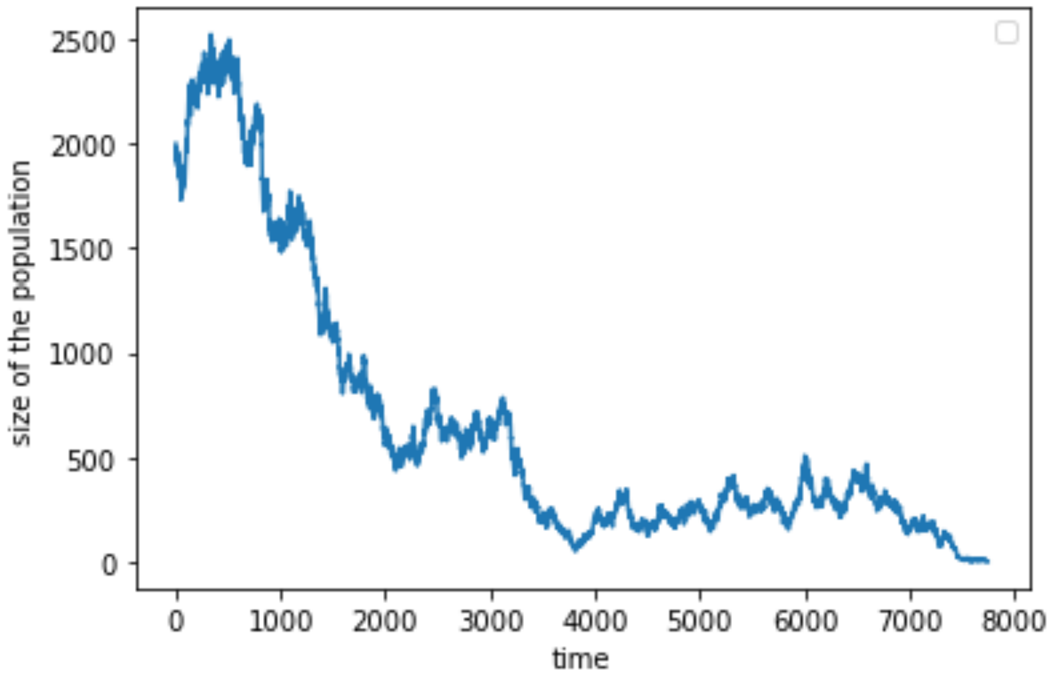

N = 2000, p = 0.05, $\sigma$ = 1




N = 2000, p = $10^{-6}$, $\sigma$ = 1

Exponential growth
This leads to the second choice for the mutation step

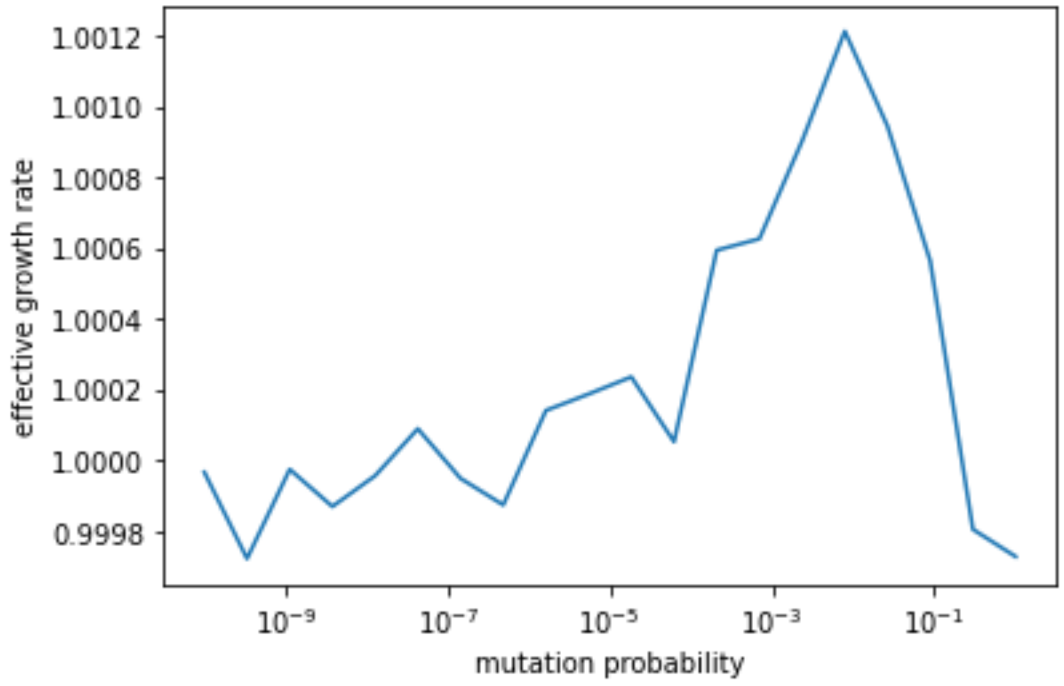
Low mutation rate, no exploration and limited growth



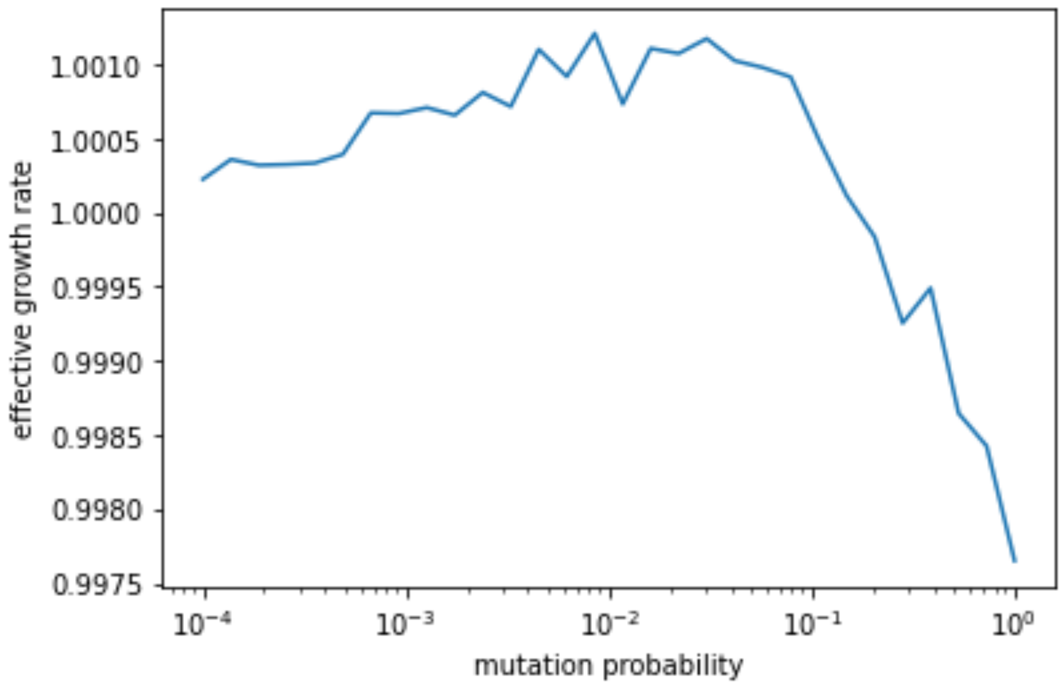

N = 2000, p = 0.3, $\sigma$ = 1

High enough mutation rate leads to the decay of the population and long term extinction.

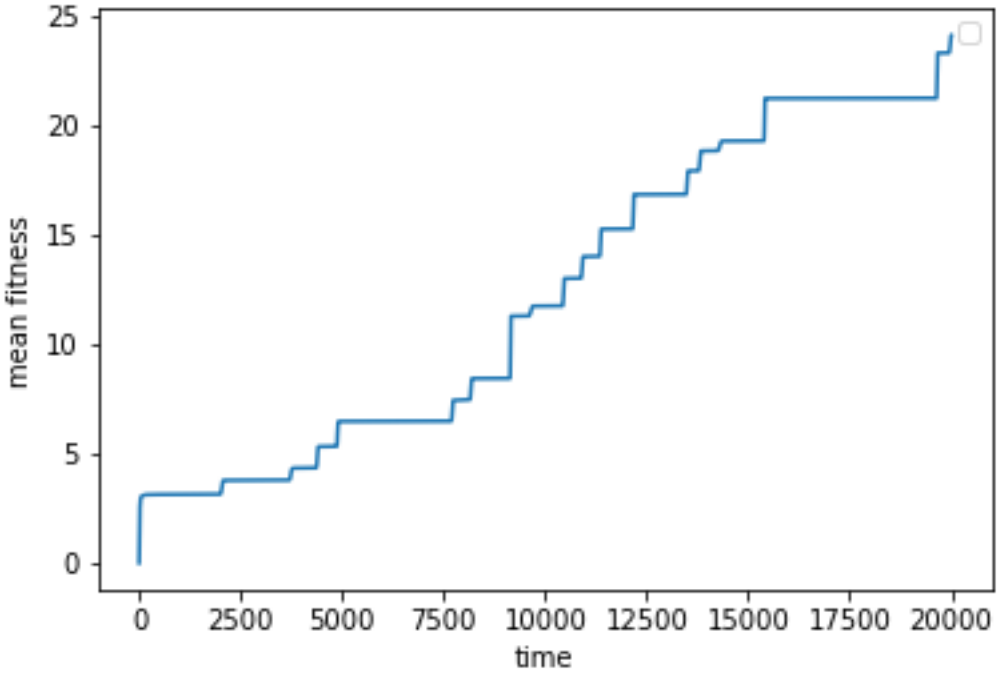
# Optimum for population growth

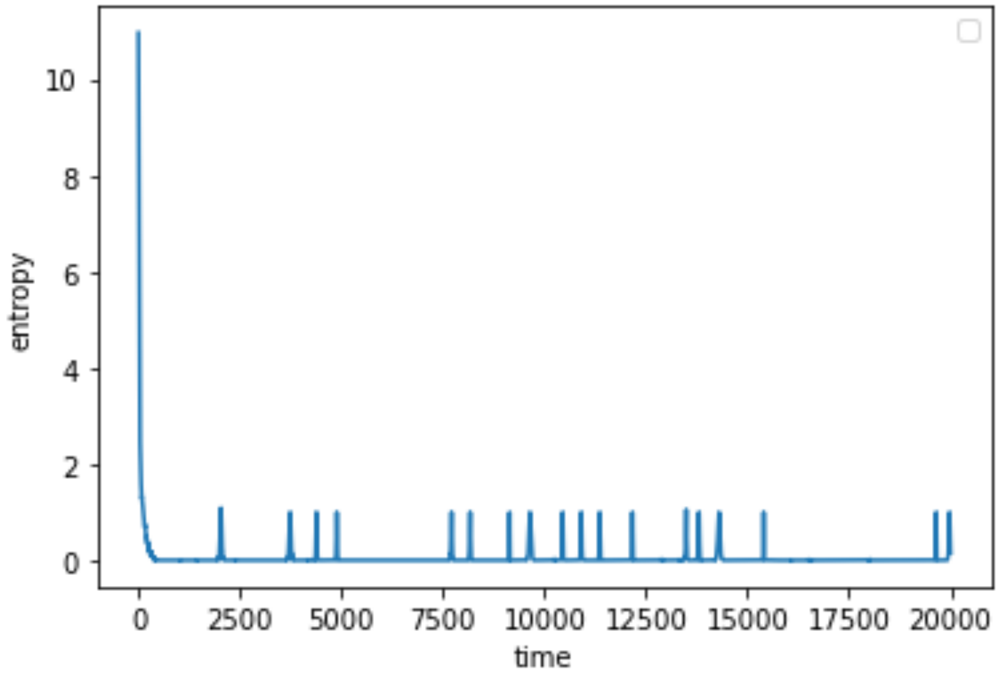



The optimal growth seems to be obtained for a mutation rate close to 0.01



# A stepwise increase in fitness



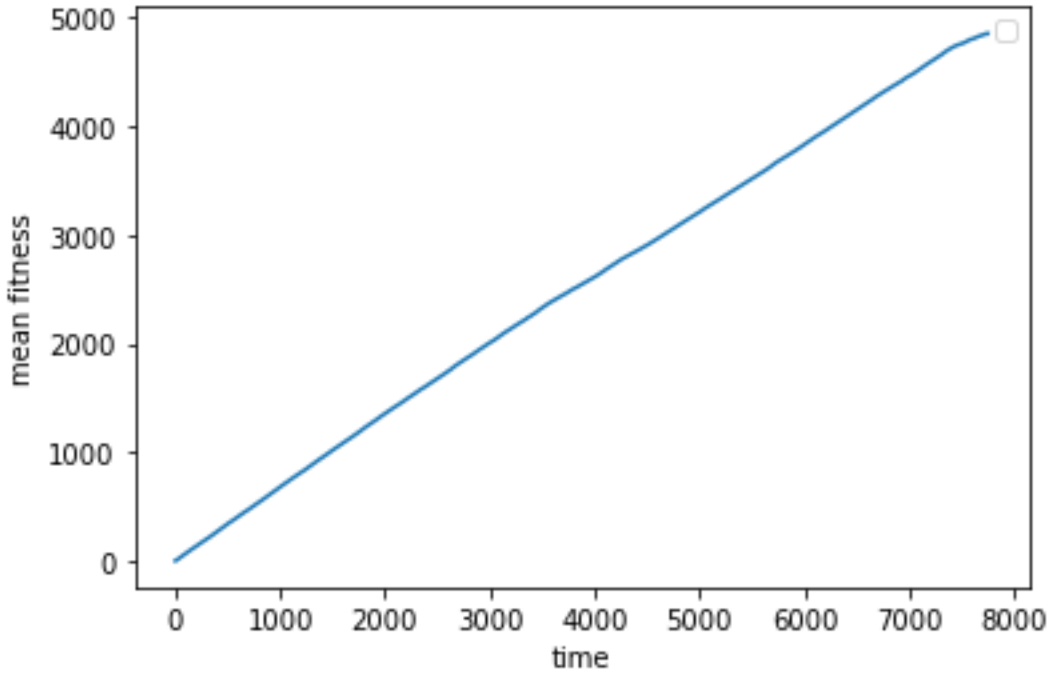




We recover this behaviour for p << $10^{-5}$
Here, p = $10^{-6}$ and all parameters stay the same.

Each instance of steep growth is visible by a brief increase in entropy. When we are close to perfect homogeneity, we have a correlation between entropy and growth

# Some issues with this first version of the model

- Exponential growth is not realistic, even if this concerns a small region of the parameters
- No stationary regime, the average fitness increases forever

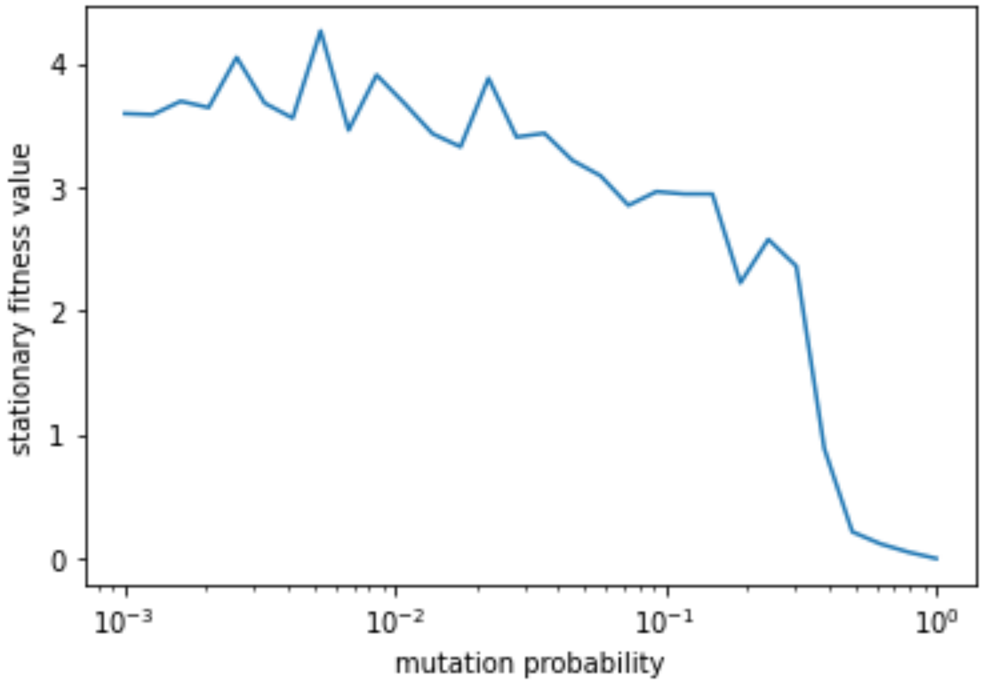



This actually corresponds to a decaying population (p=0.3) , despite the very high fitness.

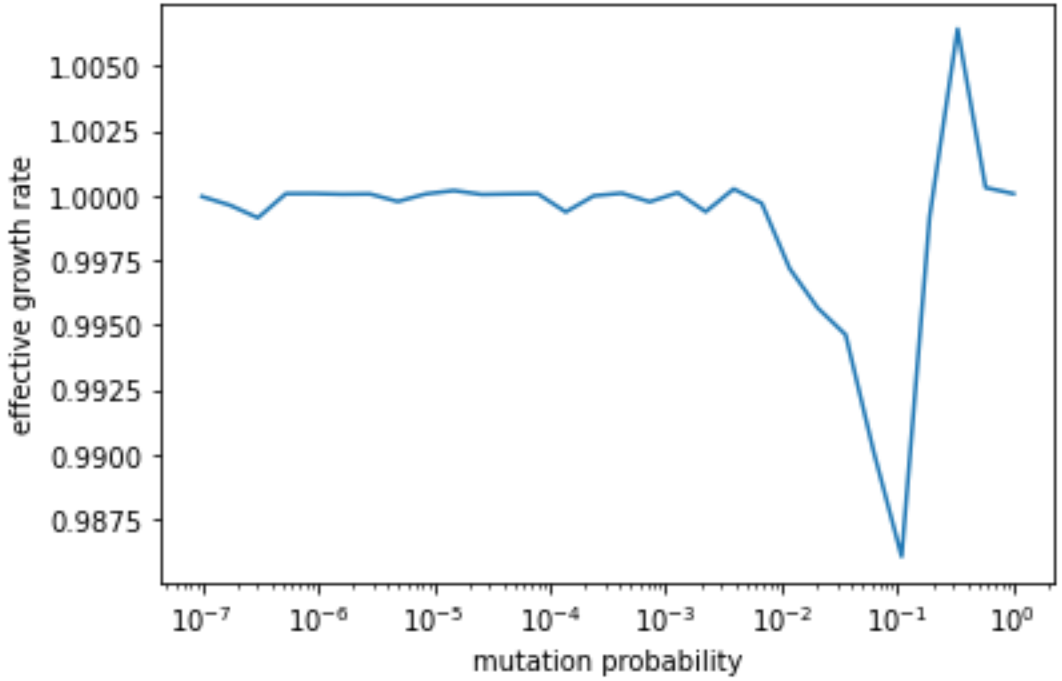
# Suppressing correlations

- The new fitness is taken around 0 instead of the previous fitness.
- This especially reduces the probability of a mutation increasing the fitness if the previous fitness was too high. More realistic
- For example, $P_r(1) = 0.16$ instead of 0.5 in the previous model.
- We also have $P_r(+\infty) = 0$ : once the fitness is too high, we cannot improve it anymore.

# Equilibrium fitness




And More …



- References :


- Van Nimwegen, E., Crutchfield, J. P., & Mitchell, M. (1997). Finite populations induce metastability in evolutionary search. *Physics letters A*, *229*(3), 144-150.
- Hermisson, J., Redner, O., Wagner, H., & Baake, E. (2002). Mutation–selection balance: ancestry, load, and maximum principle. Theoretical population biology, 62(1), 9-46.
- Baake, E., & Wagner, H. (2001). Mutation–selection models solved exactly with methods of statistical mechanics. Genetics Research, 78(1), 93-117.
- Manrubia, S. C., Domingo, E., & Lázaro, E. (2010). Pathways to extinction: beyond the error threshold. Philosophical Transactions of the Royal Society B: Biological Sciences, 365(1548), 1943-1952.