256B Project Proposal

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The Ising model of spin systems is one of the most studied systems in the field of collective behavior in condensed matter physics. By examining the free energy of the system we are able to see a phase transition as a result of the divergence of the correlation length, ξ, at a critical temperature, T_c. From these results it was discovered that several thermodynamic quantities such as the the heat capacity, $C \propto |T - T_c|^{-\alpha}$, and the magnetic susceptibility, $\chi \propto |T - T_c|^{-\gamma}$, follow power law behavior with critical exponents that are distinct for a particular class of systems. The critical behavior of systems driven by the divergence of ξ demonstrate a unique aspect of the correlation function, $g(\mathbf{r}) \equiv \langle m(\mathbf{r})m(\mathbf{0})\rangle$, which is self similarity. That is $g(\lambda\mathbf{r}) = \lambda^p g(\mathbf{r})$. It has been shown that the information theory description of the entropy density, $h_\mu \equiv \lim_{L\to\infty} \frac{H(L)}{L}$, is up to a multiplicative constant equivalent to the thermodynamic entropy density [cite], which is proportional to the integral of the heat capacity. The renormalization group, RG, is a semigroup due to the lack of an inverse, which is due to the loss of information by rescaling.

a) The primary goal of my project is to investigate possible relationships between scale invariant thermodynamic quantities and information theoretic quantities in a 1D/2D Ising model. I would like to learn if there is any relationship between the information lost by rescaling and the scaling factor or on the rescaling of h_μ. I would like to see if there is any universality to Shannon Entropy and see if there are correlations between the loss of information and the spin correlations.

b) The 1D/2D spin chain/lattice is dynamic and non linear due to spin fluctuations. The coupling between spins changes with changes in order parameter. This creates a non linear dynamic for the system where, near the critical temperature perterbations can induce chaotic behavior. This can be seen by the charecteristic exponent of the linearized dynamics near a fixed point.

c) The dynamics I will explore is the formation of domains in an Ising spin system at different temperatures followed by re-evaluation of these quantities with decimation and renormalization.

d) The intrinsic computation properties I will investigate are mutual information between local magnetic configurations as the system evolves. As well as the effect of lost information in a local environment on correlation at a the same length scale.

e) I will use kinetic Monte Carlo to evolve an Ising spin system. This method is appropriate because it simulates the stochastic dynamics of a coupled system.

f) At this point I have no idea what I think the result should be.

g)
- Develop simulation and record relevant properties time: 1-2 days.
- Read literature on the topic and develop a better understanding of what these results might mean in information theory. time: 2 weeks
- Run simulations. time: 2 weeks, concurrent with reading.
- Process and analyse data. time: 1 week.
- Write report and create presentation. time: 1 week.