Project Proposal

1. Goal: What is your primary project goal? What would you like to learn?

My primary project goal is to apply what I learned in this course about nonlinear dynamical system and computational mechanics to freshwater ecosystems I work with. In ecology, it has been long suspected that chaos arising from nonlinear species interactions may be widespread in nature (Hasting et al., 1993). However, very little solid evidence from natural ecosystems supports the existence of chaos. Using data collected from freshwater ecosystems, and synthetic data produced by dynamical models, I would like to employ methods from computational mechanics to calculate entropy rate, excess entropy, and structure complexity of different time series, and compare whether entropy rate is equal to Lyapunov exponent. Through this project, I hope to find a workable method to convert data collected or produced from continuous time and continuous variable systems to discrete binary sequences that can be directly used by ε machine to compute information metrics.

2. System: Describe how the dynamical system is nonlinear and time-dependent. What's the state space? What's the dynamic? Why is the system behavior interesting?

I plan to study two systems: one is a three-species food chain model (Hasting and Powell, 1991), and the other is long-term time series data collected from a natural stream — the Hubbard Brook (NH). The three-species food chain model includes three ordinary differential equations describing predation relation among three species (Eq. (1)). The nonlinearity of the model arises from the nonlinear functional response term (i.e., $f_1(x)$ and $f_2(y)$ terms in Eq. (1)). It has been shown that this model exhibits complex dynamics, including stable equilibrium, chaotic attractors, and coexistence of multiple attractors when parameters are within different ranges (Hasting and Powell, 1991). I am interested in the chaotic dynamics of this model.

$$
\begin{aligned}
\frac{dx}{dt} &= R_0 x(1-\frac{x}{K_0}) - C_1 f_1(x) y \\
\frac{dy}{dt} &= f_1(x) y - f_2(y) z - D_1 y \\
\frac{dz}{dt} &= C_2 f_2(y) z - D_2 z
\end{aligned}
\tag{1}
$$

Where t is time; x, y, and z are biomass of the three species; R_0, K_0, C_1, C_2, D_1, and D_2 are parameters, and $f_1(x)$ and $f_2(y)$ are non-linear functional response function, which is likely the main origin of complex dynamics.

For the long-term time-series data collected from a natural stream, I am interested in exploring whether the time-series are chaotic or not. One time series is NO_3-, which is strongly affected by algae uptake and detritus mineralization, therefore showing clear seasonal patterns (low in summer and fall, but high in spring, Fig. 1a). The other time series is Cl-, which is relatively conservative in water, and less affected by biological activities. Cl- exhibits more random behaviors than NO_3- (Fig. 1b). It is so far unclear what dominates the variation of Cl-: whether the variation is deterministic or stochastic.

The reason for choosing two different systems is that I am not sure if the computational mechanics methods I learned in class can be readily applied to real-world data that is noisy. So, I plan to first apply these methods to the data generated from the three-species food chain model that I know it is chaotic and test these methods. If the methods work well, I will expand the methods to the real-world time series.

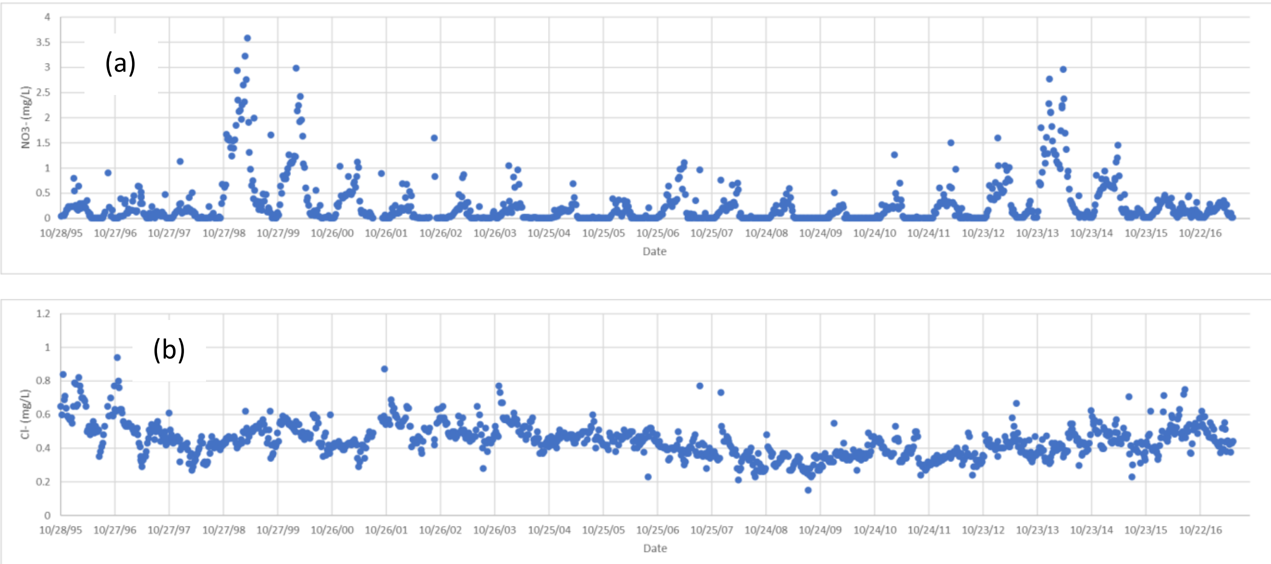

Figure 1 Observed water chemistry time series on the Hubbard Brook (NH): (a) NO_3^- and (b) Cl^-

3. Dynamical properties: What dynamical properties are you going to investigate?

The three-species food chain model has many interesting behaviors and dynamical properties, such as coexistence of chaotic attractors and limited cycles, fractal boundary of attractor basins. For this project, I would like to focus on chaotic behavior, particularly Lyapunov exponent, and compare this value with entropy rate calculated by computational mechanics using synthetic time series data generated by this dynamical model.

4. Intrinsic computation properties: What information processing properties are you going to investigate?

For both the synthetic time-series produced by food chain model and the real-world time-series, the information processing properties I will investigate are (1) entropy rate, (2) excess entropy (mutual information between the past and the future), and (3) structural complexity. I also would like to see the number of causal states each time series has and the symbol length of each causal state (i.e., depth of future morph). Interpretating causal states of ε machine ecologically will be another important task of this project.

5. Methods: What methods will you use? Why are they appropriate?

I will numerically solve the food chain model by the Runge-Kutta method. I will then plot bifurcation diagram over one or two key parameters to figure out the region with chaotic dynamics. Next, I will find a group of parameter values that produces chaotic dynamics and calculate Lyapunov exponent of each state variable. Synthetic time series will be generated for investigating intrinsic computation properties. Before using the computational mechanics package, I need to partition time series into binary words. Since it is unknown the best value for partition, I will try different values for partition, and calculate entropy rate of each partition. It is likely that the partition with the maximum entropy rate is the best partition (Strelioff and Crutchfield, 2007). After I get binary word of each time series, I will calculate word distribution of certain length, and then use computational mechanics package to find hidden Markov Chain of the ε machine. If I can find a ε machine with finite causal states to represent these time series, it will be easy to calculate these intrinsic computation properties.

6. Hypothesis: What is your current guess as to what you will find.

(1) The sum of positive Lyapunov exponents will equal to entropy rate of the time series generated by the food chain model.

(2) Structural complexity and excess entropy of the real-world time series will be larger than the time series produced by the food chain model.

(3) Structural complexity of the NO_3^- time-series may be higher than the Cl^- time-series because of the impact of biological activities on NO_3^-.

7. List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, and write up report.

Steps of investigation	**Amount of time**
Numerically solve the food chain model	1 week
Bifurcation diagram and synthetic time series generation	1 week
Partition time series and calculate word distribution	1 week
Run computational mechanics package and calculate intrinsic computation properties	1 week
Write up the report and have a presentation	1 week

8. Time: Estimate how long each step will take. Can you complete the project within one month?

I am not 100% sure I will complete all of my plans within one month. What I am mostly worried about is the result from the real-world time series. I am not sure if I can get a ε machine with finite causal states and if I can interpretate the ε machine ecologically, but it is worth a try.

Reference

Hastings, A., C. L. Hom, S. Ellner, P. Turchin, and H. C. Godfray. 1993. Chaos in ecology: is mother nature a strange attractor? Annual review of ecology and systematics, 24:1-33.

Hastings, A. and T. Powell. 1991. Chaos in a three-species food chain. Ecology, 72:896-903.

Strelioff, C. and J. Crutchfield. 2007. Optimal instruments and models for noisy chaos. Chaos, 17:043127.