Detecting Chaos in Ecology

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Ecology: Environment ⟺ Organisms

A Savanah ecosystem

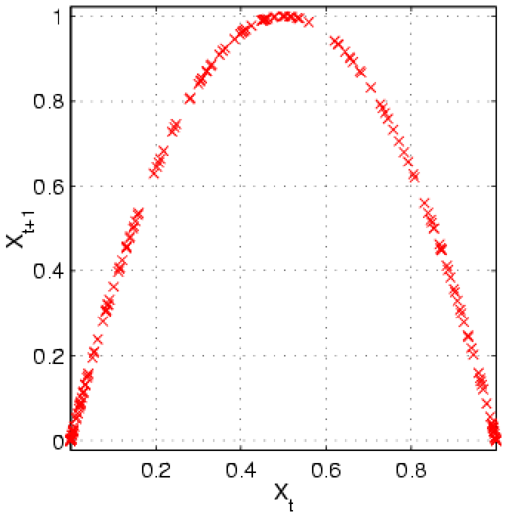
Features of ecosystems

- Many components
- Complex interaction networks
- Vary in space and time
- …

Why is chaos possible in ecosystems?

Nonlinearity

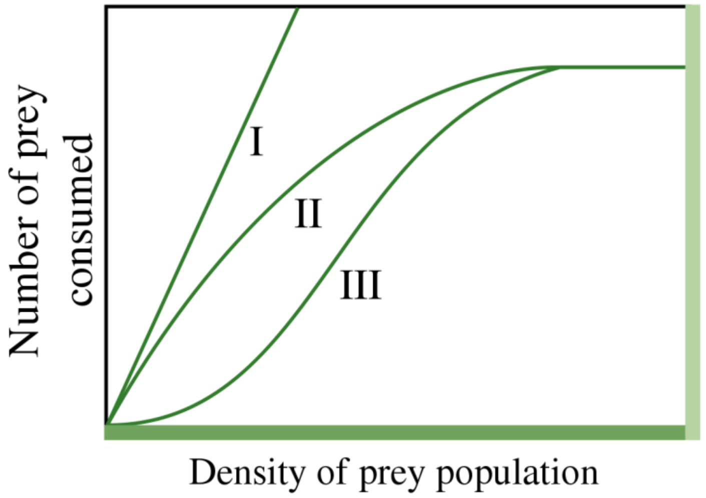
- For one species/population



Logistic map $x_{t+1} = r\, x_t\,(1-x_t)$

Growth rate (the slope of the curve) is density dependent

- For multiple species – a community



Species interaction is nonlinear

Some ecological models show **chaotic dynamics** under certain parameter ranges.

Does chaos exist in natural ecosystems?

Chaos in a long-term experiment with a plankton community

Elisa Benincà[1,2]*, Jef Huisman[1]*, Reinhard Heerkloss[3], Klaus D. Jöhnk[1]†, Pedro Branco[1], Egbert H. Van Nes[2], Marten Scheffer[2] & Stephen P. Ellner[4]

Chaotic Dynamics in an Insect Population

R. F. Costantino, R. A. Desharnais,* J. M. Cushing, Brian Dennis

Overall, the detection of chaos remains rare in nature!

Species fluctuations sustained by a cyclic succession at the edge of chaos

Elisa Benincà[a,1], Bill Ballantine[b], Stephen P. Ellner[c], and Jef Huisman[a,2]

Methods to detect chaos

1. Parameterize realistic dynamical models with observed data (most commonly used)

2. Calculate Lyapunov exponent directly from observed time series (Wolf, 1985)

3. Calculate entropy rate of the ε machine constructed from time series

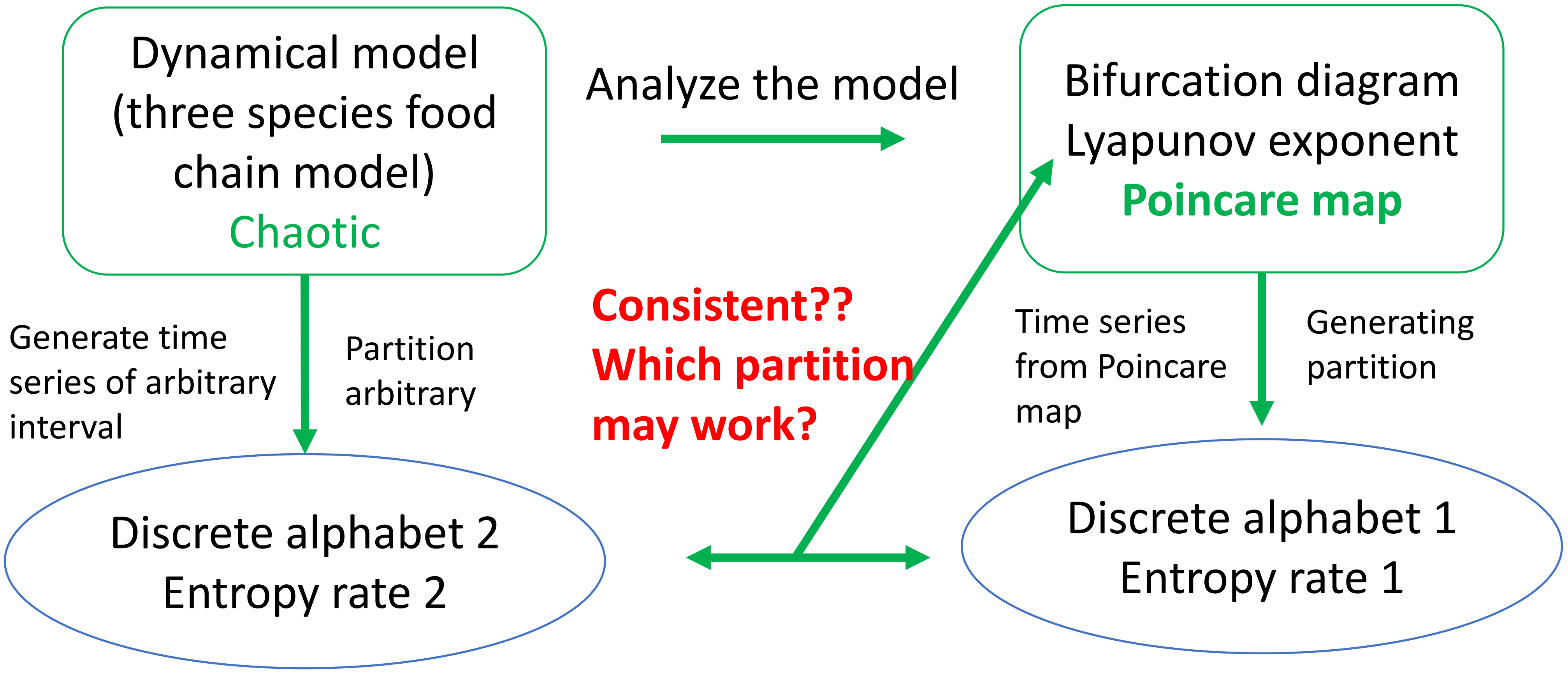
The difficulty of ε machine method

Most natural systems
continuous in time and continuous variable.

Partition?? →

ε machine
Discrete in time and **discrete** variable.

Strategy of partitioning and constructing ε machine



Three species food chain model

$$\frac{dx}{dt} = R_0 x(1 - \frac{x}{K_0}) - C_1 f_1(x) y$$

$$\frac{dy}{dt} = f_1(x) y - f_2(y) z - D_1 y$$

$$\frac{dz}{dt} = C_2 f_2(y) z - D_2 z$$

$$f_i(u) = a_i u / (1 + b_i u)$$

Species *z*

↑

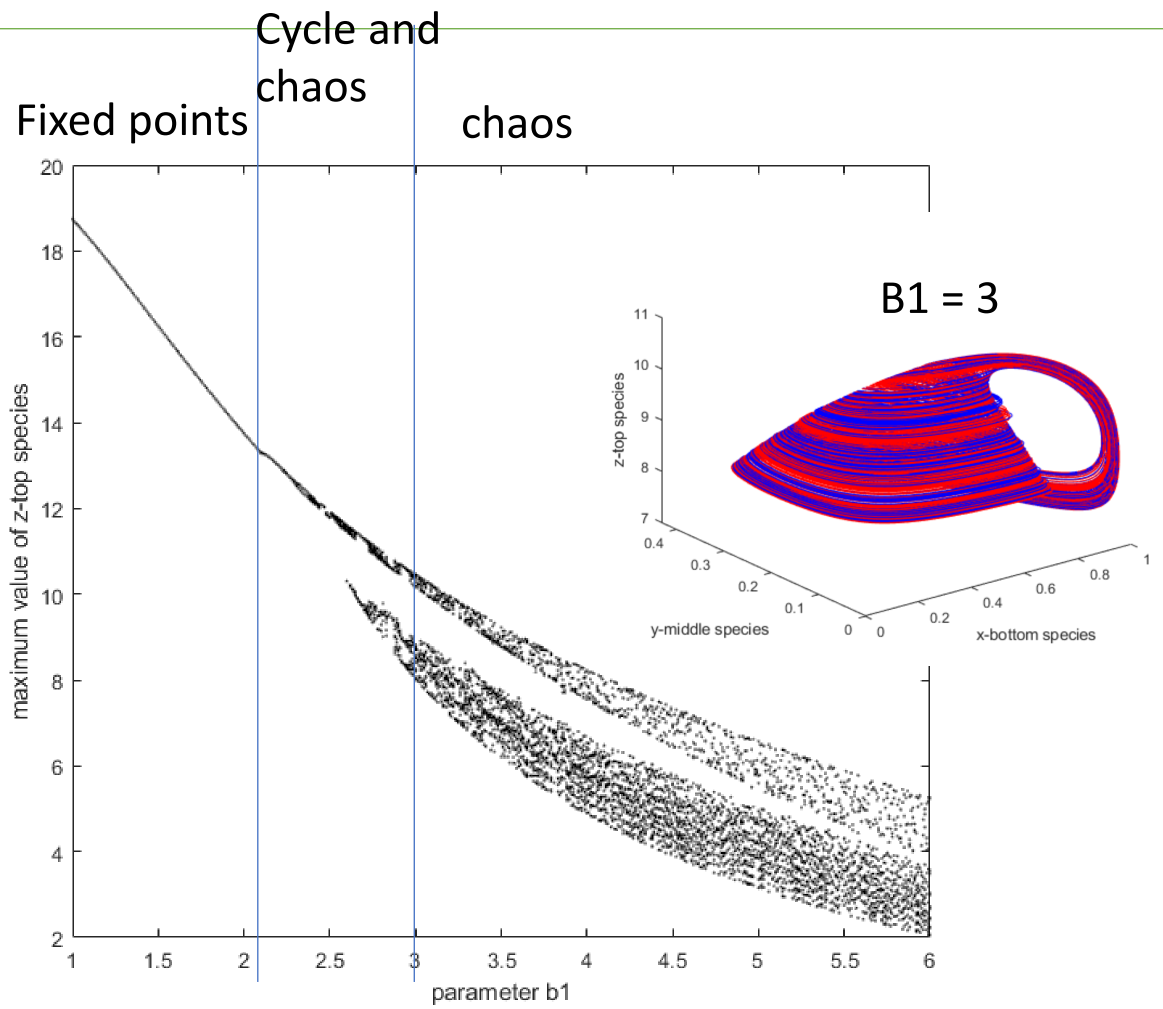
Species *y*

↑

Species *x*

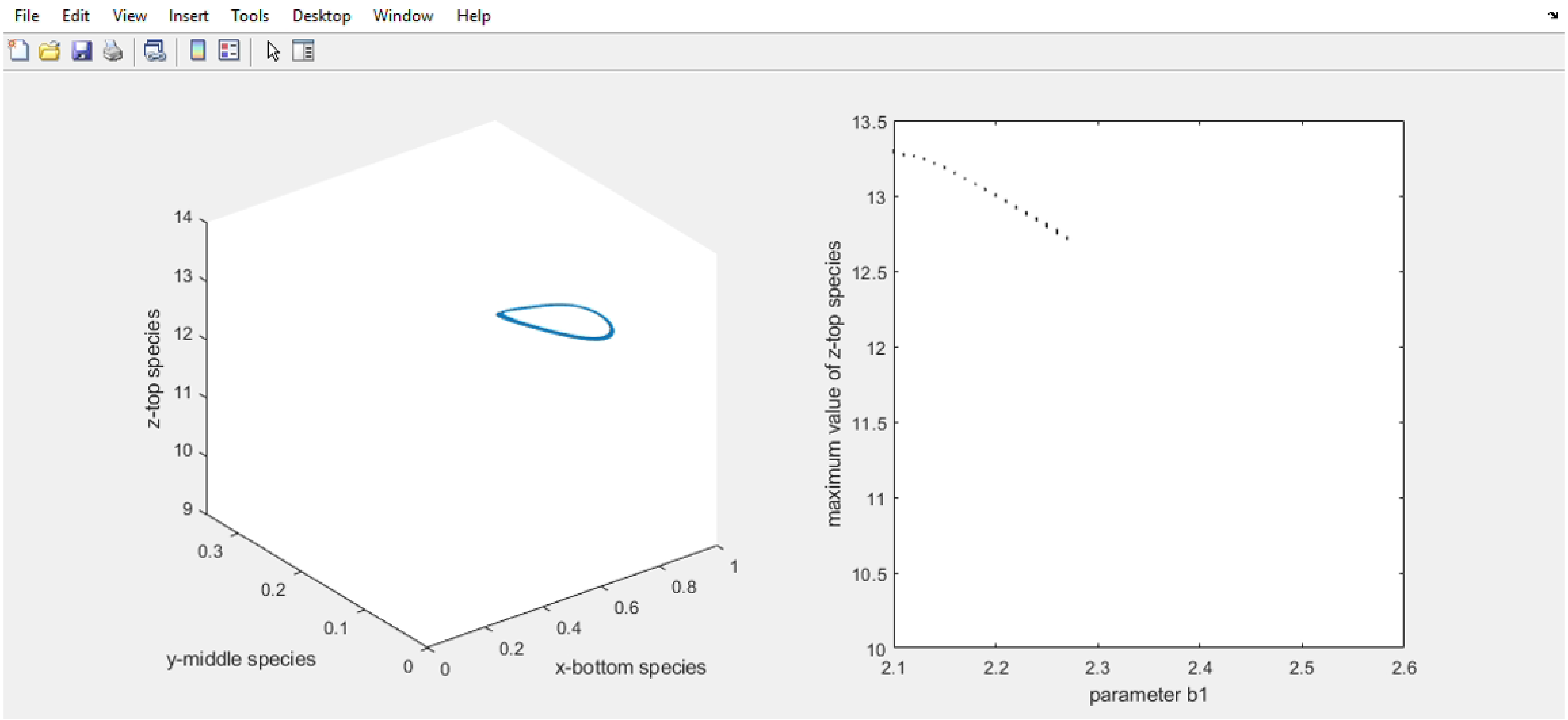
Where t is time; x, y, and z are biomass of the three species; R_0, K_0, C_1, C_2, D_1, and D_2 are parameters, and $f_1(x)$ and $f_2(y)$ are non-linear functional response function.

(Hasting and Powell, 1991)



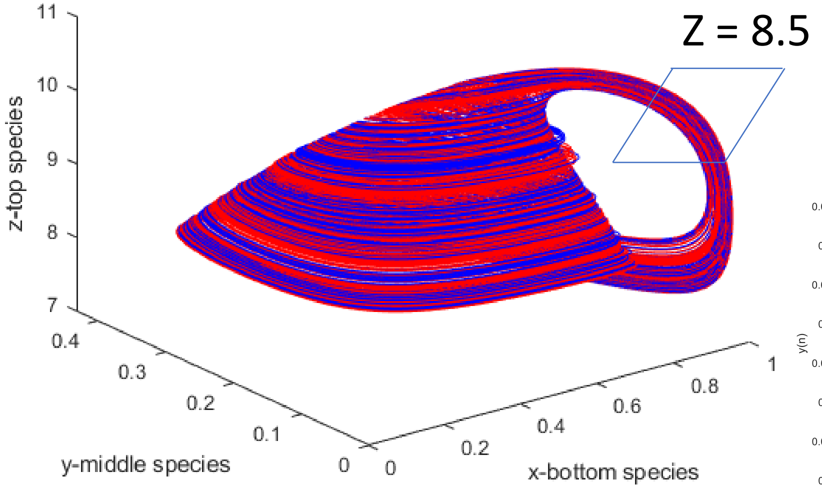
Bifurcation diagram

Detailed bifurcation structure

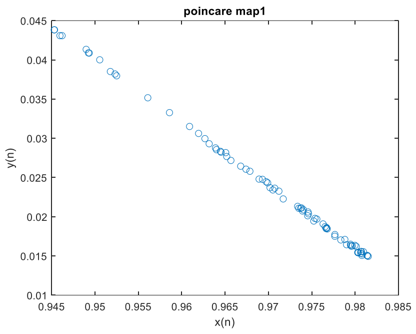


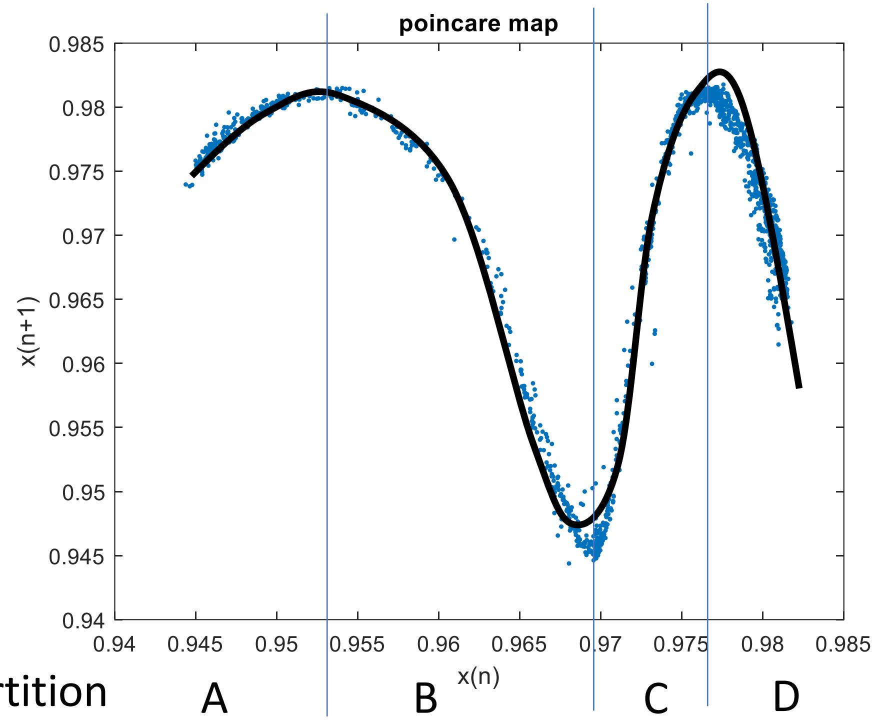
Poincare map

Lyapunov exponent = 1.7132



Poincare section





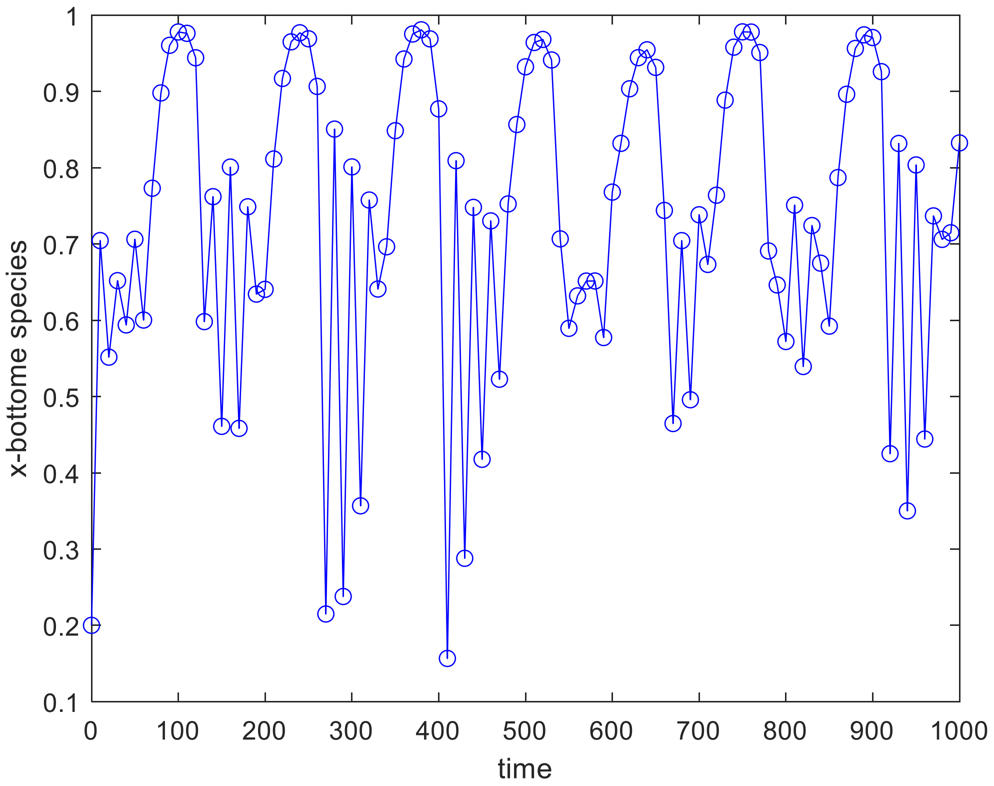
Generating partition

Entropy rate of Markov chain of 4 states: 1.683

5 states: 1.869

6 states: 1.963

Entropy rate of time series of arbitrary interval



Interval of output time step = 10

Entropy rate of Markov Chain

Markov chain	2 states	3 states	4 states	5 states	6 states
2 Partitions	0.547	0.541	0.512	0.475	0.422
3 Partitions	0.855	0.754	0.677	0.603	0.571
4 partitions	0.99	0.90	0.868	0.932	1.235
5 partitions	1.097	1.008	1.095	1.476	-- out of memory

Recall: Lyapunov exponent from Poincare map ≈ 1.7132

Future work

- Construct reliable ε machine on supercomputer
- Try different partitions and ε machine of more states.

Thank you for your attention!