(1g)Answer The asymptotic entropy rate h_μ using its closed-form expression for unifilar Markov chains is calculated as:

$$h_\mu = -\sum_{v\in V} \pi_v(\infty) \sum_{s\in\{0,1\}} \sum_{v'\in V} T^{(s)}_{vv'} \log_2 T^{(s)}_{vv'}$$

The asymptotic state is:

$$\langle\pi| = (p_A = 0, p_B = \frac{1}{3}, p_C = \frac{2}{3})$$

The calculated asymptotic entropy rate is:

$$h_\mu = 2/3 \text{bits per symbol}$$

The statistical complexity is defined as the Shannon entropy of the asymptotic distribution of the causal states:

$$C = -\sum_v p_v \log_2 p_v$$

$$C = 0.918$$

(1h)Answer The recurrent states are B, C with the asympototic probabilities of these states being $(p_B = \frac{1}{3}, p_C = \frac{2}{3})$.

Observerations at asymptotic times

1. A pair of 0s are not allowed by the given process.

In [0]:

Problem 2. Write up your Project Proposal with the following sections. The result should be 2-3 pages long.

2a. *Goal*: What is your primary project goal? What you would like to learn?

2b. *System*: Describe how the dynamical system is nonlinear and time-dependent.

What's the state space?

What's the dynamic?

Why is the system behavior interesting?

2c. *Dynamical properties*: What dynamical properties are you going to investigate?

2d. *Intrinsic computation properties*: What information processing properties are you going to investigate?

2e. *Methods*: What methods will you use? Why are they appropriate?

2f. *Hypothesis: What is your current guess as to what you will find?
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2g. *Steps*: List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, write up report, and so on.

2h. *Time*: Estimate how long each step will take. Can you complete the project within one month?

In [0]:

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Classical random walks on graphs have proved to be a powerful tool in developing stochastic algorithms. They have also provided valuable insights in Physics, Biology, Economics and Finance. Quantum Walk serves as a quantum analog of the classical random walk. Quantum walks are an important tool in developing quantum algorithms such as the Grover's algorithm. Quantum walks differ from their classical counterparts as the particle performing quantum walk can be in a superposition of states. Other quantum effects such as entanglement and quantum statistics and phase of the particles become relevant when there are multiple quantum walkers which are of particular interest in studying many body physics. It was recently shown that correlations emerge between two bosons performing quantum walk in an optical lattice. Increasing the interaction between the two bosons led to complex dynamics[1].The primary goal of the project would be to study the dynamics of two bosonic quantum walkers on discrete 1D lattice and evaluate the Von Neumann Entropy and quantum analog of mutual information and other information measures to provide an insight into the complex dynamics.

I would like to learn how asymptotic probabilities are calculated for quantum walks on linear and circular graphs. I would like to use them to evaluate Shannon entropy and entropy rates(or the quantum analogues). I am also hoping to learn how information measures are applied to quantum systems and in particular what they can tell us about entanglement, correlations and quantum statistics.

The Hamiltonian describing the bosons performing a quantum walk on a discrete lattice is given by the Bose Hubbard Hamiltonian that is stated below.

$$H_{BH} = \sum_{\langle ij \rangle} J a_i^\dagger a_j + \sum_i \frac{U}{2} n_i(n_i - 1) + \sum_i iEn_i$$

Here, J gives the amplitude of tunneling in the $x-$ directions while U is the on-site repulsive term and E is energy shift associated with each site i. $a_i^\dagger$ and a_i are the bosonic creation and annihilation operators, respectively, and $n_i = a_i^\dagger a_i$ gives the atom number on site i.

The non-linearity in the dynamical system comes from the repulsive on site interaction. The system evolves unitarily in time under this Hamiltonian.

The state space of this system is all the points on the discrete 1D lattice. Depending upon the interaction strength of the bosons observation of repulsively bound pairs and fermonization was reported in[1].

This system can help us not only understand many body physics but the dynamics of the bosons performing random walks is reminiscent of cellular automata. Quantum Cellular automata are considered an important tool in simulating physical systems. Understanding the correlation between multiple particle quantum walk and QCA is another question I am interested in looking at.

I would like to evaluate the asymptotic probability distributions starting with different initial conditions for the discrete 1D graph. I would also like to look at the particle densities at each lattice site with time for different interaction strengths and initial conditions which gives rise to Quantum Cellular automata.

I would like to investigate the entropies and mutual information between the bosons performing the random walks.

I will be using the methods outlined in[2,3] to obtain the asymptotic probabilities on quantum walks on graphs. I will also numerically compute the particle densities at each lattice site using the method in [1].

I am hoping that different initial conditions and interaction strengths between particles performing quantum walks leads to interesting dynamics that can be further explored.

Steps:

1. Literature review and discussion - 1st week
2. Evaluate the asymptotic probabilities using the formalism in [2,3] - 2nd and 3rd week
3. Numerically compute particle density at each site using the formalism in [1] - 3rd and 4th week
4. Compile results and finish report- 5th week

Above, I have outlined the steps and time for each step and would like to believe that I can complete it in one month(by may be overlapping and updating the provided time line).

[1] Preiss, P.M., Ma, R., Tai, M.E., Lukin, A., Rispoli, M., Zupancic, P., Lahini, Y., Islam, R., Greiner, M.: Strongly correlated quantum walks in optical lattices. Science 347(6227), 1229–1233 (2015)

[2] Nayak, A., Vishwanath, A.: Quantum walk on the line. quant-ph/0010117

[3] Venegas-Andraca, S.E. Quantum walks: a comprehensive review. Quantum Inf Process 11, 1015–1106 (2012). https://doi.org/10.1007/s11128-012-0432-5 (https://doi.org/10.1007/s11128-012-0432-5)

In [0]: