Problem 2. Write up your Project Proposal with the following sections. The result should be 2-3 pages long.

2a. *Goal*: What is your primary project goal? What you would like to learn?

2b. *System*: Describe how the dynamical system is nonlinear and time-dependent.

What's the state space?

What's the dynamic?

Why is the system behavior interesting?

2c. *Dynamical properties*: What dynamical properties are you going to investigate?

2d. *Intrinsic computation properties*: What information processing properties are you going to investigate?

2e. *Methods*: What methods will you use? Why are they appropriate?

2f. *Hypothesis: What is your current guess as to what you will find?
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2g. *Steps*: List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, write up report, and so on.

2h. *Time*: Estimate how long each step will take. Can you complete the project within one month?

Goal

The project is aim to understand the behavior of the quantum system by using the ϵ-machine in computational mechanics. In this project there are general two aspects I would like to learn: quantum and computational mechanics. For the quantum part, I need to learn how to express the quantum states, their time evolution and underlying math structure. For computational mechanics part, I would like to apply this to the quantum system in order to solidify my understanding of the ϵ-machine and its related stuff.

System

The system we will study is a D-dimension quantum system. In Dirac notation, any state in this space can be written as

$$|\psi\rangle = \sum_{i=1}^{D} c_i |\psi_i\rangle.$$

And states $|\psi_i\rangle$ and $|\psi_j\rangle$ are equivalent if they are equal up to a complex number. So the states are the rays in D-dimension complex space which are in the complex projective space ($\mathbb{CP}^n$) by its definition. We can write any state by a D-dimension complex array Z^α and Hamiltonian H by a $n \times n$ Hermitian matrix H^α_β. The dynamics is governed by the Hamiltonian H. After we consider the equivalent relation we have the modified Schrödinger equation which is called projective Schrödinger equation:

$$Z^{[\alpha} dZ^{\beta]} = iZ^{[\alpha} H^{\beta]}_\gamma Z^\gamma dt.$$

For ordinary quantum mechanics in textbooks we do not emphasis this equivalent relation and treat the state space as $\mathbb{C}^n$. The main advantage of taking account of this equivalent relation is that $\mathbb{CP}^n$ has a finite volume after equipped with a specific metric and is easy to visualize. For the simplest quantum system is spin-1/2 system of which space is

$$\mathbb{CP}^1 = S^2$$

which is called Bloch sphere. And we can visualize any evolution in spin-1/2 system as a trajectory on S^2. Furthermore we can study mixed state under this scheme. A pure state is a point on S^2 and a mixed state can be treated as a probability density function on S^2. And in the complex projective space, quantum mechanics is almost the same as the classical mechanics. Like in the ordinary dynamics theory we want to study whether in this system attractors, limited cycles and chaos exist or not. The metric we use in this space is Fubini-Study metric induced from natural distance in complex projective space:

$$\cos^2 d(\psi, \phi) = \frac{\langle\psi|\phi\rangle\langle\phi|\psi\rangle}{\langle\psi|\psi\rangle\langle\phi|\phi\rangle}.$$

If we can find chaos in the quantum system we can use symbolic dynamic to study its information processing property. The reason why this is appropriate is that under this scheme the quantum mechanics can be studied by dynamics theory. Another possible project is that given a quantum process can we use he ϵ-machine to find out the optimal prediction of future.

My current guess is that if we can carefully choose a Hamiltonian we may have chaotic attractor in quantum system. And for the prediction project, we may study some quantum processes and try to use computational mechanics to study the quantum processes.

Plan

Steps: Reading literature (Almost Finished) $\rightarrow$ write simulator / do math analysis $\rightarrow$ estimate properties from simulation $\rightarrow$ write up report/presentation

Time:

write simulator / do math analysis

estimate properties from simulation : 2 or 3 weeks in total

write up report: 1 week

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