Problem 2. Write up your Project Proposal with the following sections. The result should be 2-3 pages long.

2a. *Goal*: What is your primary project goal? What you would like to learn?

2b. *System*: Describe how the dynamical system is nonlinear and time-dependent.

What's the state space?

What's the dynamic?

Why is the system behavior interesting?

2c. *Dynamical properties*: What dynamical properties are you going to investigate?

2d. *Intrinsic computation properties*: What information processing properties are you going to investigate?

2e. *Methods*: What methods will you use? Why are they appropriate?

2f. *Hypothesis: What is your current guess as to what you will find?
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2g. *Steps*: List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, write up report, and so on.

2h. *Time*: Estimate how long each step will take. Can you complete the project within one month?

ϵ-Machine reconstruction and information study of 2D Ising model

The primary goal of this project is to build a simple Ising model using Metropolis Monte Carlo algorithm under a low temperature and the absence of an external magnetic field, observe how the system changes over time, and build an ϵ-Machine of hidden Markov chain model to describe the periodic change of the total magnetization. From this study, we hope to figure out if ϵ-Machine and hidden Markov chain are able to decribe the simplest model that's related to phase transitions. If yes, we're interested in what are the states of the system in terms of transient and stable states, or if there could be other state types that can be extended from the basic model. We also want to find out if there are certain patterns which associate with the phase transitions of the system or the formation/deformation of the domain of magnetization in our system.

The Ising Model is a mathematical model that doesn't correspond to an actual physical system. It plays a big role as the Drosophila of phase transitions. It's a huge (square) lattice of sites, where each site can be in one of two states. We label each site with an index i, and we call the two states -1 and +1. To say that the i'th site is in the state -1, we write sigma_i = -1. The configuration of all the spins in the lattice fully depends on the configuration of out lattice in the last time step. In details, the spin of a lattice site in the next time step is set to be determined by the spins of its neighboring sites in the current time step. People usually start with all spins being up's or down's, which gives a conformation of zero entropy, and with time evolving the total entropy of our system starts to grow with the total energy begins to fluctuate.

In the language of our course, the state space consists of all the possible spin conformation of our system. However, with a 2D square lattice of 10x10 dimension, there are 10^2 sites and 2^(100) possible conformations, with symmetrical double-counting's, that contribute to our state space, which will be a huge mess to look into. In this project, I'll look for another possible measure to represent states in order to shrink the size of consideration, promptly I will try using the total energy or Euclidean distances to categorize states. On top of that, we may build models/machines to investigate the dynamical property of the energy distribution, the configuration distribution, magnetization domain distribution, phase, and system entropy, and if there are patterns among them.

For each modeling trials, the histogram of outcome will be made and there for the probabilities for each possible outcome will be calculated as well as the conditional probabilities, in order to construct the parse tree and figure out the morphs. Also, at each time step, the block entropy and the entropy rate will be calculated to study the asymptotic behavior of the entropy growth. These numbers will also be used to evaluate the model in terms of information processing properties.

The model will be a classic Ising model with Metropolis Monte Carlo method for each decision-making process. The experiments will mainly focus on spin evolutions under low temperature (~kBT = 1) in order to explore the patterns since the lattice will have a totally random picture of the spin distribution at high temperatures. I would expect to see a clear ϵ-Machine with transient states, relative stable states and more stable states, each representing the initialization, optimization, and finalization of the system. We might also find out that the transition between states being a function of system size and temperature. And the magnetization patterns formed by spin configurations could also be related to certain states. \

Steps and time: \ Literature research: 2 weeks \ Model building and tuning: 1 day \ Experiments: 3 days \ Data processing and interpretation: 3 days \ Presentation: 1 night \ Report: 1 week