# HMM and epsilon-Machine Reconstruction of Simple 2D Ising Model

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# 2D Ising Model

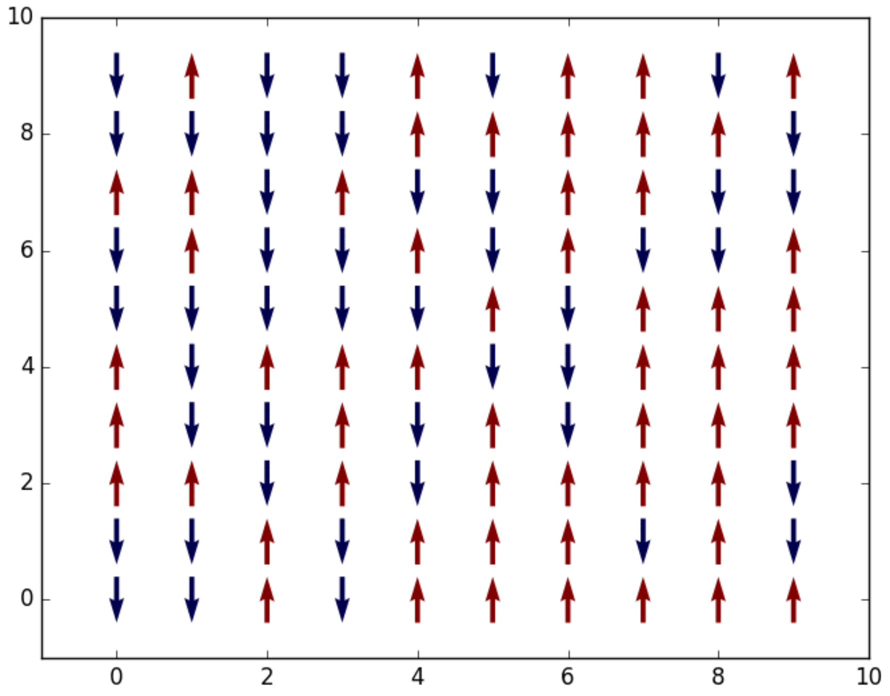
NxN matrix (lattice) with each element being +1 or -1 representing spin up and spin down
Hamiltonian:

$$\mathcal{H} = -h\mu_0 \sum_i \sigma_i - J \sum_{adj.} \sigma_i \sigma_j$$

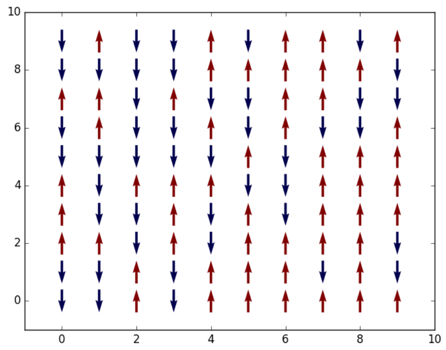
$\sigma$ is the direction of the ith particle's spin which can be either "up" (+1) or "down"(-1), h is the magnitude of the applied magnetic field(not Planck's constant), and J is the spin-spin coupling term between adjacent spins on the lattice.

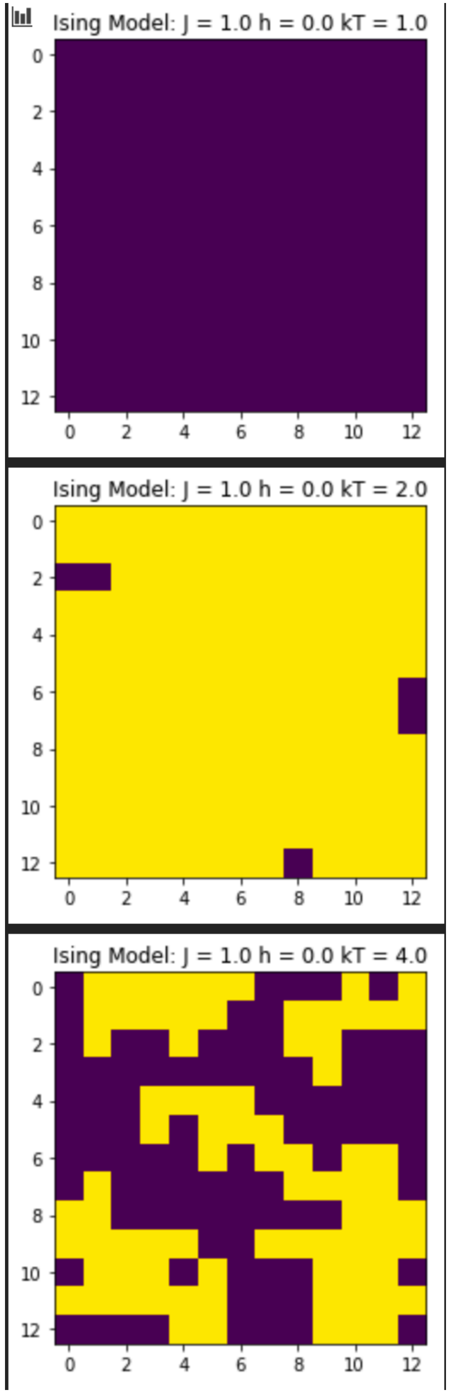
Our simplification:

1. h = 0, no external magnetic field, no preference on becoming all up or down, more chaotic.
2. All spins up initialization, ferromagnetic.
3. Choose a high temperature.



# Monte Carlo Simulation






How the system evolves over time?

At each time step i:

- randomly pick a pixel from NxN grids
- flip its spin and calculate the new system energy
- if new E < old E:
  - keep the change
- else:
  - calculate Boltzmann factor, exp(-dE/kT),
  - being the transition probability

# Connection to Dynamic Systems

The Ising model can be expressed by a Markov process:

Next state only depend on the previous one

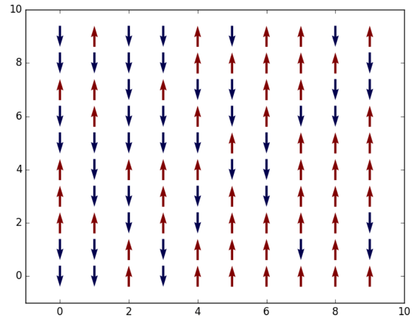
Previous work:

Encoding the success of flipping a random cell at each time step as 1 or 0 to make our word distribution.

Ignored the geometric distribution of spins (magnetic domain)

The overall sign/average magnetism/voting result is not the direct observable/prediction.

# What I’m Trying to Do




Choose a temperature to observe the overall sign of the system per 5000 steps.

Encoding: > 0, 1; < 0, 0

Let the system to evolve and make observation per each period of time and then collect word distribution.

Hidden states representation, epsilon-machine reconstruction, look at the order of the Markov process.

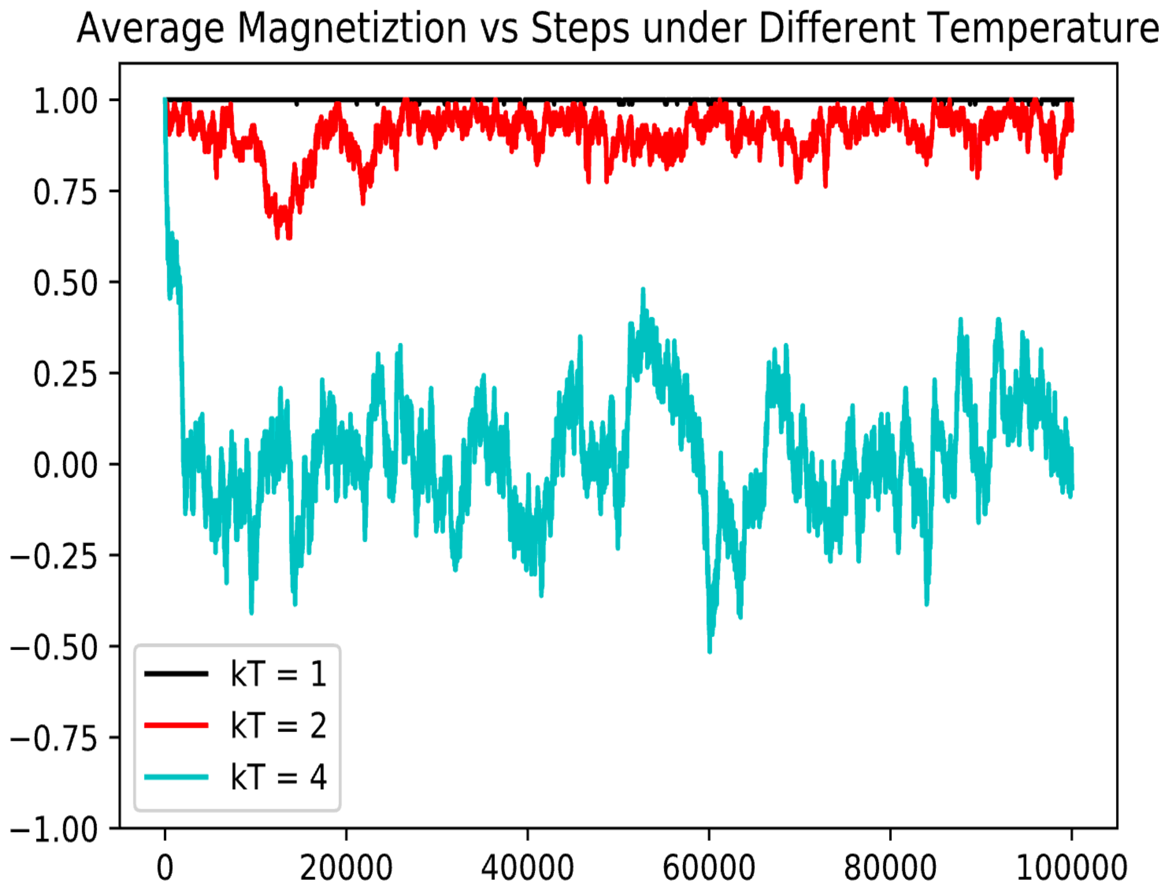
Compare information measures with the actual model.

# Motivations

No stable conformation for a small system.

Ignoring specific conformations.

Fluctuations and patterns.



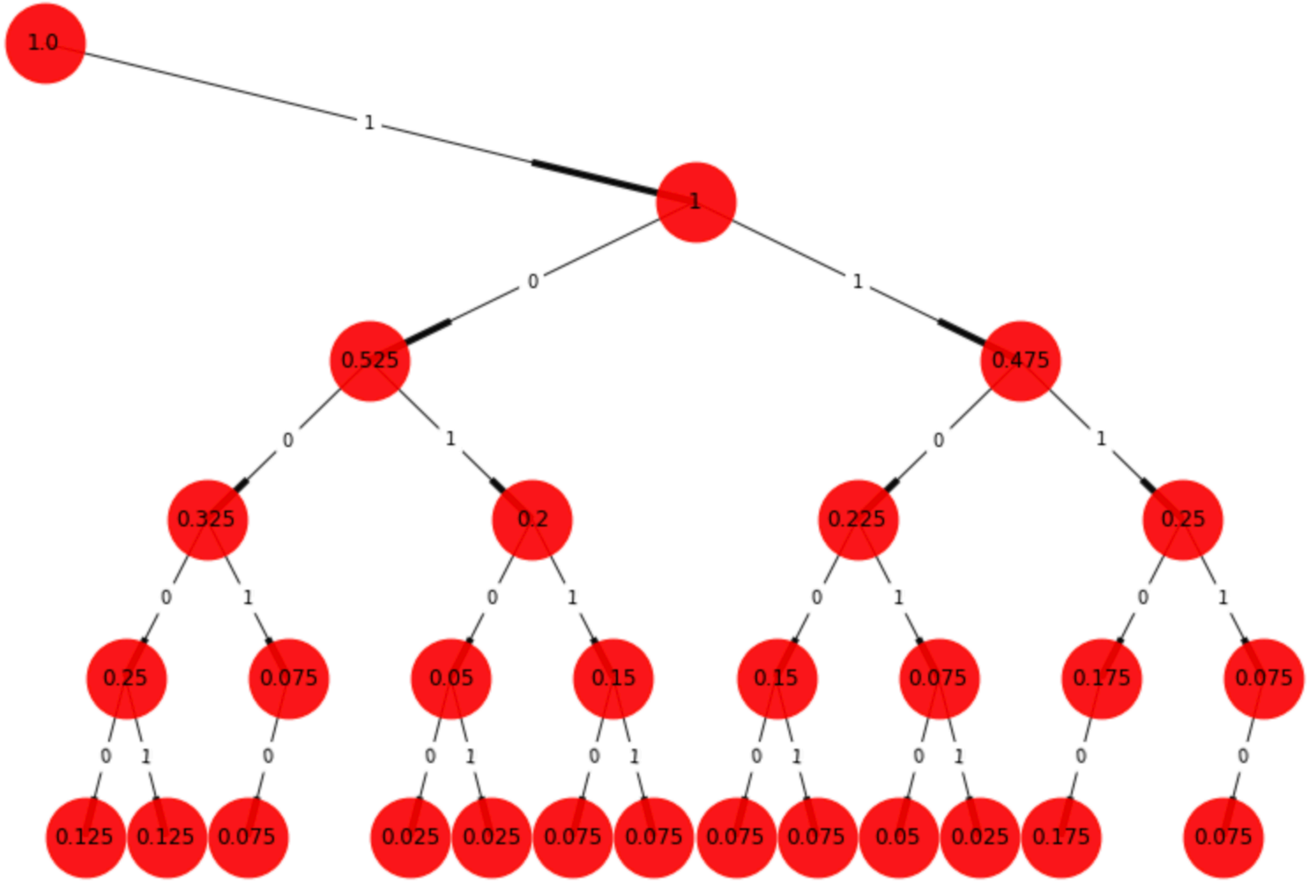
# Experiments

Experiments of 25000 time steps, segmentation to make different word length.

State representation.

Inferring HMM

Evaluation of entropy rate, excess entropy.



# Future Work

1. More careful encoding to include geometric distribution.
2. Temperature dependance of the structure.
3. Markov order of the system.
4. Difference b/w Markov and Hidden Markov.