Problem 2. Write up your Project Proposal with the following sections. The result should be 2-3 pages long.

2a. *Goal*: What is your primary project goal? What you would like to learn?

2b. *System*: Describe how the dynamical system is nonlinear and time-dependent.

What's the state space?

What's the dynamic?

Why is the system behavior interesting?

2c. *Dynamical properties*: What dynamical properties are you going to investigate?

2d. *Intrinsic computation properties*: What information processing properties are you going to investigate?

2e. *Methods*: What methods will you use? Why are they appropriate?

2f. *Hypothesis: What is your current guess as to what you will find?
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2g. *Steps*: List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, write up report, and so on.

2h. *Time*: Estimate how long each step will take. Can you complete the project within one month?

2a

My primary goal is to to explore how the loss of synchronization that occurs in nonunifilar models can be analyzed and quantified. Since this project will require a review of preexisting studies on synchronization, my goal more broadly is to examine the process of synchronization in general and present a concise review of the established tools and techniques used to study it.

2b

I will be primarily concerned with nonunifilar hidden Markov models (HMMs). Nonunifilar HMMs are interesting because it is possible for an observer to lose synchronization with the model after having been previously synchronized. This phenomena occurs "naturally" in nonunifilar HMMs in the sense that there is a nonzero probability of desynchronization resulting from simple time evolution. In contrast, a synchronized observer of a synchronizable unifilar HMM will remain synchronized for all time (assuming the observer does not "artificially force" desynchronization by deleting its memory, for example).

2c & 2d

Some questions I would like to investigate are:

- Can I write down a general, closed-form expression for the average time between desynchronization events?
- How effectively can an observer anticipate an upcoming desynchronization event?
- Can I define relevant information quantities pertaining to desynchronization? For example, is there a well-defined rate of synchronization information loss during and after a desynchronization event? Is there a well-defined "resynchronization cost" to quantify how much information an observer needs to extract in order to become resynchronized?
- Are certain desynchronization events harder to recover (resynchronize) from than others? Can this be quantified?
- Can the "locality" of a desynchronization event be quantified? For example, can a desynchronization event which results in the state uncertainty concentrated on a small number of strongly connected states be said to be "more local" than an event which results in the state uncertainty spread out over a large number of states?
- What does desynchronization of the nonunifilar model look like on a unifilar presentation (ϵ-machine, mixed state presentation, etc.) of the model? Can we use a unifilar presentation to calculate some of the quantities proposed above?

2e

Most of the work in this project will be theoretical, but I anticipate relying mostly on `CMPy` for computing information measures and ϵ-machines or mixed state presentations as needed. I will need to further familiarize myself with `CMPy` 's capabilities. On the theoretical side, I am not yet sure what methods will be required outside of those introduced in class. I have been thinking that z-transform analysis may help in describing and analyzing the time dependence of state uncertainty during a sync-desync-resync process, but I still need to assess the utility and applicability of this method.

2f

I am guessing that most of the questions I posed above have some form of positive answer since , to me, none of them seem particularly ill-defined or impossible to answer.

2g

1. Literature review. I have already collected a few papers to start out with.
2. Theoretical development of answers to the questions above
3. Apply theoretical results, measure synchronization quantities for some example nonunifilar models
4. Write report
5. Hopefully be proud of what I've accomplished (tentative step)

2h

1. 7 days
2. 13 days
3. 4 days
4. 7 days

In [0]: