Optical ponderomotive force

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Abstract

Ponderomotive force is a classical EM force, the motion of charged particle by this force is well studied. For my project, we built optical analogy of ponderomotive force that describing the motion of a small particle trapped by the laser and gravity. We build the inhomogeneous oscillating field and obtain equation of motion. The solution could be stable or unstable. The chaotic behavior can be obtained by get the motion plot with the chosen parameter.

1 Introduction

Study the motion of the particle by optical pondermotive force is my previous project. I mainly do the experimental part, which is mainly get the particle levitated in environment. I was so surprised when I see the real particles get levitated by laser and gravity. I have done the cold atom experiment before. I know how atom of trapped by optical lattice. Small particle has much larger size than the atom and the energy of them are much more complicit than the atom. When I successfully get a particle . I engaged myself to this project. Due to the large thermal effect in vacuum environment, we didn't get the excepted data. Another reason that I do this project is that I spent too much time on experiment before and I want to make up for theoretical part, which I didn't do it before.

I think first get the small particle levitated is very cool. Another interest is that since gravity play an important role in this project, so like the mass spectrum of the ions, we cab build mass spectrum of the particle. I will talk about it later.

In this project, We build the inhomogeneous oscillating field by study the the intensity of the beam and get it modulated. Then study the equation of motion by scattering force and gravity. By the given equation, we study the stable solution and get it plotted. Right now, I just finish first step. The further should be done which I will talk in conclusion.

2 Background and Dynamic System

Ponderomotive force is defined as a nonlinear force that describing the motion of a charged particle in an inhomogeneous oscillating electromagnetic field.

$$F = -\frac{q^2}{4m\omega^2}\vec{\nabla}|\vec{E}(r,t)|^2 \tag{1}$$

To study the motion of the charged particle by ponderomotive force, instead of electron ponderomotive force, we build a system with "optical ponderomotive" force by levitating a microsphere in a vacuum environment. As you can see in Fig4, particle is trapped by laser and gravity .

2.1 Intensity of Gaussian Beam and modulation

Gaussian beam is applied to levitated the particle and its intensity can be obtained by solving the Helmholtz equation in cylinder coordinates [3] is

$$I(r,z) = I_0(\frac{\omega_0}{\omega})^2 e^{-2r^2/\omega^2(z)} \tag{2}$$

where

$$\omega(z) = \omega_0\sqrt{1+(z/z_R)^2} \tag{3}$$

r is the radial distance from beam center axis, z is the axial distance from the center of beam (waist), $\omega(z)$ is the waist radius. z_R is the boundary of the Rayleigh range. Details are shown in figure.1.

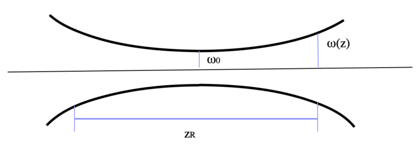

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Figure 1: The side view of propagating Gaussian beam. z_0 is the center of the Rayleigh range, which is the smallest cross section area. We assume that all the motion of the particle are bounded by Rayleigh range.

To simplify the calculation, we set $r = 0$. Therefore, we obtain the equation of intensity

$$I(z) = I_0(1+(z/z_R)^2)^{-1} \tag{4}$$

To get the intensity around the center, we apply the first-order expansion of Eq.4 and obtain

$$I = I_0(\frac{1}{\hat{F}} - \frac{2\sqrt{\hat{F}-1}}{\hat{F}^2}\bar{z}) \tag{5}$$

where

$$\bar{z} = \frac{z-z_0}{z_R} \tag{6}$$

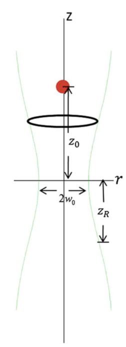

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Figure 2: Trapped particle balanced by scattering force and gravity [1]

$$\hat{F} = 1 + (\frac{z_0}{z_R})^2 \tag{7}$$

Then the next step is to add oscillation into the field. It can be easily done by using electro-optical modulator. Modulation depth is define as the variation of intensity divide the initial intensity.

$$\gamma = \frac{A}{I_0} \tag{8}$$

So

$$I(\bar{z}, t) = I_0(\frac{1}{\hat{F}} - \frac{2\sqrt{\hat{F} - 1}}{\hat{F}^2}\bar{z})(1 - 2\gamma \cos 2\omega t) \tag{9}$$

where the ω is the modulation frequency.

2.2 Scattering Force and Equation of motion

Here, we use the spherical non-absorption particles, thus we can ignore the gradient force due to temperature difference between different parts of the particle [4]. We consider the scattering force of laser, , which can be treated as radiation pressure. Scattering force is obtained by Mie theory [2]

$$F_{scattering} = \frac{I(z)\pi a_p^2}{c} Q \tag{10}$$

wherea_p is the radius of the spherical particle, Q is a dimensionless parameter calculated from Mie theory and c is the speed of light.

As shown in Fig. 4, equation of motion can be written as

$$m\ddot{z} = \frac{\pi a_p^2 Q I}{c} - mg \tag{11}$$

By substituting the Eq.9 into Eq.11, we obtain,

$$\frac{d^2 z}{d\tau^2} + a(1 - 2\gamma \cos 2\tau)z = -2\gamma \cos(2\tau) \tag{12}$$

$$a = 2\frac{\sqrt{\hat{F} - 1}}{\hat{F}} \tag{13}$$

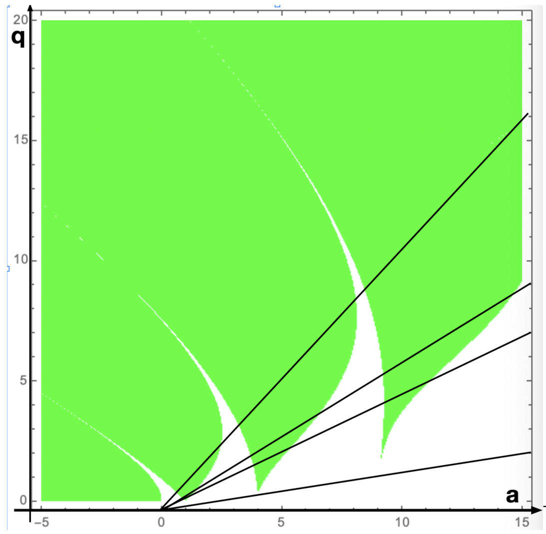

Figure 3: Stability diagram of homogeneous Mathieu equation. Area of green is unstable region and area of white stable region of parameters a, q. Straight line is $q = a\gamma$. Different slopes corresponding to different values of γ. All the a,q chosen should lie into the straight line.

$$q = a\gamma \tag{14}$$

Note here $\frac{\pi a^2 Q I_0}{c\hat{F}} = mg$, corresponding to the initially particle is balanced by gravity and scattering force at the equilibrium point. It is obvious Eq.12 is an inhomogeneous Mathieu Equation.

2.3 Mathieu equation and stability diagram

To get the motions of the particle plotted, we should build the simulation of this system to determine the stable and unstable region with given parameters$a$, q. Here we introduce the homogeneous mathieu equation [5]

$$\frac{d^2 z}{d\tau^2} + a(1 - 2\gamma \cos 2\tau)z = 0 \tag{15}$$

The analytic solution of homogeneous mathieu equation is the combination of cosine elliptic $ce(a, q, \tau)$ and sine elliptic $se(a, q, \tau)$. And the integral of over ce, se and $cos(2\tau)$ is convergent. So the stability of Eq.12 should exactly same as Eq.15.

The stability diagram of mathieu equation shown in Fig.2.3. The parameters in the green area will result in the unstable solution, which means the particle will escape. And the parameters in the white area will gives us a stable solution of the particle, i,e, the particle will oscillate around the equilibrium position.

3 Method

This project or the first step of this project is to build the equation and get the motion of particle plotted. The majority work is mathematical analysis of the particle motion in an

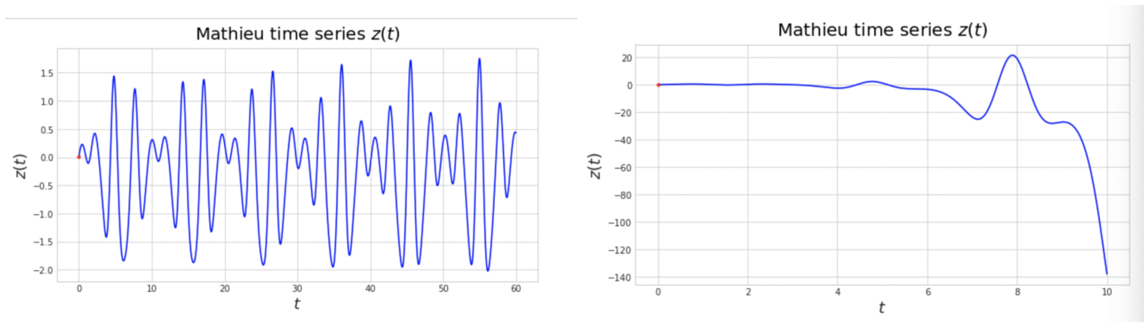

Figure 4: The motion of the particle: Left one, $a = 7.5$, $q = 2.5$, we get a bound periodic motion. Right one, $a = 7.5$, $q = 7.5$, the particle escapes the bound region quickly. Both of follow the stability diagram shown above.

nonlinear oscillating field, which already shown above. The simulation is much more straightforward. The first thing is to build stability diagram and then with the given parameters to get the motions plotted.

Stability diagram is built by Mathematica, Mathematica can solve the mathieu equation directly. So I supply a range of a and q and solve mathieu equation directly. I set a very large number, may be 10000. If the absolutely value of z is bounded by 10000. I treat it as stable. If the value for some t is large than 10000, that will be treated as unstable. This works but I can only get roughly picture. It is hard to obtain the exactly boundary is. If we want to get the boundary value of the region, we still need to do analytic analysis of mathieu equation.

Once parameters given, I rescale the parameter so that Eq.12 can be written as three separate first order derivative equation

$$\dot{x} = a(1 - 2\gamma cos2t)z - 2\gamma \cos 2t$$
$$\dot{z} = x$$
$$\dot{t} = 1$$

With the help code in coclac, i get the plot of position as time. The short of this method is that due to large time plot, the simulation runs very slowly.

4 Result

So far, what I can show you is the motion of particle shown in Fig.4. I try different values of parameters in stable and unstable region. What I get is almost same thing, i.e, a periodical oscillating wave with different period and rapidly decay wave. So getting the motion plotted is not enough to describe the feature of the motion. To distinguish the feature of motion to different parameter, I will go back to analyze the Eq.12. If we see Fig.4. The solution includes oscillation part and non-oscillating part. To get the non-oscillating part, we can do the average the z with given time interval and compare it with the given parameter. I have not figured out a code to do time average of z. This should be my next step of this project.

5 Conclusion

The work done so far is obtain the equation of motion and figure out its stable region and unstable region of parameters, which restricted parameters we can choose. And the next step is to get time average of z with different parameters numerically. Mathematically, I get the solution of non-oscillating part is a function of a and q(I get this result with help of my friends, I am not going to show it without his approval). Since both q and z are functions of a. And a is a function of mass of the particle. Thus, I think the mass spectrum can be built if we get this relation.

References

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