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256B Term Paper. Demonic Variations, or Szilard's *other* Engine

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BACKGROUND

Since James Clerk Maxwell first proposed an intelligence that could, by using precise observations of particulars molecules, apparently violate the second law, the idea has been turned over countless times. In his 1872 book on heat, he first formally introduced the seeming paradox: a "finite being" that could, in essence, capture individual thermal fluctuations in order to extract macroscopic amounts of work from a heat bath [1] in violation of the second law. A couple of years later, William Thomson dubbed these beings "Maxell's Intelligent Demons" [2]. Thus, the paradox of "Maxwell's Demon" was born. Over the next few decades, many of the resolutions of the paradox had to do with purely physical limitations imposed by how a particular demon acted on its observations (a particular method of sorting). Thomson makes this point quite explicitly in a lecture given before the Royal Institution in 1879, where he closes his abstract with the following

"The conception of the "sorting demon" is merely mechanical, and is of great value in purely physical science. It was not invented to help us to deal with questions regarding the influence of life and of mind on the motions of matter, questions essentially beyond the range of mere dynamics." [3]

This seems to point to two important distinctions that early ideas about the demon made. First, that the demons primary job was that it was able to physically sort microscopic particles by their individual characteristics. Second, that Maxwell's Demon (MD) cannot shed light on the influence of "mind" on the motion of matter. It was not until 1929, when Leo Szilard published his seminal paper [4] that a direct connection between what we would now call "information" and a thermodynamic cost was established. In this paper, Szilard showed that both of Thomson's assertions could be relaxed. His examples do not involve manipulation (sorting) of individual molecules, but always involve observation (measurement) of them. Szilard's genius was taking a new definition of "mind", linking it inextricably with a physical instantiation (which, of course, any memory need have). Though the biological phenomena that govern the working of a "finite being" are beyond the scope of physics, Szilard attempted to distill the minimal capabilities a mind would need to ceate MD-like behavior and then created physical machines with these abilities. His conclusion: a physical memory's interaction with a system must come with entropy production, if we are to assume that the second law is valid.

The Szilard paper is referenced in nearly every work on the subject, a notable exception being R. Landauer's 1959 paper [5], in which he seems to respond directly to Szilard's conclusion (that measurement has an inherent entropy cost) by showing explicit costs from erasure. While Szilard's paper is not explicitly referenced here, it is likely that Landauer was at least aware of it. That irreversible erasure of a measurement is the source of the compensating cost was taken to be the resolution of the paradox for the better part of a half century [6, 5] until recent developments in information theory and non-equilibrium thermodynamics allowed for more precise accounting; the result of which is that, at least in some cases, measurement and erasure act as a conjugate pair in which a decrease in the cost of one increases the cost of the other [7].

Szilard describes three different examples of machines, in an attempt to account for how the flow of heat, work, and information(though he doesn't use this word) drives each step of a process. However, since then, the term "Szilard Engine" has come to refer to his first example only. In light of the recent developments that allow new treatments of information engine behavior, it is pertinent to look back at this foundational work and see if we can find additional insight from another Szilard machine. Below, we retrace Szilard's steps in the construction of his second device, and investigate his reasoning with more contemporary ideas and techniques for dealing with chaotic systems, information flow, and the energetics of non-equilibrium thermodynamic states.

SZILARD'S MODEL

Take an ensemble of particles contained in a cylindrical tube in contact with a thermal reservoir of temperature T. Each particle inside of this cylinder is defined by two variables: a particle type variable,$x \in \{A, B\}$, and a variable that relates to the demons knowledge about the particle ,$y \in \{0, 1\}$. The particles can be converted "monomolecularly" (total particle number is conserved) from one type to the other, so as to maintain a particular favored equilibrium distribution. The probability of being one type is given by δ_A and the other $\delta_B = 1 - \delta_A$. This equilibrium distribution can be imposed by there being a specific ΔE between the particle types, or perhaps by spin statistics (as in the case of ortho and para hydrogen [8]). Thus, is it not necessary that the energy of the two particle types differ significantly. We will assume that they do differ in energy for the sake of generality, but not in mass for the sake of clarity. As such, we define the equilibrium hamiltonian to be $H_0 = \epsilon_A N_A + \epsilon_B N_B + \sum_{i=1}^{N} \frac{p_i^2}{2m}$.

The walls of the cylinder are totally impermeable to either type of particle, but there are four pistons inside. Two of these are also impermeable (Szilard calls them A and B), and are initially set some distance ℓ apart. The other two are permeable each to a different of the two particle types and each is set just inside of the the impermeable pistons (A' being set next to B and B' next to A). These pistons are set so they can slide within the cylinder, keeping the distance between A and A' and B and B' fixed at ℓ. It is convenient for the purpose of clarity to consider this a system of two nested cylinders that can slide relative to each other. Each having three impermeable sides, with one base being semi-permeable.

Initially, the distribution over particle type is given by the aforementioned equilibrium distribution, hereafter denoted as δ, and the distribution of the memory variable is uncorrelated to particle type. At some moment, the type of each particle is imparted to the memory variable such that each type $A(B)$ particle has its y variable set to 0(1). Here, the initial distribution over memory state is changed from an initial distribution $\rho(y)$ to a condition distribution $\rho(y|m)$; this construction is very general to any type of measurement. It is also where the concept of some type of "information" cones into play: in order for this measurement to happen, we need to both observe the x variable of each particle and then store a y variable in some type of memory. Schematic diagrams of this and the following processes can be found in figures 1 and 2.

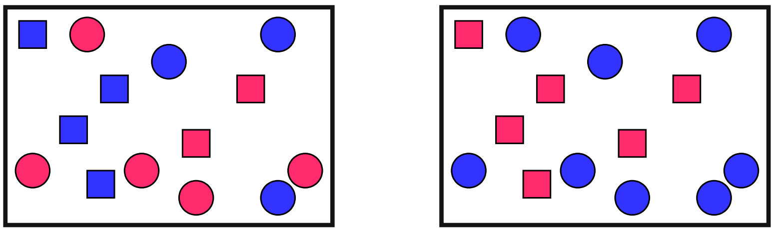
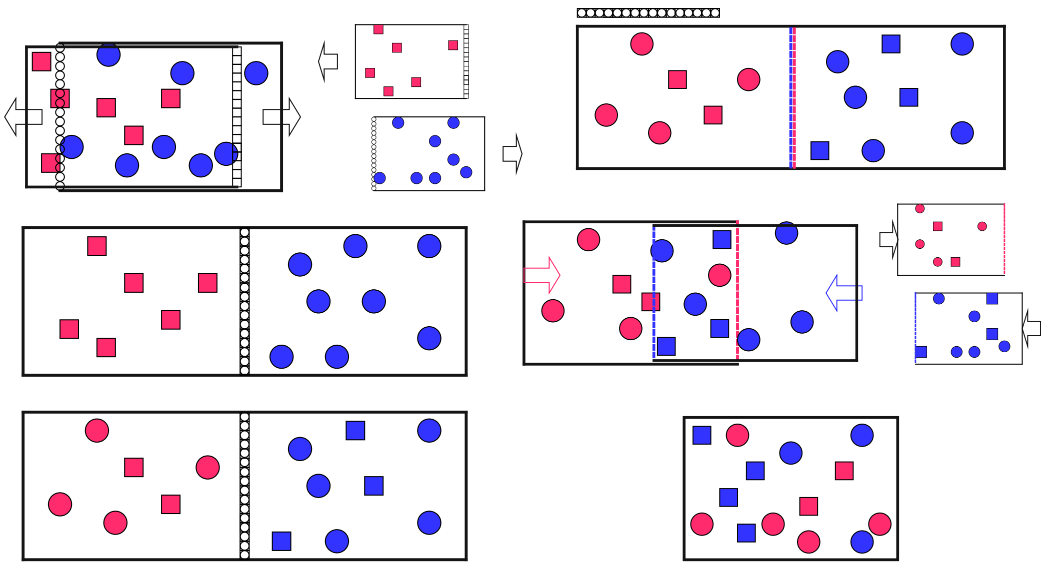


Figure 1: Schematic diagram of Szilard's measurement process. The particle type variable $x \in \{A, B\}$ is represented by shape, and memory variable $y \in \{0, 1\}$ is represented by color. We see that the measurement corresponds to the establishment of a correlation between color and shape (square→red, circle→blue) from the initially uncorrelated state(on the left).

Now, the two cylinders are pulled apart so that we separate the particles by type. This is done without any input of work or heat, because from the perspective of each particle its container is just being translated (as demonstrated in the top line in figure 2a). We must make a note about time scales here: this separation must happen slowly enough so that the gas will always be in equilibrium with respect to the spatial volume of the container, but fast enough that no particles will transition types during the process. This is not a very serious constraint though, as the time-scale for a gas to fill its container is generally very quick. Now, each type of particle exists independently in a container of the same size as the initial container. There is no longer equilibrium with respect to the type variable within the containers. We can, in principle, recover the equilibrium distribution ρ_0 individually within the containers by waiting long enough or by externally driving the process. Szilard now claims that the "entropy has certainly increased". Let us investigate his claim in detail.



(a) (b)

Figure 2: a) The separation process, membranes that are semipermeable to the circles(squares) are represented as a line of squares(circles)– as they are, in essence, walls for the squares(circles) only. b) The process of reintegrating to the initial state, by replacing the type membranes with color(memory state) membranes.

The change in entropy from the initial state to the final state in which the particles have re-achieved equilibrium can be found using the Sakur-Tetrode equation for an ideal gas

$$S = Nk\ln\frac{V}{N} + \frac{3}{2}Nk\ln\left(\frac{4\pi mU}{3h^2N}\right) + \frac{5}{2}Nk$$

We can instantly neglect many of the terms, because they obviously will not change. Cursory inspection reveals that ΔS will be determined by, at most,

$$Nk\ln\frac{V}{N} + \frac{3}{2}Nk\ln\left(\frac{E}{N}\right)$$

In our case, the energy density term will also drop out– since both the start and end state have achieved the equilibrium distribution $N_A = \delta_A N$, $N_B = \delta_B N$, and so $\frac{U}{N} = \delta_A\epsilon_A + \delta_B\epsilon_B + KE_{avg}$ for any number of particles. Finally, since each container has the same volume, the volume term can't contribute to the change in entropy. Therefor the change in entropy is

$$\begin{aligned}
\frac{\Delta S}{N} &= -\delta_A \ln N\delta_A - \delta_B \ln N\delta B + \ln N \\
&= -k\left(\delta_A \ln\delta_A + \delta_B \ln\delta B\right) \\
&= S(\delta)
\end{aligned}$$

Therefore, we have increased the entropy of our system. If we are to drive the re-establishment of the equilibrium distribution in a reversible way, instead of spontaneously, then there must be a corresponding decrease in entropy in the reservoir equal to $-S(\delta)$. Now, this is business as usual. We should absolutely expect this kind of process to increase the entropy of the system, since we have increased the effective volume of the gas. We will not be able to easily put the cylinders back into each other now, since there are particles of both types on each side of the permeable membranes.

Of course, we have yet to use the y variable. If we assume the timescale in which the y memory degrades is significantly longer than any of our other timescales (any physical memory has a finite lifetime, so we must at least be be aware of time scales), it is safe to assume that each particle in the left container still has $y = 0$ and those in the right have $y = 1$. If the demon now exchanges the particle type membranes with membranes that are semi-permeable to the memory states, then we are able to play the exact same game as when separating the particles to bring them back into the same volume (figure 2b). Again, we can do this work-free given that the process is done on the proper time scale. Within the original volume, we no longer have correlation between the x and y variables, but in the process of destroying the correlation– we have managed to bring the entropy of the system back to its initial value. Since this was done without any need to interact with the heat bath, we have only the entropy decrease in the environment to contend with.

Here Szilard relies on the validity of the Second Law, stating that "If we do not wish to admit that the Second Law has been violated, we must conclude that . . . the measurement of x by y, must be accompanied by a production of entropy" [4]. The careful reader will notice a number of fuzzy areas that require further investigation and definition. Szilard does not specify a mechanism of this increase of entropy, nor does he investigate the work required to drive the reversible process he postulates. Additionally, one may note that the final distribution over the y variable, while not correlated with x at the end of the cycle, will necessarily be distributed so that $N\delta_A$ particles are in the 0 state and $N\delta_B$ particles are in the 1 state. That is, unless we include an additional erasure step that resets y to some arbitrary initial distribution. All of these issues are discussed below; we shall see that, while the selection of the initial distribution over y is arbitrary, the choice does have implications for the thermodynamic costs of measurement and erasure. We start by looking at the work required by the process that drives out separated gasses back to particle-type equilibrium.

To look at the process that goes from a non-equilibrium state, ρ to an equilibrium one ρ_0 (figure 2a), we use recent developments in information theory and non-equilibrium thermodynamics [9, 10, 11]. We are able to connect the statistical entropy function $S(\rho) = -k\sum_x \rho(x)\ln\rho(x)$ to the energetics of the process in question. First, we instantaneously shift the hamiltonian from $H_0 \to H_\rho = -kT\ln\rho$; ρ is now the equilibrium distribution since the canonical equilibrium probability distribution is $\rho_{eq} = e^{-\beta H_\rho}$. Since we have shifted the hamiltonian, this process requires work given by

$$W_{\Delta E} = \langle H_\rho\rangle_\rho - \langle H_0\rangle_\rho$$

Now, we quasi-statically shift the hamiltonian back to H_0. During the qui-static process, the system is in equilibrium the whole time, so the work to drive this can be calculated using traditional equilibrium thermodynamics. Conservation of energy implies that $\Delta U_{res} + \Delta U_{sys} = W_{QS}$. If the volume of the system and the reservoir are to remain constant, we can then calculate $\Delta S_{res} = \frac{Q_{res}}{T}$ by noting that $Q_{res} = \Delta U_{res}$,

$$T\Delta S_{res} = W_{QS} - \Delta U_{sys}$$

Thus, the total entropy change, including both the system and the reservoir must then be

$$T\Delta S_{res} + T\Delta S_{sys} = W_{QS} + T\Delta S_{sys} - \Delta U_{sys}$$

The difference of $T\Delta S_{sys} - \Delta U_{sys}$ is nothing more than the change in free energy of the system, $-\Delta F_{sys}$. Thus, if we are to choose the reversible process ($\Delta S_{tot} = 0$), then the work to drive the quasi-static quenching to H_0 is:

$$W_{QS} = F(\rho_0) - F(\rho)$$

Adding up W_{QS} and $W_{\Delta E}$ yields the the total driving work for both components of the process,

$$\begin{aligned} W_{drive} &= \langle H_\rho \rangle_\rho - \langle H_0 \rangle_\rho + \big(\langle H_0 \rangle_{\rho_0} - TS(\rho_0)) - (\langle H_\rho \rangle_\rho - TS(\rho) \big) \\ &= \langle H_0 \rangle_{\rho_0} - \langle H_0 \rangle_\rho + TS(\rho) - TS(\rho_0) \end{aligned}$$

In our case, Each term is very simple. The expectation values of the energy for both distributions are the same:

$$\begin{aligned} \langle H_0 \rangle_{\rho_0} &= N\epsilon_A \delta A + N\epsilon_B \delta_B \\ \langle H_0 \rangle_\rho &= N\delta_A(\epsilon_A \delta_A + \epsilon_B \delta_B) + N\delta_B(\epsilon_A \delta_A + \epsilon_B \delta_B) \end{aligned}$$

Our non equilibrium statistical entropy is 0, since each region is deterministic in terms of particle type. The equilibrium statistical entropy is simply $S(\rho_0) = NS(\delta)$.

$$W_{drive} = -TS(\rho_0) = -NTS(\delta)$$

It is now clear that this process can be considered an engine; the driving work is negative, signifying that there is an opportunity to extract work form the heat bath. We can also verify that the entire process is indeed negative by adding up the entropy production of both steps,

$$\begin{aligned} \Delta S_{tot} &= \Delta S_{sys} + \Delta S_{res} = \Delta S_{sys} - \frac{Q_{drive}}{T} \\ &= S(\rho_0) - S(\rho) + \frac{W_{drive}}{T} \\ &= S(\rho_0) - S(\rho_0) = 0 \end{aligned}$$

where $Q_{drive} = -W_{drive}$ because there is no change in internal energy. We see here that, indeed, the process is reversible. We could continue along the same lines, to calculate the thermodynamics of the information flow as well. However, let's turn to a more explicit model of the system in which these calculations will be much easier

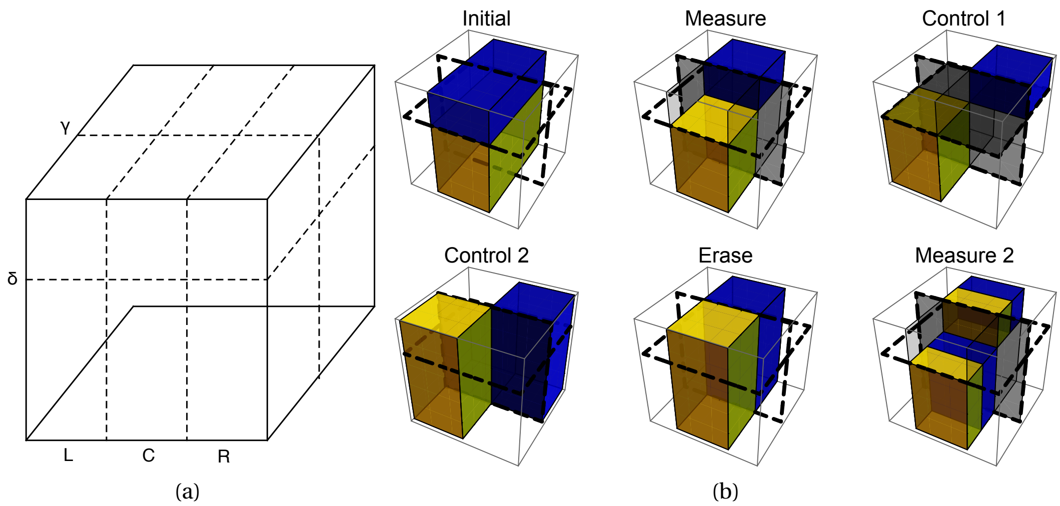


Figure 3: (a) Partition of a 3D unit box. The lower(upper) partition corresponds to particle type $x = A(x = B)$. The forward(rear) partition corresponds to the memory state $y = 0(y = 1)$. γ and δ parameterize the sizes of the partitions, specifically, $\delta = \delta_A$ if the initial distribution is uniform along the vertical direction. The width of the partitions in the horizontal (LCR) dimension is ℓ. (b) The action of the second Szilard engine broken down into individual steps on an initially uniform distribution. The color illustrates which particles start as which type.

The Szilard Map

Here we follow a schematic model diagrammed roughly in [6] and recently investigated much more thoroughly [7]. It is well known that information can be stored in the physical position of particles [5], so we need not distinguish between the container of the particles and the demon. We construct a 3-dimensional unit box, that can keep track of particle type and memory states by sorting the particles into different pre-assigned regions within itself as in figure 3a. With this specific incarnation of Szilards model in mind, we can describe the process as the series of steps shown in figure 3b.

The gas of particles can be treated as an ideal gas, and the steps of the processes can be represented by sliding barriers to compress or expand the volume of the gas. We can use this to verify agreement with the calculations detailed above. For the "control" process, we can find the work to drive the process as $-\int P\mathrm{d}V$ with $P = \frac{NkT}{V}$. For the step "Control 2", that becomes:

$$
\begin{aligned}
W_{drive} &= -\int_{\ell\delta\gamma}^{\ell\gamma} \frac{NkT}{V}\mathrm{d}V - \int_{\ell(1-\delta)(1-\gamma)}^{\ell(1-\gamma)} \frac{NkT}{V}\mathrm{d}V \\
&= NkT\left(\ln\delta + \ln(1-\delta)\right) = -NTS(\delta)
\end{aligned}
$$

Which is in accordance with calculated value above. Thus we see that this specific model achieves the bound on efficiency shown above. We can now calculate the costs of the measurement and erasure process. In these processes, the internal energy of the gas remains fixed– so $Q_{sys} = -W_{sys}$. To investigate the energy that is dissipated in the heat bath, we can draw a relation between Q_{sys} which is positive when heat flows into the system from the bath, and $Q_{diss} = -Q_{sys}$ which is positive when heat is being dissipated into the heat bath. For the measurement process

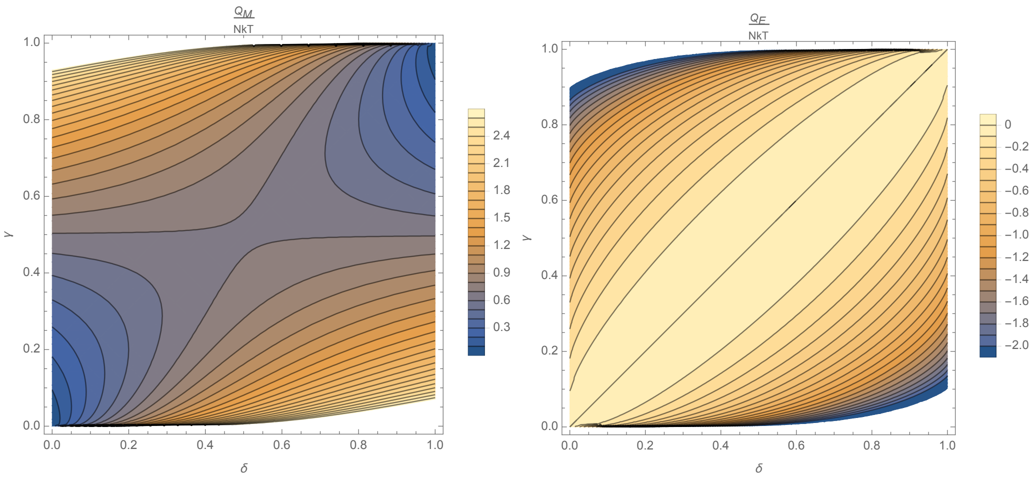
$$
\begin{aligned}
Q_M &= -\int_{\ell\delta}^{\ell\delta\gamma} \frac{N\delta kT}{V}\mathrm{d}V - \int_{\ell(1-\delta)}^{\ell(1-\delta)(1-\gamma)} \frac{N(1-\delta)kT}{V}\mathrm{d}V \\
&= NkT\left(-\delta\ln\gamma - (1-\delta)\ln\left(1-\gamma\right)\right) \\
&= NkT\left(\delta\ln\frac{1-\gamma}{\gamma} - \ln\left(1-\gamma\right)\right)
\end{aligned}
$$

The contour plot of Q_M in figure 4a shows that the thermodynamics do depend on the partition parameters γ, δ and that heat will always be dissipated in this measurement process. The efficient process, then, would seem to be to choose the set of parameters that minimizes the heat dissipated in measurement. Before making this claim, we must investigate the erasure process. Here, we address the aforementioned concern as to the ambiguity of the initial state. It should be clear, by inspection of the erasure step in figure 3b, that in order for us to begin another measurement we must remove the partition that separates the two memory states that was inserted during the measurement process. This is the true source of the cost of erasure, as the translation of the boxes back to the central cell does not require any input or output of energy. The removal of this barrier will not, in general be entropy neutral because the density of the gas on each side of the partition is determined by the values of γ and δ, just as the distribution over $0,1$ in our piston-cylinder model was determined by δ_A. The change in entropy of mixing two identical gasses at different densities depends only on the part of the entropy given by $Nk\ln\frac{V}{N}$. Initially, we have two separate gasses with the relevant entropy components: $S_A + S_B = Nk\delta\ln\frac{L\gamma}{\delta N} + Nk(1-\delta)\ln\frac{L(1-\gamma)}{(1-\delta)N}$. In the final state, we have a single gas $S_F = Nk\ln\frac{L}{N}$. The difference between them gives us:

$$\begin{aligned}\frac{\Delta S}{Nk} &= \ln\frac{1}{N} - \delta\ln\frac{\gamma}{\delta N} - (1-\delta)\ln\frac{1-\gamma}{(1-\delta)N} \\ &= -\left((1-\delta)\ln\frac{1-\gamma}{1-\delta} + \delta\ln\frac{\gamma}{\delta}\right)\end{aligned}$$

It is not surprising that the entropy cost of erasure vanishes when $\delta = \gamma$, since in this case the densities of the two gasses will be equal before removing the barrier. Interestingly, figure 4b, shows that the erasure is not a cost; the increase in entropy of the system corresponds to a negative $Q_E = NkT\left((1-\delta)\ln\frac{1-\gamma}{1-\delta} + \delta\ln\frac{\gamma}{\delta}\right)$. Instead, the erasure appears to provide yet another opportunity for extraction of energy from the heat bath. However, by looking at the sum $Q_M + Q_E$ we can see that choosing the parameters to maximize the energy extraction in erasure will increase the cost of measurement commensurately. The sum is algebraically independent of the parameter γ,

$$
\begin{aligned}
\frac{Q_M+Q_E}{NkT} &= \left((1-\delta)\ln\frac{1-\gamma}{1-\delta}+\delta\ln\frac{\gamma}{\delta}\right)+\left(\delta\ln\frac{1-\gamma}{\gamma}-\ln(1-\gamma)\right)\\
&= -\left((1-\delta)\ln(1-\delta)+\delta\ln\delta\right)+\left((1-\delta)\ln(1-\gamma)+\delta\ln\gamma\right)+\left(\delta\ln\frac{1-\gamma}{\gamma}-\ln(1-\gamma)\right)\\
&= -(1-\delta)\ln(1-\delta)-\delta\ln\delta =
\end{aligned}
$$



(a) (b)

Figure 4: The heat cost of (a) measurement and (b) erasure.

Thus, the total combined cost of measurement and erasure will depend only on δ as $NTS(\delta)$ and is exactly the amount necessary to compensate for the work extracted from the heat bath during control. Since the choice of γ and δ will not affect the total work extracted form the heat bath, it is pertinent to choose $\gamma = \delta$ so that erasure is cost neutral, and all of the extracted work comes from the control process. In this way, we only need consider the "cost" of the measurement and the "profit" from control. Of course, there is no net profit in this game–even in the most efficient system. It is interesting to note that, just as in [7], the distinction between measurement and erasure turns out to be, in some sense, arbitrary. We may increase or decrease the cost of one, but we do so at the expense of the other. This harkens back to

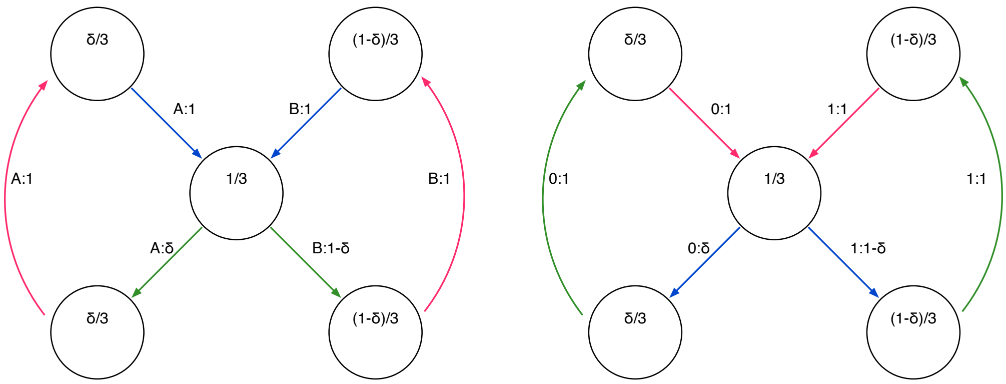
Szilard's original work, where he puts the entropy production on the measurement, and then goes on to demonstrate with a specific measurement apparatus that it is the erasure step which increases the entropy. Szilard was not much concerned about where the entropy was produced, as that the production had to be associated with the process of establishing and destroying correlation between particle type and the memory state.

Information theory

Now, by this point, it is clear that the LCR dimension of our box has become redundant at our current level of abstraction. This dimension is a useful construction to institute a barrier that stops particles that corresponded to the different memory states from interacting when we were considering Szilard's initial problem statement, but now that we are storing the memory and type states in positional coordinates already– the barrier that we use to compress the gas into the "measure" step of the process can serve this purpose itself. This is evinced by the fact that parameter for this dimension, ℓ, dropped out of all the above considerations. Additionally, the action in the LCR dimension is completely deterministic. Thus, we do not need a 3-dimensional box to model the system's information and thermodynamic action. We can, instead look at the action of this map on any vertical cross section of figure 3b. The result is almost identically to the map in [7], which is constructed by considering the single particle Szilard engine. The only differences between the maps is a different initial state distribution. Of course, the second Szilard engine could have been constructed to have the same initial state; for the purpose of illustration, we shall investigate the information flow of the map under this default memory state. We fully expect that the results will agree with [7] in every fundamental sense.

The two dimensional map is, clearly, the bakers map [12]. We find that when $\delta = \gamma$, the distribution over state space is both uniform and constant over repeated interations of the map. Instituting the Markov partition suggested by figure 3a, we can develop information transducers to capture and measure the information processing aspects of the engines operation. figure 5 shows the transducer for each dimension separately and figure 6 shows the joint process. Following the steps in [7] closely, we recover many of the same results. All three machines are counifilar, so the processes are not cryptic. The entropy rate is $\frac{1}{3}H(\delta)$ per map step (consistent with the analytical result for the bakers map, from Pesin's theorem). However, there is slight variation in terms of the statistical complexity, C_μ, which is the information in the machines causal state distribution $\{\mathbf{S}\}$. It is immediately clear from figure 5 that C_μ^y and C_μ^y will be equal to each other. This was not the case in [7]; thus, we can see that the different choice of initial state symmetrizes the stored information with respect to particle type and memory state. The calculations for C_μ are staightforward, for example:

$$
\begin{aligned}
C_\mu^x = C_\mu^y &= -\left[\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\left(\delta\log_2\frac{\delta}{3} + (1-\delta)\log_2\frac{1-\delta}{3}\right)\right] \\
&= \log_2 3 + \frac{2}{3}H(\delta)
\end{aligned}
$$



(a) (b)

Figure 5: The transducers for a) the particle type (x) and b) the memory state (y) subsystems. The protocol steps are designated by color as (control, measure, erase)→(blue,green,red). The numbers inside of the states correspond to the asymptotic distribution, and the transition notation $s : p$ corresponds to emitting the symbol s with probability p.

Similarly, we find that $C_\mu^{joint} = \frac{4}{3}H(\delta) + \log_2 3$. These are quantitatively different from the results in [7]; howerver, this is where the differences end. If we consider the relationship between the three, we recover that $C_\mu^{joint} = C_\mu^x + X_\mu^y - \log_2 3$. So, we see that the two models have the same information related to synchronization of the two subsystems. In fact, we can explicitly check that all other information correlation measures agree with [7]. In doing so we recover: the asymptotic communication rate, $\lim_{L\to 0} \frac{I[X_{0:L}; Y_{0:L}]}{L} = \frac{1}{3}H(\delta)$; the correlation rate, $\lim_{L\to 0} \frac{I[S^x_{0:L}; S^y_{0:L}]}{L} = \frac{1}{3}H(\delta)$; and the dependence of the correlation during the protocol steps, which yields only $I[X_0 : Y_0|M] = H(\delta)$. This makes sense because the measurement step is where the single-symbol correlation is established.

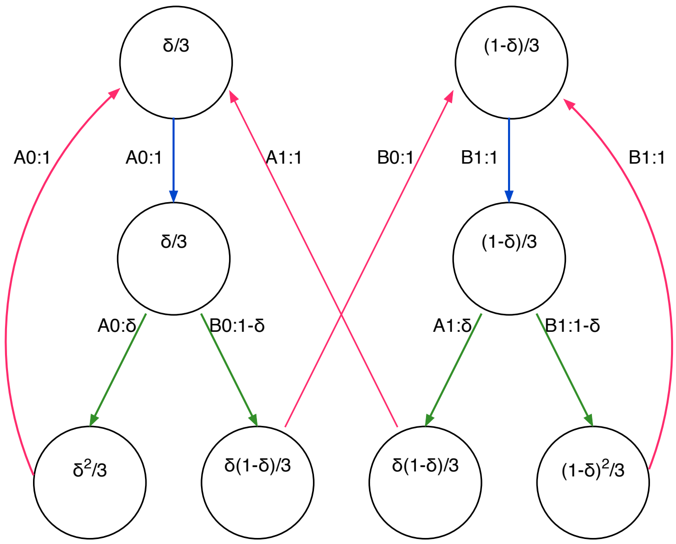


Figure 6: The transducer for the joint system

Conclusion

Since the 1929, the concept of a nanoscale "being" that interacts with heat and information reservoirs has only become less abstract, as modern computing emerged and microprocessors were invented and then miniaturized by nano-fabrication techniques. Thus, understanding the workings of tiny machines that interact with such reservoirs (information engines) is only more relevant than it was when Maxwell first conceived the idea.

The connection between MD and chaos has been discussed in many different settings, for example in [7] and [13] to name just two; current works point towards this route as being incredibly powerful for precise accounting of simultaneous flow of energy and information. As the body of work in the field is vast and varied, it is unsurprising that the topic would interface with the study of chaotic systems. However; the connection is, perhaps, fated. A machine that exhibits demon like behavior is a machine that takes microscopic thermal fluctuations and amplifies them to macroscopic effect. Chaotic systems are systems that are sensitive to small variations in initial conditions. With this in mind, it is evident that while there have been myriad approaches to resolving myriad MD's, the current models feel especially suited.

To Do

The analysis above is far from refined and complete. As far as next steps go, I would like to investigate the relationship between the equality $e^{-S_1/k} + e^{-S_2/k} = 1$ and current information theory and thermodynamics, specifically through the lens of [14] and [10]. It seems to me that this equation, which is central to Szilard's arguments, points almost directly to the relationship between the physical entropy and the statistical Shannon entropy. I also am interested in investigating Szilar'd third example, in which he demonstrates a model of measurement and erasure (without usage of the information) where the erasure bears the full brunt of the entropy cost. Szilard does not seem to concerned with keeping measurement and erasure as two different things, and possibly had already noted that it was possible for either to produce or consume resources– something that I think has only been taken seriously recently. Finally, I would like to use some other framework to analyze this model and see if it is analogous to the single particle model under that framework as well. Again, possibly the work in [14] and [10] as well as additional insight from [11] and related papers by Boyd and Crutchfield over the past couple of years.

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