SZILARD'S OTHER ENGINE

VARIATIONS ON A DEMONIC THEME

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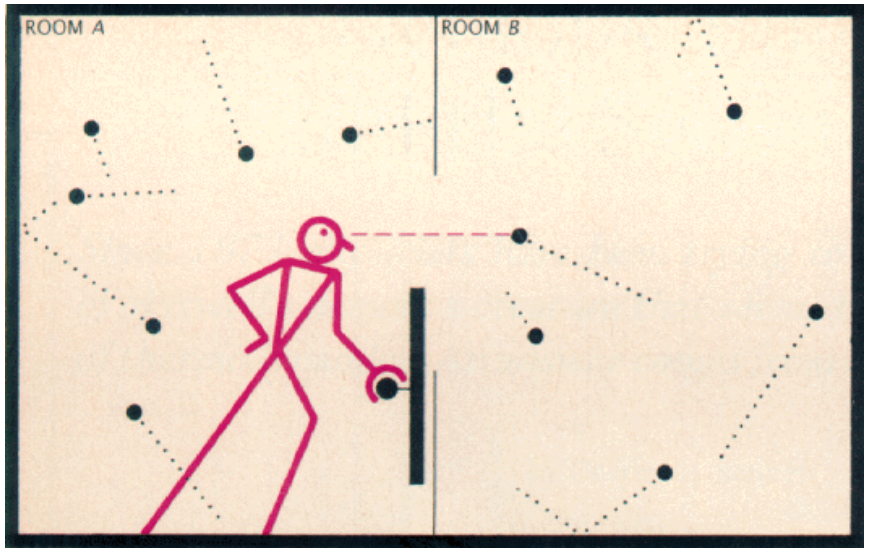
”The Szilard Engine”

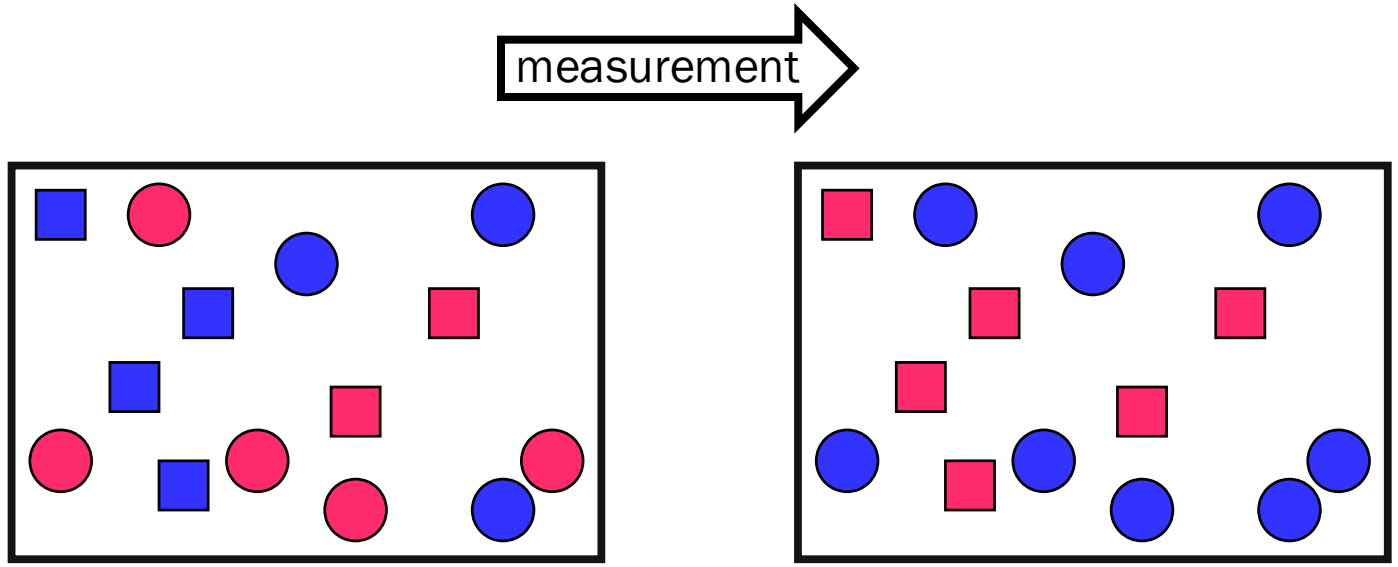
- In 1929, Leo Szilard attempts to account precisely how Maxwell’s Demon avoids violating the second law
- The term “Szilard Engine” has come to mean only one particular machine from his paper
- Let’s investigate his second machine
  - As originally conceived
  - Why an engine?
  - Chaotic Map, Symbolic Dynamics



http://www.pynchon.pomona.edu/entropy/demon.gif

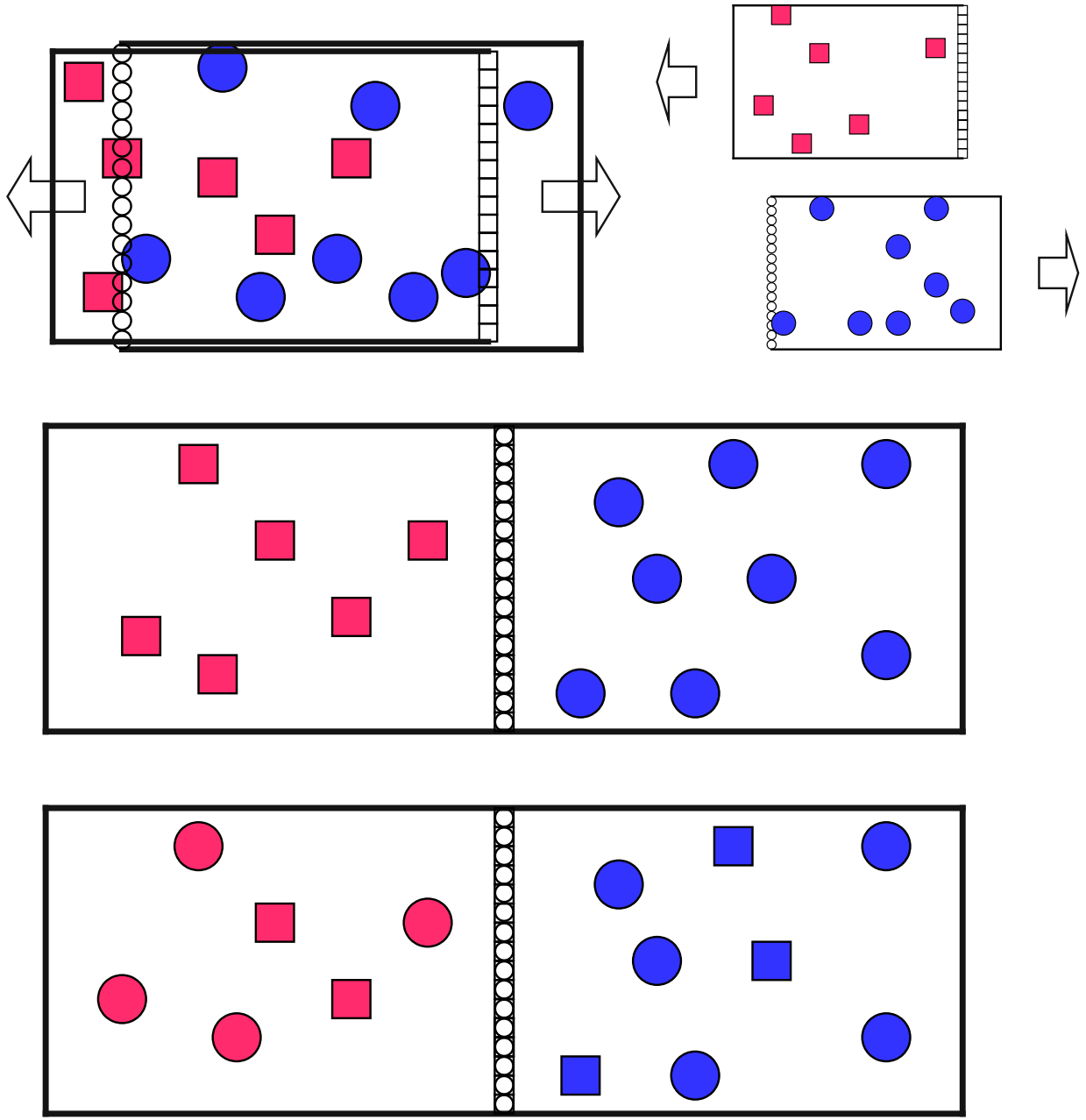
Szilard, measurement

- Two variables for each particle: Type and “Memory(color)”
- There is a equilibrium distribution over particle type, particles can convert between type



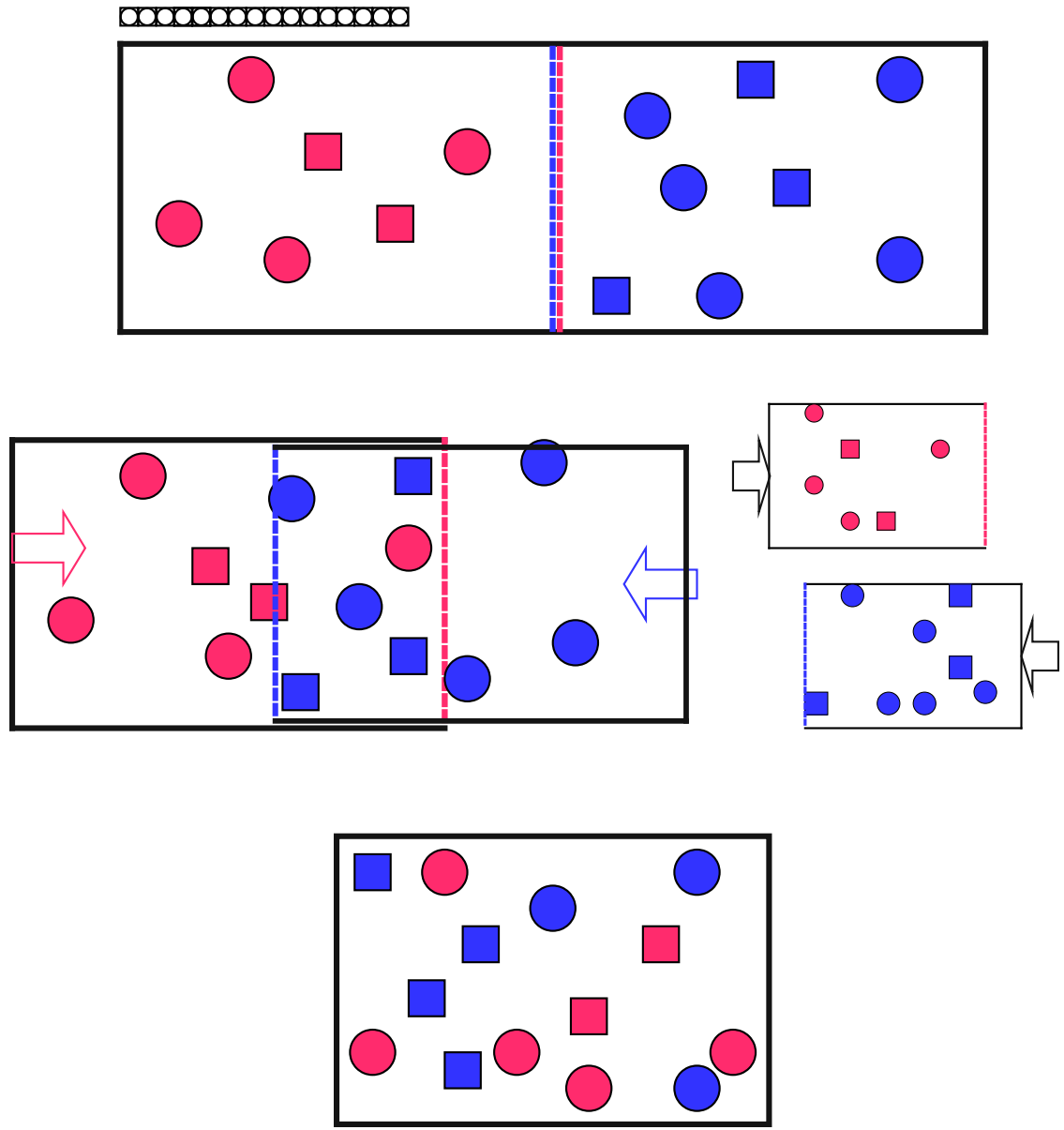
Szilard, control

- Semi permeable membrane for each particle type
- No work, no heat. We are just translating two boxes



Szilard, erasure

- Remove the type membranes and replace them with “color” membranes
- No work, no heat. We are just translating two boxes
- We recover a distribution in the original volume, where the color (memory) is not correlated to the type



Szilard, conclusion

- By step 3, we have changed the entropy:

$$S = Nk\ln\frac{V}{N} + \frac{3}{2}Nk\ln\left(\frac{4\pi mU}{3h^2N}\right) + \frac{5}{2}Nk$$

$$\frac{\Delta S}{Nk} = -\omega_A \ln N\omega_A - \omega_B \ln N\omega B + \ln N$$

$$= -(\omega_A \ln\omega_A + \omega_B \ln\omega B)$$

- In fact we have increased it.

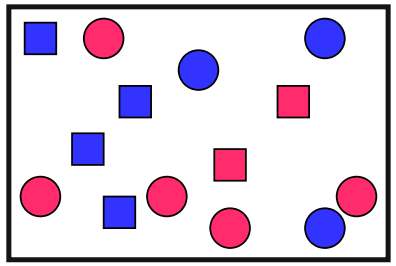
- If done reversibly, that means an an equal decrease in the environment.

- The entire cycle is entropy negative, If we ignore the internal mechanism of the demon.
- Demon must create entropy $\geq -(\omega_A \ln\omega_A + \omega_B \ln\omega B)$

1

Equilibrium

$n_A = \omega_A N,\ n_B = \omega_B N$



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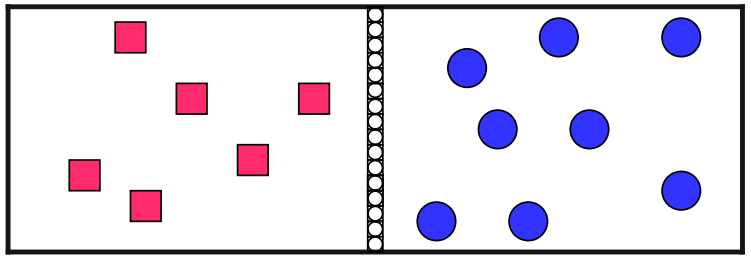
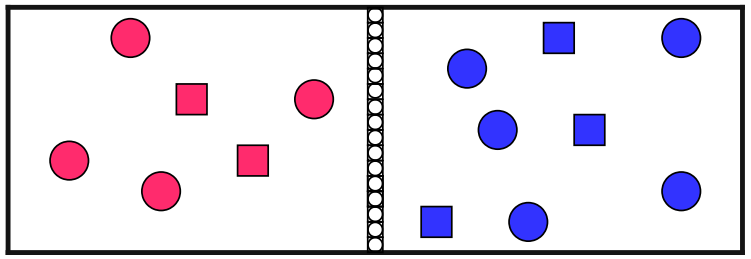
Non-Equilibrium



Reversible

3

Equilibrium



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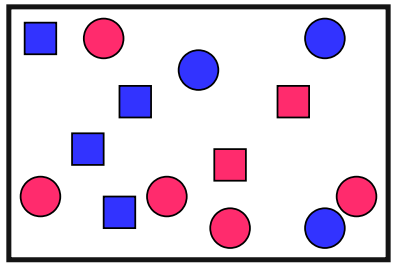


Beyond Szilard

- Why did you call this an engine?

Non Equilibrium Process

- Reversible process from a non-eq. distribution, ρ , to an eq. distribution, ρ_0.
- First, we instantaneously shift the Hamiltonian

$$H_0 \rightarrow H_\rho = -kT \ln \rho$$

- This will take work equal to

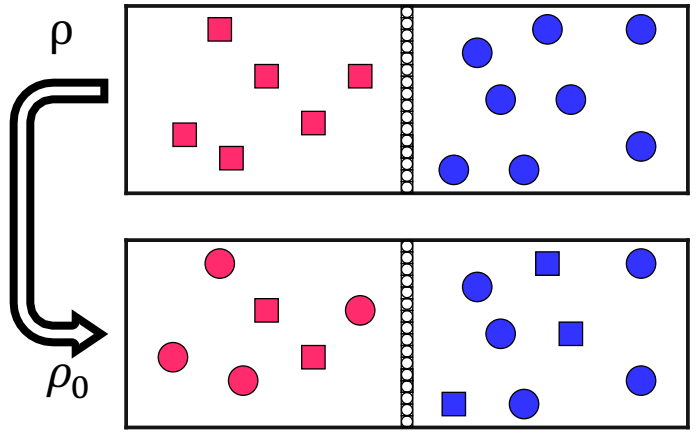
$$W_{\Delta E} = \langle H_\rho \rangle_\rho - \langle H_0 \rangle_\rho$$

- Then we quasi-statically shift back to H_0, which will take work equal to

$$W_{QS} = F(\rho_0) - F(\rho)$$

- Thus, total work to drive the process is:

$$W_{drive} = W_{\Delta E} + W_{QS} = \langle H_0 \rangle_{\rho_0} - \langle H_0 \rangle_\rho + TS(\rho) - TS(\rho_0)$$



$$\langle H_0 \rangle_{\rho_0} = N\epsilon_A \omega A + N\epsilon_B \omega_B$$

$$\langle H_0 \rangle_\rho = N\omega_A(\epsilon_A \omega_A + \epsilon_B \omega_B) + N\omega_B(\epsilon_A \omega_A + \epsilon_B \omega_B)$$

$$S(\rho) = 0$$

$$S(\rho_0) = -Nk(\omega_A \ln \omega_A + \omega_B \ln \omega_B)$$

$$W_{drive} = -TS(\rho_0) = NkT(\omega_A \ln \omega_A + \omega_B \ln \omega_B)$$

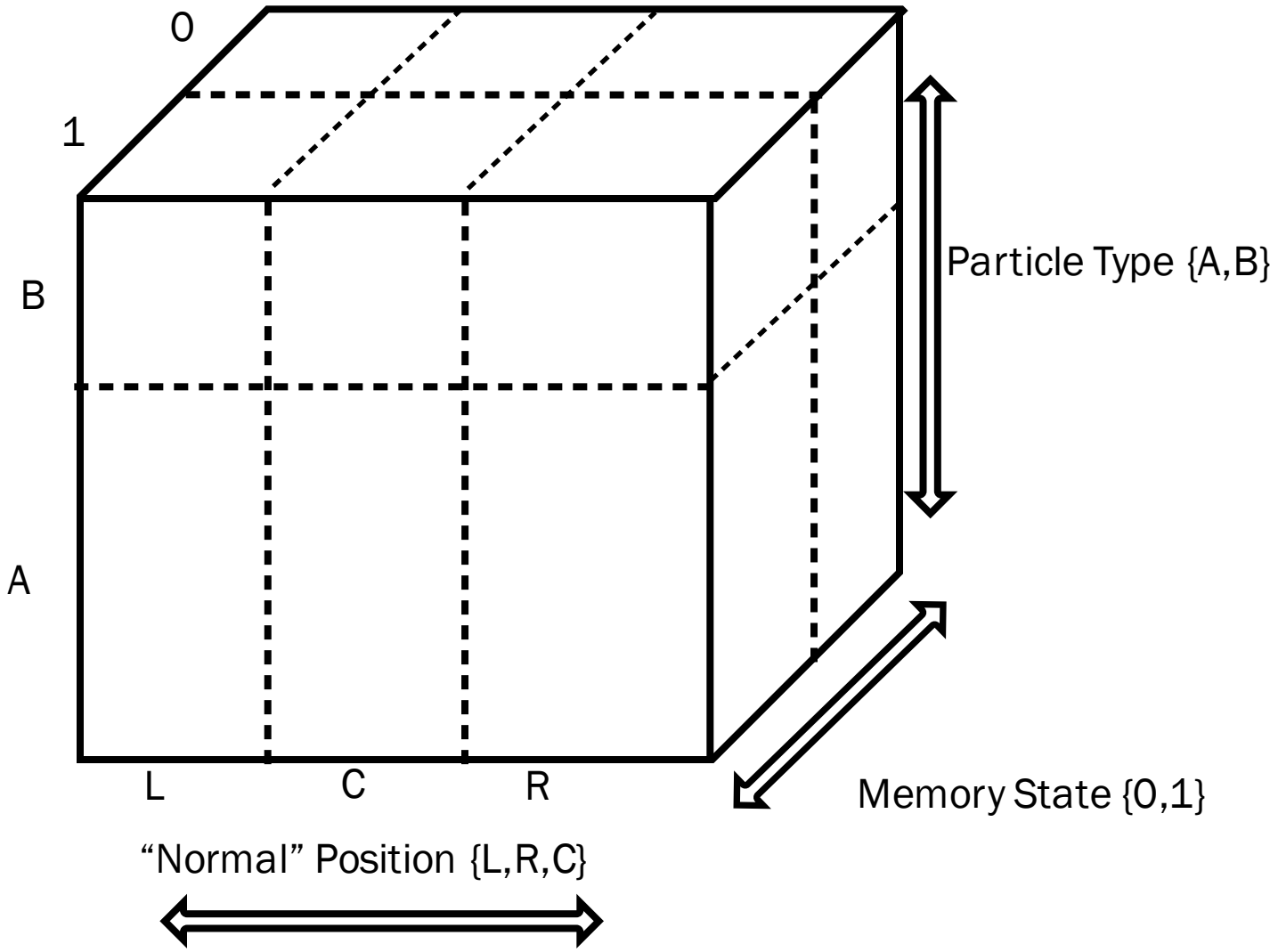
$$\Delta S_{tot} = \Delta S_{sys} - \frac{Q_{drive}}{T} = S(\rho_0) - S(\rho_0) = 0$$

What about the Demon?

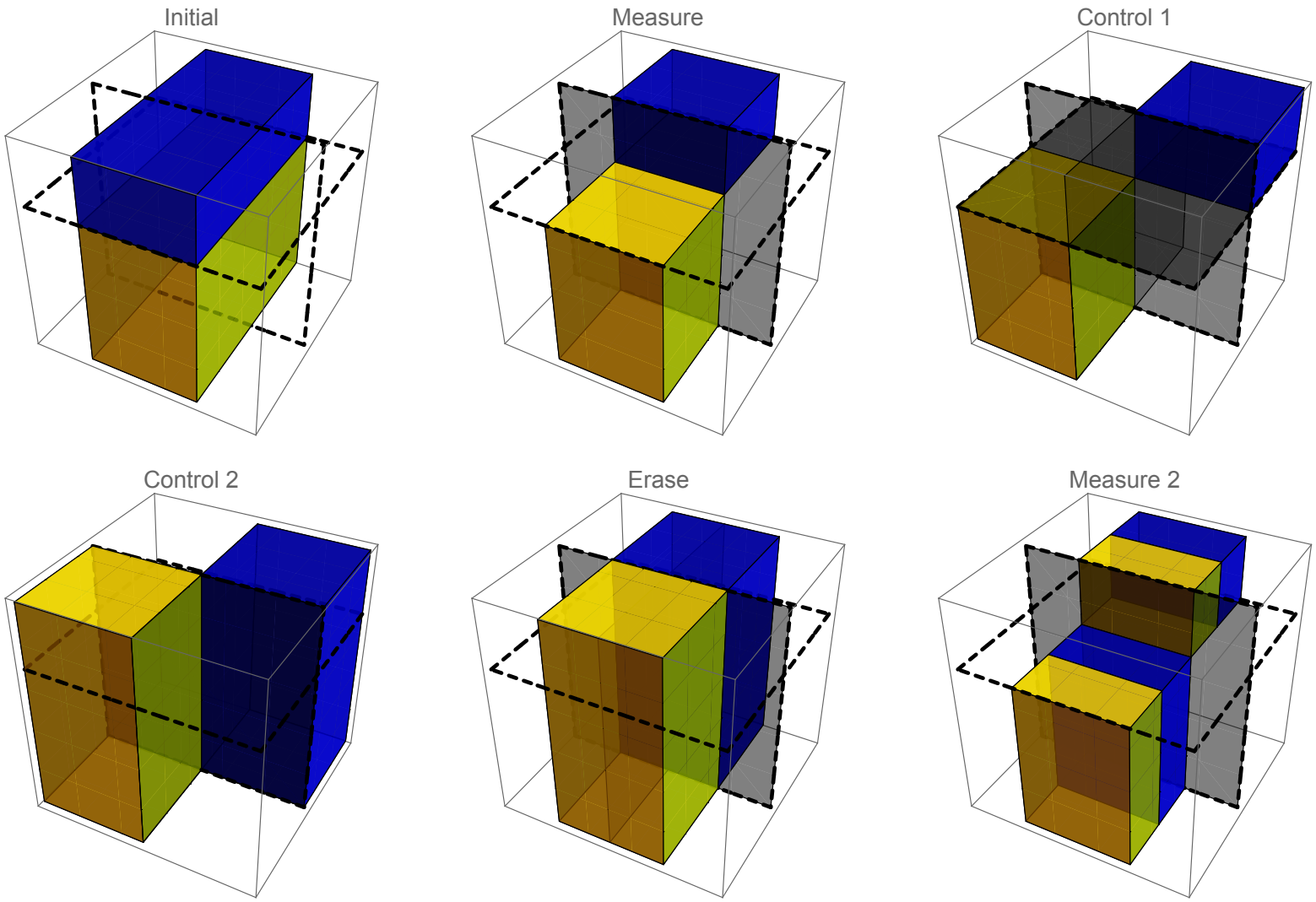
- In order to consider the workings of the demon, we'll need to have an explicit model
- The box IS the demon

The Demon Box

- The demon keeps track of particle type and memory state by pushing the particles using sliding barriers.
- We can calculate thermodynamic quantities easily, treating the particles as an ideal gas contained.



The Other Szilard Map



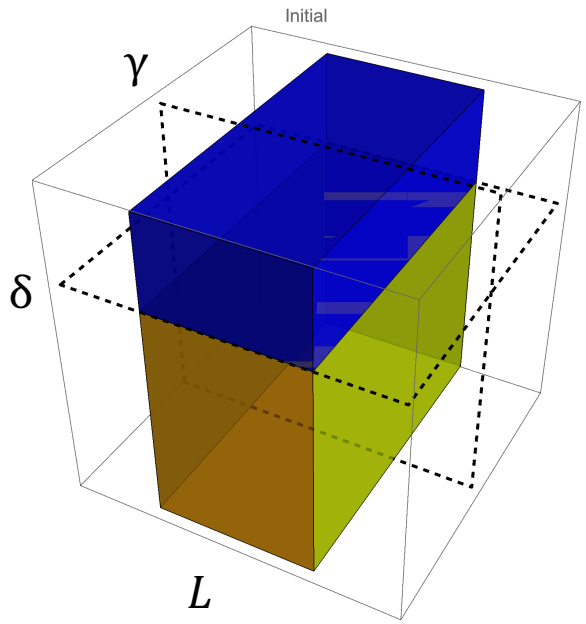
The Other Szilard Map

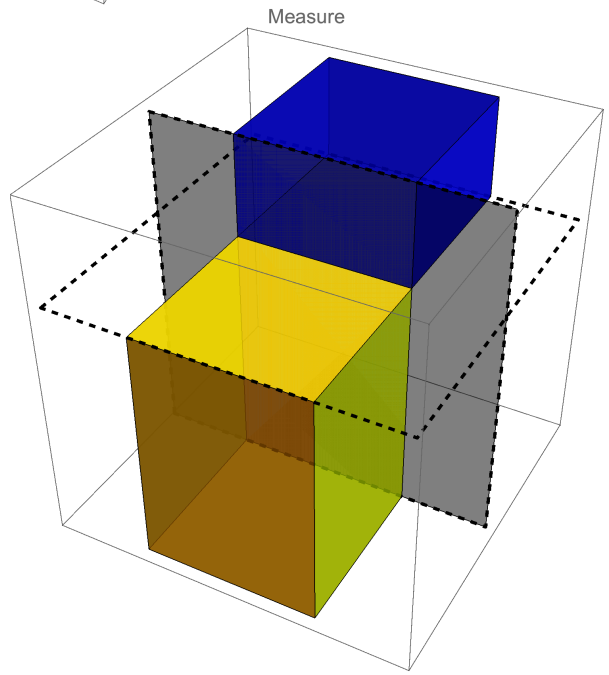
Now, we can calculate the cost of measurement:

$$W_M = -\int_{L\delta}^{L\delta\gamma} \frac{N\delta kT}{V}\mathrm{d}V - \int_{L(1-\delta)}^{L(1-\delta)(1-\gamma)} \frac{N(1-\delta)kT}{V}\mathrm{d}V$$

$$= NkT\left(-\delta\ln\gamma - (1-\delta)\ln(1-\gamma)\right)$$

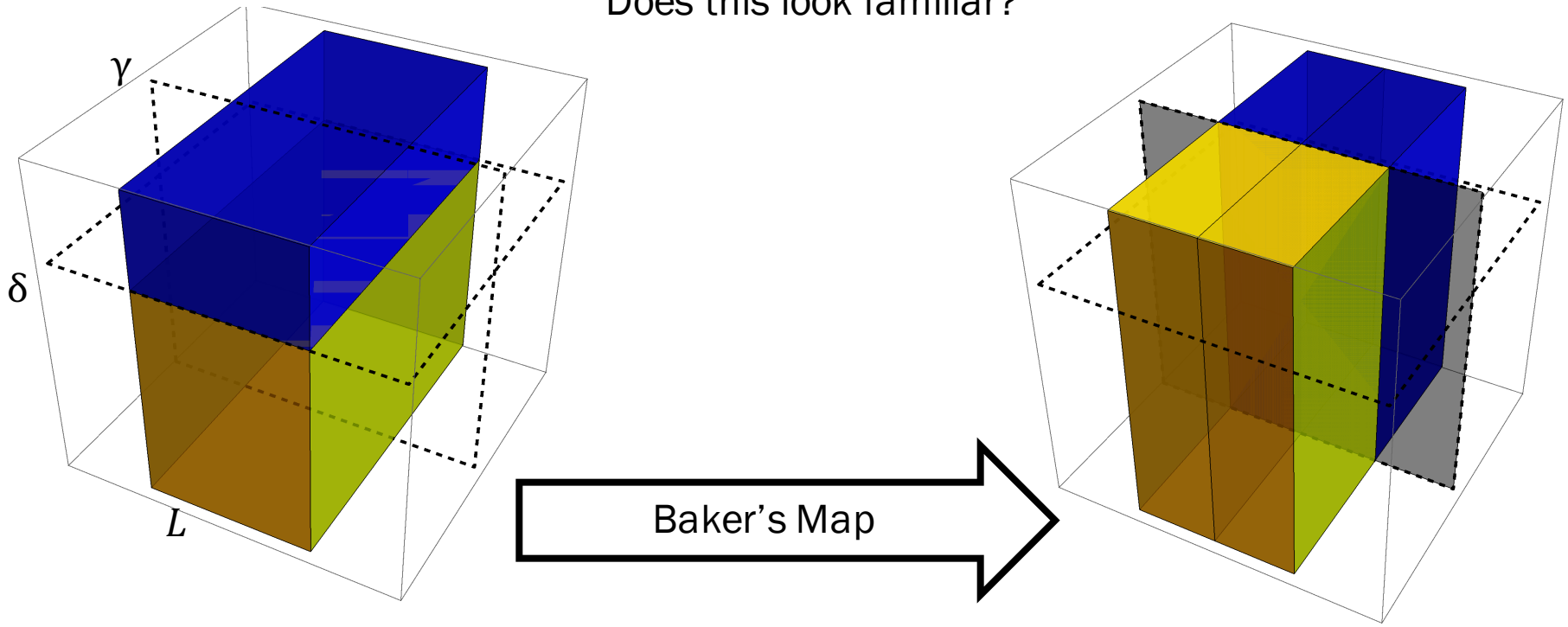
$$= NkT\left(\delta\ln\frac{1-\gamma}{\gamma} - \ln(1-\gamma)\right)$$





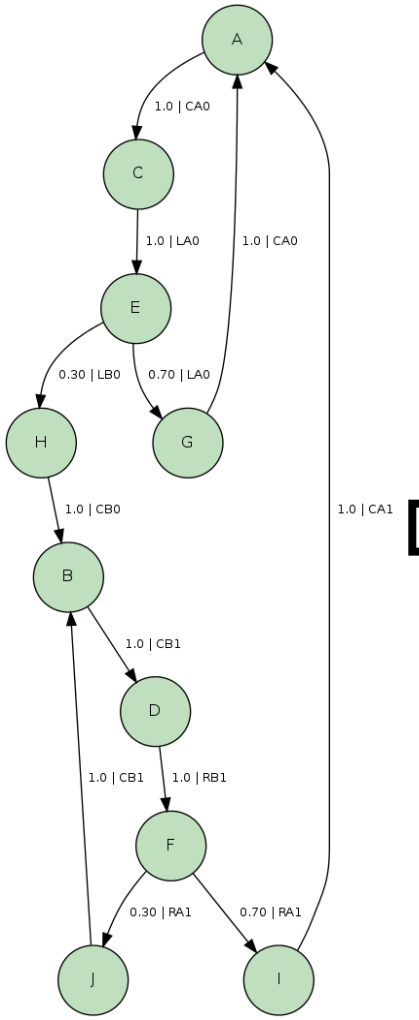
The Other Szilard Map

Does this look familiar?

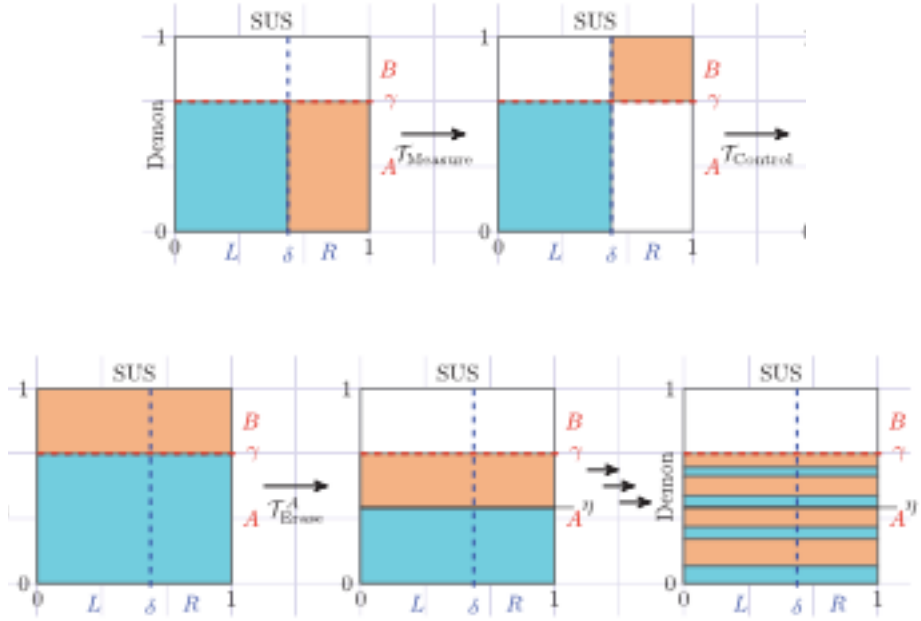


Now we can calculate "anything"

ϵ-Transducer



REMOVE THE LRC DIMENSION



Boyd and Crutchfield (2016)

Closing Remarks

- Kind of a null result, but not necessarily an obvious one
- Traditional Szilard engine is a single-particle engine, so this might be easier to implement while capturing all the same essential information engine thermodynamics
- There is a third machine that Szilard describes, that also might merit investigation

References

[1] Szilard Leo. On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings. *Behavioral Science*, 9(4):301–310.

[2] Charles H Bennett. The thermodynamics of computation?a review. *International Journal of Theoretical Physics*, 21(12):905–940, 1982.

[3] Juan MR Parrondo, Jordan M Horowitz, and Takahiro Sagawa. Thermodynamics of information. *Nature physics*, 11(2):131, 2015.

[4] Alexander B Boyd and James P Crutchfield. Maxwell demon dynamics: Deterministic chaos, the szilard map, and the intelligence of thermodynamic systems. *Physical review letters*, 116(19):190601, 2016.