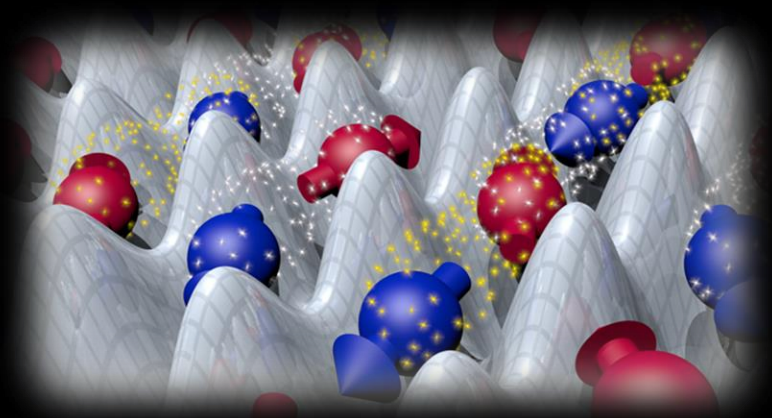
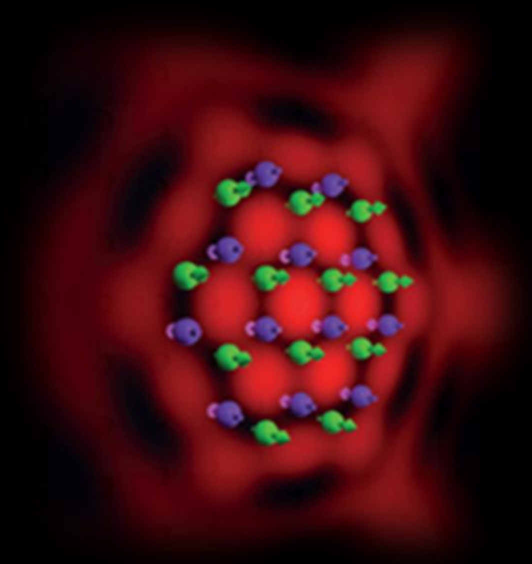
PHY 256B Presentation

Machine Learning Architectures & Many-Body Physics

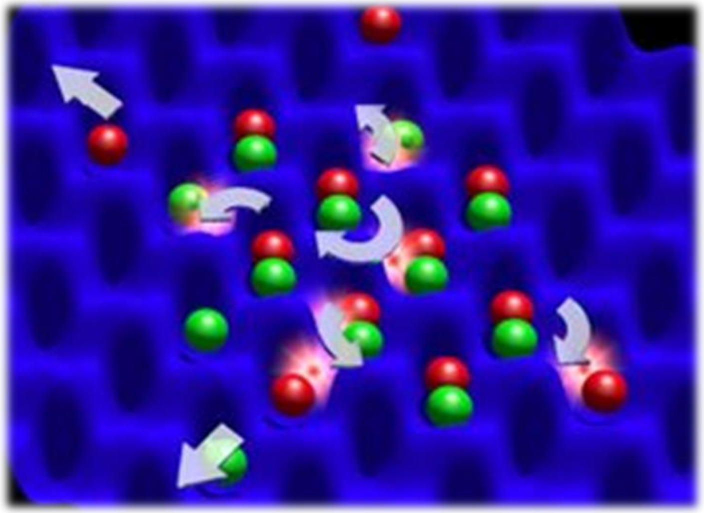




Zhe Wei KHO

7th June 2017

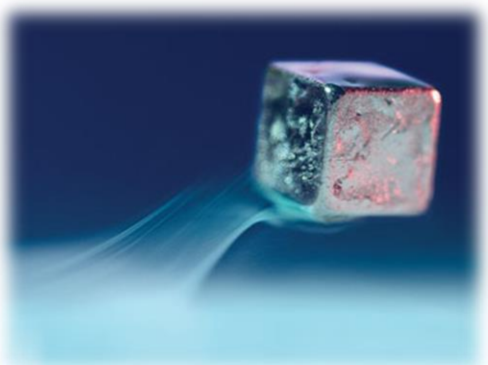
More is Different



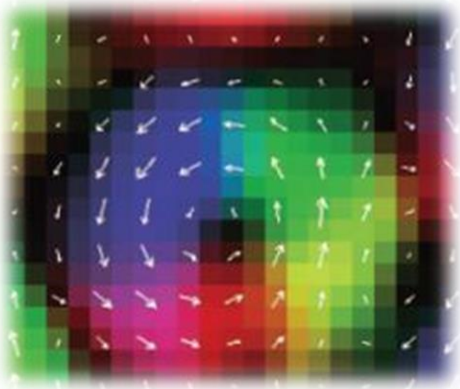
"The behaviour of large and complex aggregations of elementary particles, it turns out, is **not to be understood in terms of a simple extrapolation of the properties of a few particles**.

Instead, at each level of complexity **entirely new properties appear**, and the understanding of the new behaviours requires research which I think is as fundamental in its nature as any other."

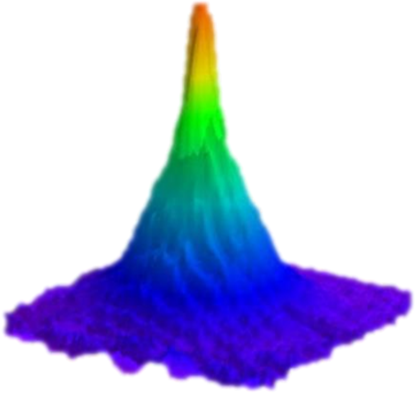
Philip W. Anderson, *More is Different*



Superconductivity



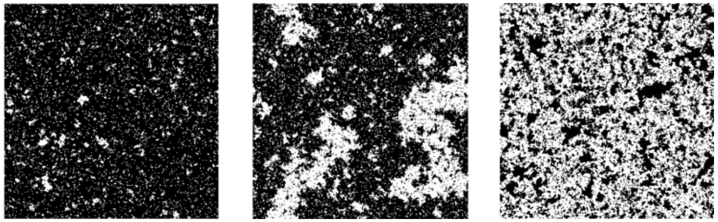
Quantum Magnetism



Bose-Einstein Condensation

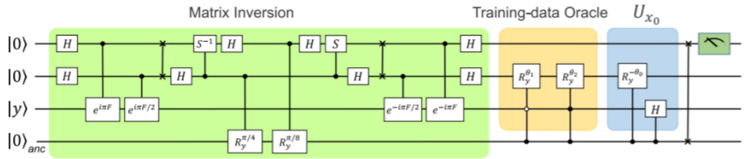
ML & Many-Body Physics

Phase Classification



J. Carrasquilla & R.G. Melko
Nat. Physics **13** 431 (2017)

Quantum ML

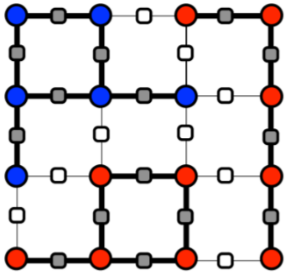


Quantum speed-up of classical ML algorithms

J. Biamonte et al. Nature **549** 195 (2017)

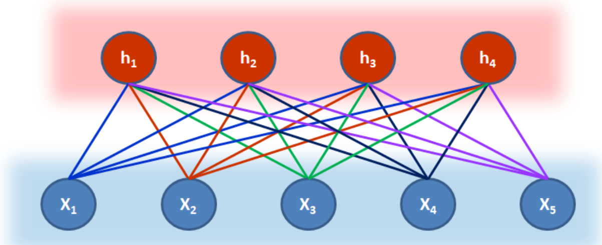
Recommender Systems

Deep learning algorithms can discover efficient Monte Carlo updates



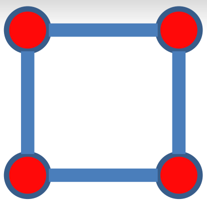
L. Wang. PRE **96** 051301R (2017)

Efficient Repr. of State

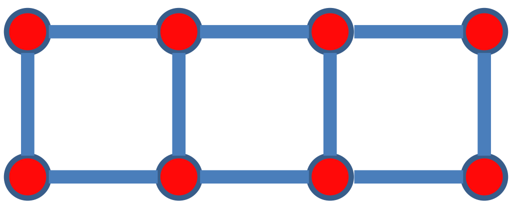


G. Carleo & M. Troyer. Science **355** 602 (2017)

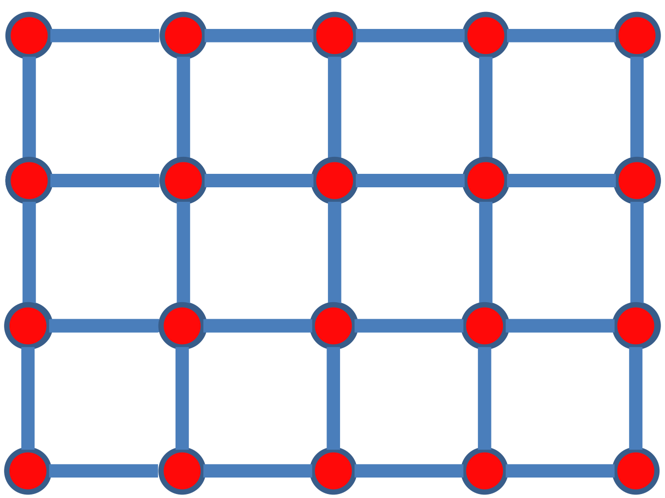
More is Exponential



D = 8



D = 256



D = 1048576

Quantum many-body systems are in general **not solvable analytically**

Dimensionality of Hilbert space **increases exponentially** with the size of the system

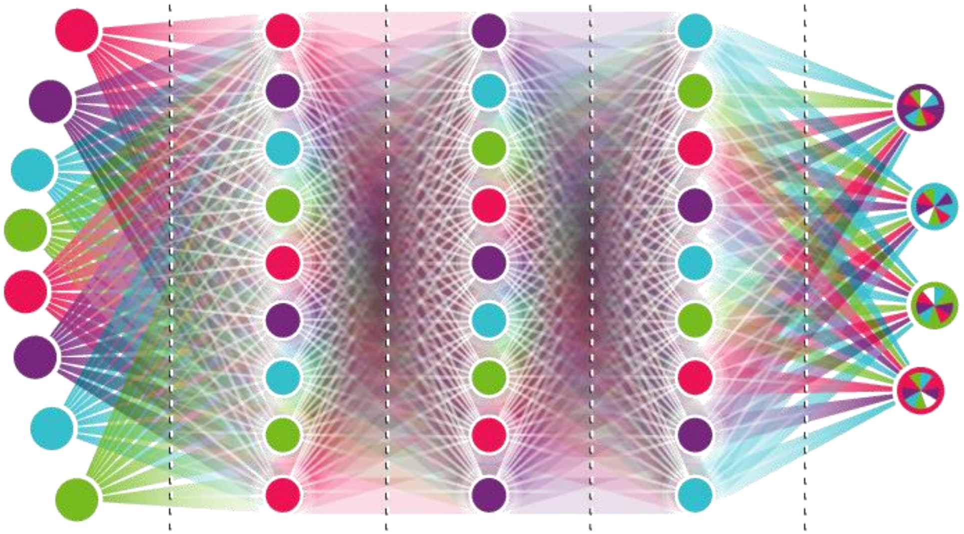
Computationally restricted to **unrealistically small** systems

Can we find an efficient way to represent the states of interest?

Deep Learning

Consists of layer(s) of nonlinear processing units with the aim of extracting **features (macroscopic behaviour)** from the underlying **data (microscopic details)**

Works surprisingly well with a **relatively small number** of layers or nodes (as compared to the **exponential state space** of the inputs)



neuralnetworksanddeeplearning.com - Michael Nielsen, Yoshua Bengio, Ian Goodfellow, and Aaron Courville, 2016.

"Why does deep and cheap learning work so well?"

H.W. Lin, M. Tegmark & D. Rolnick
J. Stat. Phys. **168** 1223 (2017)

Physics ↔ Machine Learning

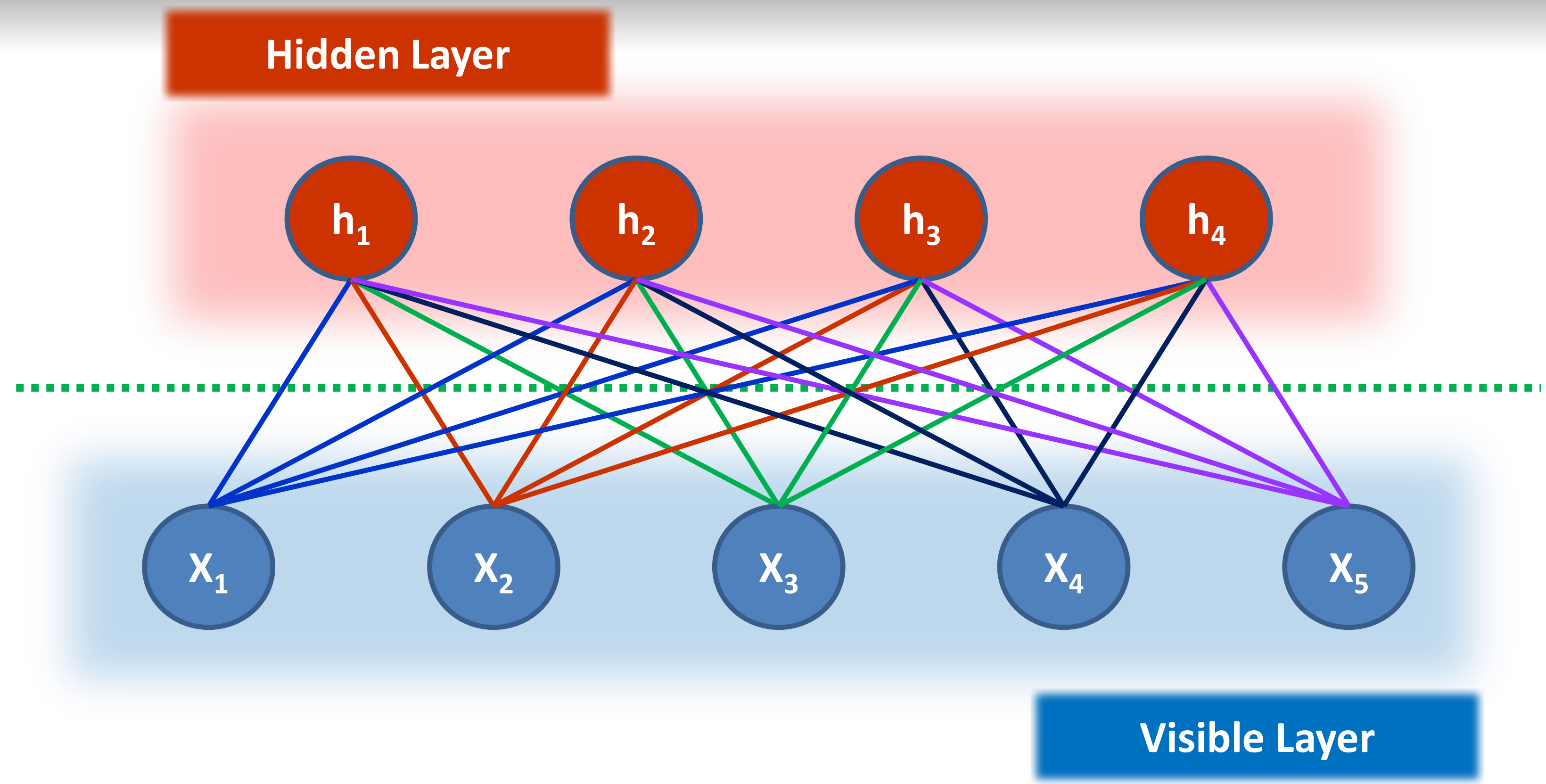
Physics
Hamiltonian
Quadratic Hamiltonian
Partition function
Spin
Relevant Operator
Irrelevant Operator

Machine Learning
Surprisal (Self-Information)
Gaussian Probability Distribution
Softmax function
Bit
Feature
Noise

"The success of shallow neural networks depends not only on mathematics, but also on **physics**, hinging on **symmetry**, **locality**, and **polynomial log-probability** in data from or inspired by the natural world."

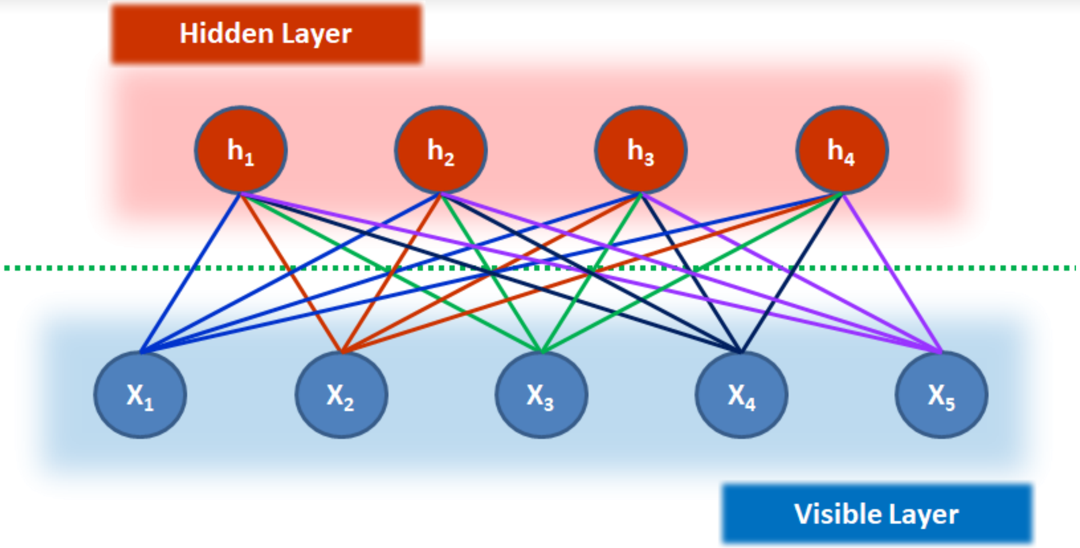
H.W. Lin, M. Tegmark & D. Rolnick
J. Stat. Phys. **168** 1223 (2017)

Restricted Boltzmann Machine



$$E(\mathbf{X}, \mathbf{h}) = \sum_i a_i X_i - \sum_i b_i h_i - \sum_{ij} w_{ij} X_i h_j$$

Restricted Boltzmann Machine



Each layer is **conditionally independent** (essentially Markov shielding)

Activation Probabilities

$$P(X_i = 1|\mathbf{h}) = \sigma \left[a_i + \sum_j w_{ij} h_j \right]$$

$$P(h_i = 1|\mathbf{X}) = \sigma \left[b_i + \sum_j w_{ij} X_j \right]$$

Energy function

$$E(\mathbf{X}, \mathbf{h}) = \sum_i a_i X_i - \sum_i b_i h_i - \sum_{ij} w_{ij} X_i h_j$$

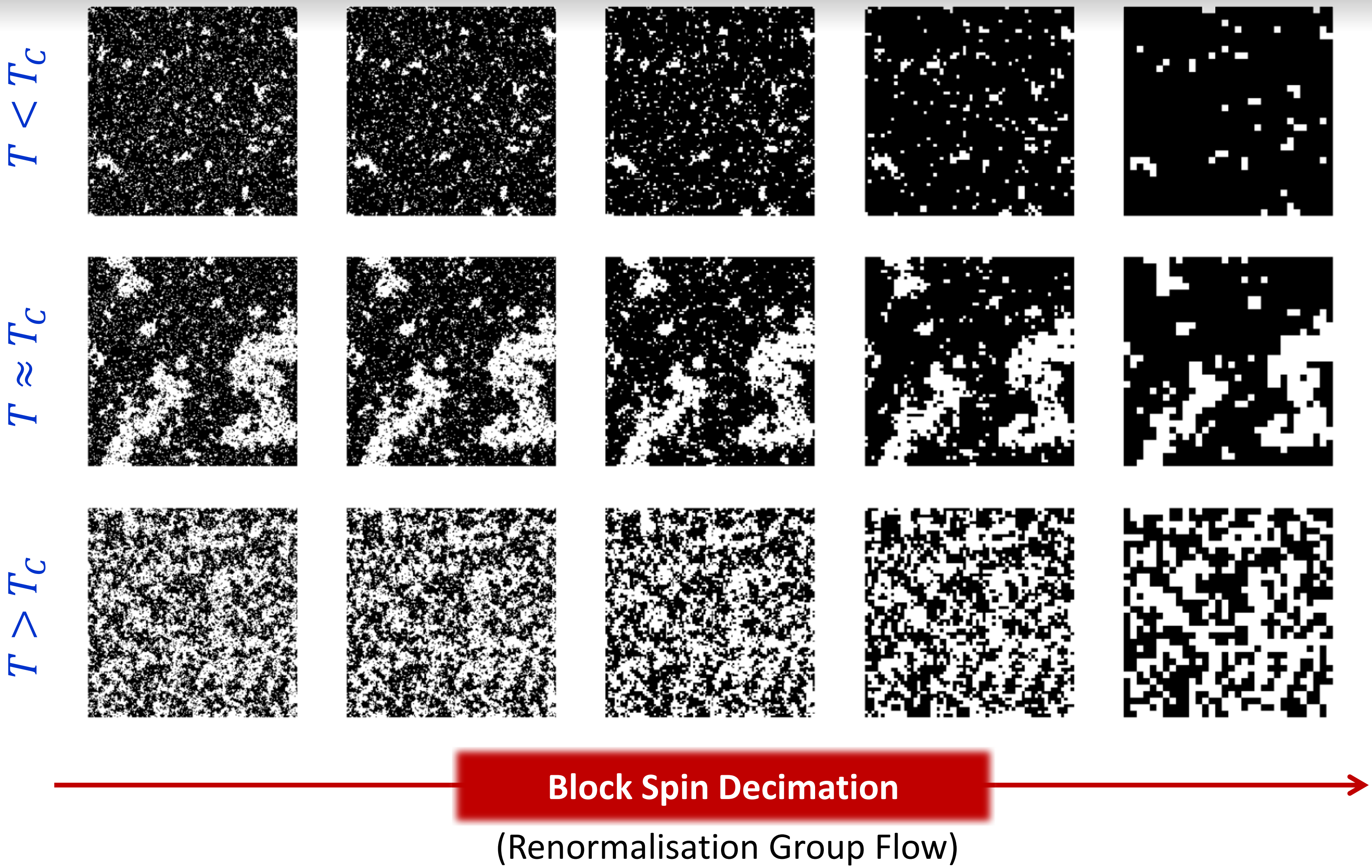
State Probability

$$P(\mathbf{X}, \mathbf{h}) = \frac{1}{\mathcal{Z}} e^{-E(\mathbf{X}, \mathbf{h})}$$

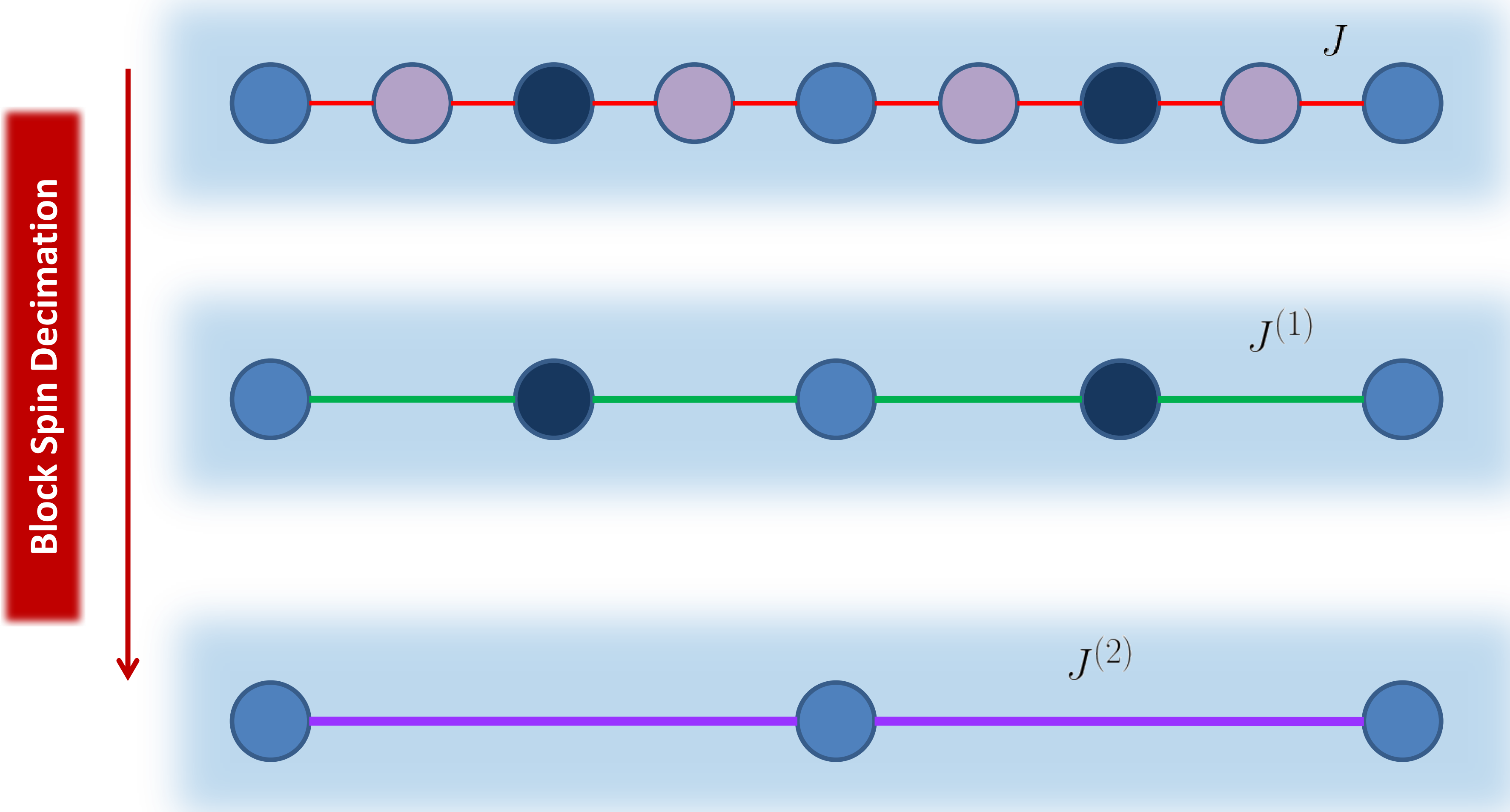
Connection exists between RBMs and Renormalization Group!

P. Mehta & D. Schwab, arXiv:1410.3831 (2014)

Renormalisation Group

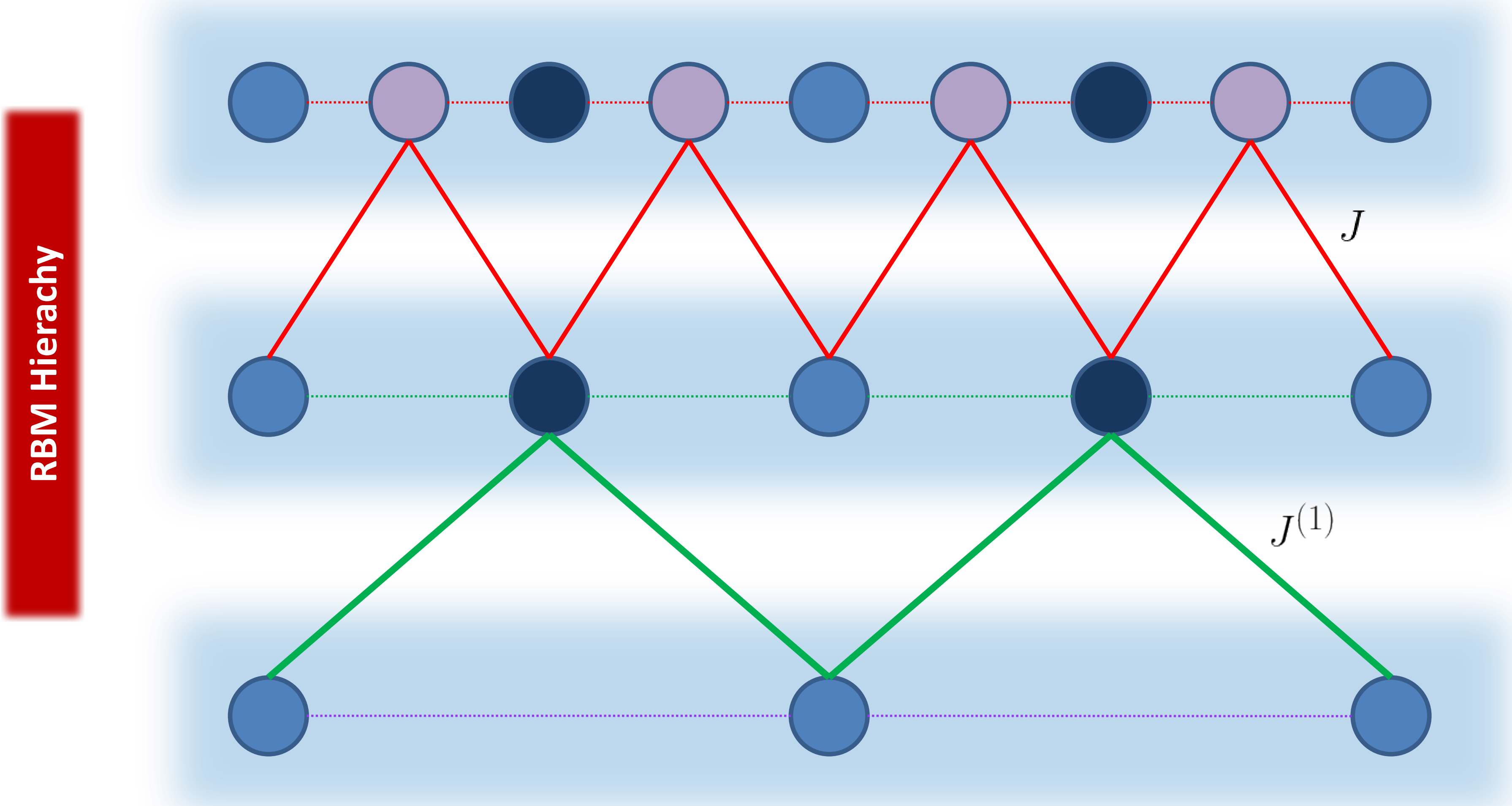


1D Ising Model



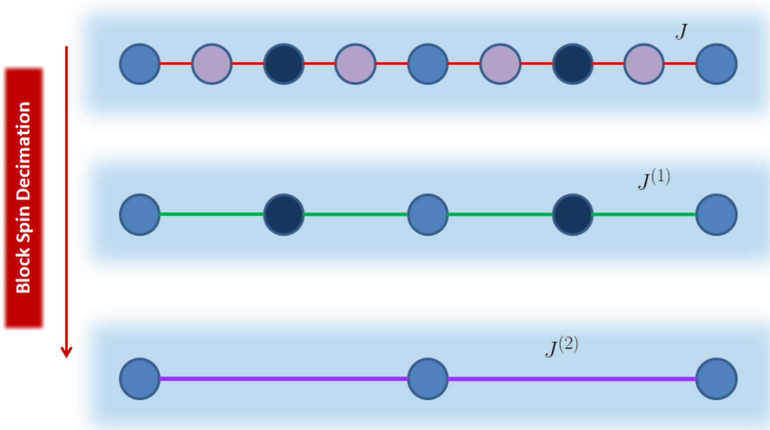
P. Mehta & D. Schwab, arXiv:1410.3831 (2014)

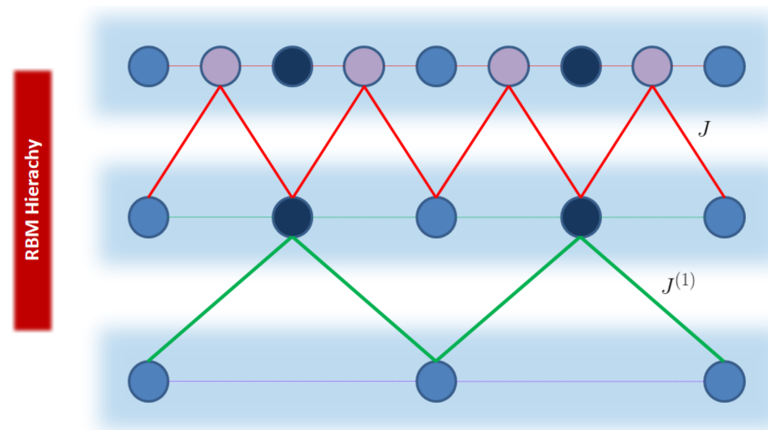
1D Ising Model



P. Mehta & D. Schwab, arXiv:1410.3831 (2014)

RG ↔ Deep Learning





Devising RG transformations is an **art** – need to identify **irrelevant degrees-of-freedom** and perform rescaling of couplings in a **tractable manner**

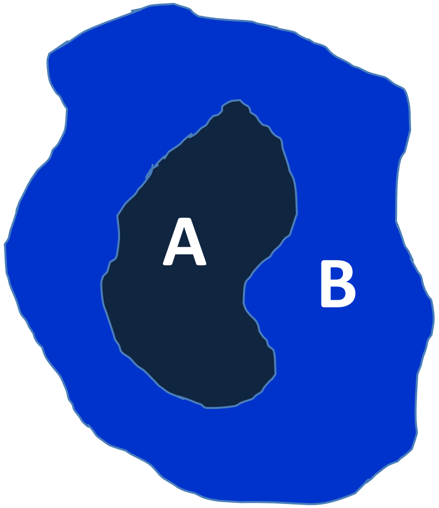
Does the DL network "know" the correct RG procedure to perform?

Yes! The "Real Space Mutual Information" (RMSI) algorithm to able to implement RG transformations in an unsupervised manner

M. Koch-Janusz & Z. Ringel
Nat. Phys. **14** 578 (2018)

Efficient Representation of States

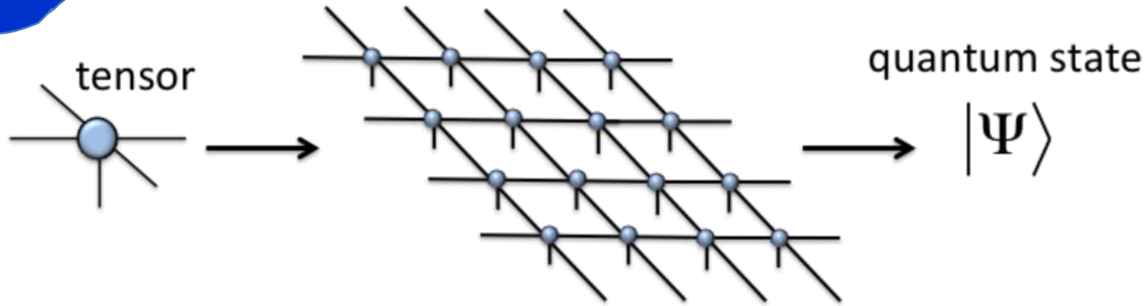
While generic states require exponential resources to describe, typical states of interest in both ML and Many-Body Systems seem to require only polynomial resources



Low-lying states obey the **entanglement entropy area scaling law**

$$S_{AB} \sim \partial A$$

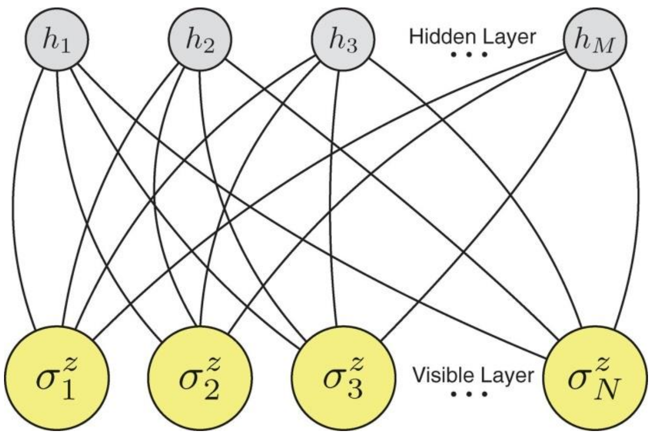
Allows for efficient representation with Tensor Network States



R. Orus. Annals Phys. **349** 117 (2014)

Efficient Representation of States

Variational Expression of Wavefunction



$$\Psi_M(\mathcal{S};\mathcal{W}) = \sum_{\{h_i\}} e^{\sum_j a_j \sigma_j^z + \sum_i b_i h_i + \sum_{ij} W_{ij} h_i \sigma_j^z}$$

G. Carleo & M. Troyer.
Science **355** 602 (2017)

RBM ↔ TNS Duality

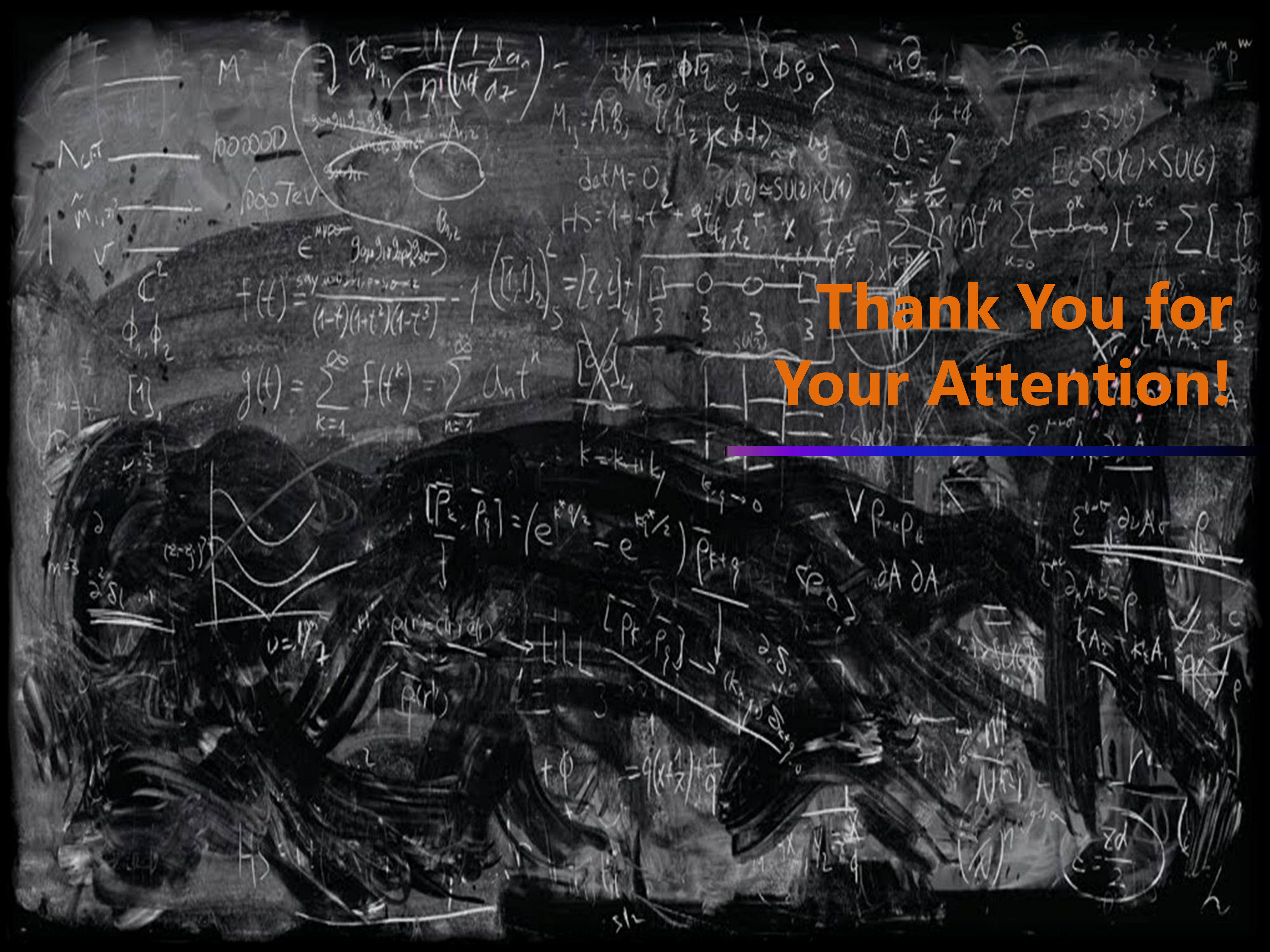
Established direct mapping of RBM to TNS

Derived conditions under which TNSs can be represented as a RBM

Idea: establish **cross-fertilization** between concepts in both domains

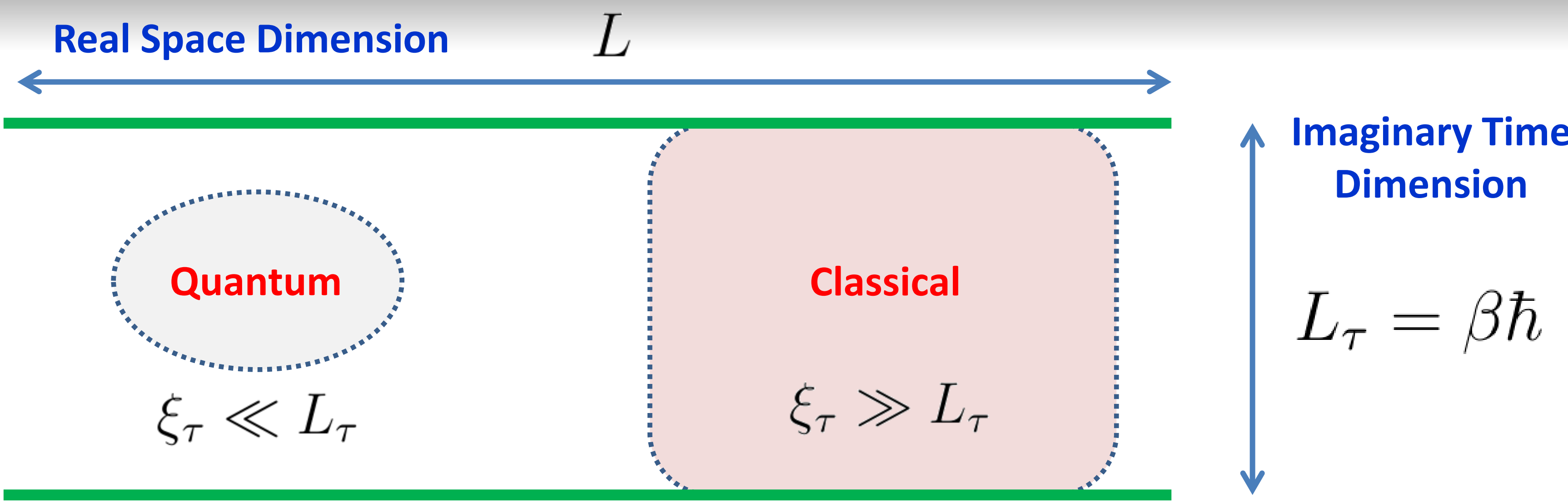
- Quantify expressiveness of RBM via entanglement entropy bounds
- Quantify complexity of quantum state via information measures of the RBM
- RBMs may offer a way to parameterise the state with fewer parameters

J. Chen et al.
PRB **97** 085104 (2018)



Thank You for Your Attention!

Quantum Phase Transitions



Coherent state path integral formalism leads to a **low energy effective action**

$$S[\varphi] = \int_0^{\beta\hbar} \mathrm{d}\tau \int (\mathrm{d}r)\, \mathcal{L}\left(\varphi, \nabla\varphi, \partial_\tau\varphi\right)$$

in the **vicinity of the critical point**

Zero temperature quantum phase transitions can be mapped to **(d+1) dimensional** classical phase transitions

Classical techniques can be employed to understand and study quantum systems

Effect of **finite temperature** corresponds to imposing a **finite system size** in one dimension