Variations in Statistical Complexity of Harmonic Oscillator

Alaina Gibbons

PHYS 256B, UC Davis

June 5, 2018

Langevin Dynamics

Langevin dynamics (LD) is a class of molecular dynamics in which two forces are added to the conservative vector field: frictional force proportional to velocity; and stochastic thermal white noise. The system of ODEs then becomes

$$\dot{r} = v$$

$$m\dot{v} = f(r,t) - \alpha v + \beta(t)$$

where the stochastic force is assumed to have statistical properties of

$$\langle \beta(t) \rangle = 0$$

$$\langle \beta(t)\beta(t') \rangle = 2\alpha k_B T \delta(t - t')$$

The relative size of the temperature and coefficient of friction are therefore what determine the statistical complexity of the system.

Basic Simulation: Simple Harmonic Oscillator

- Particle under simple harmonic oscillation
- Left and right of equilibrium correspond to 0 and 1 edges respectively
- Recorded whether the particle was in a "0" or "1" position as time progressed
- Word probabilities calculated for the different lengths along with transition probabilities
- All particles will have same period oscillation
- Temperature and coefficient of friction were varied

Transition Probabilities for $T = 0$, $\alpha = 0$

- Symmetric
- Period-3 deterministic sets
- Probabilities steadily decreasing - cannot tell limit due to not being able to analyze long enough strings yet

Transition Probabilities for $0.0 < T < 1.0$, $\alpha = 0$

- Structure is still symmetric
- Change in temperature seems to have little effect
- Probabilities might be converging to a limit more quickly

Transition Probabilities for $T = 0$, $\alpha = 1.0$

- Symmetric
- Period-6 deterministic sets, with guaranteed alternation
- Some may have longer period
- Faster convergence, possibly - could be converging to 50/50.

Transition Probabilities for $T = 1 \times 10^{-12}$, $\alpha = 1.0$

- Symmetric for first few letters of words, then becomes more and more random.
- The temperature appears to be playing a far greater role in creating statistical complexity for a larger value of friction, even though the temperature has an incredibly small relative size.