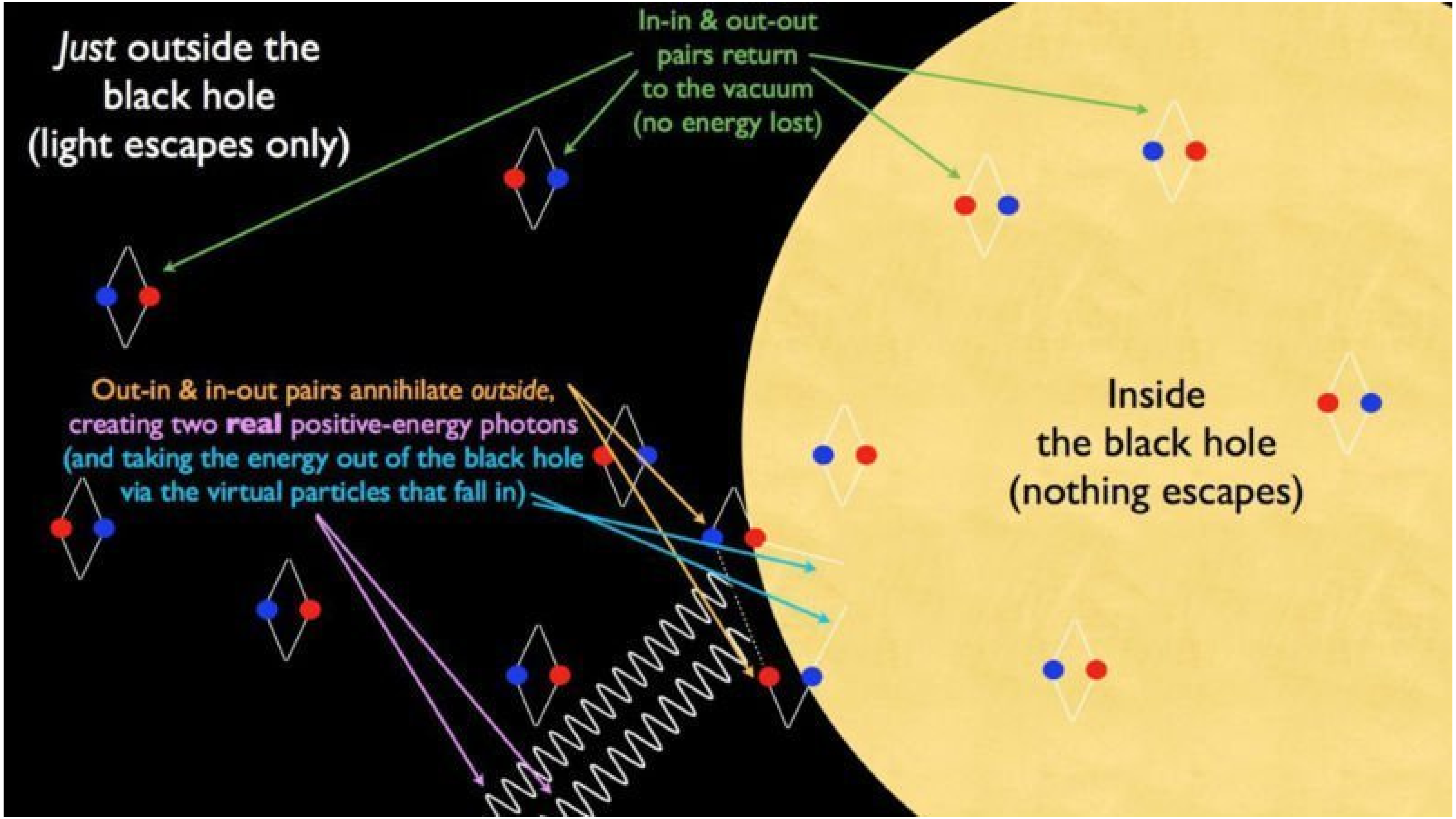
Quantum Gravity Using (Hidden) Markov Models

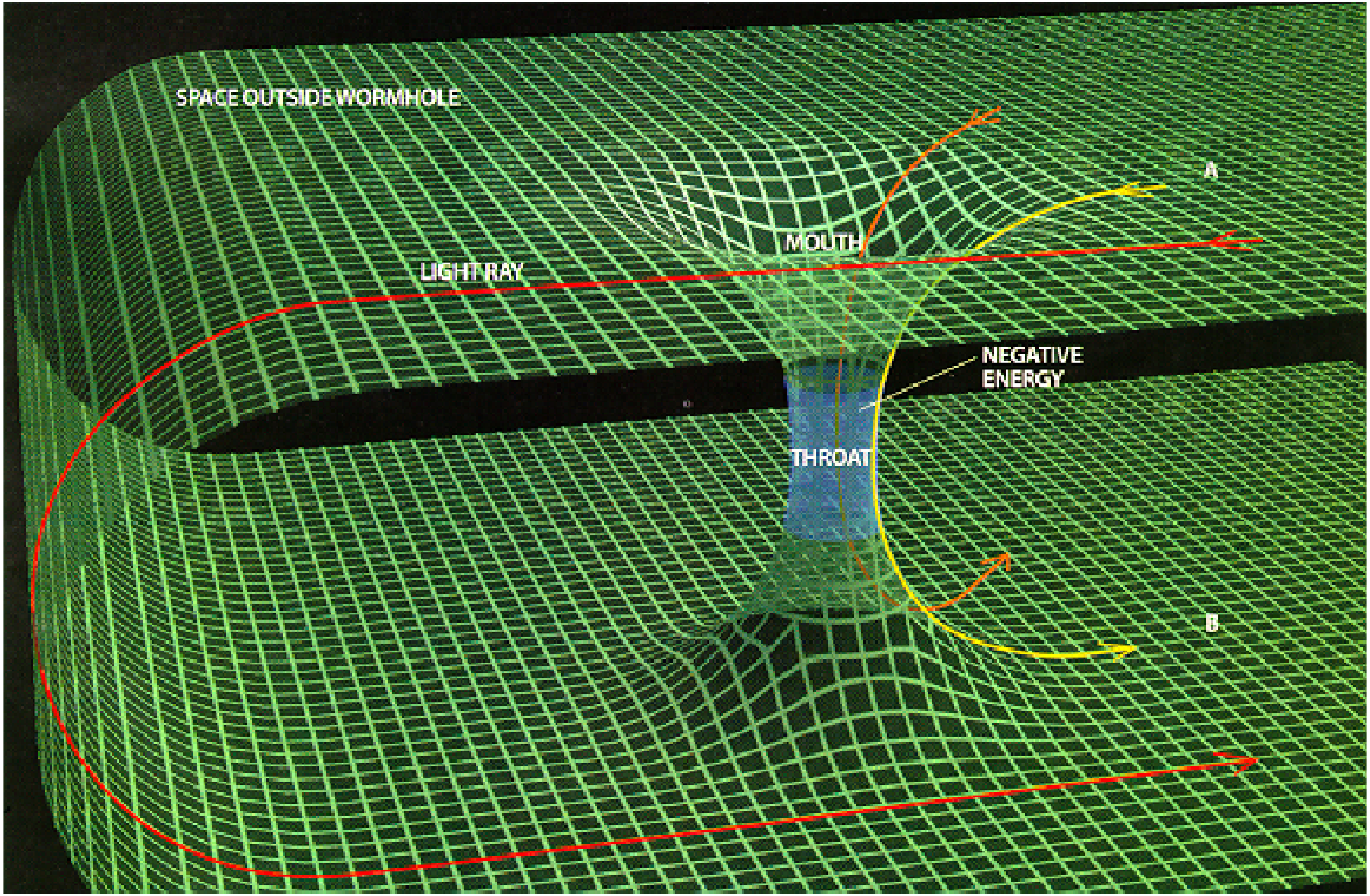
Adam Getchell
University of California, Davis
Department of Physics

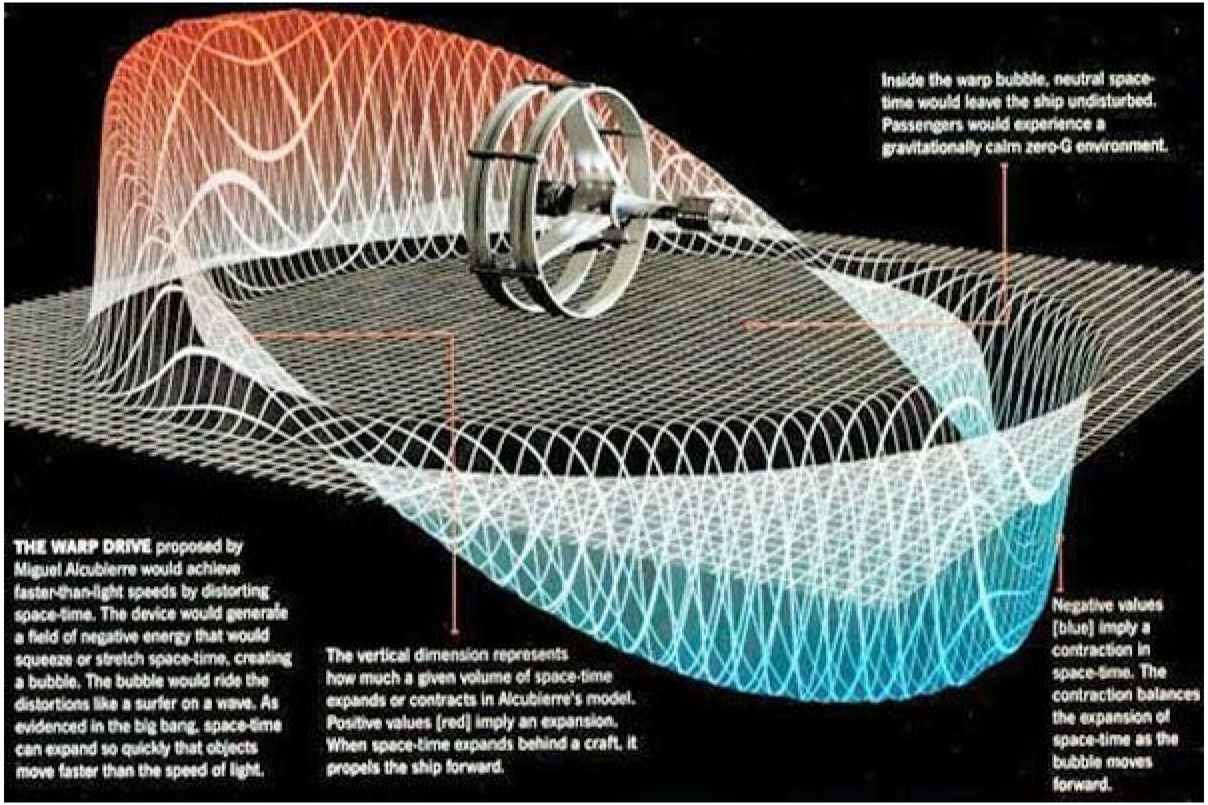
Why?

"Nevertheless, due to the interatomic movements of electrons, atoms would have to radiate not only electromagnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."

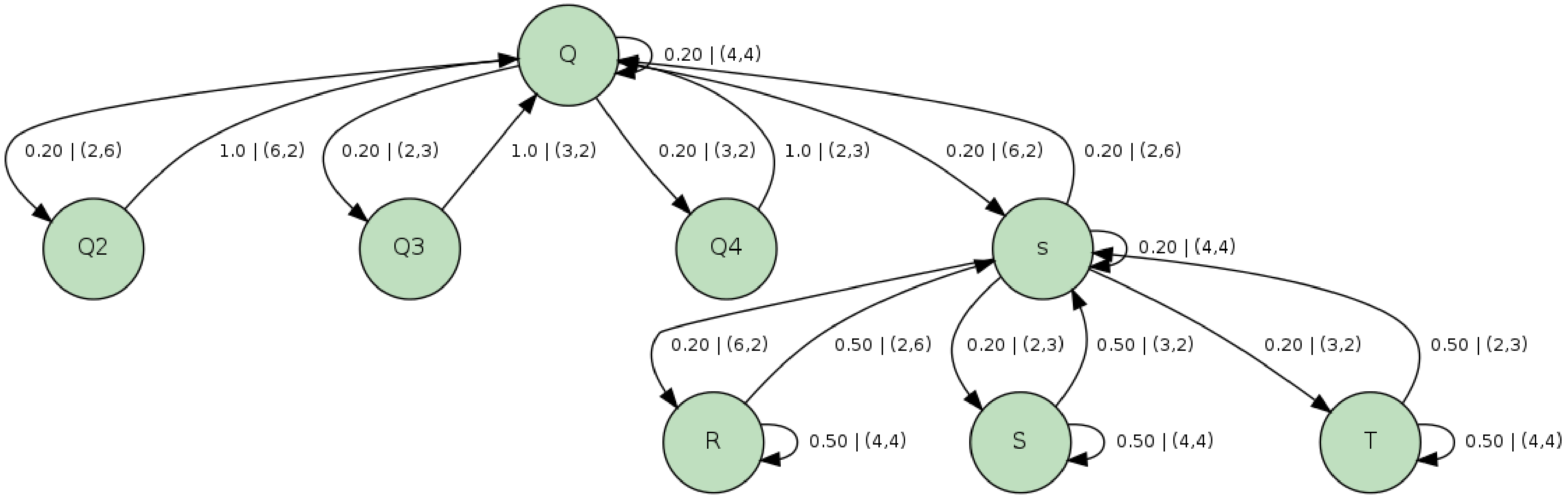
— A. Einstein, *Approximative Integrations of the Field Equations of Gravitation*, 1916


Just outside the black hole (light escapes only)
In-in & out-out pairs return to the vacuum (no energy lost)
Inside the black hole (nothing escapes)
Out-in & in-out pairs annihilate *outside*,
creating two **real** positive-energy photons
(and taking the energy out of the black hole
via the virtual particles that fall in)

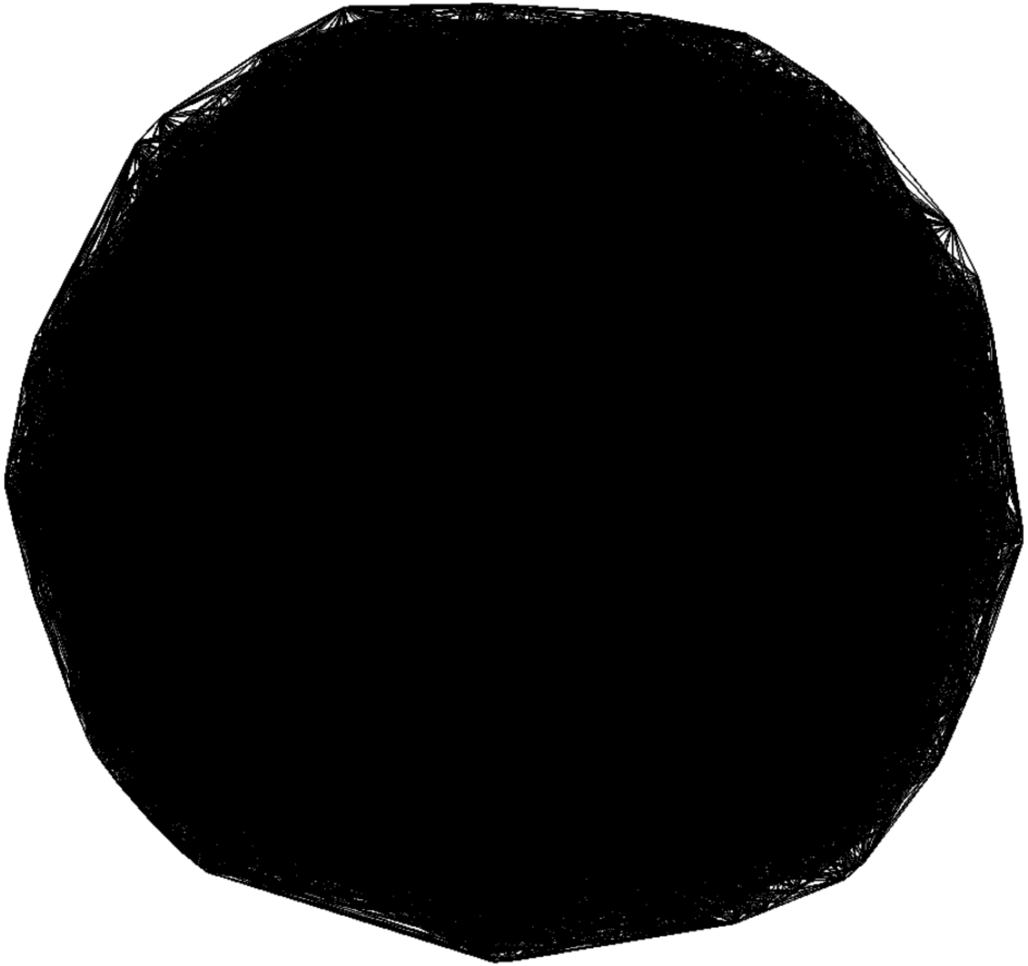

SPACE OUTSIDE WORMHOLE
A
MOUTH
LIGHT RAY
NEGATIVE
ENERGY
THROAT
B


Inside the warp bubble, neutral space-time would leave the ship undisturbed. Passengers would experience a gravitationally calm zero-G environment.
THE WARP DRIVE proposed by Miguel Alcubierre would achieve faster-than-light speeds by distorting space-time. The device would generate a field of negative energy that would squeeze or stretch space-time, creating a bubble. The bubble would ride the distortions like a surfer on a wave. As evidenced in the big bang, space-time can expand so quickly that objects move faster than the speed of light.
The vertical dimension represents how much a given volume of space-time expands or contracts in Alcubierre's model. Positive values [red] imply an expansion. When space-time expands behind a craft, it propels the ship forward.
Negative values [blue] imply a contraction in space-time. The contraction balances the expansion of space-time as the bubble moves forward.

Synopsis


Q
0.20 | (4,4)
0.20 | (2,6)
1.0 | (6,2)
0.20 | (2,3)
1.0 | (3,2)
0.20 | (3,2)
1.0 | (2,3)
0.20 | (6,2)
0.20 | (2,6)
Q2
Q3
Q4
s
0.20 | (4,4)
0.20 | (6,2)
0.50 | (2,6)
0.20 | (2,3)
0.50 | (3,2)
0.20 | (3,2)
0.50 | (2,3)
R
0.50 | (4,4)
S
0.50 | (4,4)
T
0.50 | (4,4)

Set of States

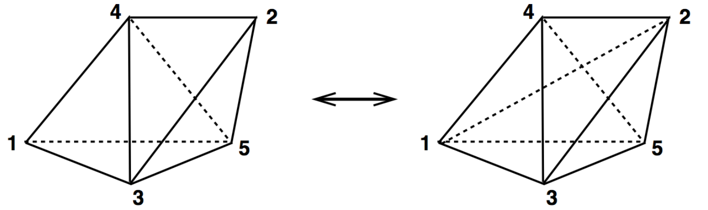


256 timeslices, 222,132 vertices, 2,873,253 faces, 1,436,257 simplices

Output Alphabet

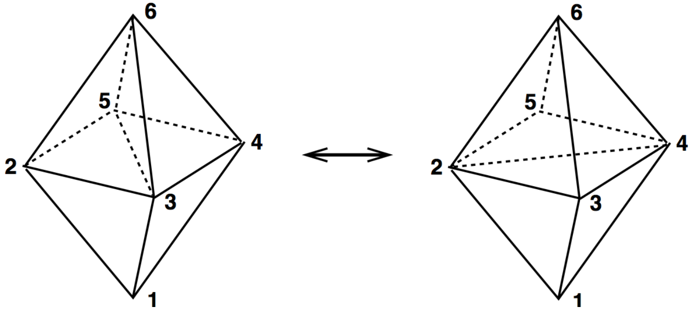
Simplices involved | Move name

(3,1) & (2,2)



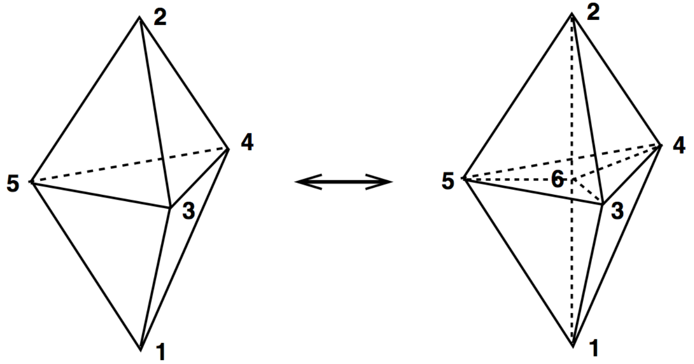
(2,3) & (3,2)

2 (1,3) & 2 (3,1)



(4,4)

(1,3) & (3,1)



(2,6) & (6,2)

Transition Probabilities

1. Pick an ergodic (Pachner) move
2. Make that move with a probability of a=a1a2, where:

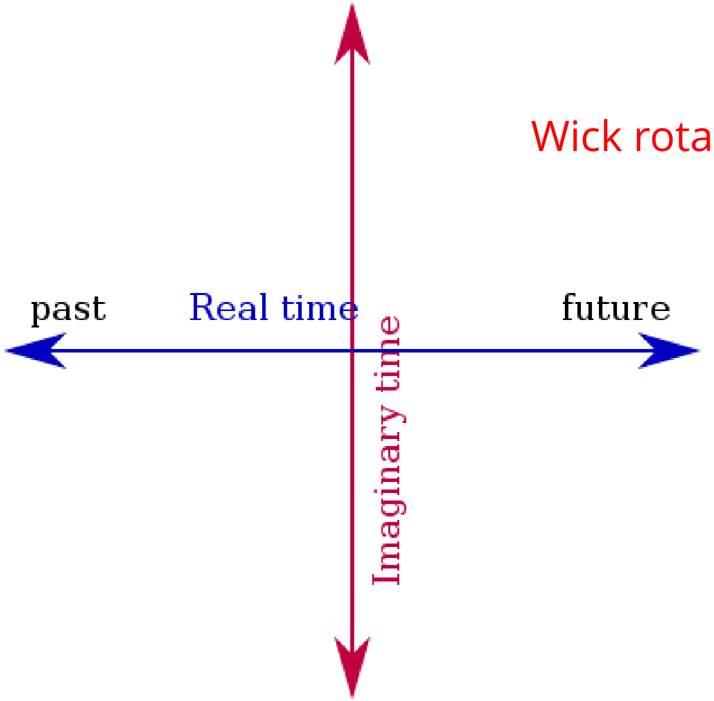
$$a_1 = \frac{move[i]}{\sum_i move[i]}$$

$$a_2 = e^{\Delta I}$$

$$I_R = \frac{1}{8\pi G_N}\left(\sum_{hinges} A_h \delta_h - \Lambda \sum_{simplices} V_s\right)$$

Output Probabilities

$$\langle B|T|A\rangle = \sum_{triangulations} \frac{1}{C(T)} e^{-I_R(T)}$$

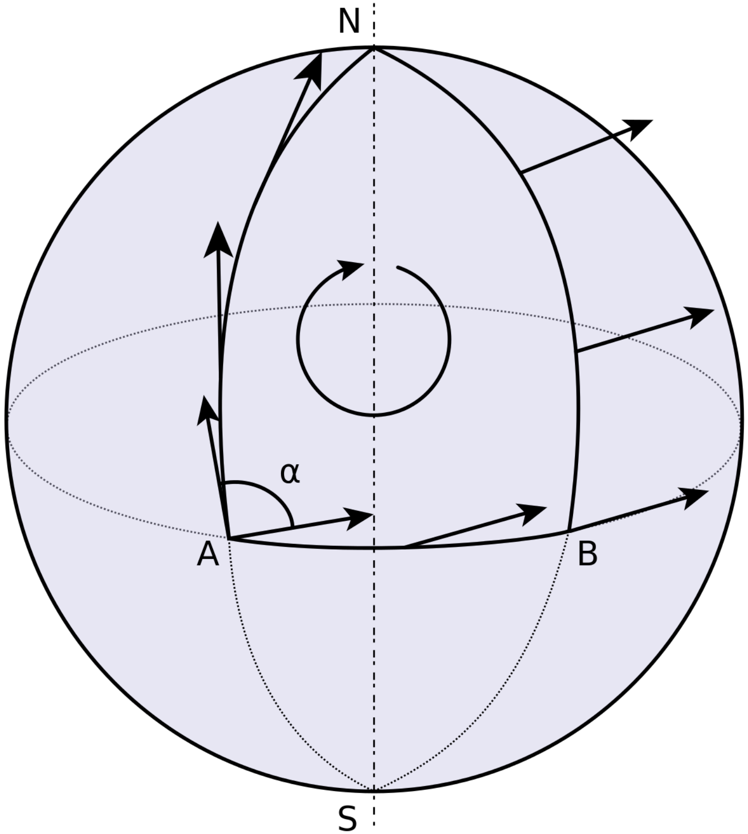


Wick rotation

Background

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Parallel Transport



$$ds^2 = e^{2\lambda}dt^2 - e^{2(\nu-\lambda)}\left(dr^2 + dz^2\right) - r^2e^{-2\lambda}d\phi^2$$

$$g_{\mu\nu} = \begin{pmatrix} e^{2\lambda} & 0 & 0 & 0 \\ 0 & -e^{2(\nu-\lambda)} & 0 & 0 \\ 0 & 0 & -e^{2(\nu-\lambda)} & 0 \\ 0 & 0 & 0 & -\frac{r^2}{e^{2\lambda}} \end{pmatrix}$$

Metric

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)$$

Affine connection

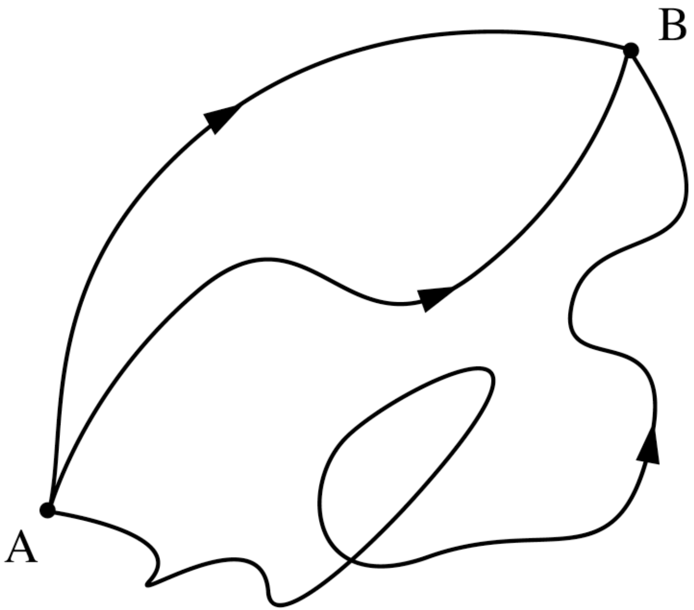
$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu\Gamma^{\rho}_{\nu\sigma} - \partial_\nu\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Riemann tensor

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu} \qquad R = R^{\mu}_{\mu}$$

Ricci tensor & Ricci scalar

Path Integral



Transition probability amplitude

$$\langle B|T|A\rangle = \int \mathcal{D}[g] e^{iI_{EH}}$$

$$I_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}(R - 2\Lambda)$$

Ricci scalar

Cosmological constant

Equations of Motion

$$\partial S = 0 \rightarrow R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Ricci tensor

Ricci scalar

Stress-Energy tensor

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General Relativity without Coordinates.

T. Regge

Palmer Physical Laboratory, Princeton University - Princeton, N. J. (*)

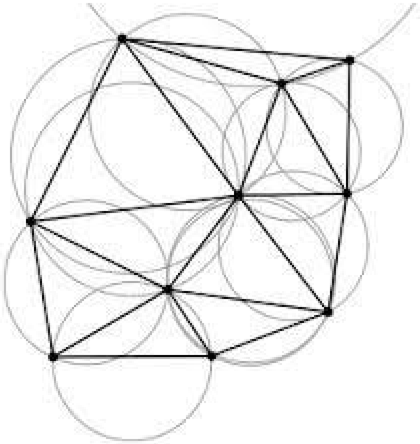
(ricevuto il 17 Ottobre 1960)

Summary. — In this paper we develop an approach to the theory of Riemannian manifolds which avoids the use of co-ordinates. Curved spaces are approximated by higher-dimensional analogs of polyhedra. Among the advantages of this procedure we may list the possibility of condensing into a simplified model the essential features of topologies like Wheeler's wormhole and a deeper geometrical insight.

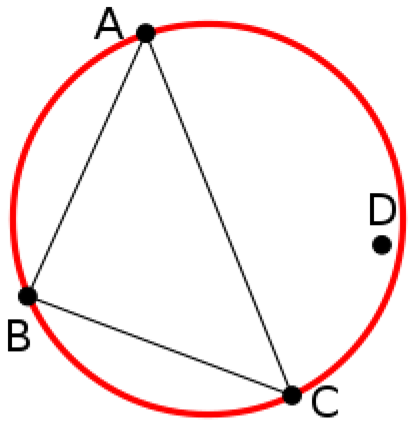
1. – Polyhedra.

In this section we shall first describe our approach for the simple case of 2-dimensional manifold (surfaces). Following Aleksandrov (1) we develop the theory of intrinsic curvature on polyhedra. A general surface is then considered as the limit of a suitable sequence of polyhedra with an increasing number of faces. A rigorous definition of limit is not given here since it would involve a treatment of the topology on the set of all polyhedra and this would carry us too far. It is to be expected however that any surface can be arbitrarily approximated, as closely as wanted, by a suitable polyhedron. The approximation will be bad if we look at the details to the picture but an ob-

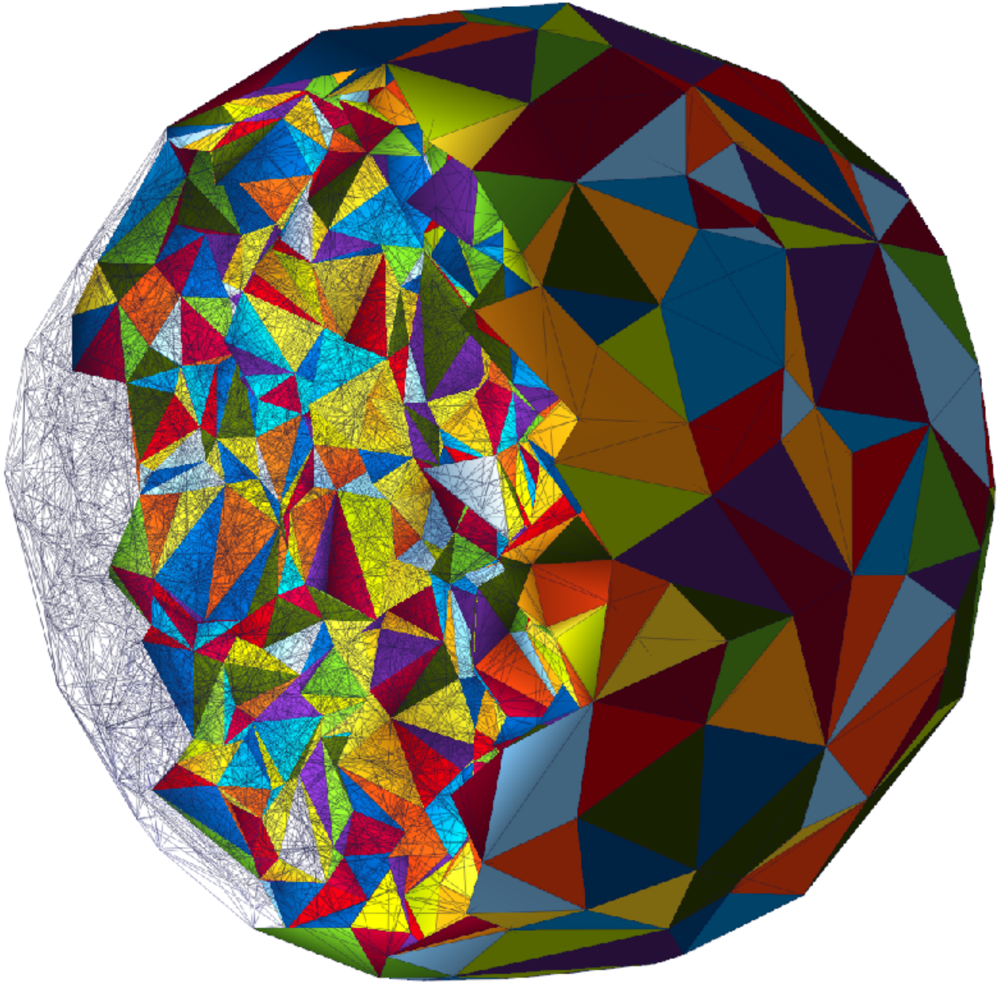
Simplicial Manifolds



Delaunay Triangulation



Not a Delaunay Triangulation



DT Path Integral

Transition probability amplitude

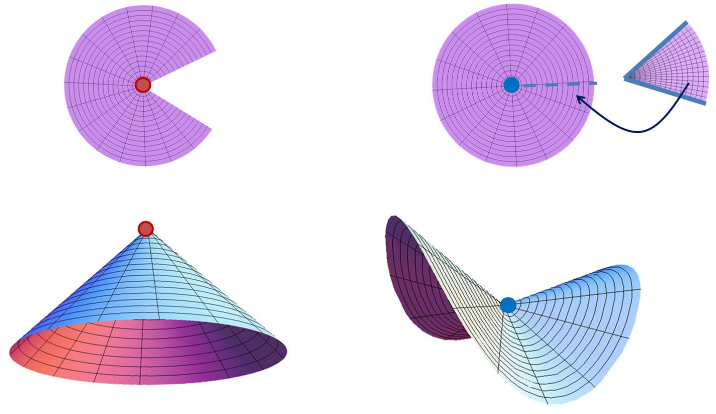
Partition Function

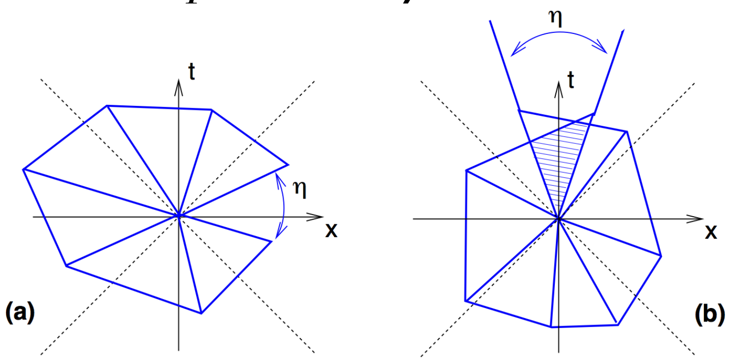
$$\langle B|T|A\rangle = \sum_{triangulations} \frac{1}{C(T)} e^{iI_R(T)}$$

Inequivalent Triangulations

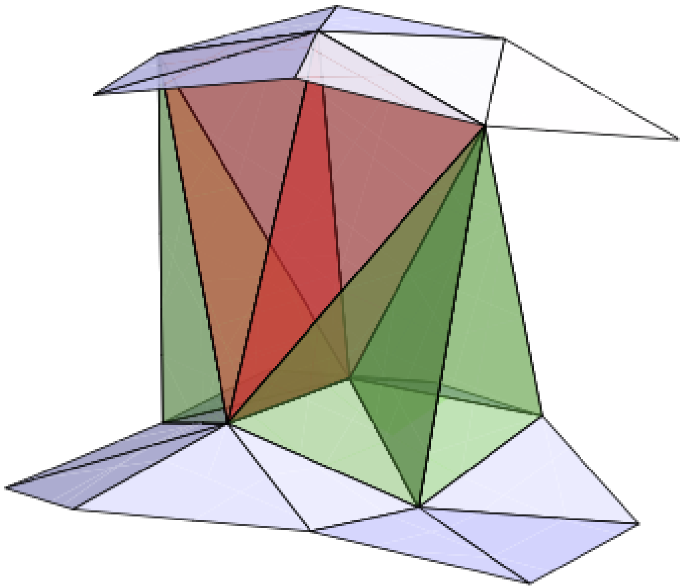
Regge Action

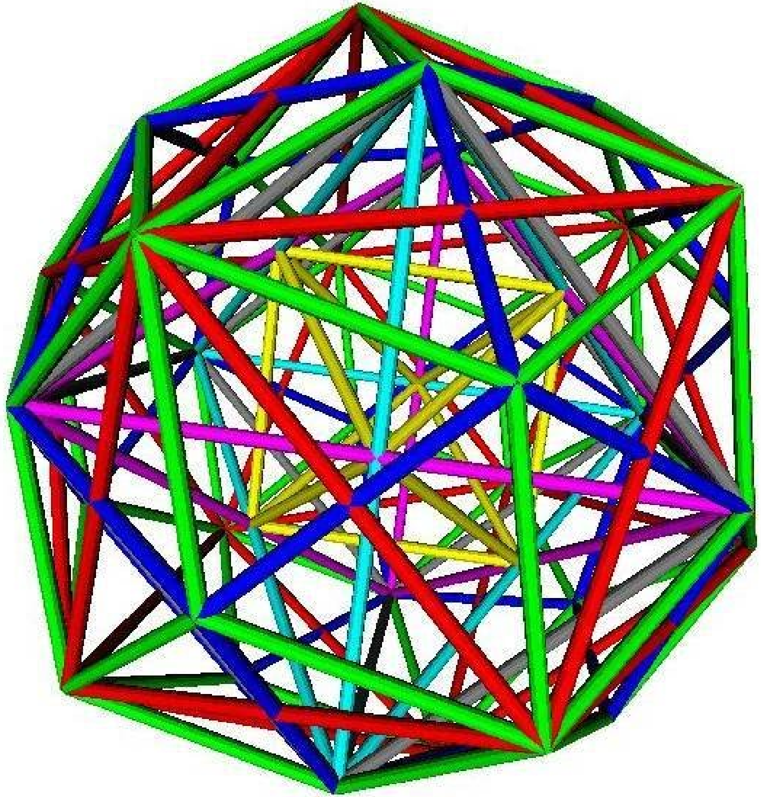
$$I_R = \frac{1}{8\pi G_N}\left(\sum_{hinges} A_h \delta_h - \Lambda \sum_{simplices} V_s\right)$$





Foliation



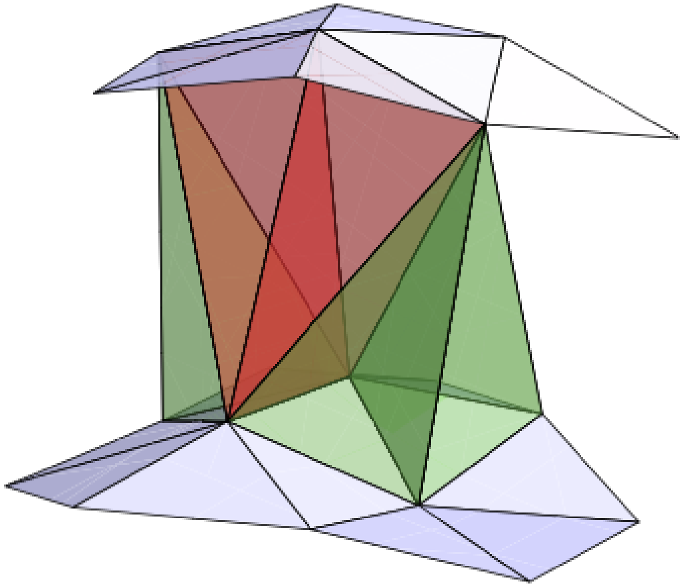


Mass = Epp quasilocal energy

$$E_E \equiv \frac{1}{8\pi G_N} \int_\Omega d^2x \sqrt{|\sigma|} \left(\sqrt{k^2 - l^2} - \sqrt{\bar{k}^2 - \bar{l}^{\,2}} \right)$$

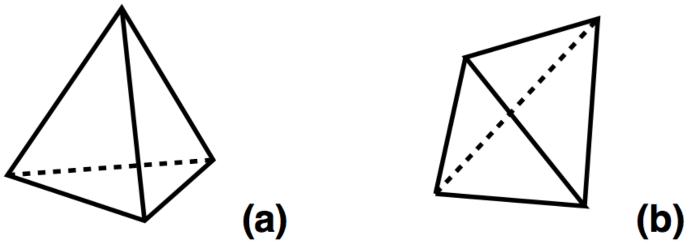
$$l \equiv \sigma^{\mu\nu} l_{\mu\nu} \qquad k \equiv \sigma^{\mu\nu} k_{\mu\nu}$$

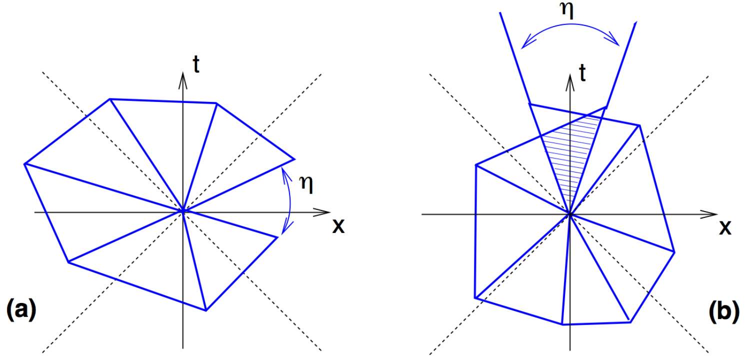
- In 1+1 simplicial geometry, extrinsic curvature at a vertex is proportional to the number of connected triangles
- In 2+1 simplicial geometry, extrinsic curvature at an edge is proportional to the number of connected tetrahedra
- In 3+1 simplicial geometry, extrinsic curvature at a face is proportional to the number of connected pentachorons (4-simplices)



CDT Action

$$
\begin{aligned}
S^{(3)} &= 2\pi k\sqrt{\alpha}N_1^{TL} \\
&+ N_3^{(3,1)}\left[-3k\,\mathrm{arcsinh}\left(\frac{1}{\sqrt{3}\sqrt{4\alpha+1}}\right) - 3k\sqrt{\alpha}\,\mathrm{arccos}\left(\frac{2\alpha+1}{4\alpha+1}\right) - \frac{\lambda}{12}\sqrt{3\alpha+1}\right] \\
&+ N_3^{(2,2)}\left[2k\,\mathrm{arcsinh}\left(\frac{2\sqrt{2}\sqrt{2\alpha+1}}{4\alpha+1}\right) - 4k\sqrt{\alpha}\,\mathrm{arccos}\left(\frac{-1}{4\alpha+1}\right) - \frac{\lambda}{12}\sqrt{4\alpha+2}\right]
\end{aligned}
$$





Metropolis-Hastings

1. Pick an ergodic (Pachner) move
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$$I_R = \frac{1}{8\pi G_N}\left(\sum_{hinges} A_h \delta_h - \Lambda \sum_{simplices} V_s\right)$$

Transition Amplitudes

$$\langle B|T|A\rangle = \sum_{triangulations} \frac{1}{C(T)} e^{-I_R(T)}$$