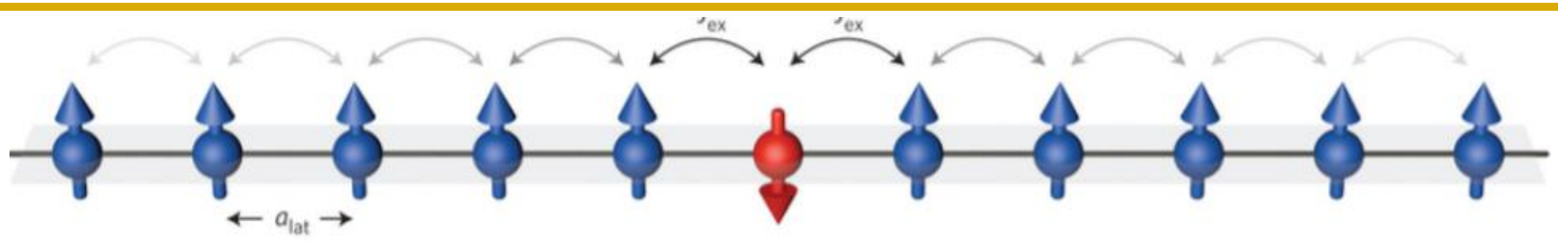
Quantum Chaos in Spin ½ Chain



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Outline

- Goals
- Background
  - Classic & Quantum Chaos
  - Spin ½ Chain & Many Body Localization
- Methods
- Results & Discussion
- Prospective

Goals

- What is quantum chaos? How could we describe it?

- How could different spin ½ chains lead to different properties, and different types of chaotic/nonchaotic systems?

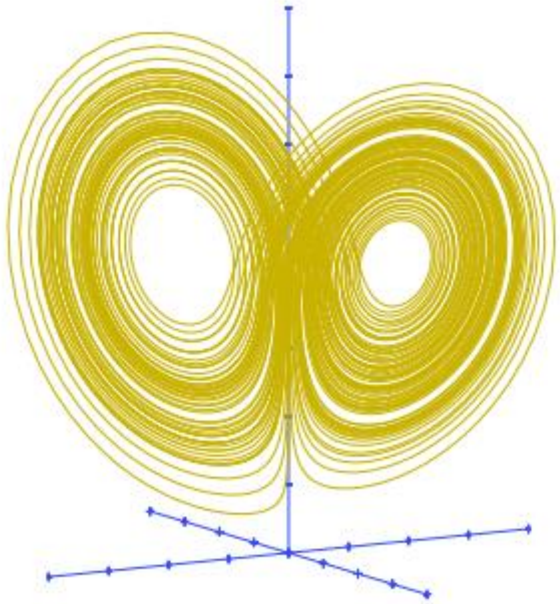
What is chaos?

"When the present determines the future, but the approximate present does not approximately determine the future."

——Edward Lorenz

Deterministic randomness:

- Fully deterministic dynamics;
- Unpredictable future;
- Exponentially sensitive to initial conditions.



Classic & Quantum Chaos

Classic	Quantum
Nonlinearity in dynamics	Schrödinger equation: linear
Continuous phase space: trajectory sensitive to initial conditions	Hilbert space; Discrete energy levels
Nonintegrable; ergodic	What is quantum integrability?

Quantum Signature of Chaos: Sensitivity

Initial condition in quantum system: a vector in Hilbert space

How to measure sensitivity to initial conditions:

- Rate of spread of an initial wave packet
- Overlap between two slightly different initial vectors

$$|\langle \psi(t) | \psi(0) \rangle|^2$$

Quantum fidelity:

- Indicator of sensitivity to small perturbations

$$\mathcal{O}(t) = |\langle \psi(t) | \psi'(t) \rangle|.$$ [1]

$H \qquad\qquad H + \delta H(t)$

Quantum Signature of Chaos: Level spacing distribution

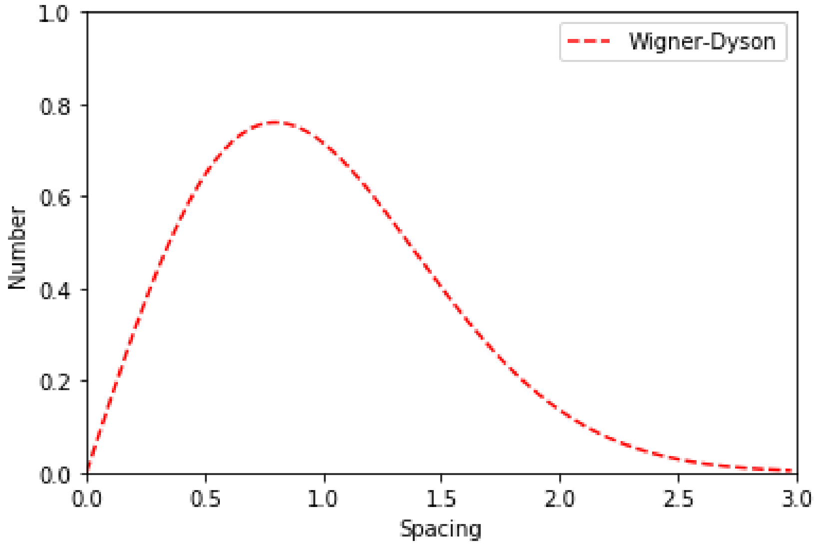
Wigner-Dyson

$$P(s) = \frac{\pi s}{2} e^{-\pi s^2/4.0}$$

Nonintegrable

Gaussian Orthogonal Ensemble

Energy levels repulsion



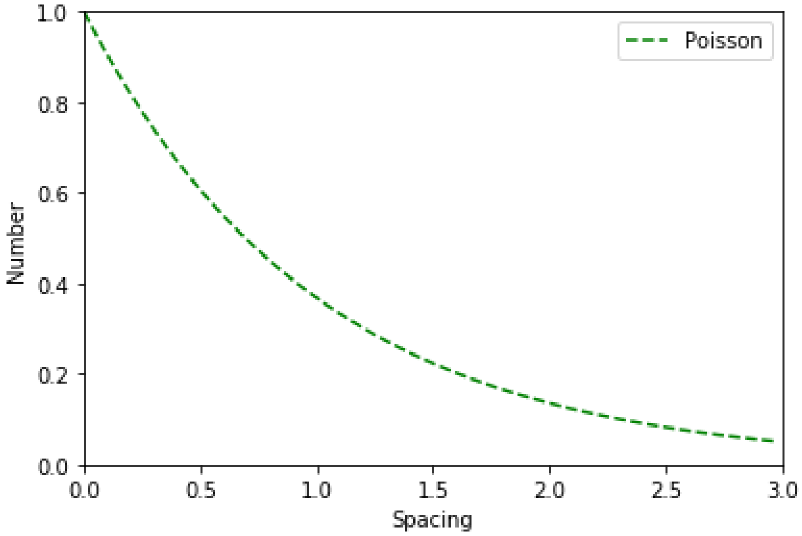
Poissonian

$$P(s) = e^{-s}.$$

Integrable

Spatially uncorrelated

Energy levels clustering



Spin ½ Chain: Hamiltonian

$$H = \sum_{i \in [1,L]} \mathbf{S}_i \cdot \mathbf{S}_{i+1} - h_i S_i^z$$

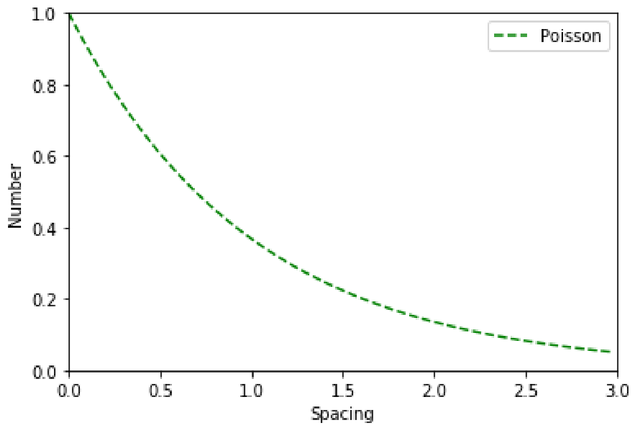
Heisenberg Interaction Model

Different Zeeman Splitting
(Defect site)

With defects, spin chain will exhibit two different phases: many-body localized phase (nonchaotic) and ergodic phase (chaotic).

Spin ½ Chain: MBL and Ergodic Phase

MBL



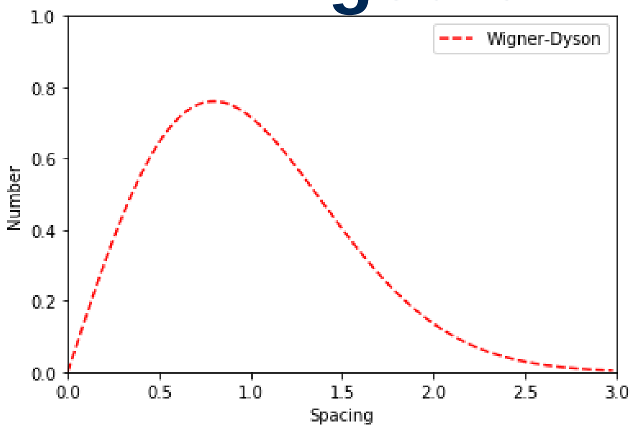
Localized: energy exchange not efficiently

Uncorrelated: energy level clustering

Integrable, not ergodic

Nonchaotic

Ergodic



Thermalized: delocalized, energy exchange efficiently

Correlated: energy level repulsion

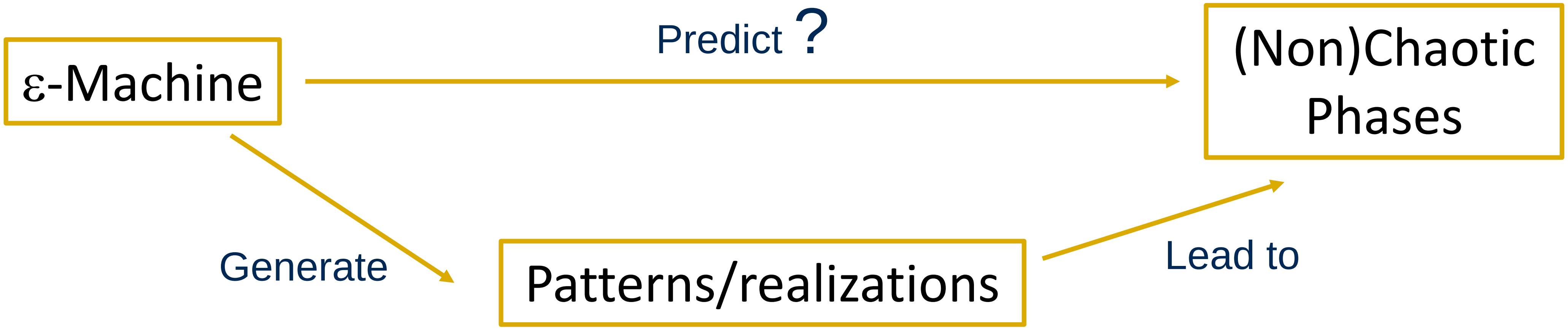
Nonintegrable, ergodic

Chaotic

Goals

- What is quantum chaos? How could we describe it?
  - Partly solved √

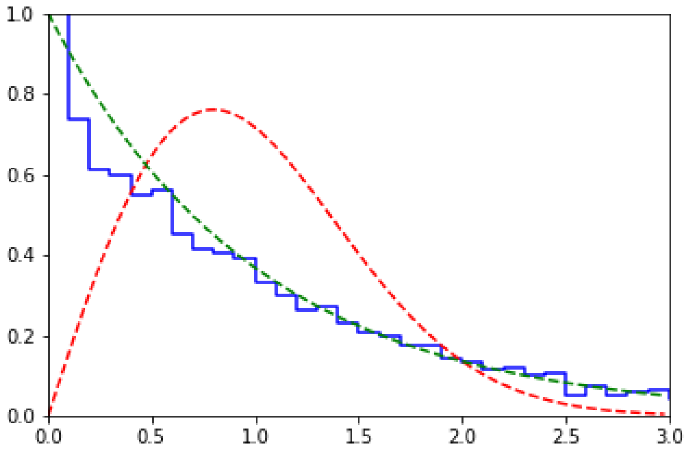
- How could different types of spin ½ chain lead to different properties, and different types of chaotic/nonchaotic systems?



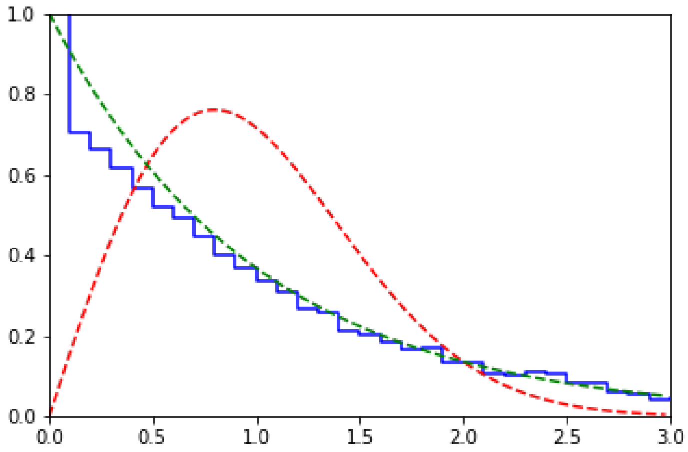
Methods

- Heisenberg model (XXX), Nearest-Neighbor approximation, Open boundary condition
- System size: L = 15 or 16 (biggest number my computer could handle)
- Up-spins : Down-spins = 1:1 (half-half) or 1:2 (one-third)
- Machines: Biased Coin (Noisy), GoldenMean, Even
- Calculate five realizations for each machine then average

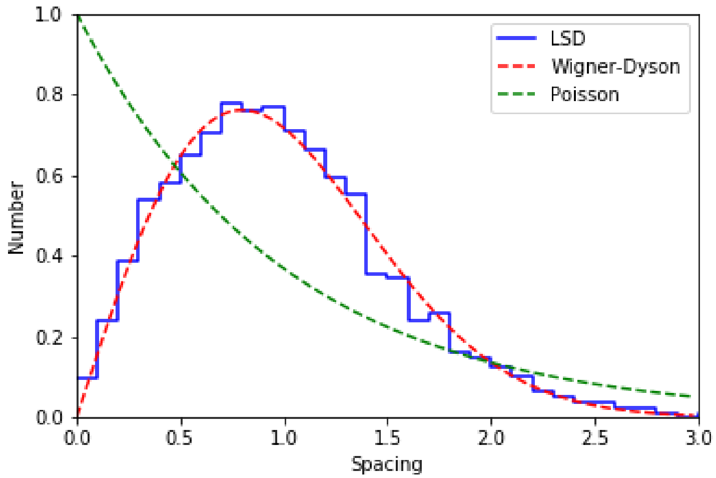
Simple realizations



8:8, no defect
Poissonian,
nonchaotic

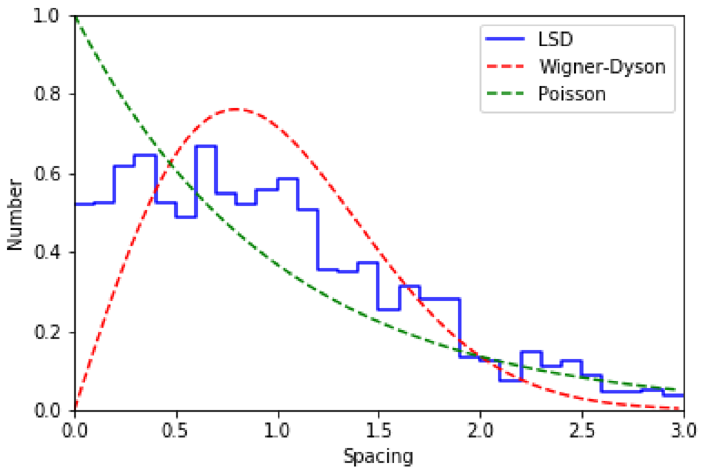


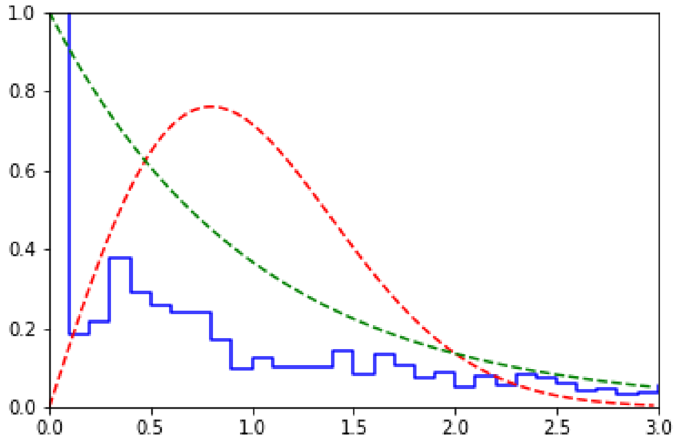
8:8, defect at
1st site
Poissonian,
nonchaotic



8:8, defect at
middle
Wigner-Dyson,
chaotic

Finite size effect



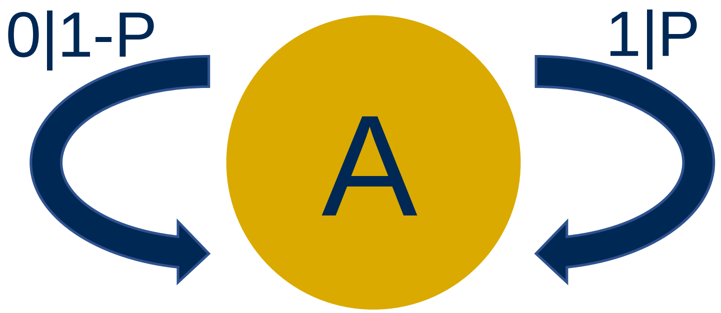


Finite size: phase transition begins gradually instead of abruptly

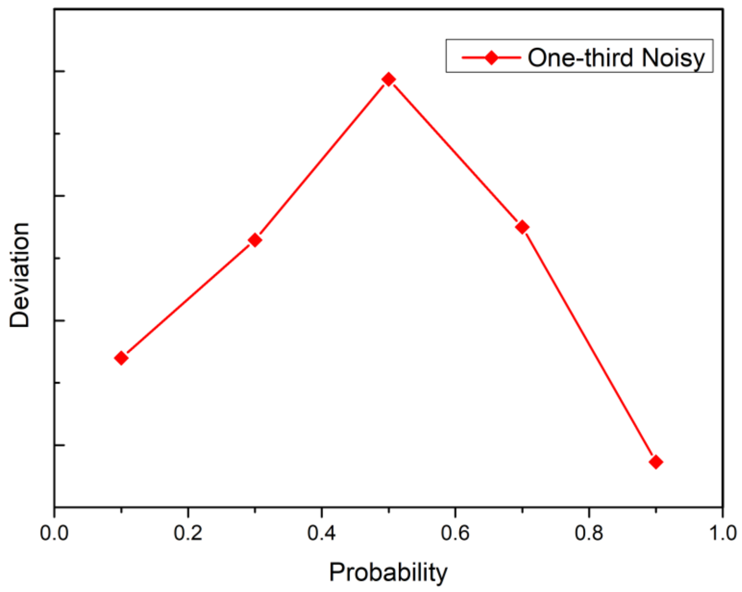
Introduce deviation: sum of distance from Poisson distribution at each sample point (bigger → chaotic)

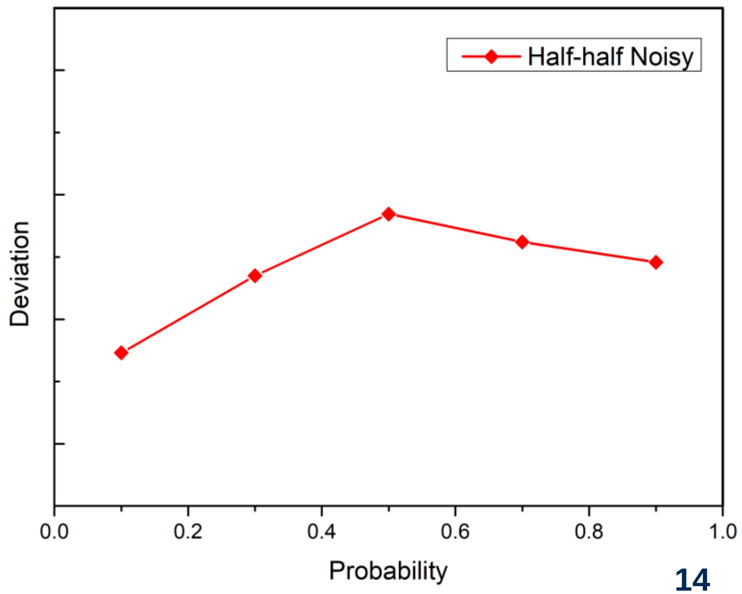
$$\sum (P(S_i) - e^{-S_i})^2 / e^{-S_i}$$

Biased Coin (Noisy)



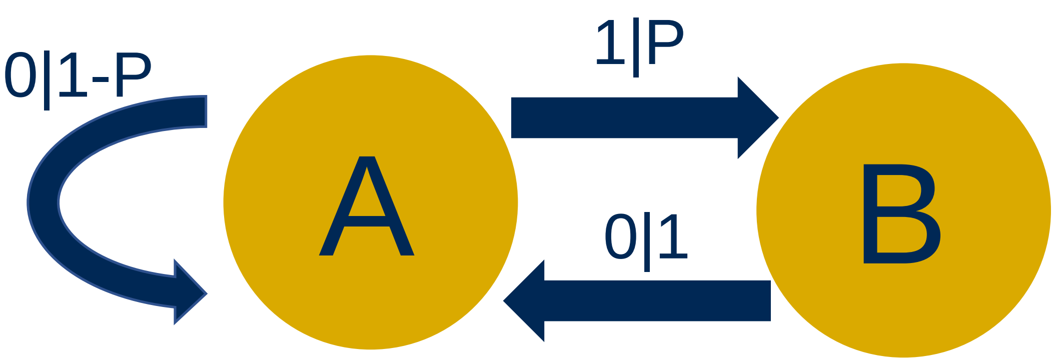
P: probability of defect at each site



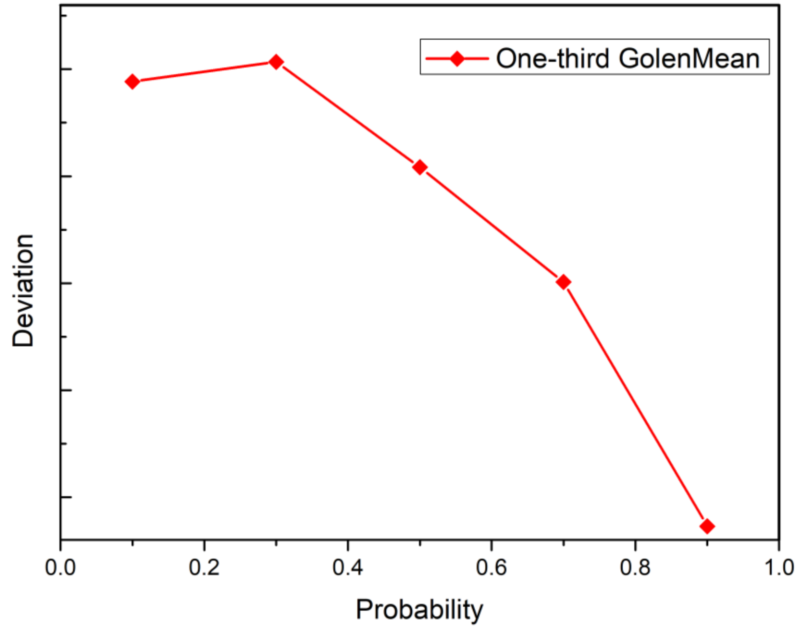


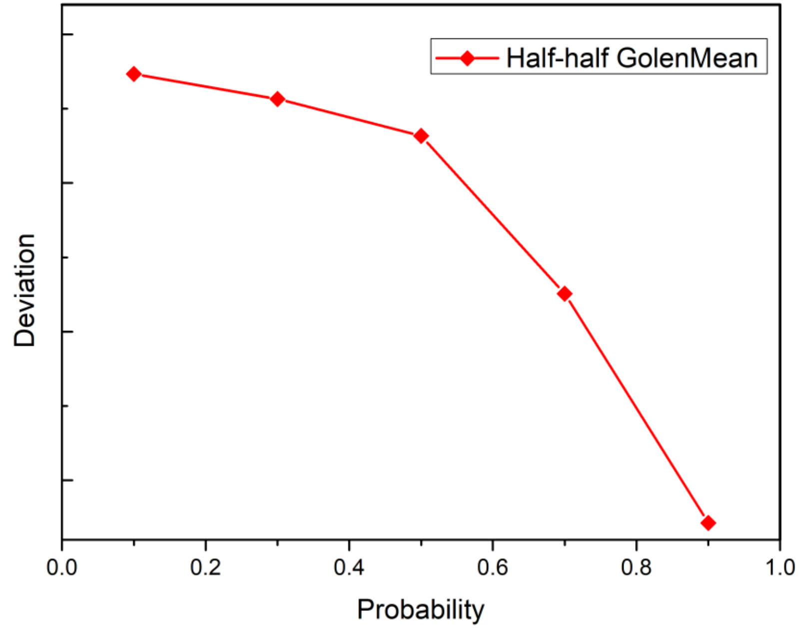
Most chaotic at P = 0.5

GoldenMean

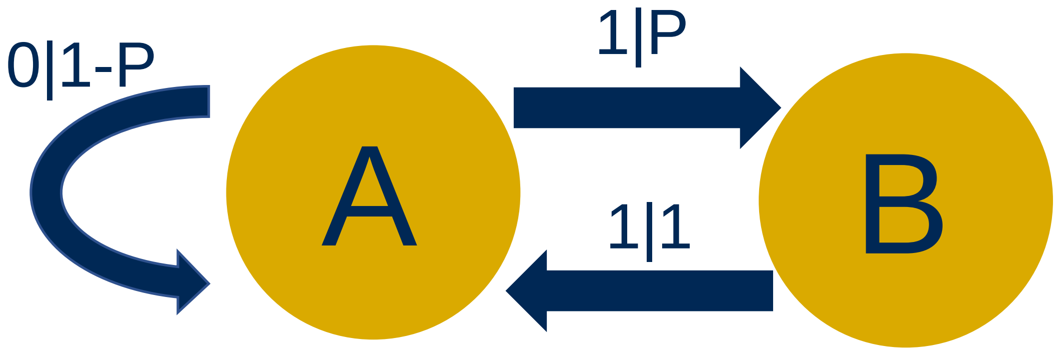


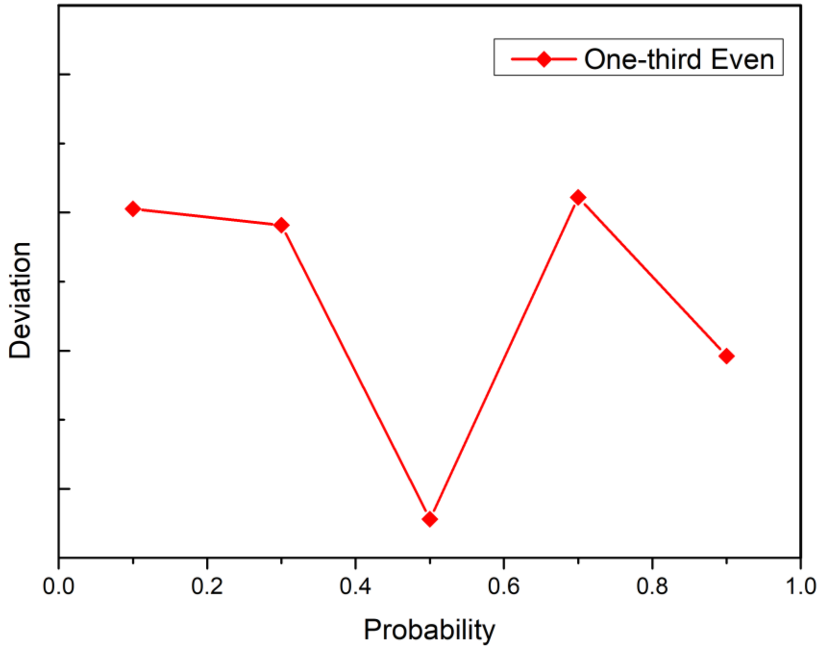
Bigger P, more occurrence of 01 pairs: symmetry!





Even



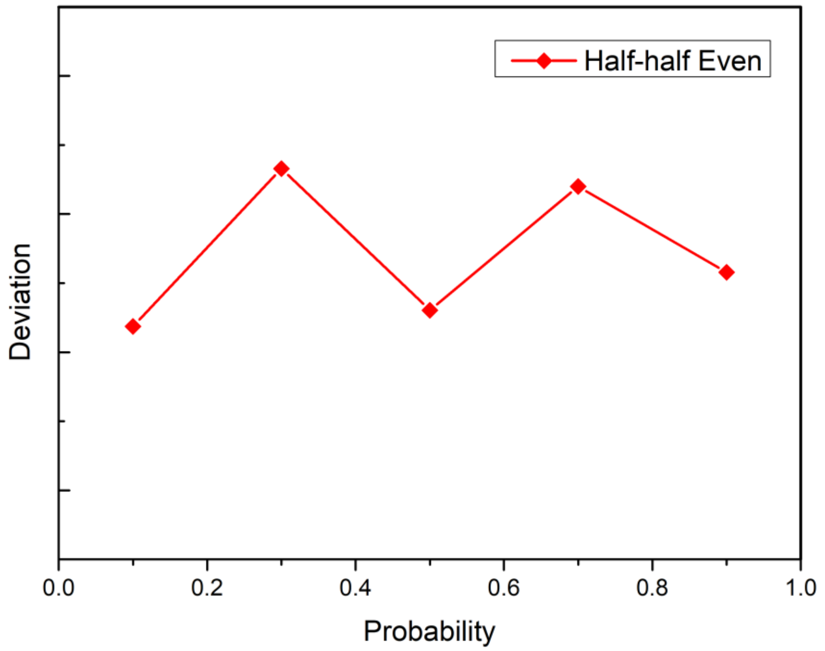


Most nonchaotic at middle:

Finite size effect?

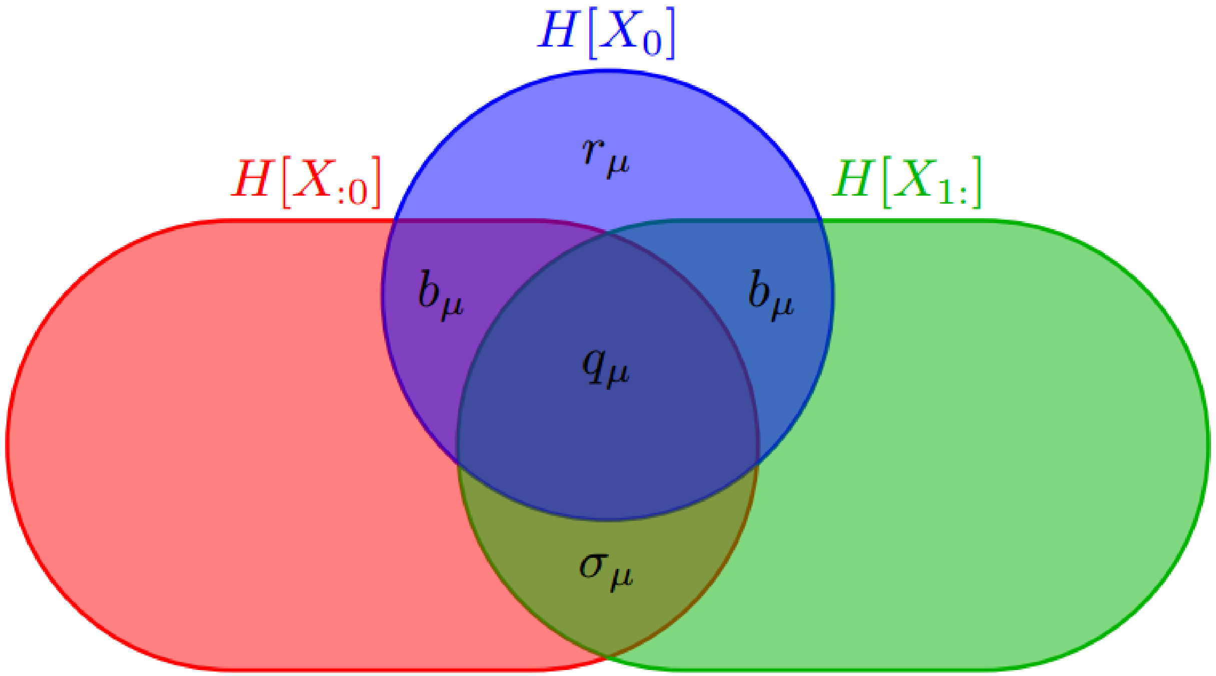
Certain occurrences of 11 could localize more?

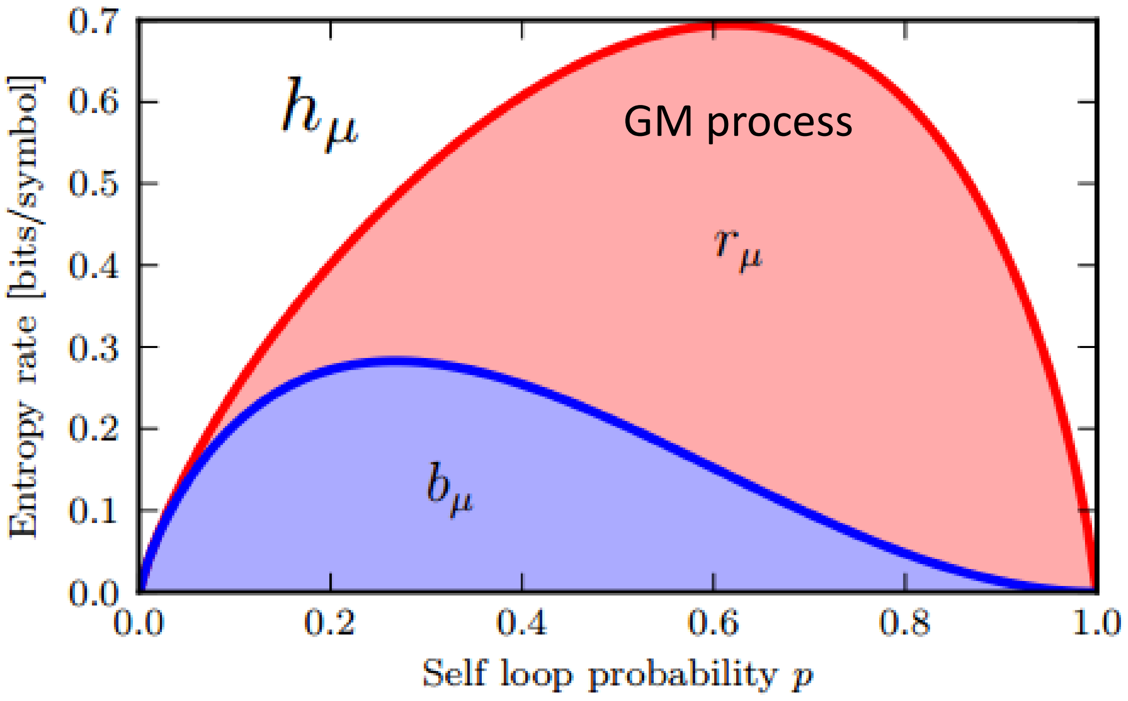
Eg.
[1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1]
[1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1]
[0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 1]



Prospective

- Different machines lead to different phases: how to describe it?
- GM and Even: different decompositions of information lead to different phase?





Thank you!

Any questions?