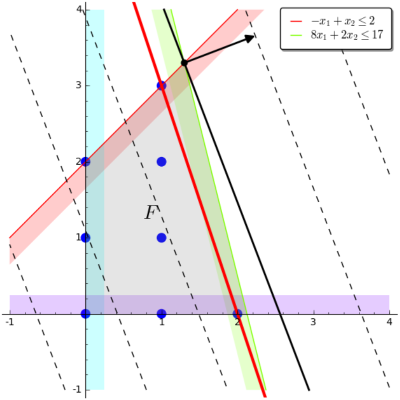


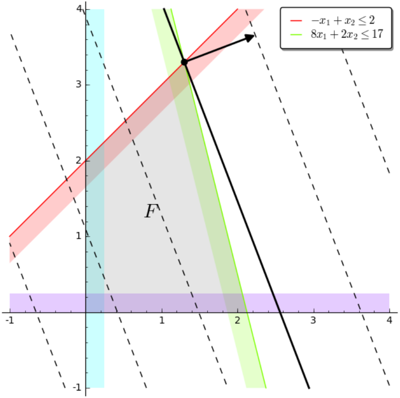
Better Guesses at Solutions to Difficult Problems: Gomory-Johnson Cut-Generating Functions

David Haley

Jun 6, 2017

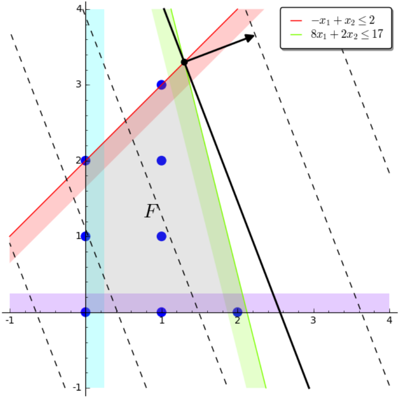
Linear Programming

$$\begin{aligned} \text{argmin}_x \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$



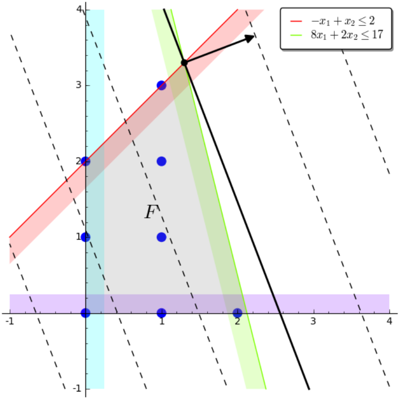
Integer Programming

$$\begin{aligned} \operatorname{argmin}_x \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \in \mathbb{Z}_+^n \end{aligned}$$



Simplex Output

$x_2 =$	$3.30 - 0.10x_4 - 0.80x_3$
$x_1 =$	$1.30 - 0.10x_4 + 0.20x_3$
$z =$	$14.08 - 0.76x_4 - 0.58x_3$



Gomory Fractional Cut

$$\begin{array}{|rl|}\hline x_2 = & 3.30 - 0.10x_4 - 0.80x_3 \\ x_1 = & 1.30 - 0.10x_4 + 0.20x_3 \\ \hline z = & 14.08 - 0.76x_4 - 0.58x_3 \\ \hline \end{array}$$

But if

$$x_2 + 0.10x_4 + 0.80x_3 = 3.30$$

and

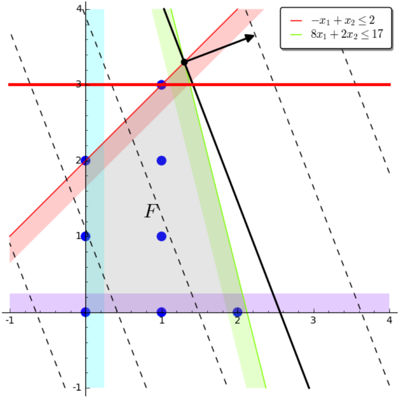
$$x_2, x_3, x_4 \in \mathbb{Z}_+$$

it must be true that

$$0.10x_4 + 0.80x_3 \geq 0.30$$

Gomory Fractional Cut

We add $0.10x_4 + 0.80x_3 \geq 0.30$ to the constraints:



Cut-Generating Functions

Take one row of the simplex tableau:

$$x_i + \sum_{j \in \mathcal{N}} r_j x_j \in f + \mathbb{Z}$$

Apply cut-generating function to get a new constraint:

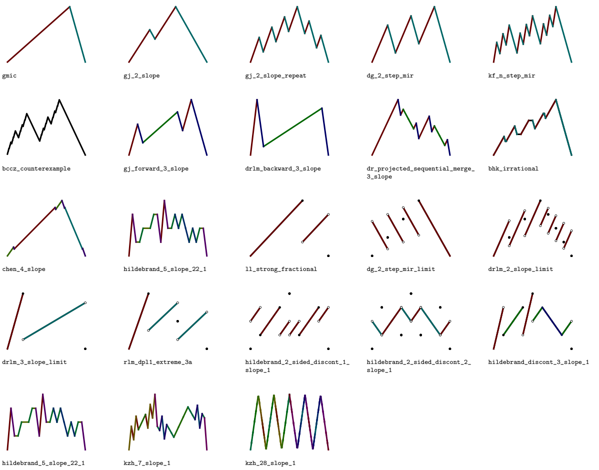
$$\sum_{j \in \mathcal{N}} \pi_f(r_j) x_j \geq 1$$

Add this constraint to the original problem and solve the modified problem.

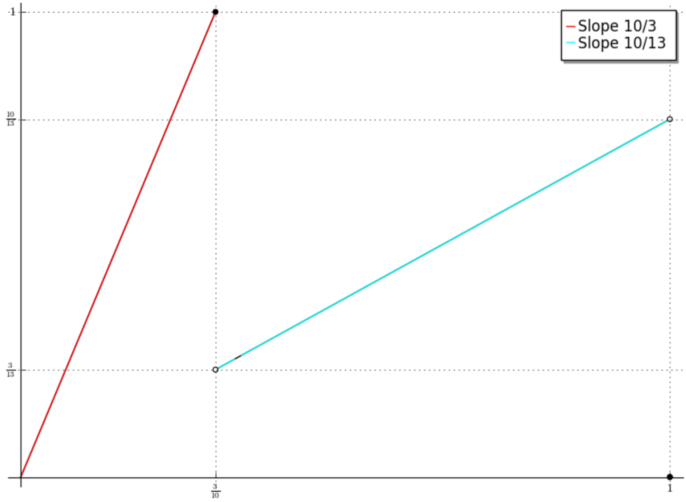
Cut-Generating Functions

3. Electronic compendium

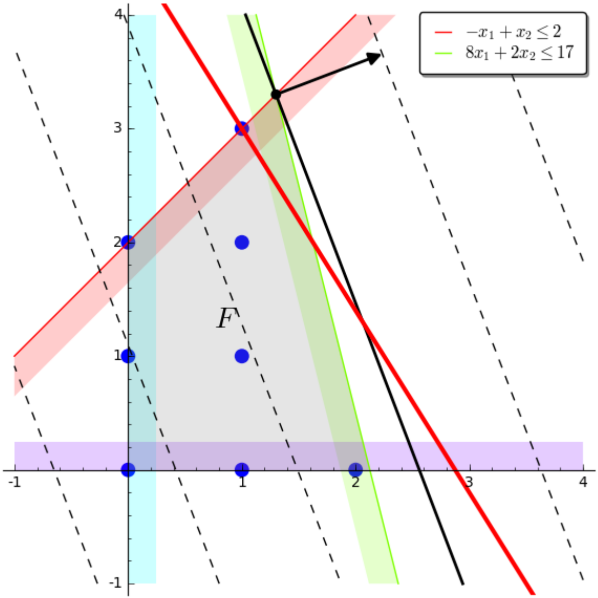
Table 1. An overview of the Electronic Compendium of extreme functions, available at `https://github.com/mkoeppe/infinite-group-relaxation-code`



Cut-Generating Functions



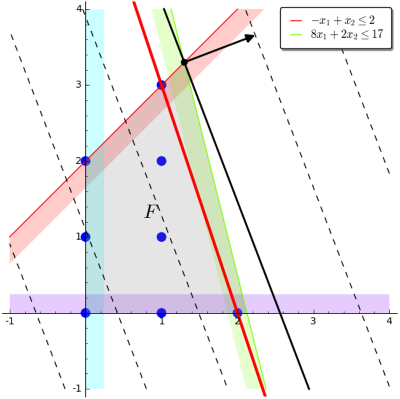
Cut-Generating Functions



Cut-Generating Functions

Which function with which tuning parameters is "best"?

The ideal cut:



Experiment

This type of cut is done in the absence of any knowledge of the other rows.

At this level, it less important to track the *identity* of the variables.

Instead, we can pay attention to the collection of coefficients.

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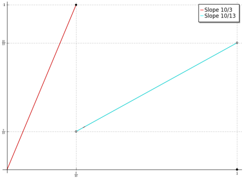
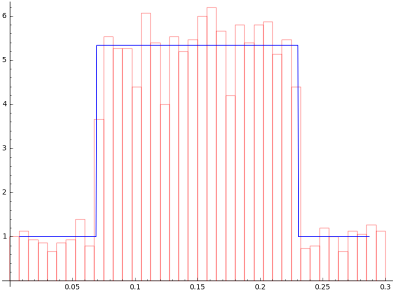
Question:

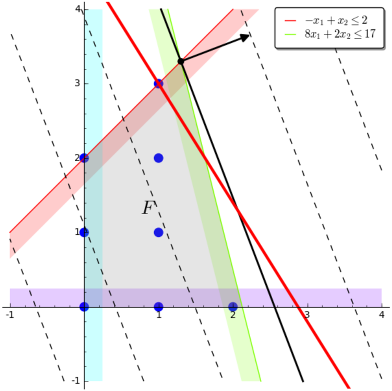
How does the distribution of (scaled) coefficients change after applying the cut-generating function?

- Is there an invariant distribution?
- How can it be interpreted?
- Can we use this interpretation to more intelligently choose the best function to apply?

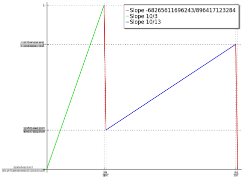
Results

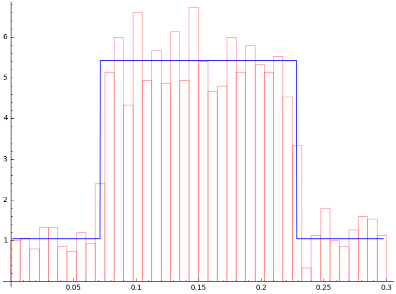

Slope 10/3
Slope 10/13

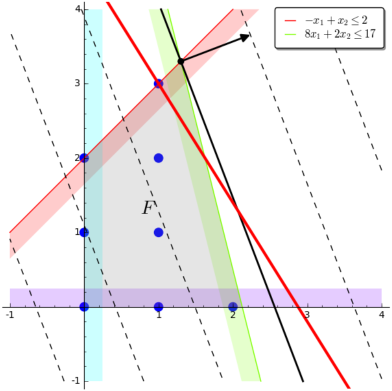

6
5
4
3
2
1
0.05
0.1
0.15
0.2
0.25
0.3


$-x_1 + x_2 \leq 2$
$8x_1 + 2x_2 \leq 17$
F
4
3
2
1
-1
-1
1
2
3
4

Results


1
Slope -68265611696243/896417123284
Slope 10/3
Slope 10/13


0.05
0.1
0.15
0.2
0.25
0.3
1
2
3
4
5
6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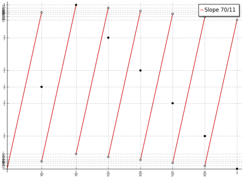
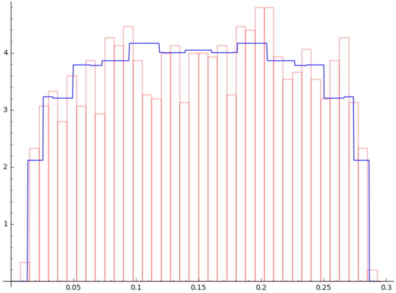
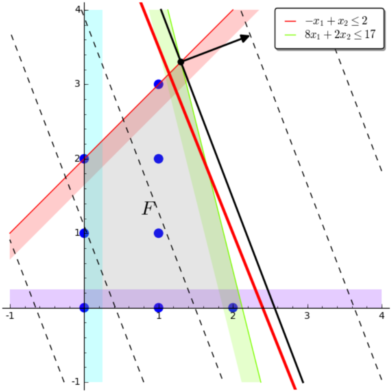
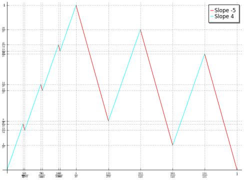


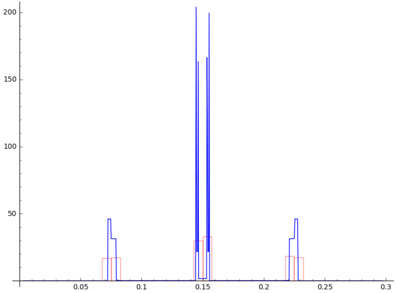




Results



Slope -5
Slope 4



200
150
100
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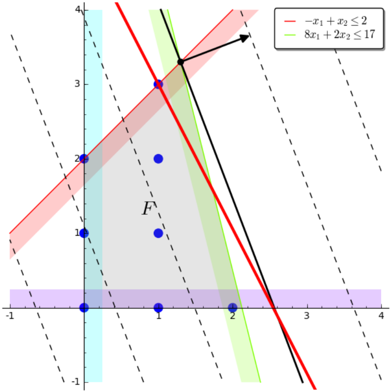
0.05
0.1
0.15
0.2
0.25
0.3



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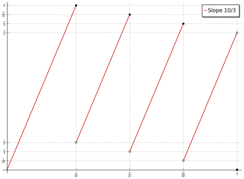
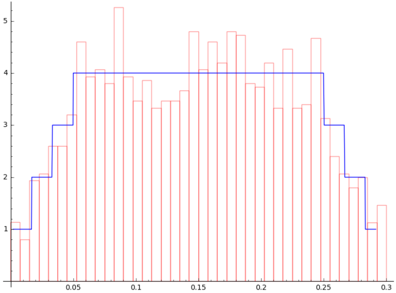

Slope 10/3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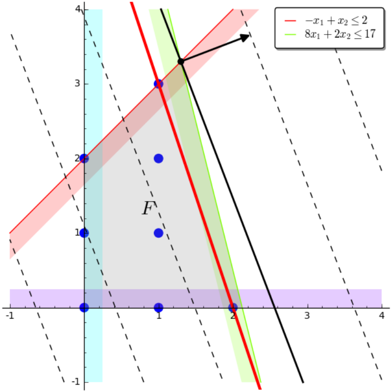
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Future Work

- Craft example problems that generate a particular distribution of coefficients in the tableau
- Leverage available information to optimize cuts over the tuning parameters

Thanks!