Complexity in the Quantum XY Spin Chain

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PHY 250 – NATURAL COMPUTATION AND SELF-ORGANIZATION

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Spin Models

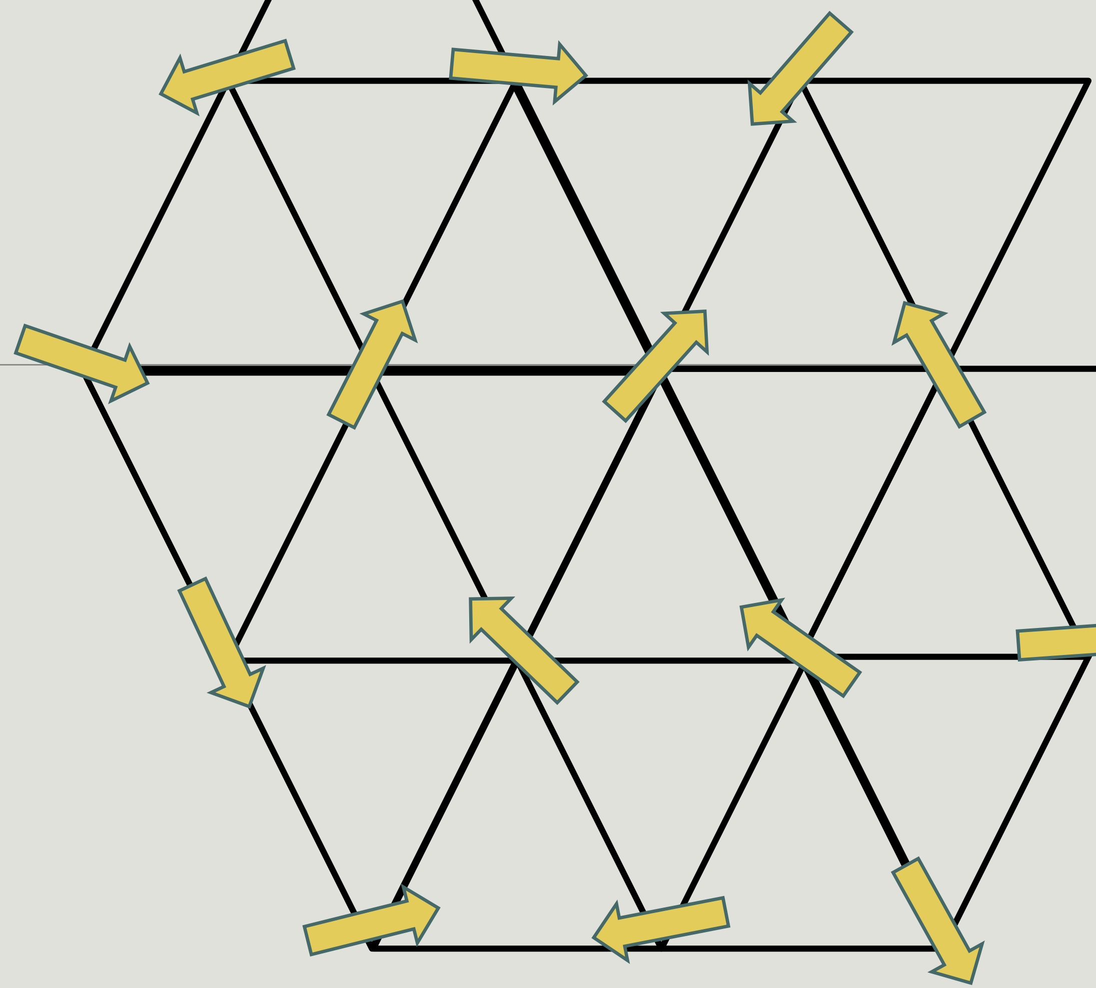
Dimension

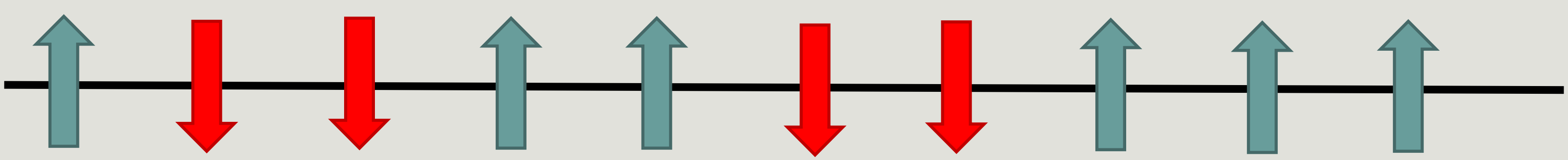
Lattice structure

Interaction

Degrees of freedom

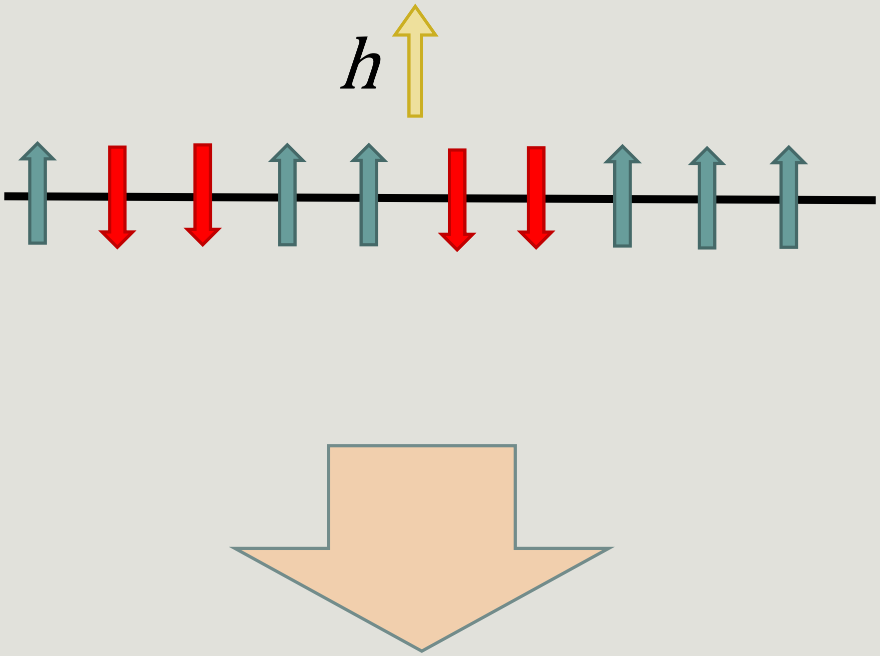
Classical or Quantum





Classical Spin Chains: Ising Model

$$H = J \sum_{<i,j>} \sigma_i \sigma_j - h \sum_i \sigma_i$$



… 1 0 0 1 1 0 0 1 1 1 …

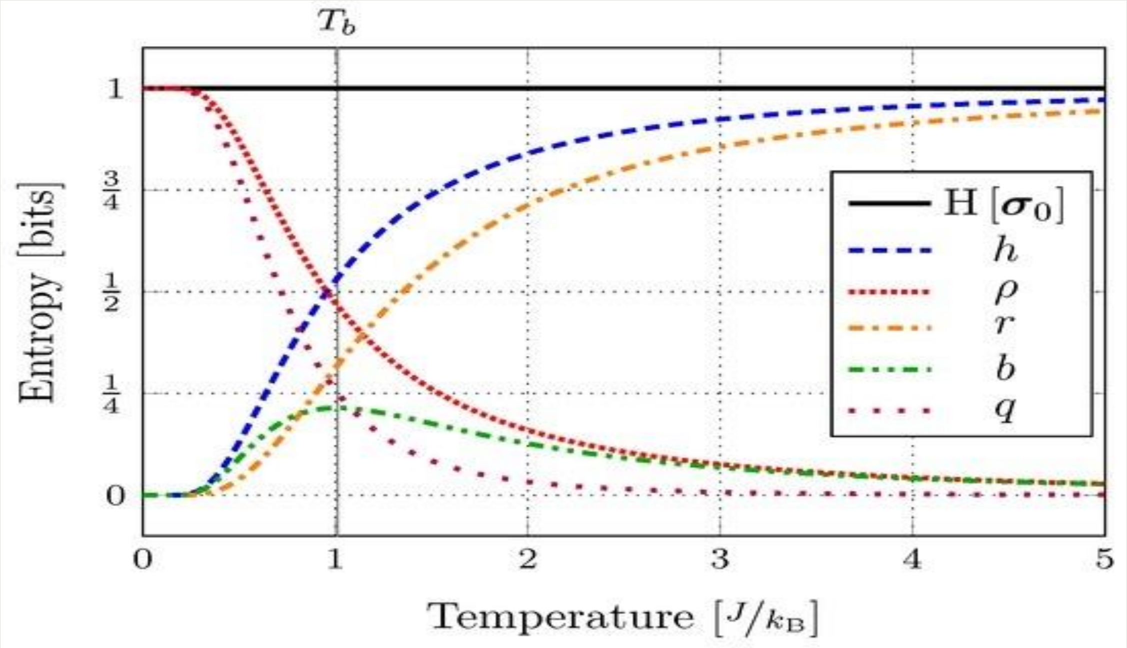
J < 0: ferromagnetic

J > 0: anti-ferromagnetic

Simulations can find favored lattice states for given temperatures

Easy to measure mutual information between spins

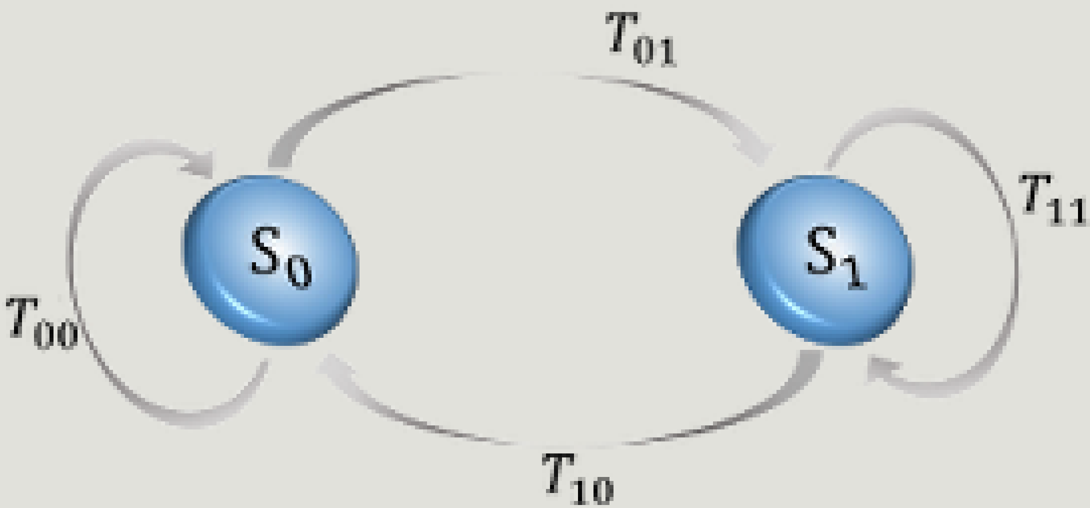
Information in Classical Spin Chains



Entropy decompositions for nearest neighbor, ferromagnetic spin-1/2 Ising chain

V. Vijayaraghavan, R. James, J.P. Crutchfield. Anatomy of a Spin, *Entropy* **2017**, *19*(5), 214; doi:10.3390/e19050214

Ising Model as Markov Chain



ε-machine for Ising chain

W.Y. Suen, J. Thompson, A. Garner, V. Vedral, and M. Gu. The classical-quantum divergence of complexity in the Ising spin chain, [arXiv:1511.05738v2]

Quantum Spin Chains

Heisenberg Model:

$$H = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j = J \sum_{<i,j>} S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z$$

XY Model:

$$H = J \sum_{<i,j>} S_i^x S_j^x + S_i^y S_j^y = \frac{J}{2} \sum_{<i,j>} S_i^+ S_j^+ + S_i^- S_j^-$$

Quantum Subtleties

1. Hilbert Space Dimension:

   $$D = 2^L$$

   L=100 → $D \approx 10^{30}$

2. In low-T limit, we only need ground state and first few excited states

3. Quantum Measurement Is Different From Classical Measurement

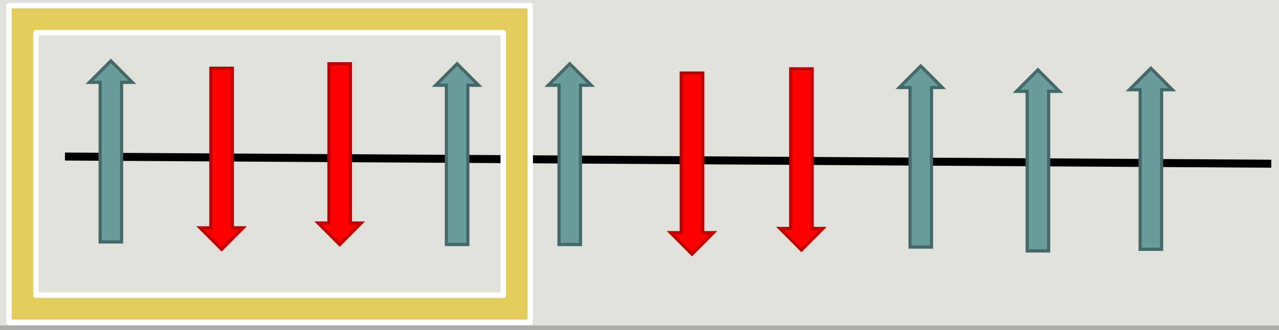
Density Matrix Renormalization Group Method (DMRG)

1. Start with one spin and add sites until Hilbert space is too large

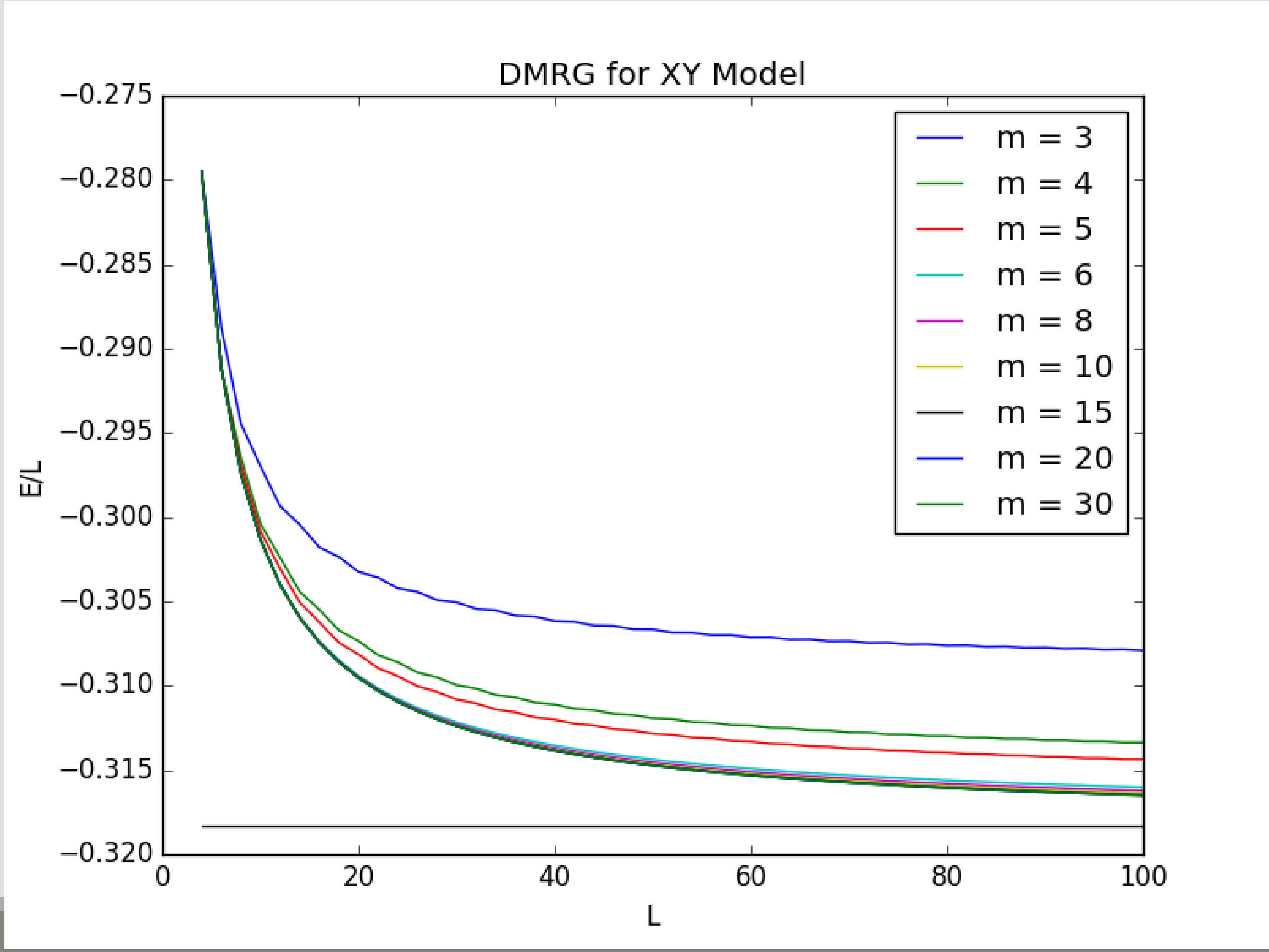
2. When Hilbert space grows to size m, diagonalize system and calculate density matrix

3. Truncate Hilbert space by keeping only largest m eigenvalues

4. Repeat until desired L is reached



Simulating Quantum Spin Chains



Measuring Quantum Spins

Different from classical measurement because:

Basis

Non-trivial measurements for DMRG final state because it is not in spin-basis

Must transform each desired observable into new basis at each DMRG step

$$(S_i^z)_j = O_j((S_i^z)_{j-1} \otimes I_d)O_j^\dagger$$

Entanglement

Measuring individual spin orientations locally returns classical spin configurations, which destroys existing entanglement

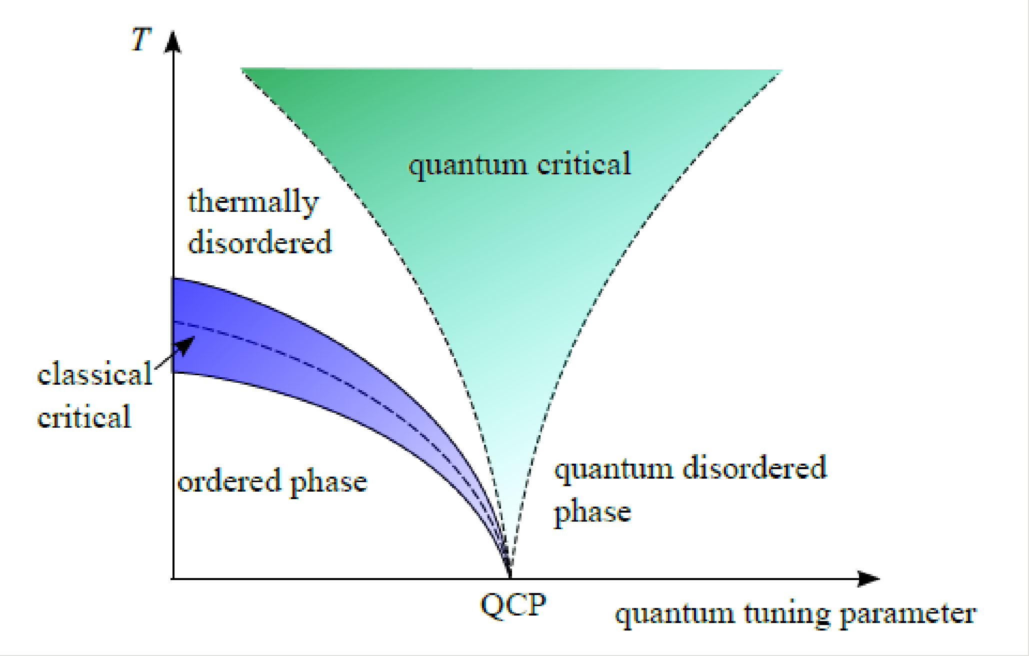
1D Quantum Phase Transitions

Change in ground state at T=0

Control Parameters: interaction strength, pressure, magnetic field, doping concentration

Driven by quantum fluctuations associated with the uncertainty principle

Effects can be measured experimentally in quantum critical region



QPT in XY Model

$$H = J \sum_{<i,j>} S_i^x S_j^x + S_i^y S_j^y + D \sum_i (S_i^z)^2$$

XY Model with Single-Ion Anisotropy [1]

$$H = J \sum_{<i,j>} S_i^x S_j^x + S_i^y S_j^y - K \sum_i S_i^z$$

XY Model with Transverse Field [2]

[1] A.S.T. Pires, Quantum-phase transition in a XY model, Physica A **373**, 387 (2007) arXiv:1511.05738v2
[2] F. Pázmándi, Z. Domański, Quantum phase transitions in XY Spin Models, Phys. Rev. Lett. **74**, 2363 (1995)

XY Model in Transverse Field

Mott Insulator

Ordered phase

Localized spins

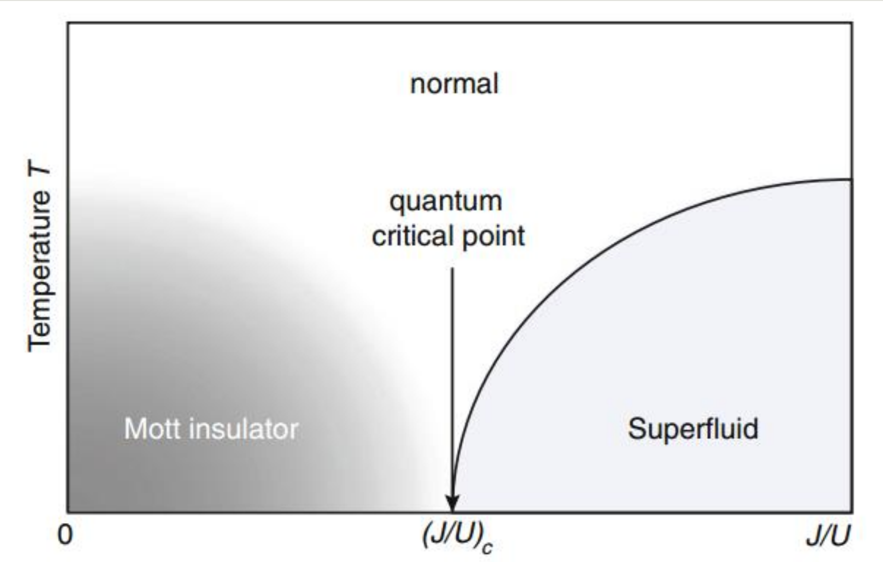
Insulates due to electron-electron interactions

Increase K

Superfluid

Fluid with zero viscosity

Exhibits long-range order



M. Endres, Probing Correlated Quantum Many-Body Systems at the Single-Particle Level, (2014)

Conclusions/Future Work

Successfully simulated 1D quantum XY model

Defining information quantities for quantum spin chains involves non-trivial measurements

Signatures of Quantum Phase Transition in information quantities can prove the quantum nature of the system is being captured

Use states from DMRG algorithm to calculate information quantities (spin-spin correlations, entropy decomposition for measurements along chain)

Calculate fully quantum information measures (entanglement entropy between blocks of chain, etc.)

ε-machines for quantum spin chains? Or purely quantum models?

Questions?

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