On aggregation of network multiplexity

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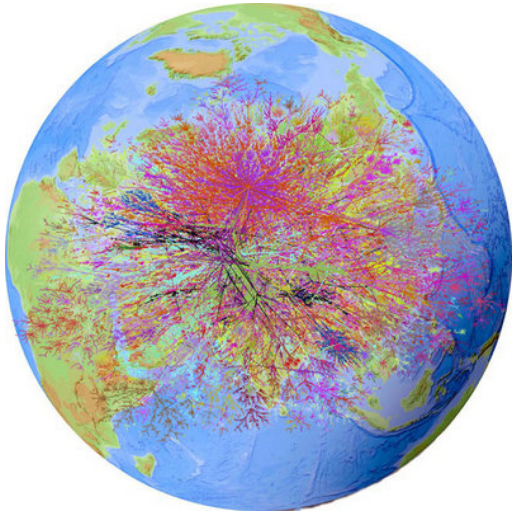
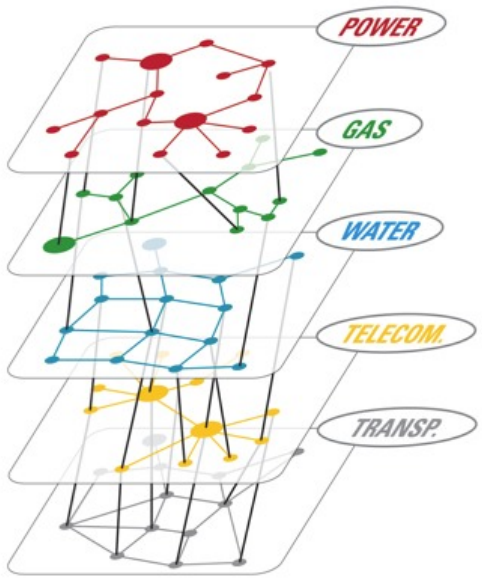
Introduction



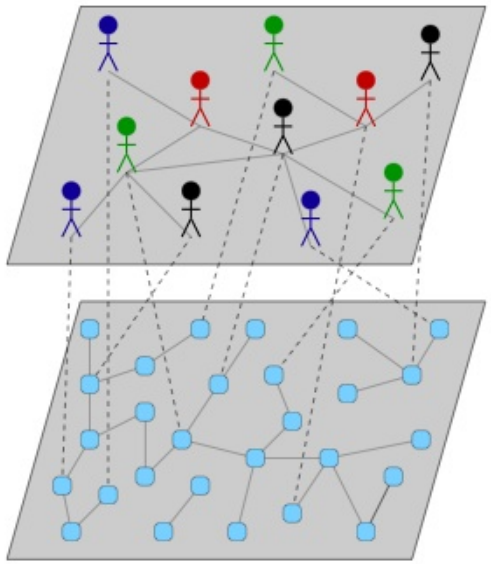
- Many real systems can be modeled as networks.
- Network include nodes and edges.
- There are some dynamics going on.

Multiplex Network



- Research showed that single layer network may not be sufficient to capture all the feature of the network.
- Different type of edges form different layers of network.
- Multiplex network is a special case of multi-layer network with same node set in all the layers.

Reduced Multiplex Network



- Many useful algorithms scale poorly with the number of layers.
- It is possible to aggregate some layers together to get a multiplex network with fewer layers while preserve most information of the original network.

Airline network

- Snapshot of US domestic flight network.
- Weighted by the number of passengers.
- Each carrier forms a layer of network.
- 60 layers in total.

Airline network

Jenson-Shannon distance

$$D_{JS}(\mathcal{N}_{L_1} \parallel \mathcal{N}_{L_2}) = \frac{1}{2} D_{KL}(\mathcal{N}_{L_1} \parallel \mathcal{N}_{L_{12}}) + \frac{1}{2} D_{KL}(\mathcal{N}_{L_2} \parallel \mathcal{N}_{L_{12}})$$

- $\mathcal{N}_L$ is normalized weight distribution over edge set.
- Aggregate two layers with smallest D_{JS}.

Airline network

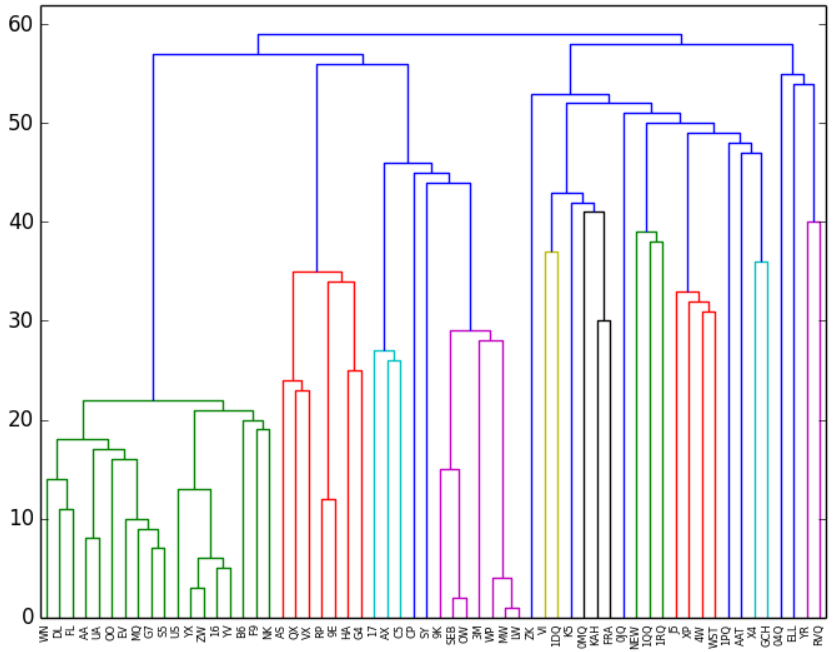


Figure: Dendrogram of Aggregation

Motivation

When is it possible to aggregate two layers together?

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- When they have similar edge properties.

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- When they have similar edge properties.
- Or when they have similar structures.

Network of Chaotic Map

Dynamics

$$x_i(t+1) = (1-\epsilon)f(x_i(t)) + \frac{\epsilon}{k_i}\sum_{(i,j)\in G} f(x_j(t))$$

- The dynamic at each node is a chaotic map f.
- Edges (i, j) represent coupling between nodes.

Multiplex Setting

Dynamics

$$x_i(t+1) = (1-\epsilon_{L(t)})f(x_i(t)) + \frac{\epsilon_{L(t)}}{k_i} \sum_{(i,j)\in G_{L(t)}} f(x_j(t))$$

- The edges in different layers take effects alternately.
- L is the function to pick a specific layer for time t.

Two Different Multiplex Network

- Independently generated two-layers, with the same coupling parameter.
- Same network in two-layers, with different coupling parameters.

Information Measure

- Convert the time series to discrete alphabet.
- Infer the ϵ-machine and calculate information measures.

Small Network

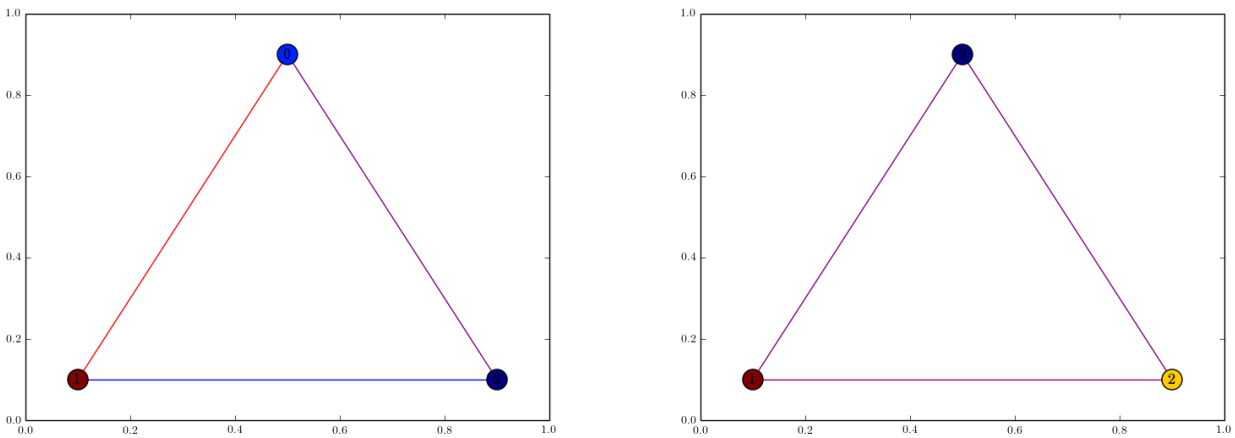

Figure: Small Network with Same Edge Property

Small Network

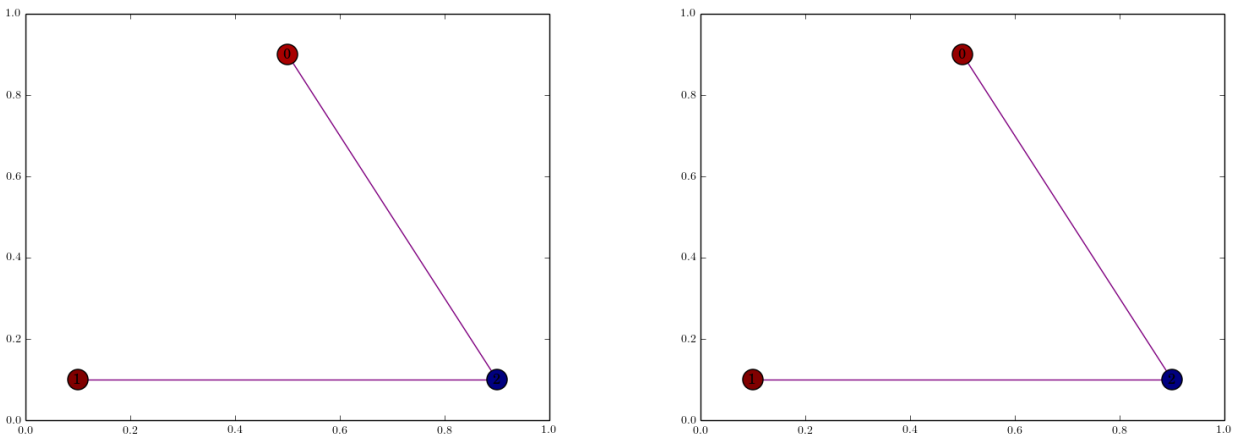

Figure: Small Network with Same Topology

Random Network

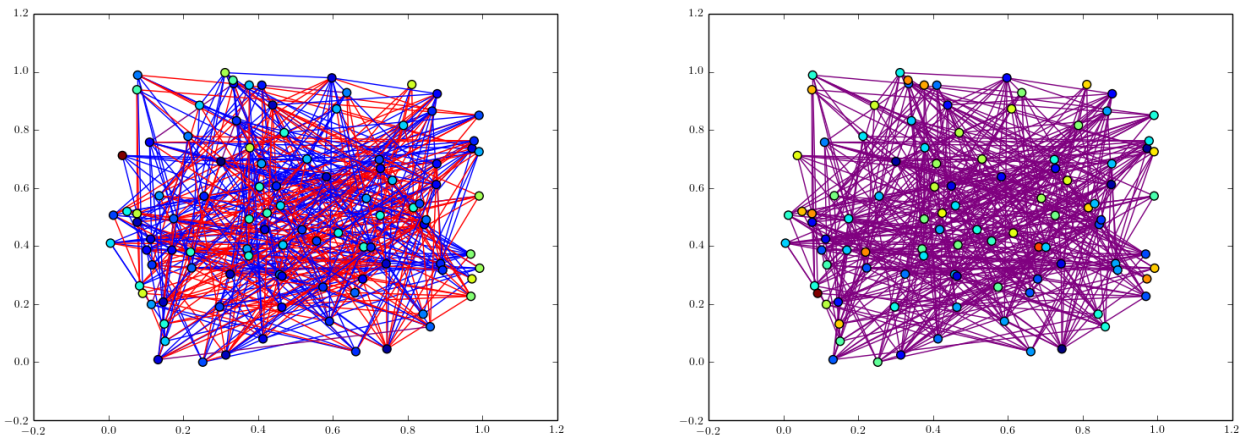

Figure: Random Network with Same Edge Property

Random Network

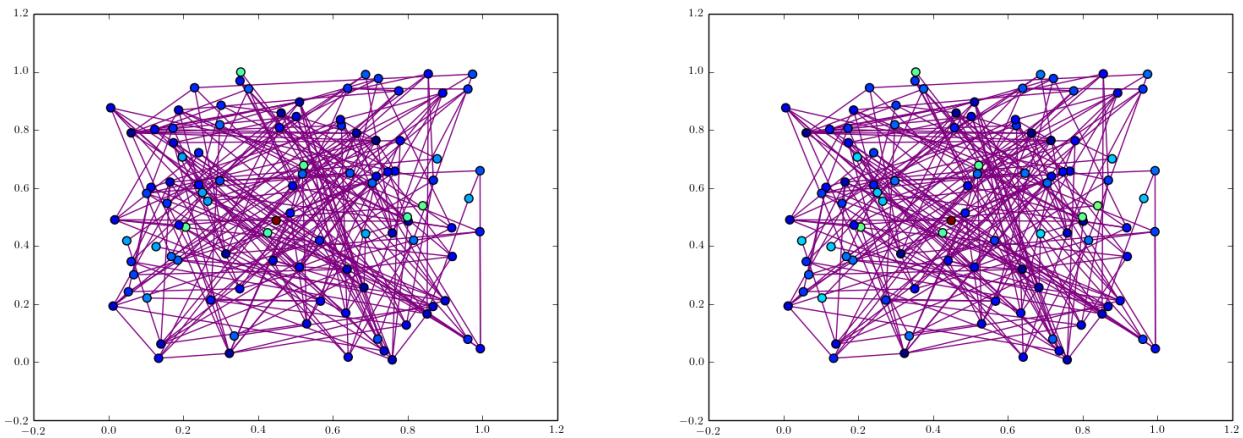

Figure: Random Network with Same Topology