Network Synchronization in Arrays of Coupled Oscillators

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PHY256: NCASO – Final Project
UC Davis Physics

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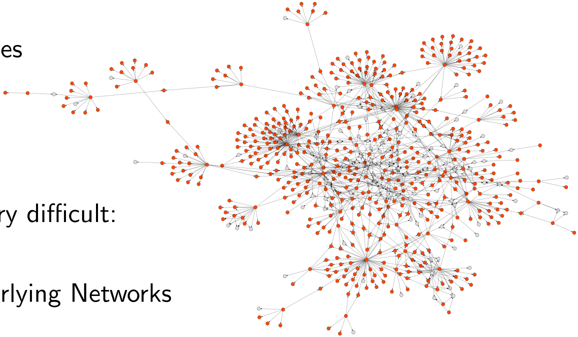
Network Control Problems

"Controlling Collective Phenomena in Complex Networks"

- Prevent Power Outages
- Save a Species
- Cure Disease
- ...

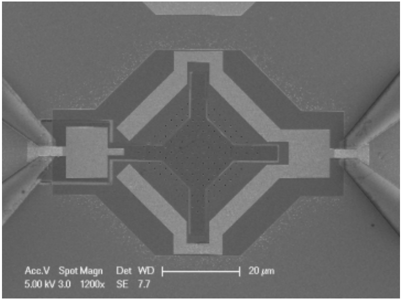
System control may be very difficult:

- Missing Information
- Large, Complex Underlying Networks
- Nonlinear Dynamics



Exciting applications, but lacking experimental verification

Experimental Opportunities



Courtesy of Roukes Group

NanoElectroMechanical Systems (NEMS)

- physically small
- scalable in number
- nonlinear
- tunable

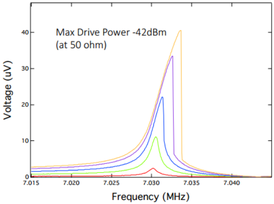
CalTech team

- Michael Roukes
- Matthew Matheny
- Warren Fon

Resonator – Physical Dynamics

Driven Duffing dynamics

$$\ddot{x}+\frac{\omega_0}{Q}\dot{x}+\omega_0^2\left(1+\tilde{\alpha}x^2\right)x=\tilde{g}\cos\left(\omega t\right)$$



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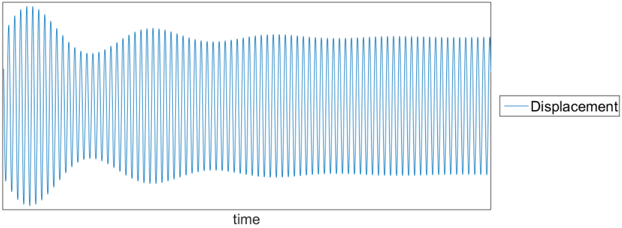


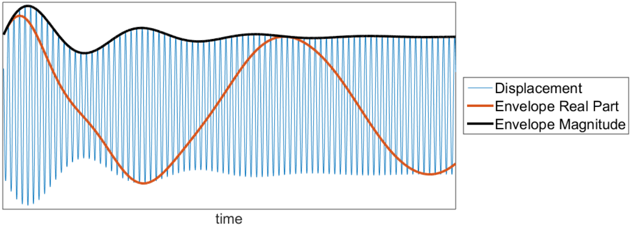
Resonator – Envelope Dynamics

Separate time scales:

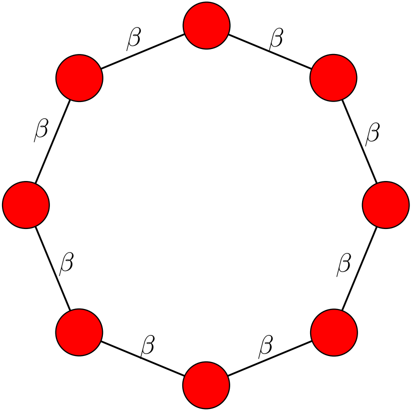
$$T \equiv \frac{\omega_0}{Q} t$$

$$x \Rightarrow x_0 \Re \left[A(T) e^{i\omega_0 t} \right], \; A(T) = a(T) e^{i\phi(T)}$$

$$\frac{dA}{dT} = -\frac{1}{2} A + \frac{i\delta}{2} A + i\bar{\alpha} |A|^2 A + g e^{i\Omega T}$$



Uniform Reactive Rings



NEMS Oscillator

$$\text{Driver: } ge^{i\Omega T} \to \frac{1}{2}\, e^{i\phi} = \frac{A}{2|A|}$$

NEMS "Network"

$$\begin{aligned} \text{System Size: } & N = 8 \\ \text{Dissipative Coupling: } & K_{ij} = 0 \\ \text{Reactive Coupling: } & \beta_{i,i+1} = \beta \\ \text{Slow-Time Frequency: } & \delta_i = \delta = 0 \end{aligned}$$

Synchronization occurs even with vanishingly weak coupling.

We want states: this is good.

URR Equations

$$\frac{dA_i}{dT} = -\frac{1}{2}A_i + i\alpha|A_i|^2 A_i + \frac{A_i}{2|A_i|} + \underbrace{\frac{i\beta}{2}\left(A_{i+1} - 2A_i + A_{i-1}\right)}_{\text{Linear Diffusion}}$$

or, with $A = ae^{i\phi}$,

$$\frac{da_i}{dT} = -\frac{a_i - 1}{2} - \frac{\beta}{2}\left[a_{i+1}\sin\overbrace{(\phi_{i+1} - \phi_i)}^{\text{Phase Difference}} + a_{i-1}\sin\overbrace{(\phi_{i-1} - \phi_i)}^{\text{Phase Difference}}\right]$$

$$\frac{d\phi_i}{dT} = \alpha a_i^2 + \frac{\beta}{2}\left[\frac{a_{i+1}}{a_i}\cos\underbrace{(\phi_{i+1} - \phi_i)}_{\text{Phase Difference}} + \frac{a_{i-1}}{a_i}\cos\underbrace{(\phi_{i-1} - \phi_i)}_{\text{Phase Difference}} - 2\right]$$

Steady States

A uniform system suggests uniform steady states:

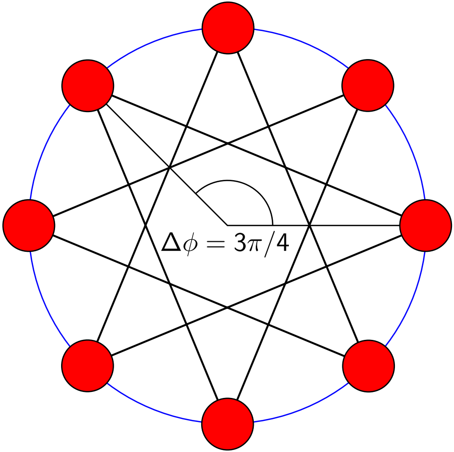
$$a_i = a = 1$$

$$\phi_{i+1} - \phi_i = \Delta\phi = \frac{2\pi n}{N},\ n \in \mathbb{Z}$$

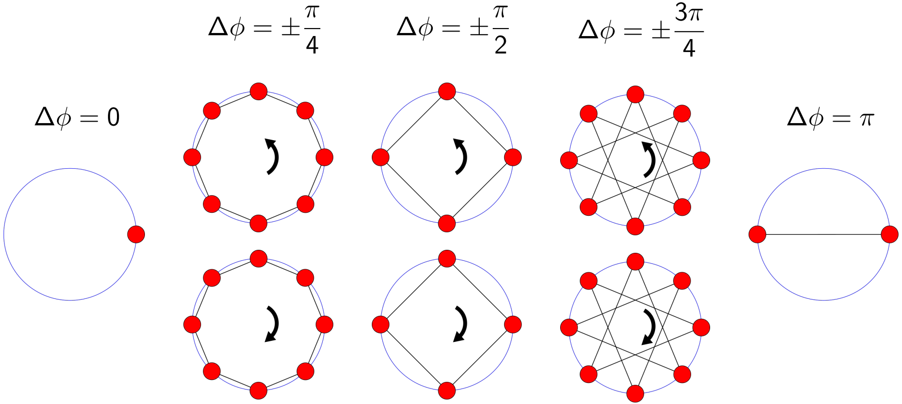
Such a steady state has fixed amplitude and constant frequency:

$$\frac{da_i}{dT} = 0$$

$$\frac{d\phi_i}{dT} = \alpha + \beta(\cos(\Delta\phi) - 1)$$



Steady State Enumeration



Steady State Linearization

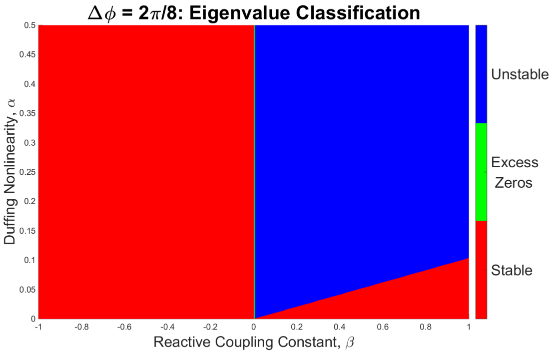
$$\frac{d}{dT}\begin{pmatrix} d\vec{a} \\ d\vec{\phi} \end{pmatrix} = \frac{1}{2}\left(\begin{array}{c|c} -\mathcal{I} - \mathcal{M}\beta\sin\Delta\phi & \mathcal{L}\beta\cos\Delta\phi \\ \hline 4\alpha\mathcal{I} - \mathcal{L}\beta\cos\Delta\phi & -\mathcal{M}\beta\sin\Delta\phi \end{array}\right)\begin{pmatrix} d\vec{a} \\ d\vec{\phi} \end{pmatrix}$$

Eigenvalues/vectors:

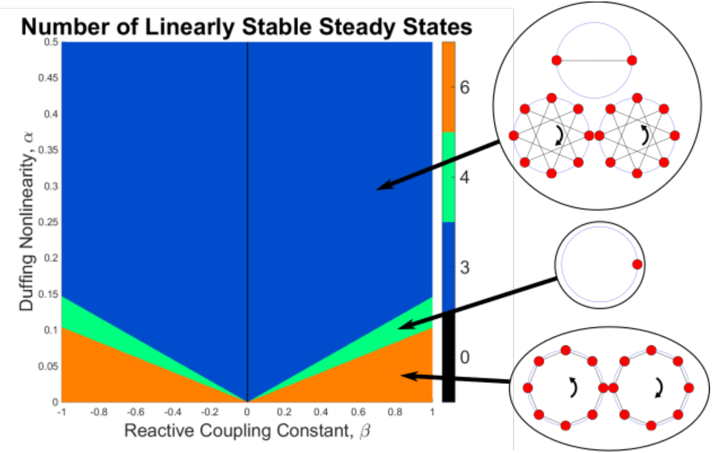
$$0 : \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \end{pmatrix}$$

$$-\frac{1}{2} : \begin{pmatrix} \mathbf{1} \\ -4\alpha\mathbf{1} \end{pmatrix}$$

$$\vdots$$



Steady State Stability

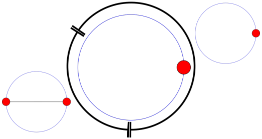


NEMS URR Control

We want to switch steady-state basins:
not move to any arbitrary state.

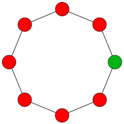
The format of control may be limited:
not an arbitrary signal.

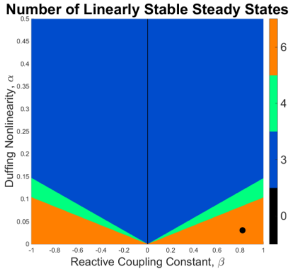
For example:
Move a single oscillator's phase
Perhaps move two? Which two?

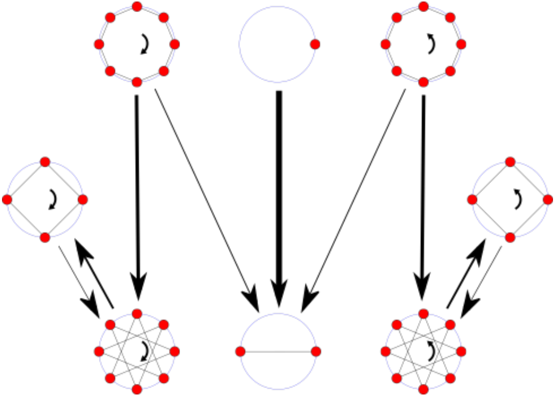


The ability to get to a particular basin from a particular steady state is represented by an edge weighted by the amount of state slice in the target basin.

Single Node Control – Strong Coupling

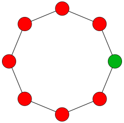


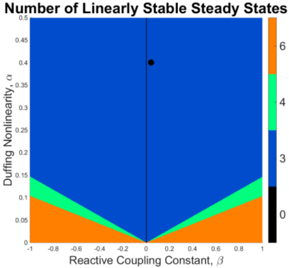




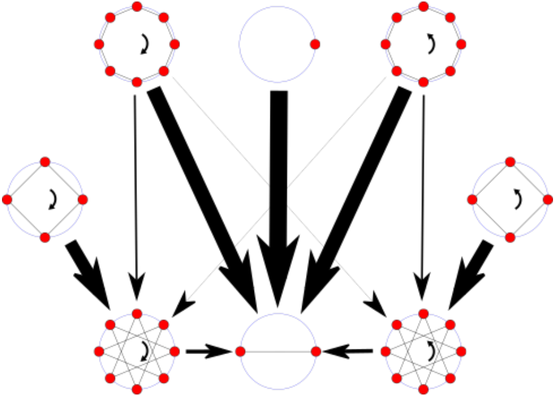
$\alpha = 0.04,\ \beta = 0.8$

Single Node Control – Weak Coupling

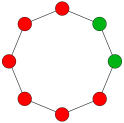


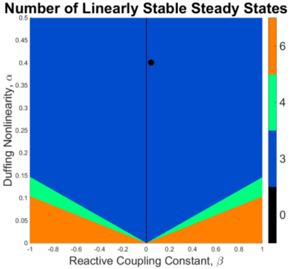


$\alpha = 0.4,\ \beta = 0.02$



Two Adjacent Node Control





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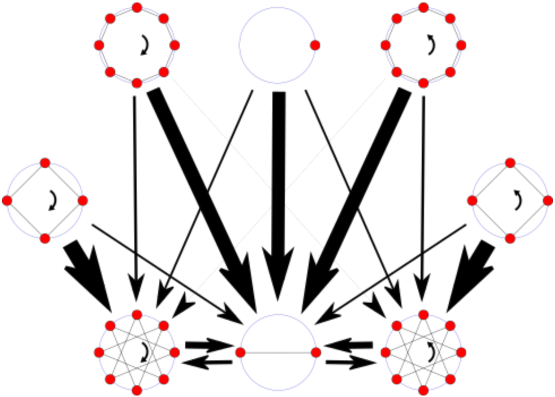
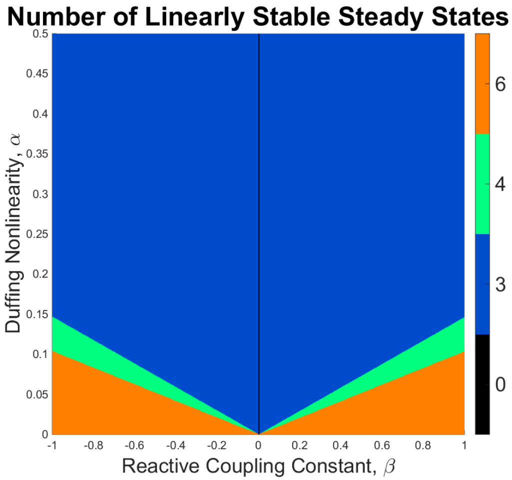


Noise-Induced Instability



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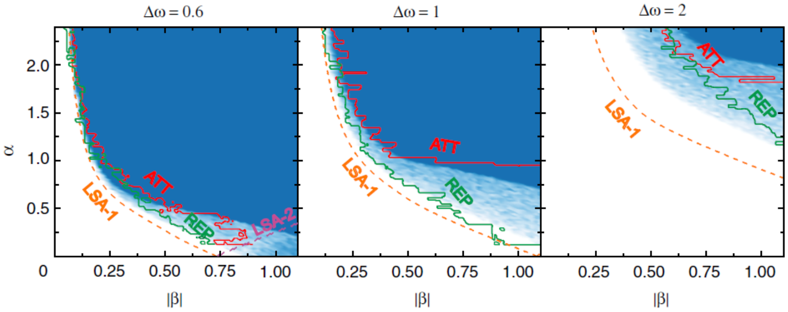
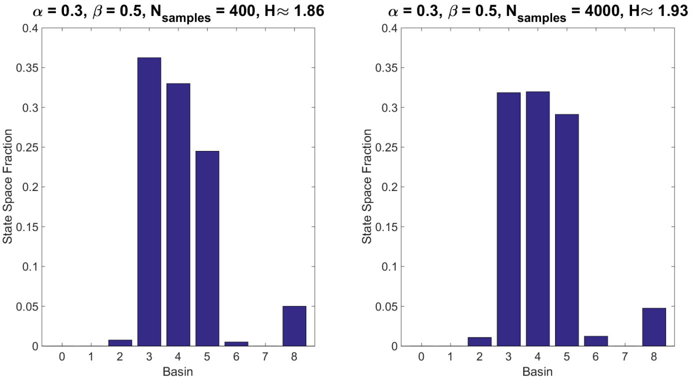
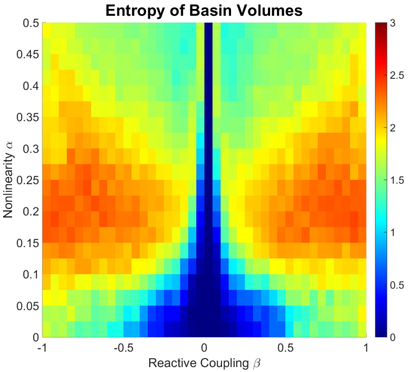


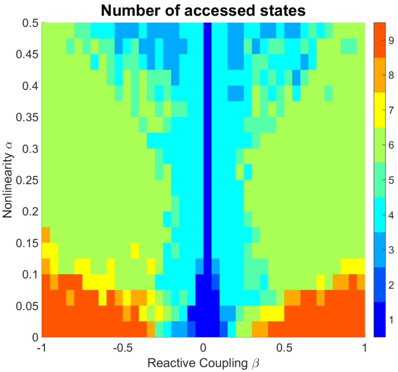
Matheny et al. PRL 2014

State-Space Basin Entropy



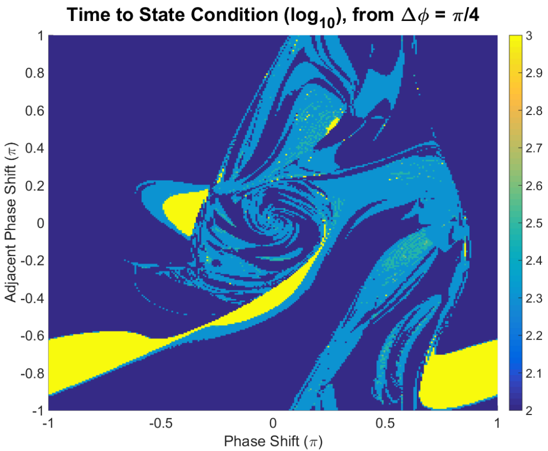
State-Space Basin Entropy





"Steady-State" Transients

Animation



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