ε-Machine Estimation and Forecasting

Comparative Study of Inference Methods

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Outline

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ε-Machines and Time Series.

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- These look way too much like time series: $\{X_t | t \in \mathbb{Z}\}$
- Let's compare with time series methods?

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  - Stock price data
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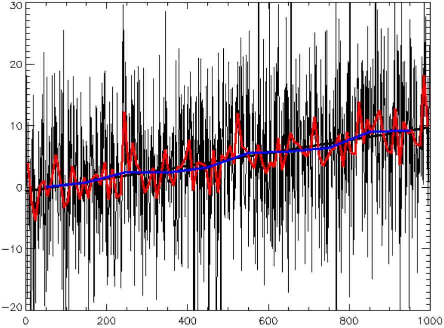

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Tools for investigating time-series data include:

- Consideration of the autocorrelation function and the spectral density function (also cross-correlation functions and cross-spectral density functions)
- Scaled cross- and auto-correlation functions to remove contributions of slow components[8]
- Performing a Fourier transform to investigate the series in the frequency domain
- Use of a filter to remove unwanted noise
- Principal component analysis (or empirical orthogonal function analysis)
- Singular spectrum analysis
- "Structural" models:
  - General State Space Models
  - Unobserved Components Models
- Machine Learning
  - Artificial neural networks
  - Support Vector Machine
  - Fuzzy Logic
- Hidden Markov model
- Control chart
  - Shewhart individuals control chart

[Dmitry + ε-machines] vs. [Time series]

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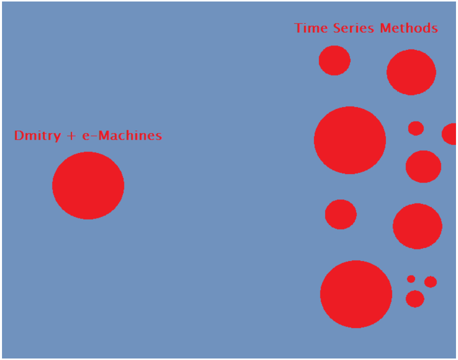

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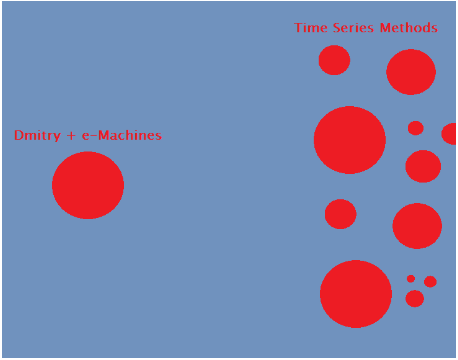

- Too many! Pick battles you can win!

Outline

Baum-Welch - What is it?

The Baum-Welch algorithm - a method for inferring the most likely parameters of a Hidden Markov Model responsible for some given string of data.

Example

Input

- Number of states - n
- Network topology and initial guess for the parameters of the HMM - θ
  - Parameters are the start distribution, state transition probabilities, and symbol emission probabilities at each state.
- A sequence of outputted symbols - $Y = \{Y_1, Y_2, ..., Y_n\}$

Example

Output

- New parameters θ' with the property that $P(Y|\theta') \geq P(Y|\theta)$.

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- Too many details:
  - Makes a forward pass to compute $\alpha_i(t) = P(''y_1, \ldots, y_t'', X_t = i|\theta)$
    - ★ Probability of seeing "word" and ending up in state i at time t.
  - Makes a backwards pass to compute $\beta_i(t) = P(''y_{t+1}, \ldots, y_T''|X_t = i, \theta)$
    - ★ Probability of having the last $T - t$ symbols be the specified "word", assuming that you ended up in state i at time t.
  - Combined these in the update step to calculate $\xi_{ij}(t) = P(X_t = i, X_{t+1} = j|Y, \theta)$
    - ★ Probability of transitioning $i \to j$ at time t.
    - ★ $\xi_{ij}(t)$ can be computed as a function of α_i's and β_i's
  - Where this eventually gets us:
    - ★ Summing $\xi_{ij}(t)$ over time, we can compute the expected number of transitions from state $i \to j$, as well as $i \to *$.
    - ★ This will allow us to compute the expected transition probabilities between states.
    - ★ And! Similarly, we can compute the expected symbol emission matrix for each state.
    - ★ Starting probabilities.
    - ★ We now have θ'!

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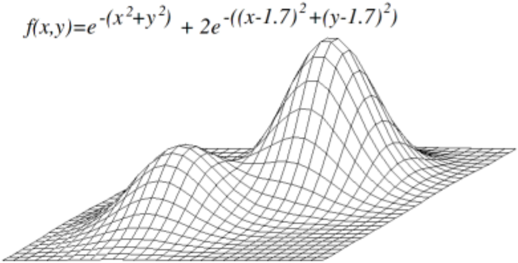

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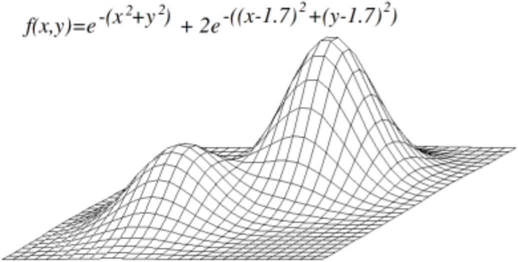

- Gets stuck in local optima, so you have to seed it and rerun it with many starting conditions.

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1. Motivation
   - ε-Machines and Time Series
   - Baum-Welch Algorithm
   - The Future, The Comparison Project

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- As fun side-project / implementation, would like to translate HMMs into some audio signal [for ex. each symbol corresponds to a pitch being emitted].
  - See if the HMMs the algorithms produce "sound" different.

Sidenote: IPython.

- IPython is great for Windows machines that have run VM's.
- It is Sage-like, here is my workflow:

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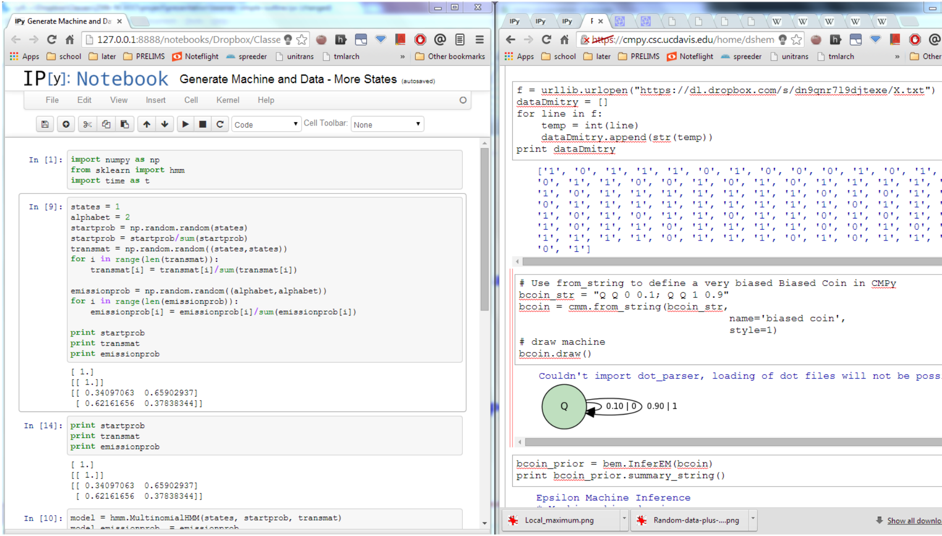

Thank you!

Thank you! Everybody!