Information and Order Parameters in the Gauge Ising Model

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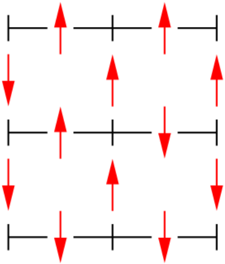
Motivation.

- Physicists work hard to find phase transitions.
- Usual way to find it is to look at the "order parameter".
- There is no theory of the order parameter. Different problems - different order parameters.
  - Can be simple (0.0001% of the models)- for example magnetization for ferromagnetic Ising model or XY model.
  - Can be hard. Rest 0.9999%.

$$H = -J \sum_{\langle i,j,k,l \rangle} S_i S_j S_k S_l \quad \text{, where sum is over the plaquet} \qquad (1)$$



- I am looking at $3D$
- $Z(T) = \sum_{S_i=\pm1} \cdots \sum_{S_N=\pm1} \exp(-H/T)$
- $\langle E(T) \rangle = T^2 \frac{\partial \ln Z}{\partial T}$
- $C_v(T) = \frac{\partial \langle E \rangle}{\partial T}$

Monte Carlo Simulation. Algorithm.

Update algorithm.

1. Calculate energy for the current state E_{old}
2. Consider flipping random spin.
3. Calculate energy for the state with flipped spin E_{new}
4. Do this flip with probability $p = \frac{1}{1+e^{(E_{new}-E_{old})/T}}$
5. Measure quantities of interest in new state(Energy, Magnetization)

Claim is that average over configurations is equal to the thermal average.

$$\langle X \rangle_{\text{over configurations}} = \langle X \rangle_{thermal}$$

Physical measurements.

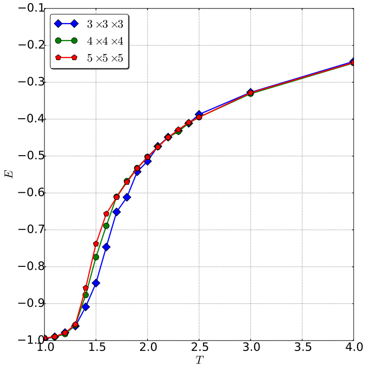


Figure : Energy as function of temperature for different lattice sizes.

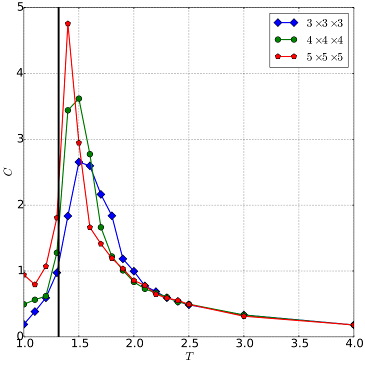


Figure : Specific Heat as a function of temperature for different lattice sizes. Black vertical line - critical temperature.

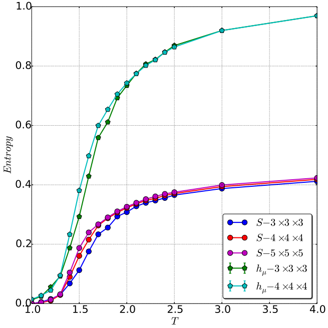
Entropy

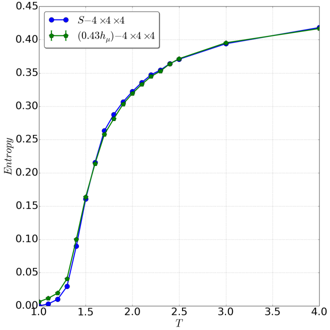
Measurements:

- We go to the next state(we filp spin) $\Longrightarrow$ 1
- We stay in the same state(we do not flip spin) $\Longrightarrow$ 0

Monte Carlo simulation tries to update spins in random order many times iterating through the lattice. So we obtain

$$\ldots 11110111100011000000001010100110110110110 11\ldots$$





Statistical complexity.

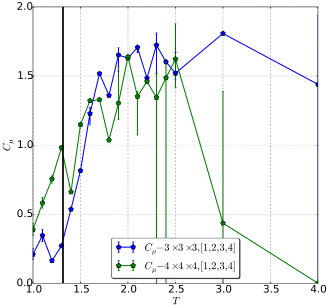


Figure : *Cmu* as a function of temperature for different lattice sizes.

Plans.

Numerical:

- Better data. (I used $6000 \times N_x \times N_y \times N_z = 162000, 384000, 750000$ which can be not enough)
- Bigger lattice sizes.
- Other quantities.
  - Excess Entropy E
  - Markov order.
  - Cryptic order.
  - Predictability Gain G.
  - Transient Information T.
  - Unanticipated and relevant information b_μ.
  - Unanticipated and irrelevant information. r_μ.
  - etc

Analytical:

- How informational theory quantities are related to the physical measurements? How to use them to indicate phase transition temperature?