Complexity and critical Behavior

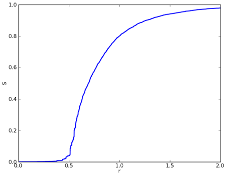
Vikram S. Vijayaraghavan

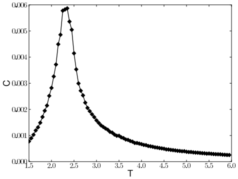
University of California, Davis.

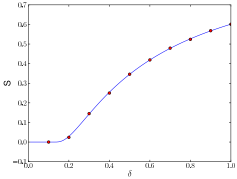
May 31, 2012

Motivation

- Phase transitions occur in a variety of systems.
- Ex. Magnetic systems, Network growth models.
- Early warning signs?

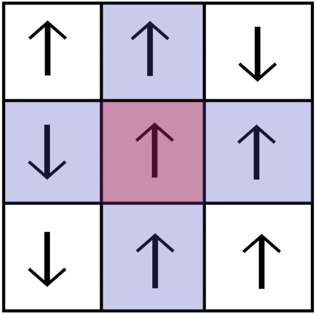






The Ising Model

- Introduced to explain phase transition in magnetic systems.
- $\mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j - B \sum_i S_i$
- Parameters : T, B.

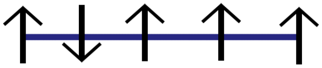


Definitions

- Magnetization $M = \sum_i S_i$
- Susceptibility $\chi = \frac{\langle M^2 \rangle - \langle M \rangle^2}{T}$
- Heat Capacity $C = \frac{\langle E^2 \rangle - \langle E \rangle^2}{T^2}$
- Two point correlations $\langle S_i S_j \rangle - \langle S \rangle^2 = e^{-l(i,j)/\zeta}$
- Correlation length ζ

- Minimize free energy $F = U - TS$.
- Energy vs. Entropy
- At low temperatures, Energy wins over Entropy.
- At high temperatures, Entropy wins over Energy.
- What is the temperature of this cross over? (T_c)

1-D Spin Chain



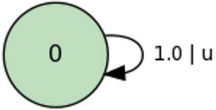
- Spin configuration shown above

$$(S_0, S_1, S_2, S_3, S_4) = (1, -1, 1, 1, 1) = (u, d, u, u, u) \tag{1}$$

- Exactly solvable.

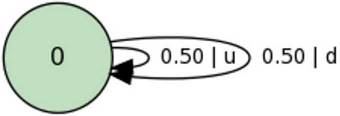
1-D Spin Chain

- Zero temperature, Non-zero magnetic field.
- All spins are aligned with the magnetic field.



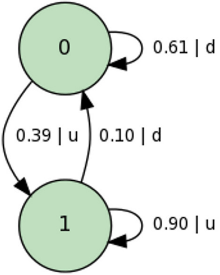
1-D Spin Chain

- Infinite temperature.
- Entropy wins over energy at all values of B.

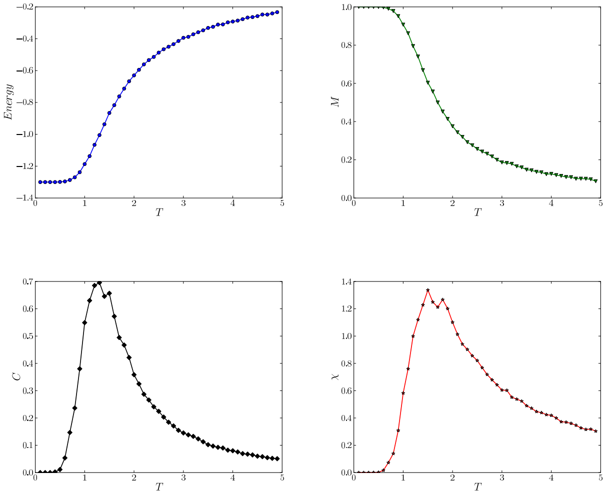


1-D Spin Chain

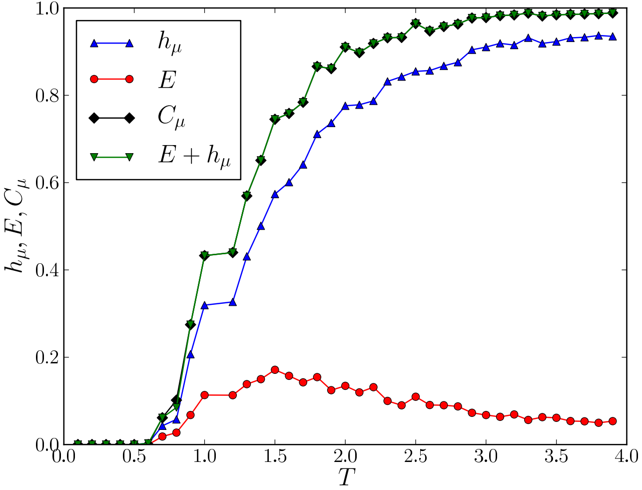
- Finite temperature.



1-D Spin Chain



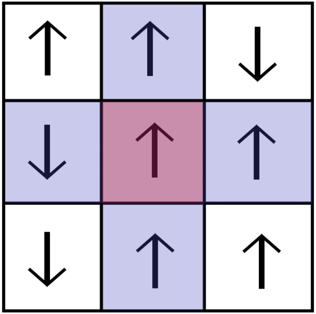
1-D Spin Chain



1-D Spin Chain

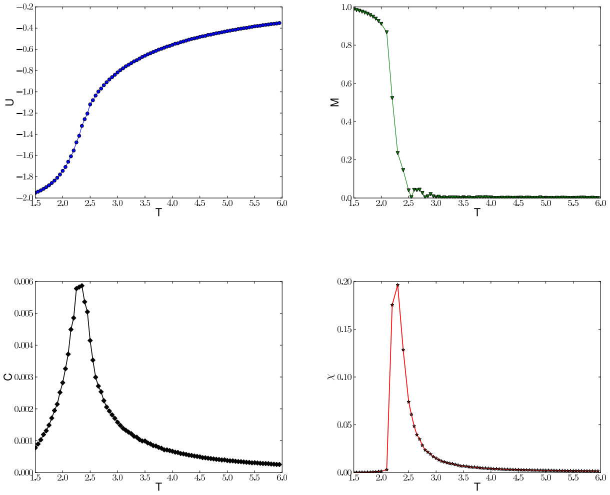
- $C_\mu = \mathrm{E} + Rh_\mu$
- E is different from other physical quantities.
- E does much better than two point correlations for detecting structure.

Moving from 1-D to 2-D



- Spontaneous ordering of spins.
- For square lattice $T_c/J = 2.26$

2-D Spin Chain



2-D Ising model

- No clear ordering of spins.
- Analogous to moving from 1-D to 2-D in calculus.
- Using 1-D strings of spin configuration captures structure of system + path.

Entropy Rate

- 1-D

$$h_\mu = \lim_{L\to\infty} \frac{H(L)}{L} \tag{2}$$

- 2-D

$$h_\mu = \lim_{(N,M)\to\infty} \frac{H(N,M)}{NM} \tag{3}$$

- The limit is taken such that N/M is a constant.
- Expressing h_μ as a conditional

Excess Entropy in 1-D

- Convergence

$$E_c = \sum_{L=1}^{\infty} h_\mu(L) - h_\mu \tag{4}$$

- *Subextensive*

$$E_s = \lim_{L\to\infty} H(L) - h_\mu L \tag{5}$$

- Mutual information

$$E_I = \lim_{L\to\infty} I[S_{-L}\ldots S_{-2}S_{-1}; S_0 S_1 \ldots S_L] \tag{6}$$

Excess Entropy in 2-D

- Convergence

$$E_c = \sum_{M=1}^{\infty} (h_\mu(M) - h_\mu) \tag{7}$$

- *Subextensive*

$$H(M, N) = E_s + E_s^x M + E_s^y N + h_\mu M N \tag{8}$$

- Mutual information

$$E_I = \lim_{M,N \to \infty} I\left[(M, N)\text{block}; (M, N)\text{block}\right] \tag{9}$$

Tasks ahead and questions to be answered

- How do the two dimensional measures vary with temperature?
- ϵ-machine representation for 2-D spatial process?
- Critical behavior and scaling relations for complexity measures?