Killing and **Collapsing**

Varying Transition Probabilities on Probabilistic Epsilon Machines

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(with great help from Ryan James)
(and guidance from Jim Crutchfield)

Varying Transition Probabilities on Probabilistic Epsilon Machines

Motivation:

- Robustness of Models
- Understanding Space of Epsilon Machines

Varying Transition Probabilities on Probabilistic Epsilon Machines

What I did:

- Parameterized the transition probabilities for the Even Process, Golden Mean Process, Telescope, Noisy Random Phase Slip, and a few other processes
- Plotted the values for **h_μ, C_μ**, and **E** over the parameter space
- Did some of thinking…

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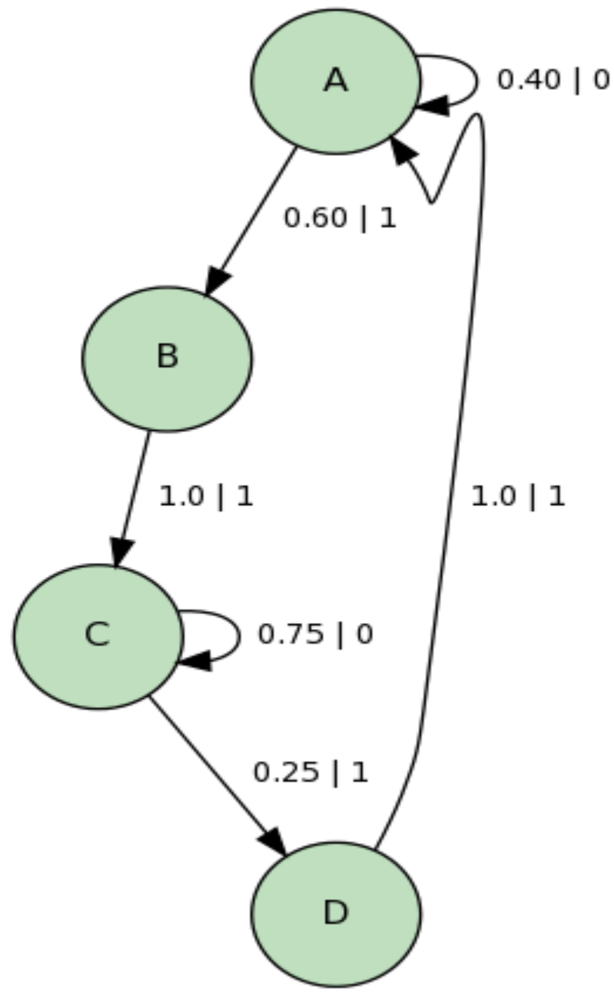
<u>A motivating example:</u>

- We start with a probabilistic epsilon machine:



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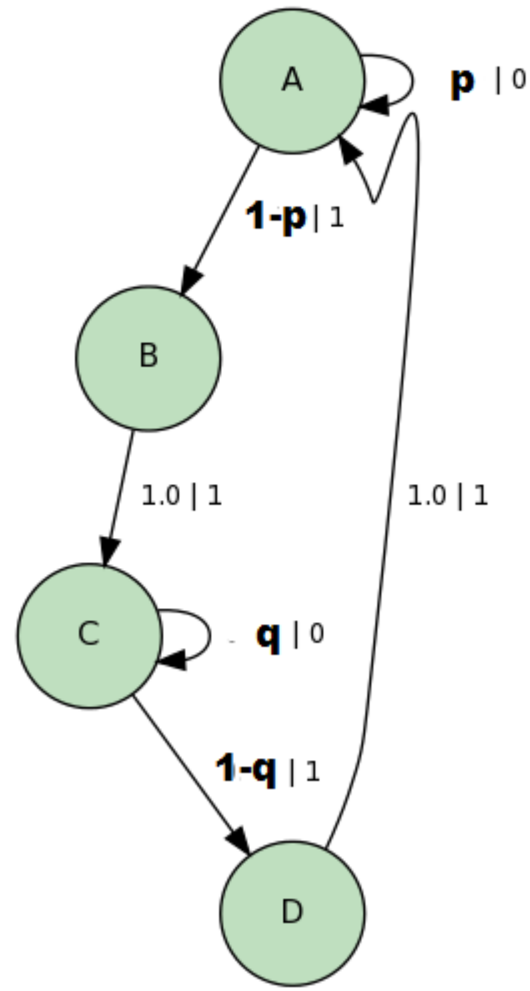
<u>A motivating example:</u>

- We then turn the

state- to state transition probabilities into parameters:



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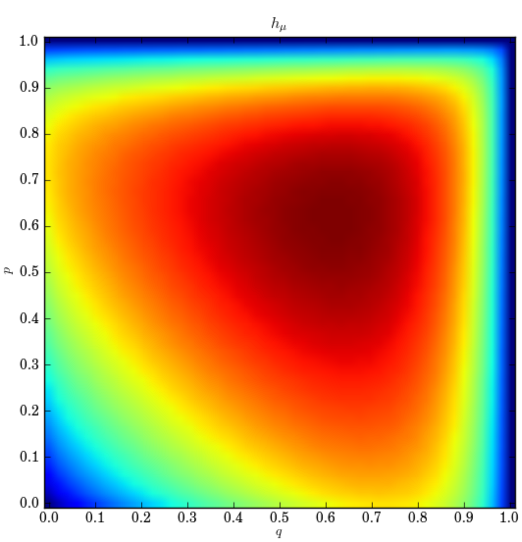
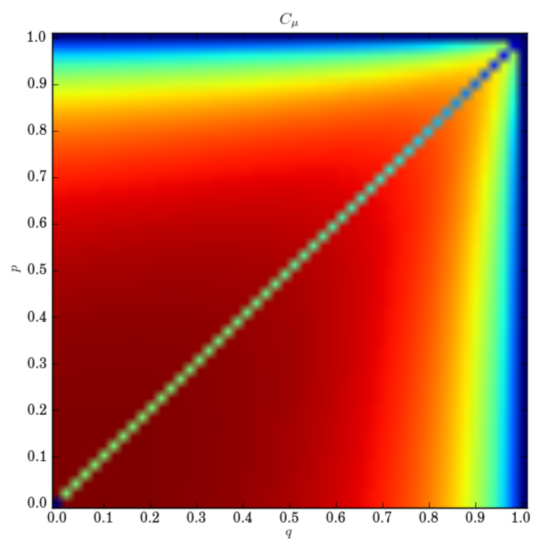
<u>A motivating example:</u>

Now we look at what happens to **hμ**, **Cμ** , and **E** as we let **p** and **q** take on all values from 0 to 1.

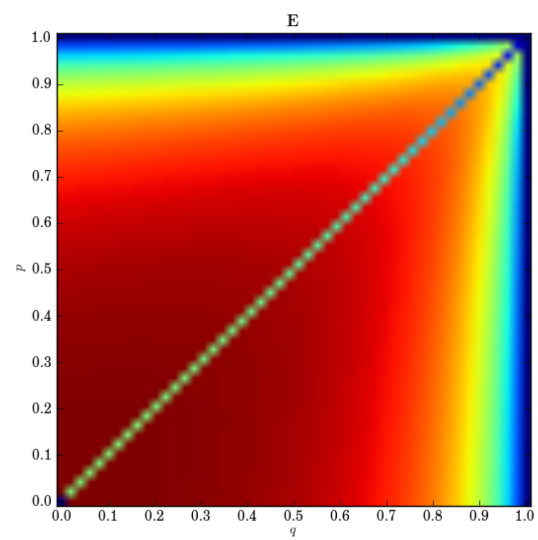
hμ



Cμ



E



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<u>Why is this important?</u>

- Robustness of model:

  *epsilon machines are generated from data

  *variability in data would lead to variability in epsilon machine presentation

- We see that in some regions, were our transition probabilities slightly off, we could get drastically different values for **hμ, Cμ, and E**.

- Even where the values of **hμ, Cμ, and E** don't vary drastically, it is nice to know how they do vary to have a bound on these quantities.

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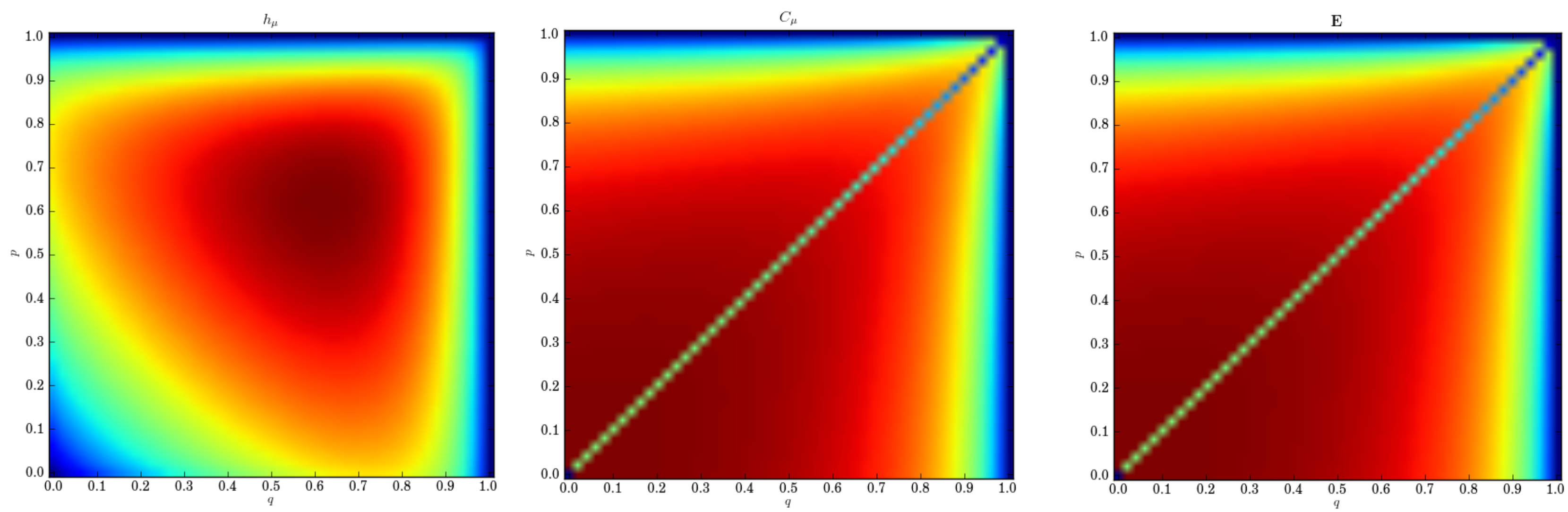
<u>Features to notice for this particular example:</u>



- Unlike **hμ**, **Cμ** and **E** do not vary continuously near the diagonal **p = q**
- The top and right boundary, where **q = 1** and **p = 1** respectively, are uniform and identical for **hμ, Cμ**, and **E.**

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What's happening there?

Reminder:

An **epsilon machine is defined as** the unique **minimal**, unifilar representation of a stationary stochastic processes whose **states are the equivalence classes of infinite histories with the same probability distribution over futures.**

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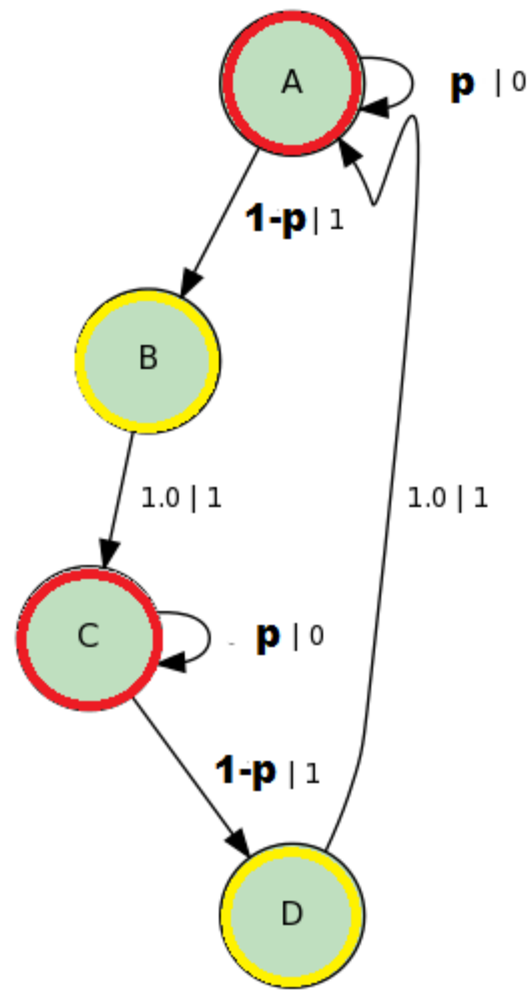
What's Happening There?
DIAGONAL: **p = q**

- When **p = q**, the states **B** and **D**, as well as **A** and **C** have the **same histories and same probability distributions over futures**.

- Because representation is **not minimal**, the machine we have is **no longer an epsilon machine**...



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What's Happening There?

DIAGONAL: **p** = **q**

But this "non-epsilon machine" does represent a stationary stochastic process, so there is an epsilon machine representation for it!

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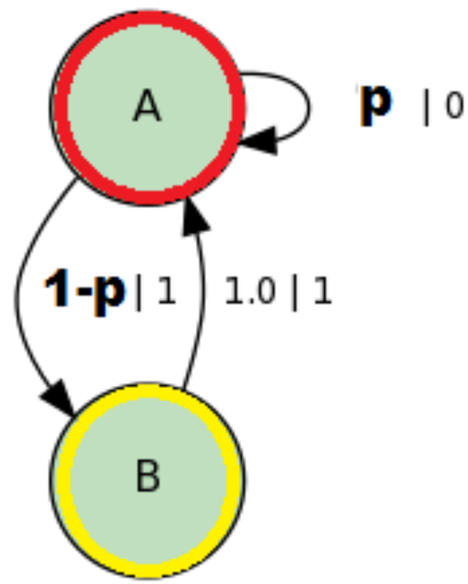
<u>What's Happening There?</u>

<u>DIAGONAL: **p** = **q**</u>

- Our problem with our "non-epsilon machine" was that it was **not minimal**
- We fix this by letting states

  B = **D** and **A** = **C**.
- This **collapses** our machine into:

The Even Process!



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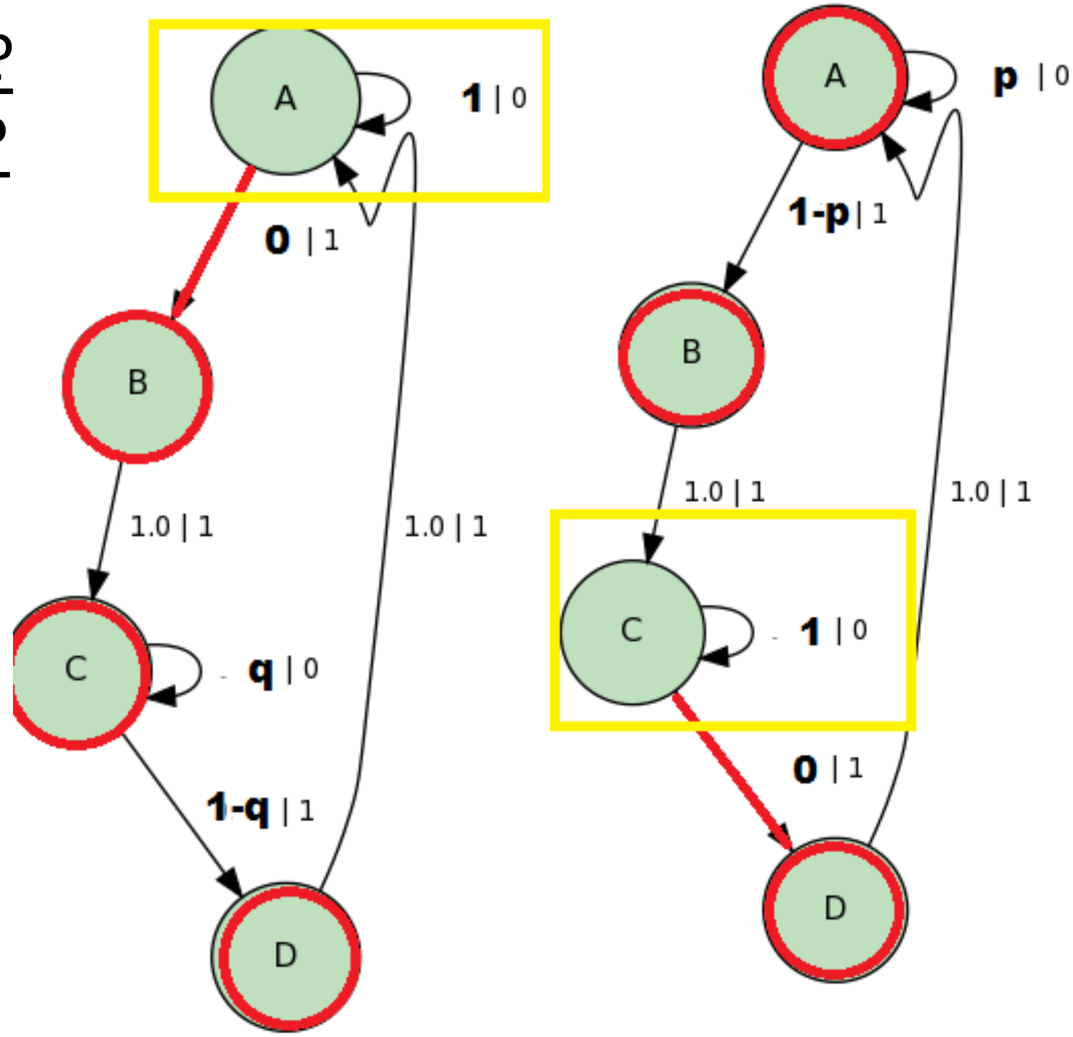
<u>What's Happening There?</u>
<u>BOUNDARY: **p = 1 (q = 1)**?</u>

- When **p = 1** (**q** = 1) we **kill** off an edge from state **A** (**C**).
- Remaining states become transient, and are **killed**.
- Results in single state that emits single symbol; thus, **hμ** = **Cμ** = **E** = **0**.



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What we've learned:

- Changing transition probabilities can **change statistical properties**

AND

- Changing transition probabilities can **change the topology** of the epsilon-machine representation

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Why is this interesting?

- Understanding the space of epsilon machines
  - Topological vs. Probabilistic Epsilon Machines

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The space of epsilon machines:

Reminder: A **topological epsilon** machine is ***defined*** as an epsilon machine where **the transition probabilities from a single state are uniform across all outgoing edges**

- Topological epsilon machines are a subset of probabilistic epsilon machines

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Theorem: Topological epsilon machines cannot be **collapsed** without first **killing**.

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Proof:

- States **collapse** when both:

  (1) entering/exiting symbols match up

  (2) entering/exiting probabilities match up

  are satisfied.

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Proof:

- **Suppose 2 things (red/blue)**: we have a **topological epsilon machine** and this topological epsilon machine has some states that have **matching entering/exiting symbols**.
- Then **that** means that there are **the same number of edges entering/exiting these states**.
- Because (by def.) **the transition probabilities of top. e- machines are uniformly distributed** ***<u>and</u>*** there are **the same number of edges entering/exiting these states**, the **entering/exiting probabilities match up**
- Then **that** means **these states would have equivalent histories and probability distributions for the future**.
- Then our **machine** would **not** be **minimal**.
- Then our **machine** would **not** be **an epsilon machine** <<< **uh-oh!**
- **<u>Contradiction!</u>** CAN'T have (**red and blue and green**)! ☺

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Conjecture: (the converse) An epsilon machine that does not **collapse** without first **killing** is isomorphic to a topological epsilon machine.

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Why this would be AWESOME!:

- Crutchfield and friends enumerated all topological epsilon machines with **n** states and alphabet size **k**
- That means that we can look at an arbitrary probabilistic epsilon machine, go through this process of **collapsing-killing-collapsing**-… and look at the "chain" of topological epsilon machines "nested" inside, or something like that
- These "chains" could possibly provide structure to the space of all epsilon machines (much more to think through!)

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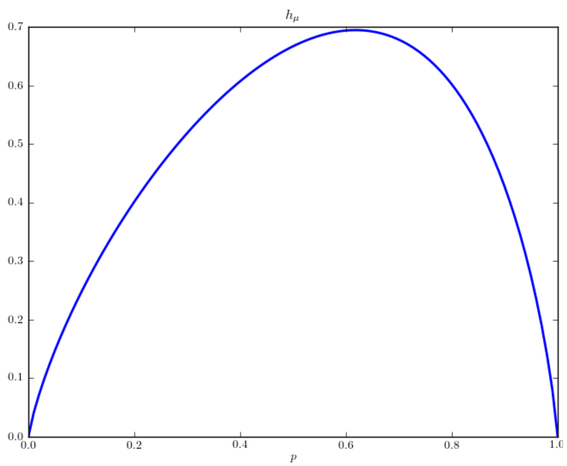
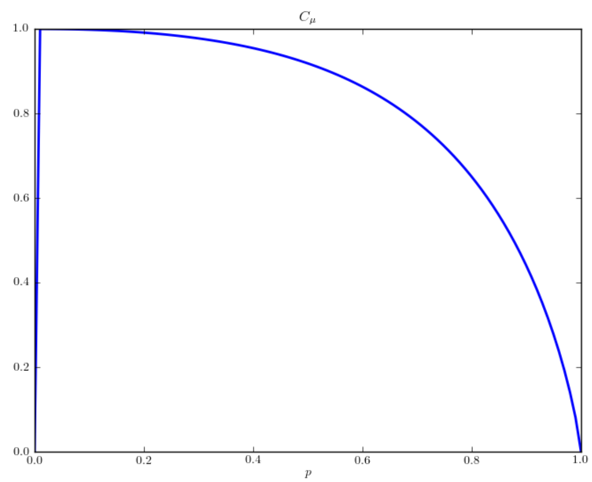
<u>Even:</u>

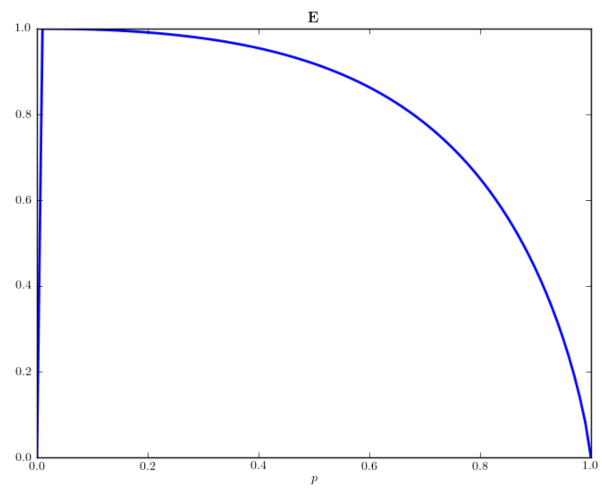
hμ



Cμ



E



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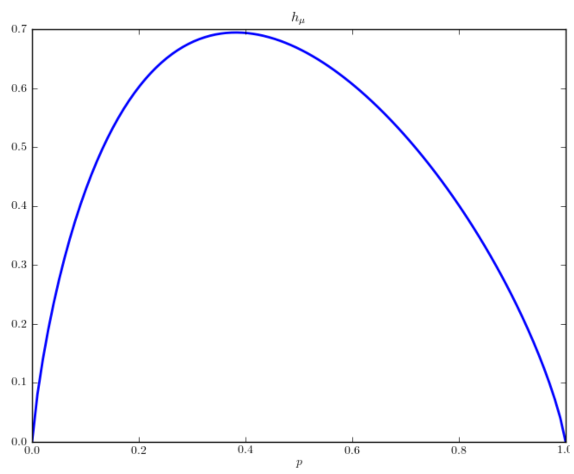
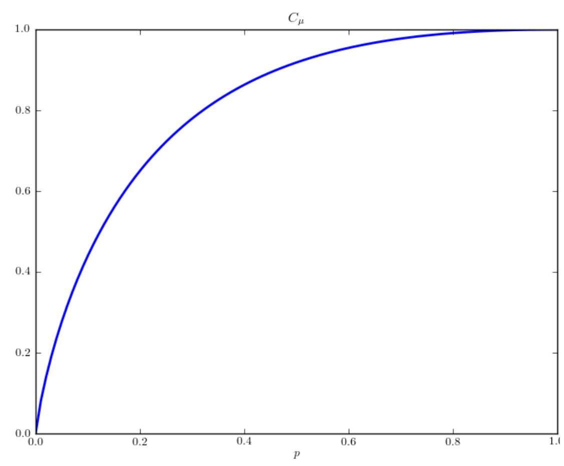
Golden Mean:

hμ



Cμ



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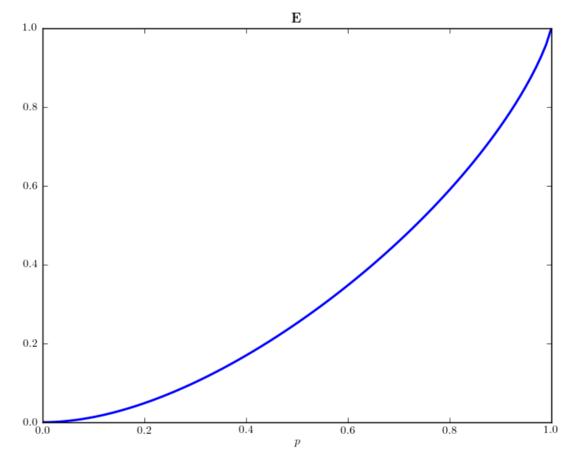

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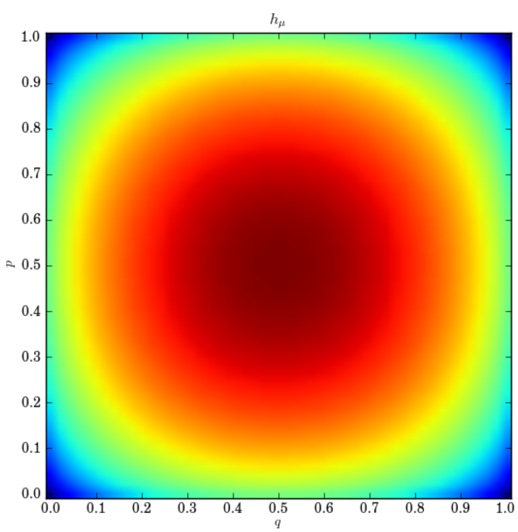
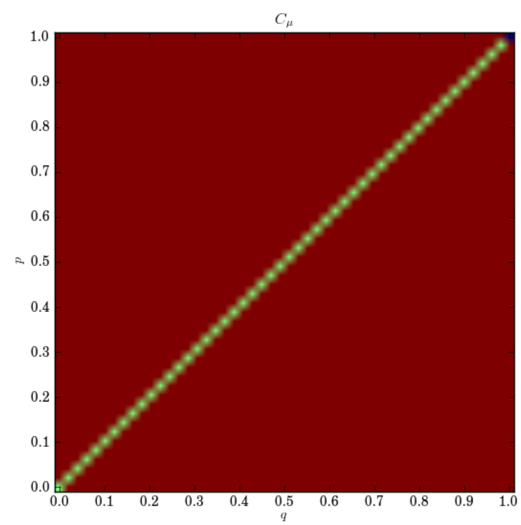
Telescope:

hμ



Cμ



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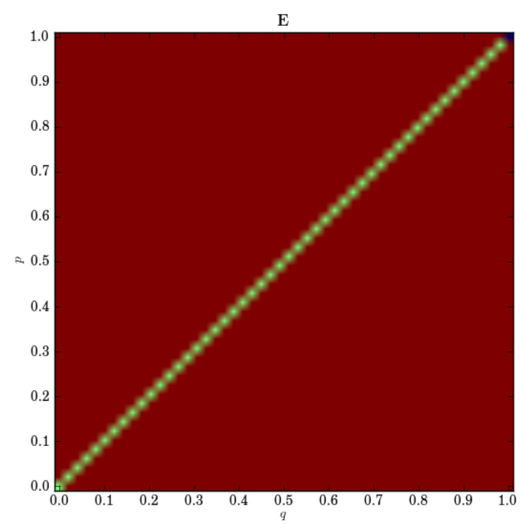

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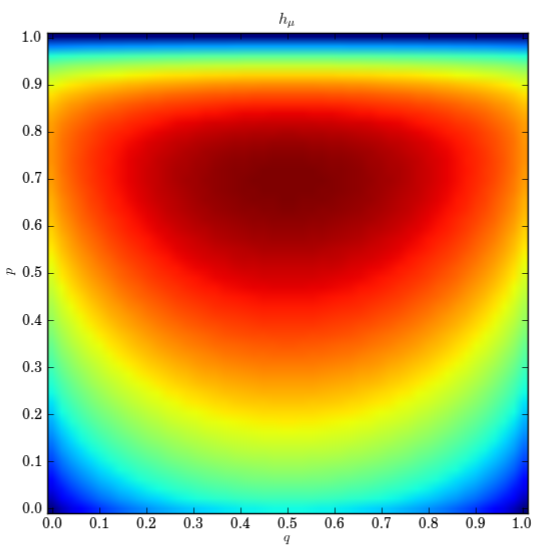
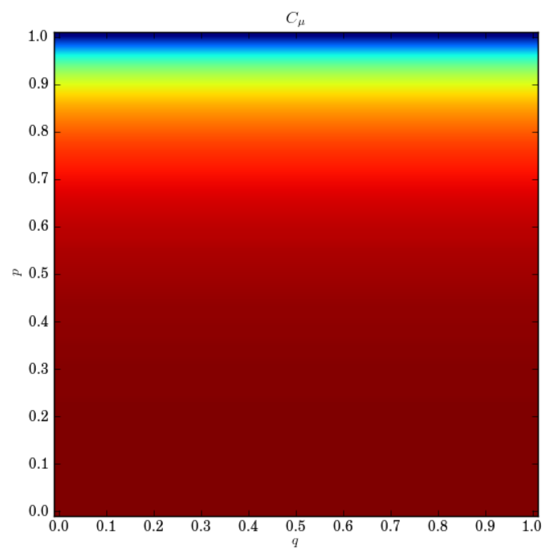
Noisy Random Phase Slip (NRPS):

hμ



Cμ



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