Nonlinear Dynamics of Passionflower Tendril Free Coiling

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Abstract:
This study challenges fundamental presumptions in the current physical knowledge of the process of tendril free coiling using the holistic interdisciplinary perspective and statistical methods of complex dynamical systems studies. Many vines have special organs called tendrils that coil around things they contact, attaching the vine to nearby surfaces, supporting its weight, and stabilizing its structure against environmental forces. Because of this function, all studies focus almost single-mindedly on contact coiling, in which both ends of the coil are fixed and the tendril is performing its primary evolved purpose. Of these, the few that address the nonlinear dynamics of contact coiling primarily use mathematical and computational models based on Kirchhoff's equations for finite rods with intrinsic curvature achieving equilibrium at a minimal energy state, in order to explain "perversions" – places where the coil shifts its handedness – in uniform helical coils. These models bear no obvious relevance to the process of free coiling – coiling without contact in open air – that produces an astonishing variety of morphologies, contrary to what would happen if there were a minimal energy state to which all coiling tends. Our study therefore attempts to integrate biochemical, physical, mathematical, and computational models in order to elucidate the complex stochastic dynamics of tendril free coiling. Using the statistical methods of computational mechanics, we analyze the morphologies of over five hundred free coils in order to measure their randomness (Shannon entropy) and structure (complexity) and determine correlated traits, in hopes of deriving a minimal model of tendril free coiling dynamics in the form of an epsilon-machine. We also plan to test our integrated model using computational simulations of free coiling, in order to compare the proximity of our model to real tendril dynamics. Better understanding the process of free coiling potentially transforms knowledge of tendril coiling in general. Simultaneously, we demonstrate the usefulness of the methods of multi-variable computational mechanics for studying patterns in biological systems.

Introduction

In addition to their beauty, passionflower tendrils display both significant regularity and differentiation in their free coiling patterns (Figure 1). This fact suggests that the ideas and tools for studying the nonlinear dynamics of complex systems – which combine both complexity (structure) and stochasticity (randomness) – may prove useful in elucidating their underlying structural dynamics and resulting morphologies. Whether one prefers to define self-organization in biological systems more generally as pattern formation (Crutchfield) or as a process whereby interactions at the "local" level without any centralized control produce patterns at the "global" level (Camazine et al), the clearly visible spatiotemporal patterns of tendril free coiling compel a far more complete and accurate explanation than currently exists.

No single scholarly article that I can find focuses primarily on free coiling in any vine species. Furthermore, many publications on contact coiling deny that "perversions" (a switch in handedness) occur in free coils at all, when in fact, our study suggests that 57% of all passionflower free coils have perversions. In general, the articles by physicists and mathematicians who write about the nonlinear dynamics of tendril coiling do not look at, study, or address real tendrils using empirical methods. Rather, after referring to the standard illustration of a contact coil drawn by Charles Darwin published in *The Movements and Habits of Climbing Plants* (1876), they proceed by assuming that contact coils form perfect uniform helices with one or more perversions in them, and they explain this ideal pattern using Kirchhoff's equations for rods with intrinsic curvature moving to equilibrium at a minimal energy state (Figures 2-3). The biological literature on tendril coiling has its own limitations owing to disciplinary trends over the last century. The shift from broad phenotypic observation (Darwin's classic study) to intricate genotypic and molecular biochemical studies presents a literary and conceptual divide between macro- and micro- approaches that for tendril coiling, at least, have yet to come together into a complete model of tendril coiling, especially one that also integrates its biomechanics.

Clearly, the field of tendril coiling studies is ripe for a new multifaceted approach that both challenges the conceptual limitations of the Kirchhoff status quo and poses a more thorough interdisciplinary model, one that attempts to consider the tendril as an interconnected biochemical, physical, mathematical and computational dynamic system. We ground our study in empirical analysis: the measure of over 500 free coiling tendrils from the passionflower vine in Christina's garden, quantifying their changing diameter (D), periodicity (P), handedness (H), "pervertedness" (Perv), angular axis rotation (A), and self-contact status (C). The data from the resulting 3,389 measurements – adding up to over 17 meters in total coil length were one to place them end to end – were compiled into a data string with the above six primary variables, subdivided into smaller subvariables. We then analyzed this string using the tools of computational mechanics, thus far running Markov chain analysis, Shannon block entropies, and mutual information, with hopes of achieving an intelligible epsilon-machine minimal model of coiling dynamics in the near future.

Simultaneously, we researched current knowledge of tendril coiling in order to try to discern a probable integrated explanation of the coiling process. *We put forward this explanatory model here not because we are certain that it is correct, but rather 1) as a reasonable hypothetical base for our simulations that study coiling* in silico, *and 2) as a way to point out what we feel are the current gaps in existing models of coiling by demonstrating the need for an*

alternative. Based upon the outcomes of this research, we selected the Turing reaction-diffusion equations, combined with equations for creating helices, as the most reasonable starting point for trying to understand and computationally simulate the variability we observe in free coils. Initial results are promising, with more refinement and results to come in the near future. These suggest not only that biochemical gradients may well play a foundational role in coiling dynamics, but also that the process of symmetry breaking may be involved. In contradistinction to scholarly assumptions that if a free coil has a perversion in it that this because it used to be a contact coil and somehow broke free, or, that in order to coil a tendril *has* to contact something at some time, internal dynamics of symmetry breaking may pose an alternative process that could result in perversions and other complex variability in free coils without ever having contact at all (Crutchfield's idea). If this seems probable and could be verified, it would transform our knowledge of the dynamics at work in contact coiling as well.

While we hope that our novel hypotheses and methods compel further verification through empirical biological studies, we primarily intend our study to serve as a demonstration of the usefulness of applying the methods of computational mechanics – pioneered by James Crutchfield and colleagues for state space analyses of complex dynamic systems – to the study of biological pattern formation. We use these statistical methods to analyze real tendril patterns as well as the patterns evident in our simulated coils, in order to compare our results across these two as a means to verify the proximity of our hypotheses and model to real tendril coiling. We also have chosen to use a multi-variable data string, owing to the complexity of tendril morphology and our desire to accurately describe all its major parameters through our data string. Our goal was to have enough description embedded in our coding process that one could come close to re-creating a particular tendril's morphology based upon its data string. Thus, instead of just having one or two parameters with 0s and 1s as our output, as has been the case with many studies using computational mechanics, we have six different major variables (D, P, H, Perv, A, C), each of which has either two or three sub-variables (d, D, 2, p, P, 3, L, R, S, 0, 1, 4, 8, 9, c, f) (Figures 4-9 offer more explanation).

We thus have the option of considering our data string as one "super-variable" (all tendril data combined), as six "variables" (the various tendril traits listed above – this is our predominant approach), or as 16 "sub-variables" (more specific tendril traits), depending on how we choose to parse or combine our data in order to answer different questions. While this permits detailed analysis of tendril patterning, including the discovery of closely correlated variables in coiling dynamics, it also exponentially increases the amount of data we have to process and complicates our analyses of it, particularly with regards to its meaning in relation to real coiling patterns. We also think we are breaking new paths with this multi-variable method (so therefore we do not yet know of a model to follow), which also adds its own exciting challenges and rewards. At least one of our means of data analysis (demonstrated in Figures 33-35, and 41-42) offers an innovative and informative plotting approach for revealing mutual information relative to the size of the block entropy of the variables involved. Finally, our initial attempts at deriving an epsilon-machine for just one of the variables was so unwieldy that we will need to use "optimal causal inference" to discern the fundamental structure beneath the noise. So, this report is partial with our work still in progress.

We begin with an overview of the biological and physical-mathematical models that describe the current state of knowledge about tendril coiling, inserting after each summary our own hypothetical working model for the biological and physical-mathematical aspects of tendril free coiling. Then, we describe our methods for empirically measuring, coding and analyzing our

sample of five hundred free coils using statistical methods of data evaluation via multi-variable computational mechanics. We first discuss the results of Markov chain analysis – primarily useful for identifying trends in the first and last coils of each tendril, as well as the probabilities of particular sequence transitions between each of the sub-variables that highlight characteristics of each variable in relation to overall coiling patterns. To these characteristics are then added the results of Shannon block entropy (L1-L5) analysis for each variable by itself and then allied with every other variable in pairwise, 3-way, 4-way, 5-way and 6-way combinations, which also includes measures of excess entropy and entropy rate. Some of our most revealing information resulted from considering the mutual information of each variable first in relation to all of the others combined and then in all pairwise combinations. While not yet complete, in the future our study will end with statistical comparison derived through computational mechanics of the structure and randomness of real coiling patterns and those of our *in silico* coils, in order to evaluate the strengths and limitations of our hypothetical models.

Biological Background and Initial Hypothesis

How do tendrils coil? The standard general answer is that cells on one side of a tendril contract causing it to become concave, while those on the opposite side elongate causing it to become convex. The tendril then curves in the direction of the concave curve.[1] This is officially termed thigmotropism (from Greek, touch-turning), whereby contact with something serves as the initial stimulus causing the response of contraction on the side of contact, wrapping the tendril around the object. More detailed descriptions, though, falter on exactly how this happens. Studies have attempted without definitive success to discern which chemical(s) initiates cell elongation or spurs cell contraction, as well as how they do this, although the best guesses are the hormones auxin and jasmonate, respectively.[2] Similarly, no study clearly explains the distribution throughout tendril tissues of the mechanical forces generated by contraction and elongation, and the effects that this transfer of forces has on overall coiling pattern. If a tendril coils only based upon touch, and if it is touched in only one place, how and which signals and forces move longitudinally through the tendril to effect a completed coiling process?

The foregoing unresolved issues pertain to contact coiling as well as to free coiling, and we would like more definitive answers. The following questions, however, that are posed primarily by considering free coiling as a result of this study, are especially interesting for the challenges they pose to the standard explanation of coiling overall as solely a thigmotropic

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[1]Accordingly, older literature on tendril coiling use the phrase "differential growth" in their descriptions, but most of the literature claims that tendrils are fully grown before they begin coiling. Growth implies both cell division and cell elongation/maturation, whereas coiling does not for the most part involve cell division as part of its geometric change. Rather, existing cells undergo contraction and elongation on opposite sides of the tendril. The only possibility of cell division and differentiation playing a key role in coiling may be the tendril's creation of gelatinous-fiber cells post-contact or perhaps as part of its general maturation. Much more research needs to be done in this area, and g-fibers cells will be discussed in more detail throughout this study.

[2] Despite its fundamental role in a multitude of plant growth processes, auxin is still directly indetectable by researchers, who have to rely on indirect methods such as _____ to gauge auxin levels within a cell. See _____.

response. Why and how do free coils coil, achieving such great variety? (cover page and Figure 1) Why do free coils have perversions? (Figure 10) Why do free coils sometimes coil in multiple distant places at once? (Figure 11) Do they really have to have been in contact with something in order to free coil, or might coiling be triggered by some internal dynamic?

The most recent advance in tendril knowledge has been the discovery in 2007 by Christoper Meloche, J. Paul Knox, and Kevin Vaughn of gelatinous-fiber (g-fiber) cells in coiling tendrils. G-fiber cells are known and studied in reaction and tension wood in angiosperms as the cells responsible for exerting extreme contraction forces in order to right trees growing in less than optimal directions, but they have not been found before in herbaceous plants. The presence of g-fiber cells in tendrils therefore raises interesting questions about how they got there evolutionarily and their possible origination within individual tendrils from self-organizing processes kick-started by contact prior to coiling. In a later article, Andrew Bowling and Vaughn claim that g-fiber cell action is chiefly responsible for the contracting forces in tendril coiling, a claim supported inversely by the fact that g-fiber cells are not found at all in the few rare tendrils that fail to coil. They assert that g-fiber cells arise spontaneously post-contact (on day 4 or 5?) in the cortical region. However, some tendrils coil much more quickly than this, so more research needs to be done to ascertain when the g-fiber cells become present or differentiated in tendril growth and maturity. Finally, g-fiber cell distribution throughout the tendril affects its directional coiling capacity. Tendrils in species that are contact sensitive on all sides and coil bi-directionally (either clockwise or counterclockwise) have a cylinder of g-fiber cells around their core, whereas those that only coil in one direction (such as cucumber tendrils) have g-fiber cells only on one side of the tendril, in an arrangement referred to as a Bronconi plate (Figure 12).

The action of g-fiber cells causes an overall net contraction and twisting in the tissue on the contact side of a tendril (Figure 13). While the literature does not explain what triggers them to begin their process, their special three-layer cell wall structure (primary, and secondary S1 and S2 layers) with its alternating microtubule orientation and g-fiber chemical composition allows them to cause significant deformation in the tissues where they are located and active (Figures 14-15). Prior to g-fiber action, the microtubules (MTs) in the S1 layer of all the g-fiber cells are generally aligned in parallel, with the microtubules of the S2 layer oriented at approximately 45-degrees to the S1 MTs (shown partially in Figure 14). At the onset of g-fiber action, the S1 and S2 layers, which are rich in lignin, shed water and shrink while simultaneously, the g-fiber layer (whose microtubules align perpendicular to the S1 and at a 45-degree angle to the S2 MTs) just to the inside of the S2 layer absorbs enough water to cause the cell overall to balloon in size. Presumably, the pressure on the S1 and S2 layers caused by this expansion ruptures the MT alignments, and depending on the variable amounts of lignin present in these layers and the stiffening that lignin causes, the newly disrupted MT orientations hold fast and even become more pronounced as the cell dries out owing to the g-fiber layer shedding the water it had absorbed. The overall effect is both a net contraction and a serious twisting force that ripples throughout surrounding tissues, causing what has been described in one article as "spiral ruptures" ___get exact phrase__ and source. The specifics of this process, however, as well as how and why the cell dries out and how long this takes, are not well explained. Bowling and Vaughn hypothesize that g-fiber action combined with variable amounts of lignin in the active contact zone may be responsible for the variety observed in tendril coiling morphologies.

While knowledge of g-fiber cell action does help explain the coiling process and its geometric morphological variability, this only seems to constitute one-half of the curvature puzzle, since cell elongation on the opposite side apparently occurs at the same time, providing

the bending action necessary for general helical formation. A straight cylinder can be transformed into a helix simply through a combined process of twist and bend (Figures 16-17). At successive intervals, if one twists a cross-section of a cylinder around the Z-axis at a set angle while also bending the length of the interval in relation to the XY-plane at a particular angle, and if these two angles of rotation remain constant, a cylinder will be transformed into a perfect helix. G-fiber cells clearly provide the necessary twist as well as an additional variable force of contraction, but bending comes from cell elongation on the opposite side. Since no article on tendrils has yet examined elongation in relation to g-fiber cell action in order to integrate it into current models of the coiling process, we turned to the literature on the phytohormone auxin outside of tendrils (primarily using studies with *Arabidopsis*) in order to better understand the role it may play in what seems like the best explanation for cell elongation in tendril coiling.

Although some disagreement exists about auxin's role as a key trigger for cell elongation in tendril coiling, more articles propose than deny its influence (_____sources___). Auxin is indole-3-acetic acid (IAA) and it is widely hailed as the most influential plant hormone, owing to its starring role in at least four major plant processes – phyllotaxis, growth via cell elongation, organogenesis, and tropic response – as well as a myriad of other processes. It is produced primarily in shoot and root apical meristem tissue (__doublecheck__) and is recognized as a self-organizing, self-regulating "morphogen" that is primarily responsible for key aspects of plant morphology. It does this through three gene families that it triggers to produce various proteins that in turn halt auxin's production; ___fill in a little more on auxin's feedback loops ___. It plays a leading role in both phototropism and gravitropism (and arguably, in thigmotropism as well), prompting cell elongation on one side of a shoot or root in order to effect optimal plant growth in relation to environmental conditions (Figures 13 and 18). **Two models of distribution predominate equally in the literature as explanations for how auxin is dispersed: the chemiosmotic/canalization theory, which proposes _____, and Turing reaction-diffusion processes establishing morphogen gradients.[3] Regardless of which model one prefers, in both __doublecheck_ it is accepted that phyllotaxis, organogenesis, and growth and tropic responses via cell elongation occur in areas of high auxin concentration (Figure 19). Gradients of auxin therefore function as thresholds for various gene activities regulated by auxin in order to effect plant morphology. __Is canalization more about vertical distribution, whereas reaction-diffusion about lateral transfer?___ If so and regardless, mention that here and describe the process of lateral transfer – diffusion, PIN carriers for active export, unidirectional movement across a tissue ___ toward auxin gradient maxima,[4] with PINOID flips in polarity (Figure 20) . Furthermore, a recent study posits that PIN polarity (and therefore auxin transport direction) and cortical MTs are both responsive to mechanical stresses (such as that of contact in tendrils), aligning themselves with each other and in parallel with the direction of the stress (Figure 21).**

Hypothetical Biological Working Model

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[3] Supporters of the chemiosmotic theory and canalization include _____; those in favor of auxin as a morphogen functioning according to Turing's reaction-diffusion model are _____.

[4] It is not clear from the literature how auxin gradients get established in order for auxin to then be transported by PIN proteins to areas of high auxin concentration. This seems to be a chicken and egg problem that needs better explanation.

Based on selections from the foregoing, our general biochemical and biophysical model of tendril coiling assumes multidirectional coiling, owing to this being the characteristic of passionflower tendrils. However, it is useful to begin by considering unidirectional coiling tendrils that have the Bronconi plate formation of g-fiber cells (Figure 12). In these tendrils, which successfully achieve helical coils (if only in one direction with a single handedness), the most possible g-fiber action is distributed around approximately one-third of the circumference of the tendril. This suggests that in multidirectional coiling tendrils, a one-third active contact zone of g-fiber cells out of the cylinder of all g-fiber cells surrounding the tendril is sufficient to achieve helical coiling. The primary differences thus are both that multidirectional coiling tendrils are contact-sensitive on all sides, and also that they can coil in any direction, reversing their handedness at any time based on the location of which g-fiber cells are active at any given time in relation to those that were active just prior. Our model thus presumes approximately a one-third circumferential active g-fiber contact zone, opposite of which active cell elongation occurs owing to a high auxin gradient.

Upon contact (if we accept this as a working assumption), something in the tendril senses the contact and likely chemically and biophysically signals the mechanical force across the tendril. While this function may be performed by Ca+2, there is no definitive explanation in the tendril literature. We propose that owing to their ability to reorient themselves based upon mechanical force, that the MTs in the cortical tissues of the tendril on the contact side align so as to communicate both this force and the signal across the tendril (Figure 21). Simultaneously, PIN proteins also align with the force direction in parallel with the MTs, thereby establishing a unidirectional auxin transfer highway oriented to the opposite side of the tendril from contact. Meanwhile, in some amount of time, the g-fiber cells originate and/or become active, beginning to twist and contract the contact side of the tendril with variable amounts of stiffness in different zones owing to differential amounts of lignin in the S1 and S2 layers of the g-fiber cells on the contact side of the tendril. On the opposite side, the auxin gradient is increasing owing to successful PIN protein transfer to this area, causing cell elongation to begin. Owing to the connectedness of cells across tendril tissues, these mechanical forces of contraction and elongation are transferred longitudinally and perhaps diagonally owing to ongoing twist+contract+bend forces with differential gradients of all three, owing to the constantly changing parameters of lignin stiffness, auxin gradients and proportional elongation, and the ongoing processes of deformation during coiling. We also propose the possibility that PINOID polarity flips in the direction of PIN transfer of auxin could offer one explanation for a shift in handedness in the direction of coiling. However this could also be achieved through changing gradients of auxin and lignin in relation to the previous contiguous coiling zone, as well as to coiling occurring at multiple locations simultaneously on the tendril that then has to negotiate its meeting points upon arriving at the same location on the tendril from opposite directions. Finally, we propose that an internal process of symmetry breaking in the biochemical and mechanical system dynamics could also explain handedness changes or even the start of coiling in the first place without contact, especially if this process occurs at different locations on the tendril that effect coiling in opposite directions.

Physical-Mathematical Background and Initial Hypothesis

Owing to their use of Kirchhoff's equations and focus solely on contact coiling tendrils fixed at both ends (Figures 2-3), current physical-mathematical models that assume fixed

uniform helical coiling fail to account for the wide variety of tendril patterns observed in free coils. Our model therefore begins with different assumptions. Rather than presuming that tendrils coil to a minimal energy state, we think that coil variations suggest that something apart from achieving equilibrium at a minimal energy state prompts observable morphologies. Because complex dynamic biological systems cross between the disciplinary specialties of those who study them, we strive to link our physical-mathematical model to our biological model based upon our research as well as upon our observations of tendril formations on the vine.

Hypothetical Physical-Mathematical Working Model

Rather than ending with a uniform helix, our model conceptually begins with the equations for transforming a long cylinder into a uniform helix, since this is the primary shape that results from coiling but then becomes varied in diameter, radii, periodicity and angle during actual free coiling. The twist and bend approach described (Figures 16-17) above can be mathematically specified using equations with sine and cosine with two fixed angles of rotation that define the dimensional parameters of a uniform helix. Our initial python code therefore could produce a variety of uniform helical forms, but none that possessed changing radii and perimeter curvatures such as is necessary to create a spiral or other variable forms that real free coils take (Figure 22).

Based upon the biological studies that posit how the Turing reaction-diffusion process might explain auxin morphogen gradients in shoot apical meristem tissues, as well as the likely possibility that auxin plays a key role in the cell elongation that results during thigmotropic response and the accessibility of the Turing reaction-diffusion equations, we chose this model as our starting point and integrated the equations with our uniform helical code (Figures 23-24).[5] In the Turing equations, **____an activator becomes its own inhibitor or ___ get the basic description of the model straight___, and then show how it conceptually might apply in tendril biological process.** Auxin is a morphogen that in fact works through feedback loops as its own activator and inhibitor. It is also possible to think roughly of the equations not as describing patterns in the gradients of auxin but rather in the gradients of lignin (although this is not accurate biologically speaking as lignin does not function as a "morphogen"). As gradients of lignin vary the stiffness of the contact zone and cause a variable limitation on the twist+contract action of the g-fiber cells (not yet integrated into our code), the effects of the Turing equations might prove helpful in approximating the twist+contract deformations on the contact side of the coil. The best model in our opinion, however, given the current state of knowledge of this system, would be one that applies the Turing equations to the side opposite contact in proportion to auxin gradient levels, while at the same time mathematically describing the actual angles and forces of twist+contract in the active g-fiber contact zone, so that these two processes would work in combination. **Although one article on tension wood suggests that ___a spiral___ or generally that MT's align at a 45-degree angle to the g-fibers and S1 layer MTs,** there is not enough specificity in these articles to mathematically specify a particular vector and force that would accurately capture the effects of g-fiber cell action on the contact side.

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[5] Actually our Turing code portion is written in reverse (after we had already achieved uniform helices), first applying the Turing equations to a "1D" cylinder (or line with added fatness) and then implementing the twist+bend sine-cosine equations, having the bending angle vary in proportion to the level of the activator/auxin at that position vertically on the cylinder.

Our next addition to our physical-mathematical model is to extend our "1D" Turing model to a "2D" one. We will do this by unfolding the cylinder of the tendril at any given cross section into a rectangle across the ends of which, as continuous boundaries, the auxin gradients can move around and up and down the tendril cylinder. We also will try to integrate some simulation of the variable twist+contract action on the contact side to work in tandem with the auxin gradient's proportional bending moment. The output of our code will be first evaluated visually to compare its observable patterns with those of real tendrils, and upon attaining a close approximation, we plan to simulate a large number of coils that we will analyze using computational mechanics in the same way we did for our real tendril sample.

Methods of Empirical and Computational Mechanics Analysis

Empirical Analysis
Beginning in April when the passionflower vine was just starting to grow after the winter, over the next few weeks we collected over five hundred free coils off the passionflower vine. While we did not try to count the calculate the percentage of coils on a vine that are free coils as compared to contact coils, owing to the vast number of coils on the vine (which is two years old and spans a fence for about thirty feet) and the rapidity with which it was growing, suffice it to say that in the first few weeks we did no damage to the vine by removing over 500 free coils. Similarly, after a month of growth when the days became windy in May, the vine became so interwoven through contact coils that free coils became harder to find, simply because I could not pry apart the vine without damaging it. The free coils we collected demonstrated a wide variety of coiling types. We began measuring our sample soon after harvesting them. Undoubtedly, some of them became skinnier upon desiccation but still retained their basic morphologies (we did not include "fatness" of the tendril as one of our measures), presumably because of the role that lignin plays in stiffening the coils. We selected a five-millimeter increment for measurement because this seemed a reasonable measure for fairly accurately describing the morphologies that appear across a tendril, based on the general rapidity with which features change. Our goal was to have our measurement data be sufficiently detailed that one could re-create the morphology of the tendril without looking at it but solely from looking at its data string (Figure 25).
In addition to measuring the free coils after removing them from the vine, we also studied them on the vine, doing some time-lapse photographic studies and pinching and marking them down one side to study their torsion as they coil. I hope to continue these studies to determine if a single free coil will coils in two directions (with opposite handedness) if it is pinched in multiple places. However, because my vine is outdoors, it is hard to control the environment in order to determine whether or not any tendril free coil I see has ever been in contact with something nearby.

Multi-variable Computational Mechanics Analysis
Standard techniques of computational mechanics include Markov chain analysis, block entropies, excess entropy, entropy rates, measures of complexity and mutual information, as well as the creation of epsilon machines that diagram the relatedness of causal states. Owing to our choice to use six major variables and sixteen sub-variables, the number of possible combinations across different means of combinatorial analysis was exceptionally large. Fortunately, out of the 324 possible symbols that arise from the combination of each of the sixteen sub-variables with

the others, almost half of them never appear (159 symbols have zero probability). The primary reasons for this is as follows. These symbols have a "c" (standing for self-contact) in them in position 5 (e.g, '012345'). As only 5% of our measurements have "c", it follows that 95% don't, so many combinations with a "c" never exist. Similarly, many non-existent symbols have an "8" (180-degree turn, full reversal) or a "9" in them (45-90-degree angular axis rotation). As only 8% of our measurements overall have an "8", and only 15% overall have a "9", these symbols rarely occur. Other symbols with zero probability have an "S" in them, which stands for "straight" (meaning, not left-handed or right-handed, but in between) but also were combined with a P (meaning, 2-4 coil periodicity) or 3 (over 4 coils periodicity). As tightly coiling tendrils cannot be straight tendrils, these combinations never appear in our real data. Symbols that combine "D" or "2" (both of which mean, large diameter coils) with either a "P" or a "3" (meaning, a high number of coils) also don't appear in real tendrils, which do not have a high number of large diameter coils in a row. The results that appear below thus are obviously those that reflect actual tendril morphology and not just our choice of variables and sub-variables.

Results

General Considerations

  Computational mechanics provides different measures of randomness and structure. *Shannon entropy* H(L) indicates the quantity of randomness in bits of a data string when considered as "words" of different length "L" (L1, L2, L3, L4, and L5). *Excess entropy* E offers a quantitative measure of the amount of mutual information in bits that is shared between the past and the future in a data string when considered at any point in the string. *Entropy rate* (the slope of E when graphed) measures the rate of unique information/surprise with each new measurement (meaning in the data string itself, or, with each jump to a new L length, say from L=2 to L=3?) when considered in succession, to the extent that steep slopes indicate a higher rate of surprise (greater unpredictability) whereas gradual slopes show a lower rate of surprise (greater predictability).

  Because we processed most of our data as variables, meaning we used the six main categories of diameter (D), periodicity (P), handedness (H), pervertedness (Perv), angular axis rotation (A), and self-contact status (C) rather than the sixteen sub-variables, when each of the above measures of randomness and structure is considered for each variable, the results are often complicated to interpret. Some even seem contradictory, with some output values suggesting that for a given variable, that there are both lesser and greater amounts of randomness depending on which measure (H(L), E, or entropy rate) one looks at. Yet if one wants to make a determination overall about how random or structured a single variable is, especially when considered in its relations to other variables and the entire data string, gauging the "weight" and importance of these seemingly contradictory indicators of randomness and structure becomes very difficult. This is clearly one of the main challenges of multi-variable computational mechanics. Our approach was to look for visible trends within single variables for each of the measures of randomness/structure (H(L), E, and entropy rate), predominantly at the L5 value, as well as trends in the results of that one variable at L5 when it was joined with each of the others in pairwise, 3-way, 4-way, 5-way and 6-way combinations. However, when one has six variables, the number of combinations with all others is 63 total (57 not counting the original 6 on their own), which when multiplied by the three main measures we considered (H(L), E and entropy

rate) at the L5 level only means there are 189 values to consider and interpret in relation to each other.

Furthermore, with regards to the results that follow, it is important to keep in mind that each of the variables has its own unique L5 entropy, meaning that when considered solely as the string of its own sub-variables tendril by tendril, each has its own unique amount of randomness. Because every symbol (e.g., "dPL04f") in our measurements is made up of six sub-variables, the fact that each variable has its own unique amount of L5 randomness means that when we move from single variable analysis to pairwise or higher variable combinations, the results must be interpreted relative to the amount of unique randomness of each variable. For example, "C" (Contact Status, with sub-variables "c" for self-contact and "f" for free) has an L5 entropy of 0.85 bits, but "D" has an L5 entropy of 5.33 bits. If we visualize these different entropy values as the area of circles (Figure 26), as if they were going to be used in a Venn diagram to show the possible overlaps or mutual information between two or more variables, their relative size places a limit on their possible amount of mutual information. For example, the maximum mutual information between "C" and "D" would be if "C" were subsumed entirely by "D" in a Venn diagram, so 0.85 bits. But a partial mutual information between "D" and "H" (where "H" has an L5 entropy of 4.90 bits) can easily be higher than the maximum between "C" and "D." Their different L5 entropies set a limit on how each variable must be considered in relation to each other and in relation to the whole, such that some data result comparisons require "normalization" and/or need to be viewed as relative percentages in order to have a fair comparison for interpretation.

Markov Chain Results

Our Markov chain analysis was primarily useful for two purposes: ascertaining the probability of each sub-variable's appearance in the first and last symbols of each tendril's data string, and well as the probabilities of each of the preceding and succeeding sub-symbol for each sub-variable within its own variable's data string. The former was made possible by the inclusion of "New Tendril" (NT) in between each tendril's data string. Our Markov chains show the probabilities of each sub-variable going into and out of "New Tendril" (Figures 4-9). This actually made for seventeen sub-variables overall for our Markov chain analysis, so the percentages below have taken this into account. These two pieces of information reveal patterns in how tendrils coil at the tips and at the end of the coils as they open up towards the base of the tendril, as well as patterns within each variable about how it generally progresses from tip to base. These are discussed below.

<u>First and Last Symbols in Each Tendril's Data String</u>

In considering the significance of these percentages, it is helpful to know the percentages overall for each sub-variable within its own variable's data string, since if a sub-variable rarely appears, then if it frequently appears in the first or last symbol, that reveals a pattern of higher significance. This information therefore appears first.

<u>Overall percentage that each sub-variable appears in its own variable's full data string:</u>
Diameter: "d" = 27.8% (0-2.99mm); "D" = 46.1% (3.0-8.0 mm); "2" = 26.1% (> 8.0 mm)
Periodicity: "p" = 53.3% (0-1.9 coils); "P" = 27.3% (2-4 coils); "3" = 19.4% (> 4 coils)

Handedness: "L" = 51.7% (Left); "R" = 38.4% (Right); "S" = 9.9% (Straight, usually in a
        perversion in between L & R)
Pervertedness: "0" = 78.1% (no perversion); "1" = 21.9% (perversion)
Angular Axis Rotation: "4" = 76.6% (0-44.99-degree angular axis change); "8" = 8.3% (180-
        degree angular axis change; full reversal); "9" = 15.1% (45-90-degree angular axis
        change)
Contact Status: "c" = 5.6% (self-contact); "f" = 94.4% (free)

First symbols at tendril's tip:
Diameter: 48% of all tendril tips start with "d" (0-2.99mm diameter); 45% start with "D" (3.0-
        8.0 mm diameter)
Periodicity: 56% of all tendril tips start with "3" (> 4 coils); 31% start with "P" (2-4 coils)
Handedness: 55% of all tendril tips start with "L" turning counterclockwise; 44% start with "R"
        turning clockwise
Pervertedness: 28% of all tendril tips start with "1" (perversion)
Angular Axis Rotation: 58% of all tendril tips start with "4" (0-44.99-degree angular axis
        change); 24% start with "8" (180-degree angular axis change; full reversal); 18% start
        with "9" (45-90-degree angular axis change)
Contact Status: 16% of all tendril tips start with "c" (self-contact)

Last symbols at coil's end:
Diameter: 69.1% of tendril sequences end with "D"; 19.5% end with "2"
Periodicity: 54.1% of tendril sequences end with "p"; 31.4% end with "P"
Handedness: 56% of tendril sequences end with "L"; 44% end with "R"
Pervertedness: 95.1% of tendril sequences end with "0"
Angular Axis Rotation: 82.9% of tendril sequences end with "4"; 8.4% end with "8"/full 180-
        degree reversal; 10.2% end with "9"
Contact Status: 95.8% end with "f"

      Free coiling tendril tips have a number of notable characteristics. Generally, they have smaller diameters than appear elsewhere through the coil, and also tighter coils with higher periodicity. Whereas diameters of under 3 mm ("d") only appear 28% of the time overall, 48% of all tendril tips begin with "d"; similarly, whereas periodicities of "3" (greater than 4 coils per 5mm increment) appear only 19% overall, 56% of all tips begin with "3". These two facts suggest a high degree of correlation between diameter and periodicity, a fact that we will return to in our block entropy and mutual information analysis below. Even more striking is the fact that 28% of tendril tips have a perversion, especially given that perversions appear only 22% overall. Realize that many scholars who are experts in tendrils believe that free coils *never* have perversions, unless they were in contact with something and got their perversion and then somehow broke free again to become a "free coil." In fact, a *majority* of free coils – 57.4% – in our sample have a perversion somewhere in their coil. The fact that 28% of all tendril tips have a perversion in them reveals that almost half of all tendrils that have a perversion in them at all have them at their tips. There are surprises with angular axis rotation and contact status as well. That 24% of all tips begin with a full 180-degree reversal ("8") is remarkable, considering that "8" appears only 8.3% of the time overall; only slightly less surprising is that 18% of tendril tips have a 45-90-degree changes ("9") when "9" appears only 15% of the time overall. Finally, tips

of tendrils are almost three times more likely to have self-contact than they are to have self-contact at all; whereas self-contact occurs only 6% of the time overall (and in fact, less overall in tendril morphology because sometimes the same contact was noted in two different measurements/symbols), 16% of tips begin with self-contact.

The pattern of how a coil ends, where it opens up towards the base of the tendril that attaches to the vine, is much more predictable than the pattern at the tip. Coils end with large diameters (89% of last symbols have diameters greater than 3 mm) and small periodicities (54% of last symbols – which represent the last 5 mm of the coil – have less than two coils). As the base of a tendril is usually straight for a few inches up to where the coil is, and the coil has to transition from being curved to becoming straight, it does so most often by loosening up its coiling pattern to broader diameters and a few number of coils. Owing to the fact that we chose a measurement increment of 5 mm, when a coil's diameter exceeded 5 mm we would begin measuring around its perimeter (since it otherwise would not fit into our increment). This means that for large diameter coils, there are a higher number of symbols in a row that mark the higher diameter and lower periodicity. The fact that these occur in the last symbols thus reflects both the fact that the coil is opening up and our measurement strategy; four of the top five most common symbols overall have large diameters (either D or "2") and lower periodicity ("p") ('2pL04f', '2pR04f', 'DpL04f', 'DpR04f'), again further suggesting a close correlation overall between diameter and periodicity as variables that share information with each other. Coils are more likely to end with left-handed directionality (just as they are more likely to begin the same way) and have no perversions, little angular axis rotation (less than 45 degrees), and no self-contact.

Most Common Predecessor and Successor to Each Sub-variable

The following analysis reveals patterns within each variable about how it transitions between its sub-variables. Only the sub-variable with the highest probability of preceding or succeeding another is listed here, except for in a few cases in which another sub-variable is almost as likely to fill that role and deserves consideration.

Diameter:
62% "d" before "d" and 62% "d" after
61% "D" before "D" and 61% "D" after
68% "2" before "2" and 68% "2" after

That each diameter sub-variable is most likely to be preceded and succeeded by itself reveals a general consistency of diameter, in that it changes slowly regardless of range or size. However, this data may also point to our measuring choice of using 5mm increments for each symbol, as noted somewhat above. If a coil falls in the "D" or "2" range (say, > 5mm), then we would switch to measuring around the perimeter of the coil, so that the number of symbols for large diameter coils greater than 5 mm are higher and more sequential owing to this switch in measuring method. This may skew the appearance making transitions look somewhat slower than they actually are, for there can be big jumps from small coils to large coils. When this occurs however, there is also usually a marked decrease in periodicity down to "p" (0-1 coils).

Periodicity:
75% "p" before "p" and 75% "p" after
43% "P" before "P" and 43% "P" after

44% "3" before "3" and 44% "3" after

As with diameter, in periodicity each sub-variable is always most likely to be preceded and succeeded by itself, showing gradual changes in periodicity overall. However, the case with "3" (> 4 coils per 5 mm increment) is a bit more interesting, in that "New Tendril" ("NT") is 43% likely to precede "3", just one percent below "3"'s likelihood. Together this shows that if a tendril is tightly coiled, that this high periodicity is likely to last beyond one measurement, and also that tight coiling is more likely to occur at the tip of the tendril.

Handedness:
71% "L" before "L" and 71% "L" after
66% "R" before "R" and 66% "R" after
49% "S" before "S" and 49% "S" after

The data for "L" (Left) and "R" (Right) show that although tendrils flip their handedness across a perversion at least once in 57% of our sample, they usually maintain the handedness they have for multiple measurements (quite some distance) before changing their directionality. The data for "S" (Straight), which we used primarily during a perversion when a coil was in between changing its directionality, reveals that most perversions are longer than 5 mm, such that the full accounting for a single perversion may fall into more than one symbol/measurement. Also, some perversions are followed immediately by other perversions, so this also helps account for sequential "S" symbols. Importantly (and not shown above), when either an "L" or an "R" succeed and "S" (meaning, follow a perversion), they occur with almost 50-50 probability: L appears 25% of the time and R appears 24% of the time following an "S". "Handedness" as a variable is therefore expected to possess a high level of randomness in its Shannon entropy and entropy rate measures, close to that of a "fair coin."

Pervertedness:
72% "0" before "0" and 72% "0" after
45% "1" before "1" and 53% "0" after (but 1 is 45% after)

As was shown in the "Handedness" data just discussed, when perversions occur, they are long enough to be preceded by a symbol that also shows the perversion; similarly, perversions can occur back to back. In the latter situation, if there are an even number of perversions in a row, then the handedness will be the same going into the perversion sequence and coming out of it. If there are an odd number of perversions in a row, then the handedness will change on either side of the perversion sequence. A symbol without a perversion ("0") is more likely, however, to follow a symbol with a perversions ("1"), likely owing to the fact that 78% of all of our symbols have no perversions.

Angular Axis Rotation:
71% "4" before "4" and 71% "4" after
42.8% "NT" before "8" and 60% "4" after
45.5% "4" before "9" and 56% "4" after

The most obvious take away from the fact that "8" is most likely to be preceded by "NT" (New Tendril) is that "8s" are most likely to start a sequence when there is an "8" in the symbol at all. As "4" (0-44.9-degree axis change) appears 77% of the time overall, it is always the most likely successor to any symbol ("8" appears overall only 8% of the time, and "9" only 15% of

the time). However, the fact that "4" usually follows an "8" or "9" shows that extreme changes in direction do not usually last beyond one measurement.

Contact Status:
42.7% "NT" before "c" and 59% "f" after
83% "f" before "f" and 83% "f" after

When there is self-contact in a tendril, it is more likely to occur at the tip than elsewhere on the tendril. We can see this from two things: "c"'s most common predecessor is NT 43% of the time, and also, considering that "c" is only present 5.55% of the time in the full datastring, the fact that tendrils start 16% of the time with a "c" shows that it appears in first symbols three times as often as it generally appears overall. This interpretation is supported by the data for first symbols in a string, discussed above.

Block Entropy Results: Block Entropy, Excess Entropy, and Entropy Rate (Figures 27-57)

For the following data, it is important to realize that as variables are combined into pairwise, 3-way, 4-way, 5-way, and 6-way combinations, that each variable appears in proportionally more of the combinations as more are combined. For example, when single variables are considered by themselves, they each occur in only 1 of the 6 (16.7% of the time). When in pairs, each variable shares in 5 out of the 15 possible combinations (33%); in 3-way combinations, 10 out of 20 (50%); in 4-way combinations, 10 out of 15 (66.7%); in 5-way combinations, 5 out of 6 (83.3%); and in 6-way combination, they all occur once (100%). This means that as the combinations contain a higher number of variables, that it becomes harder to distinguish if a particular variable falls towards one end or the other with regards to exhibiting lesser or greater randomness in any of our three major measurements (H(x), E, and entropy rate). This is because the variable is more distributed across all of the combinations. At the same time, the range of the values (in bits, or in slope) generally becomes narrower as a higher number of variables are combined (Figures 27-31). The resulting values for each of the combinations within a 3-way, 4-way, or 5-way combination sequence, as a higher number of variables are combined, become less distinct and spread apart in value, even while they are more fully distributed across all of the possible combinations.

Because of this, we are focusing our in-depth analysis on the single and pairwise variable combinations. However, the 3-way and 4-way combinations are helpful in finding patterns of randomness across all levels of combination for particular variables and symbols, which we referred to as "notable symbols" and will discuss at the end of this section (Figures 56-57). It is also useful to note a few trends before diving into more detailed analysis. For single variables and pairwise combinations, as block entropy increases, so does mutual information (with a few outstanding exceptions), prior to being adjusted relative to the size of its block entropy overall in comparison with those with which it is sharing information. For 3-way, 4-way, and 5-way combinations, as block entropy increases, generally the excess entropy does too, whereas the entropy rate tends to decrease overall (again, with a few notable exceptions).

Diameter

By itself, diameter has the highest block entropy, a mid-range excess entropy, and the steepest entropy rate (shared with handedness, at .82), indicating that it possesses a high degree

of randomness. Yet, it has the most mutual information in relation to all other variables combined (meaning, I[H(D); H(PHPervAC)], at 4.68 bits), and makes the most unique contribution overall relative to its size (12.2%, 0.65 bits).

Pairwise, diameter appears in 4:5 highest block entropies (DH at 7.97 bits, DA at 7.68, DPerv at 7.52, DP at 7.44), 3:5 highest excess entropies (DA and DP at 4.4 bits, and DH at 4.0), and 3:6 steepest entropy rates (DPerv at .85, DH at .79, DC at .76). It is also in 5:7 highest mutual information by percent relative to how much could possibly be shared between the two variables, based upon the fact that the most that could be shared would be the total block entropy of the smaller variable (DP at 48.5%, DH at 46.3%, DC at 42.4%, DPerv at 38.9%, and DA at 37.1%). When this is expressed in bits (e.g., 48.5% of P, which is the smaller variable of the two in DP, is 1.99 bits), diameter appears in 4:6 of the highest mutual information in bits (DH at 2.27 bits, DP at 1.99 bits, DA at 1.41 bits, and DPerv at 1.40 bits).

In 3-way combinations, diameter factors in 7:8 highest block entropies (Figures 46). It is part of 5:6 highest excess entropies as well as the third lowest excess entropy (Figure 47). It is part of the combination with the steepest entropy rate overall (DPervC) but also the most gradual (DPA) (Figure 48).

In 4-way combinations, diameter is part of the six highest block entropies (Figure 50), the five highest excess entropies (as well as the second lowest) (Figure 51), and 3:5 steepest entropy rates (as well as 4:5 most gradual) (Figure 52). This increased distribution occurs with all variables in 4-way combination, as each is in 67% of all of the possible combinations (although some reveal more notable patterns than others at this level). For this reason, is it not helpful to examine each variable in the 5-way or 6-way combination because they are even more distributed (83% and 100%, respectively).

In summary, diameter begins as the variable with the highest block entropy, and as it is combined with other variables pairwise through 4-way, the block entropies of combinations with diameter remain at the highest level. While its single excess entropy is mid range, as it is combined with other variables, these combinations have very high excess entropy. While it begins with the steepest entropy rate, the entropy rates of its combinations remain some of the steepest but also are some of the most gradual. Most importantly overall, however, is diameter's leading role as the variable with the most mutual information and also the most unique contribution overall to tendril pattern randomness. Diameter shares most of its information percent-wise with periodicity, a fact that we noted during our measurements and as a result also of our Markov chain analysis, and most of its information in bits with handedness, with periodicity a close second. (In both of these categories, it is only beat by the mutual information between handedness and pervertedness, which share 70.3% of pervertedness's information at 2.53 bits). Finally, the fact that diameter generally changes slowly across a coil, but that it can have marked dramatic changes (Figures 10, 11, and 22) may correspond to its mixture of high block entropies, high excess entropies, and generally steep entropy rates.

Periodicity

As a single variable, periodicity shows the third highest block entropy, the highest excess entropy by far, and the second most gradual entropy rate, which together suggest that it has a relatively high amount of structure (or alternately, a low level of randomness). It has the third highest mutual information in relation to all other variables (meaning, I[H(P); H(DHPervAC), at 3.82 bits), and it makes a mid-range unique contribution overall relative to the size of its block entropy (6.8%, 0.28 bits).

Pairwise, periodicity appears in the mid-range to higher block entropies (though not the highest) (PH at 7.58 bits, PD at 7.44), 3:4 of the highest excess entropies (PA at 4.8 bits, PD at 4.4, PH at 4.2), and 5:7 of the most gradual entropy rates (PC and PA at .37, PD at .60, PH at .65, PPerv at .68, although these are not that gradual of slopes other than the first two, which are the most gradual of all) (Figures 39-41). It appears in 4:7 of the lowest mutual information by percentage relative to the amount possible (PC and PPerv at 26.4%, PA at 34.2%, PH at 35.1%), but when paired with diameter, as already mentioned, PD is second highest overall for percent shared information (48.5%). When mutual information is expressed in bits, relative to the percent taken from the variable with the smallest block entropy, periodicity generally falls in the low to mid-range, except for two notable pairs: I[H(P);H(D)] at 1.99 bits, and I[H(H);H(P)] at 1.44 bits.

In 3-way combinations, periodicity has a full distribution range across block entropies (but not the highest or the lowest), has 9:12 of the highest excess entropies, and 9:11 of the most gradual entropy rates (Figures 46-48).

In 4-way combinations, periodicity has 3:4 of the highest and 2:3 of the lowest block entropies, 8:10 of the highest excess entropies, and the seven most gradual entropy rates.

In summary, periodicity across all combinations consistently exhibits higher block entropies (although a few low ones in 4-way combination), the highest excess entropies, and the most gradual entropy rates. Generally periodicity changes slowly, as shown by the Markov chain analysis, so its higher amount of structure and predictability as shown by its excess entropies and entropy rates makes sense. Of particular note are its high mutual information with diameter and with handedness.

Handedness

As a single variable, handedness possesses the second highest block entropy, a low to mid-range excess entropy, and the steepest entropy rate (shared with diameter, at .82). It has the second highest mutual information in relation to all the others combined (meaning, I[H(H);H(DPPervAC)], at 4.58 bits) but makes the third least unique contribution relative to its size (6.5%, 0.32 bits).

Pairwise, it has 2:3 of the highest block entropies (HD at 7.97 bits, HP at 7.58 bits), low to mid-range excess entropies, and 4:7 of the steepest entropy rates (HPerv at .80, HD at .79, HC at .78, HA at .72). When its mutual information relative to the amount it is possible for the two variables to share is considered, it generally falls in the middle range except for the fact that it has the highest overall (HPerv at 70.3%). When mutual information is viewed in bits, handedness fares better, having 3:4 of the highest amounts of shared information (HPerv at 2.53 bits, HD at 2.27 bits, and HP at 1.44 bits).

In 3-way combinations, handedness demonstrates the full distribution range across block entropy and excess entropy, including having the highest and the lowest block entropy and the top 3 highest and single lowest excess entropy. Similarly its entropy rates span the full range, with handedness factoring in the combinations with the second steepest and the second most gradual slopes.

In 4-way combinations, this split distribution continues, with handedness being part of the highest and lowest block entropies, the highest and lowest excess entropies, and the highest and lowest entropy rates.

In summary, handedness begins with high block entropy and steep entropy rates for single and pairwise combinations, and then in 3-way and 4-way combinations, it tends to the extremes (while still maintaining full distributions). What is notable though (and this will be

mentioned again below in the "Notable Symbols" section) is that the combinations with H demonstrate two particular and consistent patterns in pairs and 3-way and 4-way combination (Figure 56-57). Combinations with H that have high block entropies (and therefore high randomness) also have high excess entropy and gradual entropy rates (both lower randomness categories), whereas combinations with H that have low block entropies (and therefore less randomness) have low excess entropies and high entropy rates (greater randomness). Finally, handedness offers a mid-range level of unique contribution overall, but has three of the highest mutual information quantities when paired. That HPerv is the pair with the highest mutual information makes perfect sense, since handedness changes only at the moment when there is a perversion, so these two are absolutely linked.

Pervertedness

　　　　Pervertedness exhibits a mid-range block entropy (3.6 bits, though this is second lowest overall) but has the lowest excess entropy by far (0.125 bits), and the second highest entropy rate (.69, compared to H and D at .82). It has a mid-range mutual information in relation to the others combined (though the second lowest), and also makes the second least unique contribution relative to the size of its block entropy (3.1%, 0.11 bits).

　　　　This trend continues through pairwise and 4-way combinations. For example, pairwise its block entropies are mid-range except for PervC (4.26 bits), which is the second lowest; its excess entropies are mid-range too but it has 3:6 of the lowest (PervC at .75 bits, PervH at 2.0, PervA at 2.5), and it maintains the three steepest entropy rates (PervD at .85, PervH at .80, PervA at .79). Its mutual information in pairs is generally low, with 3:4 of the lowest percentages of shared information relative to the amount possible (PervC at 21.2%, PervA at 23.1%, and PervP at 26.4%), although it has the distinction also of sharing the most information overall in a pair (PervH, at 70.3%, 2.53 bits). A similar patterns persists when its mutual information is considered in bits.

　　　　In 3-way combination, pervertedness factors in 3:4 of the lowest block entropies and the four lowest excess entropies, and persists in having the three steepest entropy rates.

　　　　In 4-way combination, however, a surprising new trend occurs alongside the previous pattern. Pervertedness still is part of the four lowest block entropies, the six lowest excess entropies, and the six highest entropy rates. But suddenly it also appears in 3:4 of the highest block entropies, 3:4 of the highest excess entropies, and 3:4 most gradual entropy rates. In these cases, it is variously combined with D, P, H, and A (see discussion of handedness above, and also the "Notable Symbols" section below), which strongly follow the patterns across levels of combination of high block entropies, high excess entropies, and low entropy rates, so perhaps these combinations are so strong in their tendency that pervertedness is just along for the ride at the 4-way combination level. This may be so especially because it maintains its usual pattern of low block entropy, low excess entropy, and high entropy rates also at the 4-way combination level.

　　　　In summary, these qualities of having low block entropy (and thus, a low level of surprise) combined with low excess entropy and high entropy rates (both indicators of greater randomness) fit well with the Markov chain data for pervertedness. As 57.4% of all free coils have a perversion in them, this can almost be called a standard feature of their coiling pattern. However, when viewed in relation to the whole, pervertedness only appears in 22% of the symbols in our data string, whereas 78% of the 5 mm incremental measurements of coils have no perversion. This means that when a perversion occurs (other than at the outset, the tip, where

roughly half of all perversions reside), it is surprising, even if the fact that it occurs is not. Similarly, the fact that it is so closely correlated with handedness may add some element of structure to its overall pattern.

Angular Axis Rotation

From a culinary standpoint, angular axis rotation is like eggplant in its seeming capacity to absorb the flavors of other variables, as well as in the fact that it lends a highly unique contribution relative to its overall quantity in a dish. On its own, is it a bit blasé, having a mid-range block entropy (3.8 bits), a mid-range excess entropy (1.25 bits), and a mid-range entropy rate (0.50). Unsurprisingly, it also has a mid-range mutual information rate in relation to all the others combined (3.37 bits), but when this is "normalized" and viewed in relation to the size of its block entropy, angular axis rotation suddenly stands out for offering the second most unique contribution overall (11.3%, .43 bits), with only diameter being a touch higher (12.2%, .65 bits).

Pairwise, it occupies the full range of block entropies, including the lowest (AC at 4.21 bits) and the second highest (AD at 7.68). It has mid- to high excess entropies, including the two highest (AP at 4.8 bits, AD at 4.4 bits), and 3:5 of the most gradual entropy rates (AP at .37, AC at .45, AD at .65). It has a wide range of mutual information percentage values relative to the amount possible, the most notable of which is third highest (AC at 48.2%). However, when this percentage is considered as bits relative to the size of the block entropy of contact status, AC becomes the sixth lowest (.41 bits), since contact status's block entropy is the smallest of all.

In 3-way combination, angular axis rotation has 4:5 of the highest block entropies (as well as the second lowest), 5:7 of the highest excess entropies, and 5:6 of the most gradual entropy rates. In 4-way combination, it occupies 4:5 of the highest block entropies (as well as 2:3 of the lowest), 3:4 of the highest excess entropies (as well as 3:5 of the lowest), and 5:6 of the most gradual entropy rates (as well as 2:3 of the steepest).

Thus, angular axis rotation at the higher levels of combination is comfortable at both extremes – much as people either hate or love eggplant – and, not surprisingly, it is one of the variables at play in three out of four of the randomness patterns in the "Notable Symbols" category.

Contact Status

Unlike angular axis rotation, contact status is highly predictable. It has the smallest block entropy by far (0.85 bits), the second lowest excess entropy (0.385 bits, with only pervertedness being lower at 0.125), and the lowest entropy rate by far (0.09). It has the lowest mutual information in relation to all others combined (.82 bits) which, when viewed as a percentage and bits relative to its size, reveal that it makes the least unique contribution overall in bits (.03 bits, which is 3.5% of its block entropy).

Pairwise, contact status factors into the five lowest block entropies (CA at 4.21 bits, CPerv at 4.26, CP at 4.65, CH at 5.45, CD at 5.82), and 4:5 of the lowest excess entropies (CPerv at 0.75 bits, CH at 1.6, CD and CA at 2.0). It is split in its entropy rate appearances, having 2:3 of the most gradual entropy rates (CP at .37, CA at .45) and others that are fairly steep (CD at .76, CH at .78). It demonstrates a broad range of variability in mutual information contribution to pairs, but in percentages it factors in the third highest (CA at 48.2%) and fifth highest (CD at 42.4%). However, when this is adjusted to bits relative to its size, it appears in all five of the lowest pairs for mutual information.

In 3-way combination, the nine lowest block entropies all contain "C", as do 10:11 of the lowest excess entropies and 7:8 of the steepest entropy rates (as well as the third most gradual). The latters shows a switch in its entropy rate trend overall, having favored gradual rates at lower levels of combination. Similarly, in 4-way combination, contact status occupies 10:11 of the lowest block entropies, the nine lowest excess entropy positions, and the seven steepest entropy rates.

In sum, the predictability of contact status as a variable is relieved only at higher levels of combination, where it suddenly factors into combinations with steep entropy rates (high levels of randomness with each added measurement) rather than into those with gradual slopes and more predictable futures.

Notable Symbols

To discover "notable symbols," each variable was traced horizontally across all three of the measurement categories (block entropy, excess entropy and entropy rate) at a given level, and if it appears at all three categories (as one of the highest or one of the lowest for that category), it became "notable." Similarly, these were also traced vertically through different levels of combination to see if they appear in combination with each other at the next higher level of combination (although it was the horizontal, and not the vertical, criteria that allowed them to be classified as notable). As it turns out, particular variables in particular combinations do in fact combine in consistent patterns moving up through higher levels of combination. There are two dominant patterns, and three much less prevalent patterns.

As block entropy, excess entropy and entropy rate all reveal greater or lesser amounts of randomness depending on whether their values are high or low, these were then arranged so as to have the measures of lesser randomness together (low block entropy, high excess entropy, and low entropy rate) and the measures of greater randomness together (high block entropy, low excess entropy, and high entropy rate). There are nine possible combinations (or transitions) in this way of viewing the measures of randomness, so the fact that only two of these predominate, with only three others making a tiny appearance, says something significant about the two primary patterns.

The main patterns both possess a combination of greater and lesser randomness (Figures 56-57). Notable pattern #1 occurs across pairwise, 3-way and 4-way combinations (11 total) of handedness (H), periodicity (P), diameter (D), and angular axis rotation (A), and these all have high block entropies (therefore high randomness), and high excess entropies and gradual entropy rates (both lower randomness categories). Notable pattern #2 has occurs pairwise, 3-way and 4-way combinations (9 total) of handedness (H), pervertedness (Perv), and contact status (C); all of these have low block entropies (therefore low randomness) and have low excess entropies and high entropy rates (both greater randomness categories). Compared to these two, the other notable patterns are insignificant, especially because they only appear in single variable or one pairwise combination, not in multiple combinations across all four levels of combination.

What do we learn from notable patterns #1 and #2? …. To be considered with fresher eyes in the near future, especially by looking not just at the results for each variable on its own (its particular trends) but also at mutual information results… I'm just too tired to do this right now.

Epsilon Machines (Figure 58, partial view)

Our current epsilon machines from our data, run on only one variable at a time, are completely unwieldy, having over one hundred or many more causal states. We need to use optimal causal inference in order to minimize the noise in our data so as to be able to see the structure beneath the noise. I look forward to this, especially given that we now have some expectations about what some of the structure might be given the analysis here from Markov chains, block entropies, excess entropies, and entropy rates.

Still to Come (Figures 59-61)

We have yet to complete our "2D Turing" *in silico* tendril coiling simulation (which we are hoping will provide us with perversions, as our 1D model does not accomplish that). We also need to play with parameter adjustment in order to see how close we are (does our simulation approximate the morphologies of free coiling tendrils?). Then, we plan on using computational mechanics to analyze the simulated tendrils' dynamics for comparison with the results presented here, in order to make some evaluation of the effectiveness of using the Turing reaction-diffusion equations and other measures to capture the variability we observe in real tendril free coils.

Also, we have yet to measure complexity, rather than randomness, and also given Jim Crutchfield's idea that symmetry breaking might be a factor in handedness directionality, I am curious whether we can find evidence for symmetry breaking, perhaps via bifurcations, in our data.

Conclusions

I will write these out further in the near future.

About tendril biology:
I've learned a lot about tendril molecular biology and beyond… making a comprehensive interdisciplinary model of tendril free coiling has been a very fun but difficult challenge.

About tendril patterning (I will compile these from the preceeding and summarize what we've learned, which I think is a lot)
Tips and Bases, and Preceding and Succeeding sub-symbols
Variable traits across combination levels, and mutual information
Notable patterns #1 and #2

About multi-variable computational mechanics:
We recommend the usefulness of Markov chain analysis adding in "New Tendril" in order to observe patterns at the tip and base of the coil. We could have strung our data together overall, but we maintained its capacity to be analyzed by the coil, as well as by the characteristic, and both of these sets of information have revealed a lot about the patterns of tendril free coiling.

We learned that range distributions at higher combinations include more of the variables in each combination; the data becomes less revealing at the same time that the range decreases. I did not intuit this ahead of time though it makes sense given the increasingly higher percentage that each variable plays in the combinations. This means that the higher combinations are less useful overall.

I have learned a few tips on how to make multi-variable computational mechanics data analysis more manageable. I know better now how to look for trends across combinations vertically and horizontally, so this could be done by code in the future (to find notable symbol patterns). Similarly, the percentage adjustments for the variable's "area" representing the size of its block entropy in relation to the whole and in relation to pairwise combinations could be done with code. As these are some of our most revealing plots (using both the percentages relative to size, and then calculating this in bits), they could well be useful in other applications of multivariable data analysis.

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