Course Graining Cellular Automata

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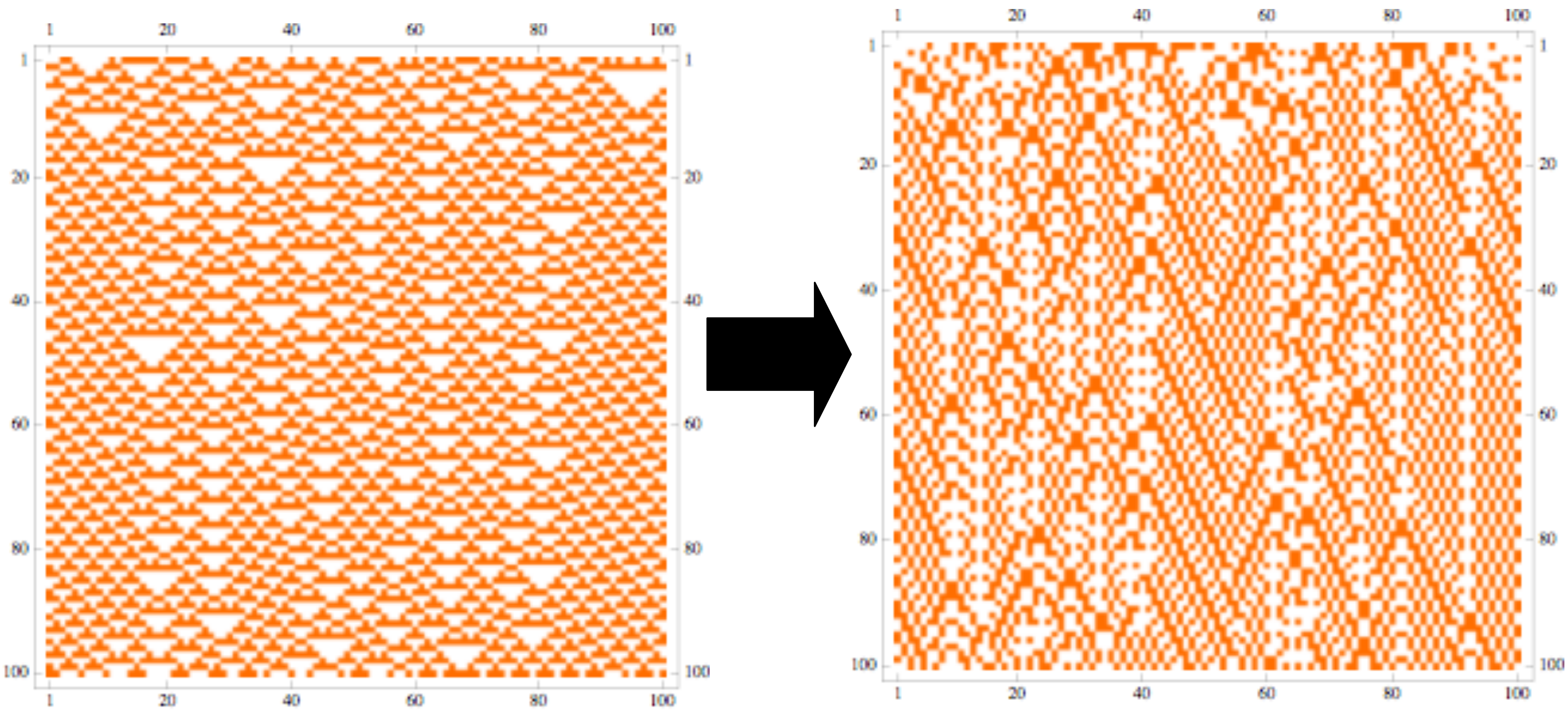


Natural Computation Spring 2012

Motivation

-When we observe a system, we are dependent on a measuring device.

-Measuring devices often neglect many details of the system, and we are left to try to understand the system from reduced data.

-How does course-graining a system affect the behavior of that system?

-Consider the simple model of 1D cellular automata.

Overview of 1-Dimensional CA

Start with a rule:

Rule 18: (000,001,010,011,100,101,110,111)

(0,1,0,0,1,0,0,0)

Apply rule to current state (string of 1’s and 0’s).

time

…00101000…

…01000100…

…10101010…

Course Graining Over Two Cells

For each time step, take every two cells, and apply a transducer to get a string of half the length.

Transducer rule 4:

(00,01,10,11)

(0,0,1,0)

Translate each string:

…0101001011…

…0 0 0 1 0…

The Rules

There are 16 possible transducer rules.

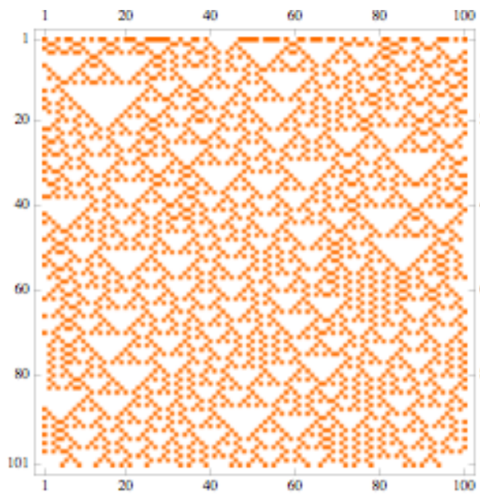
- -Note 0000 and 1111 are uninteresting.
- -Note that the entropy rate of systems are 1-0 symmetric.
- -Assume that the systems are right-left symmetric.

This reduces the total number of rules to the following five:

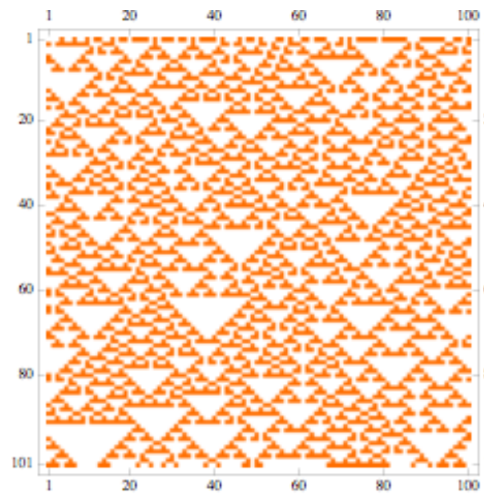
(0001,0010,0011,0110,0111)

Apply to the Interesting Automata

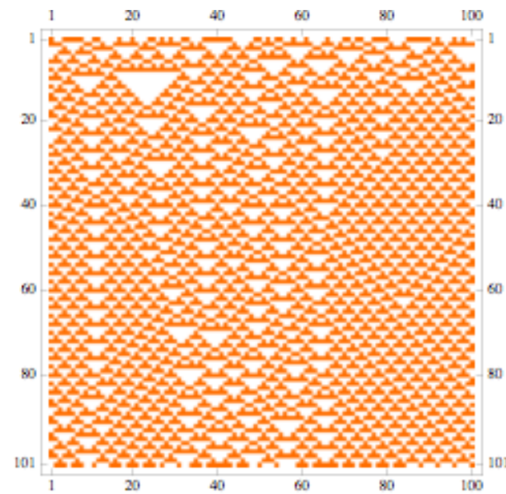
Rule 18



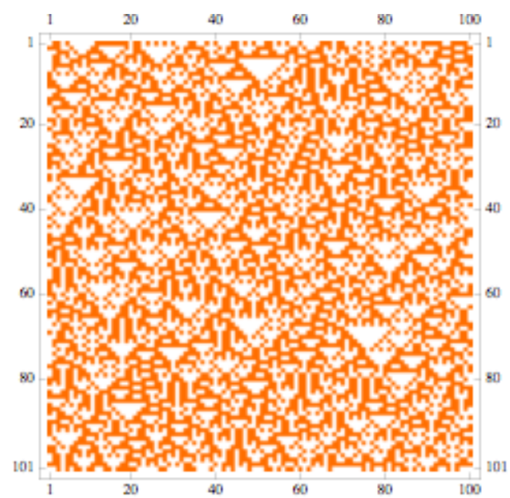
Rule 22



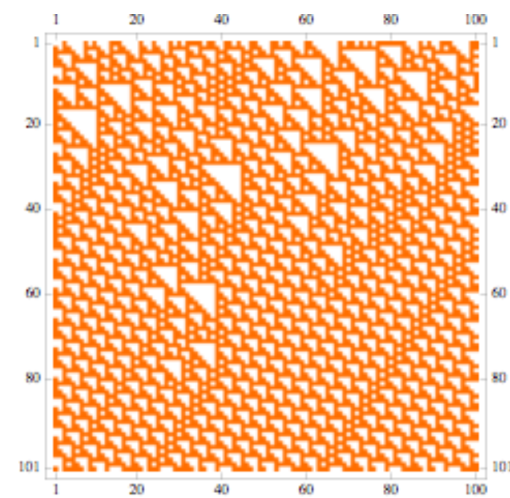
Rule 54



Rule 90



Rule 110



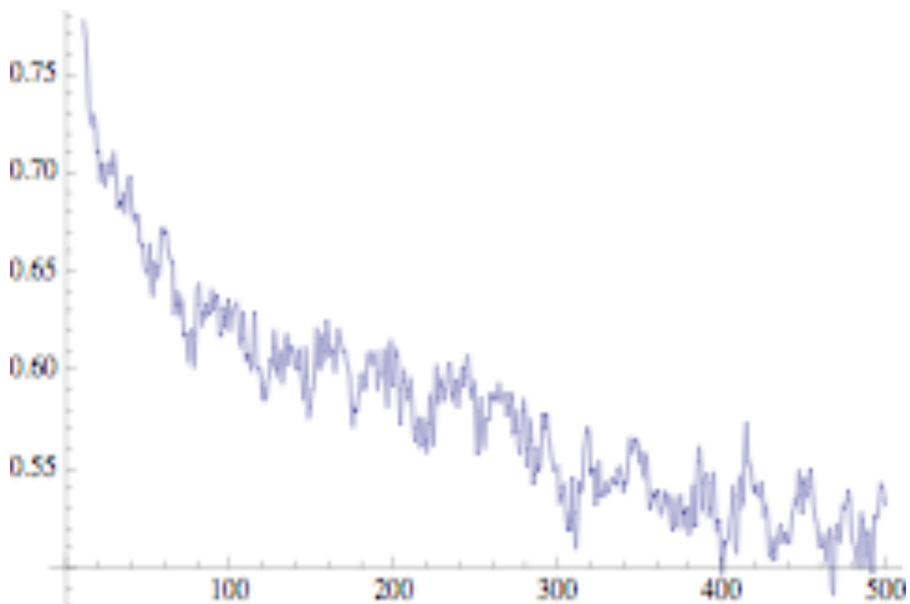
A Metric: Entropy Rate

The entropy rate of the string at a given time is like the entropy of the system, because it tells us the uncertainty in the next bit.

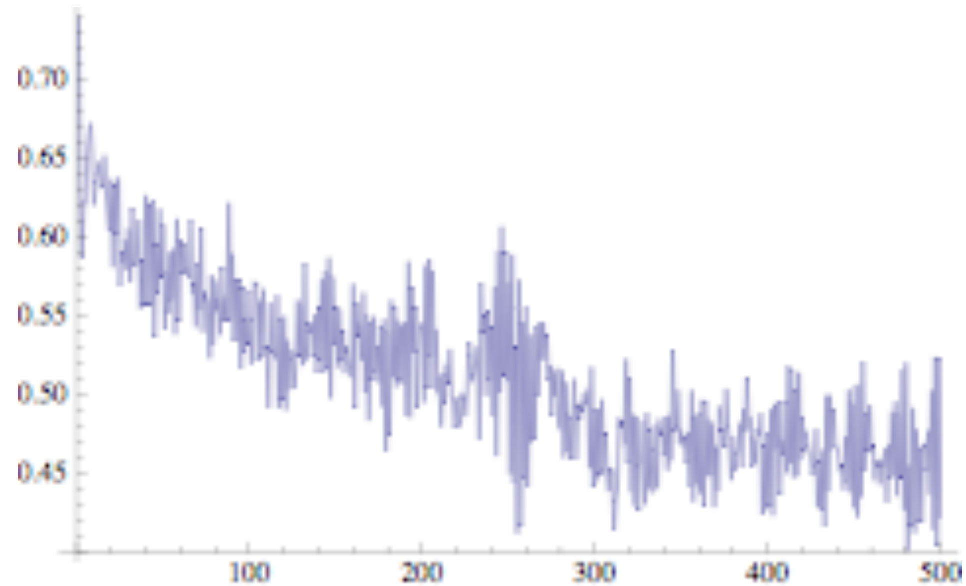
Compare how the entropy rate evolves for a CA with how it evolves for a course-graining of that CA.

Entropy Rate: First Impressions

Rule 54

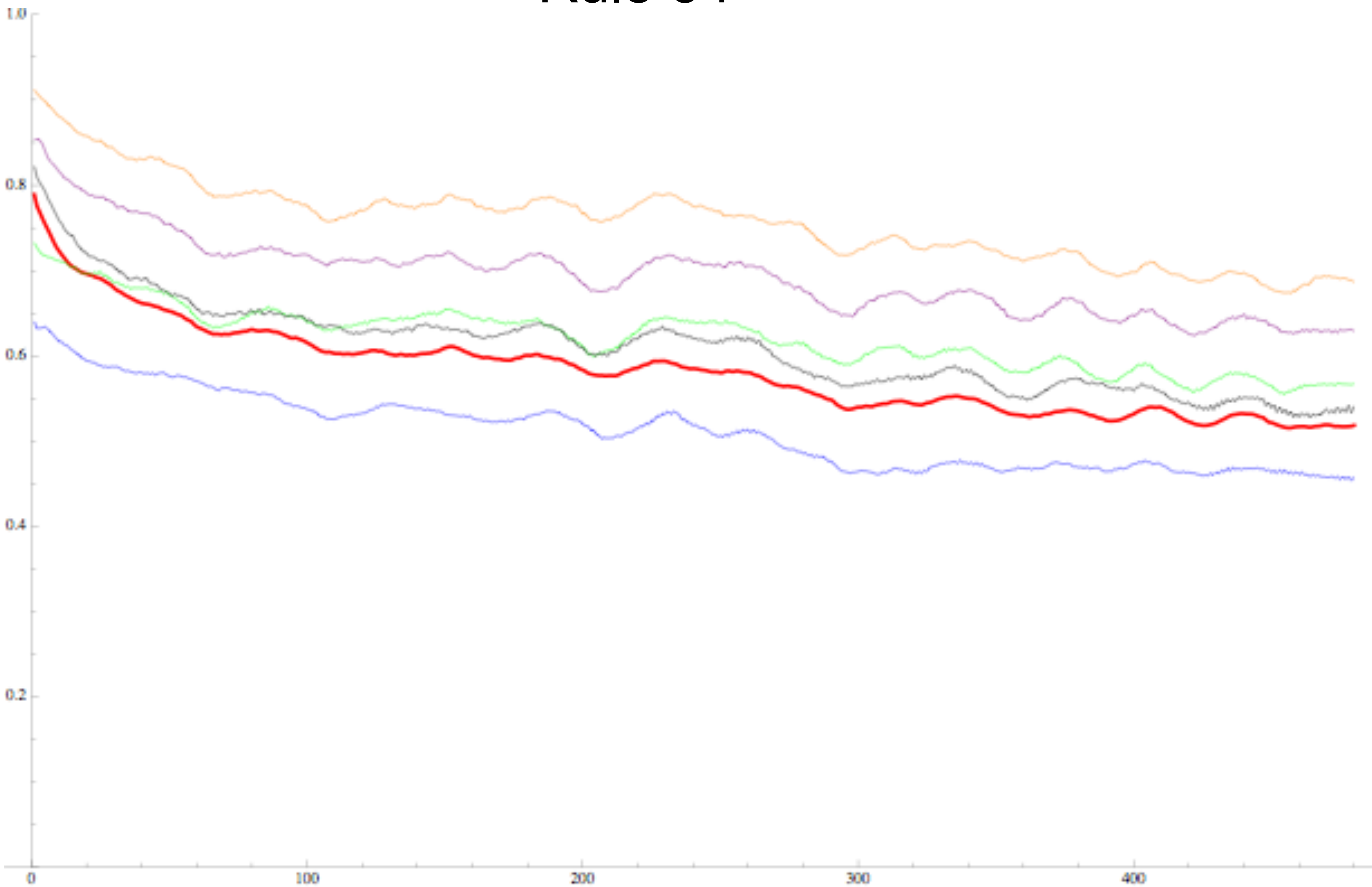


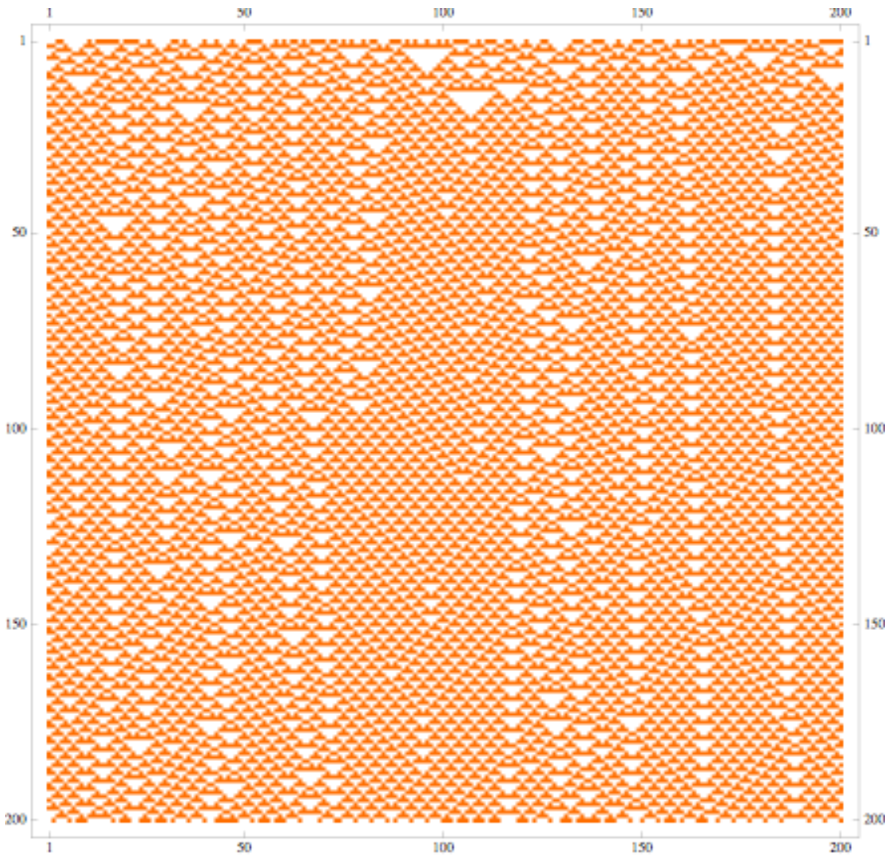
Rule 54 Course-Grained (0010)

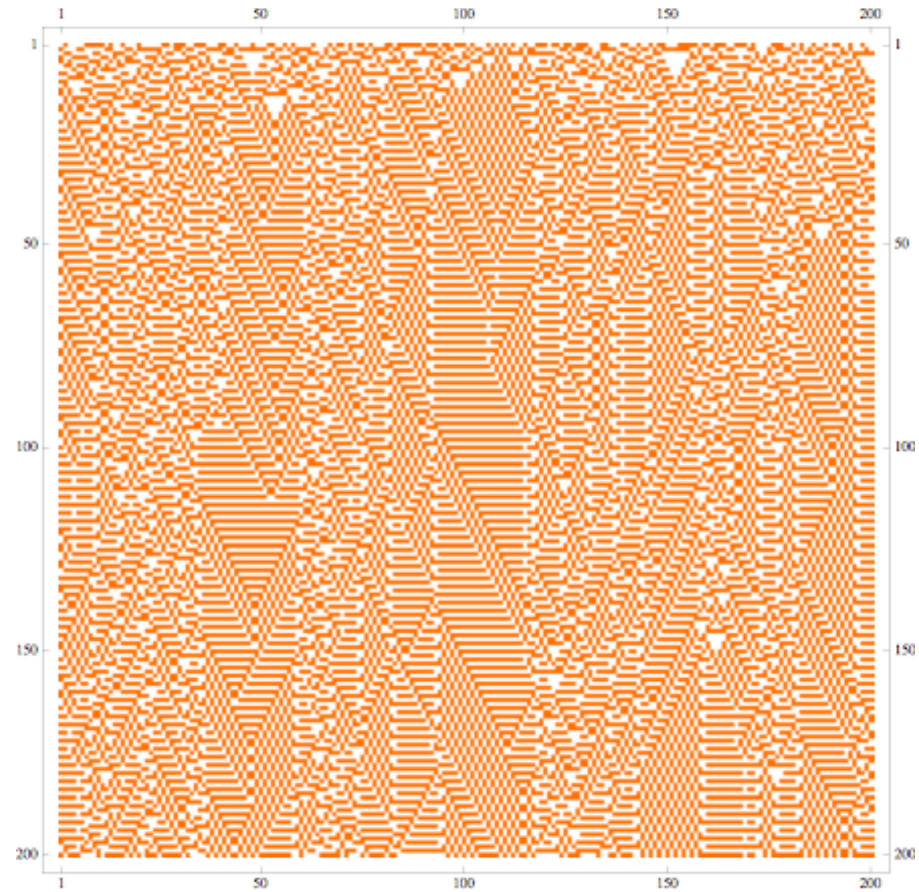


-Decreasing with similar features.
-More noise for course-grained.

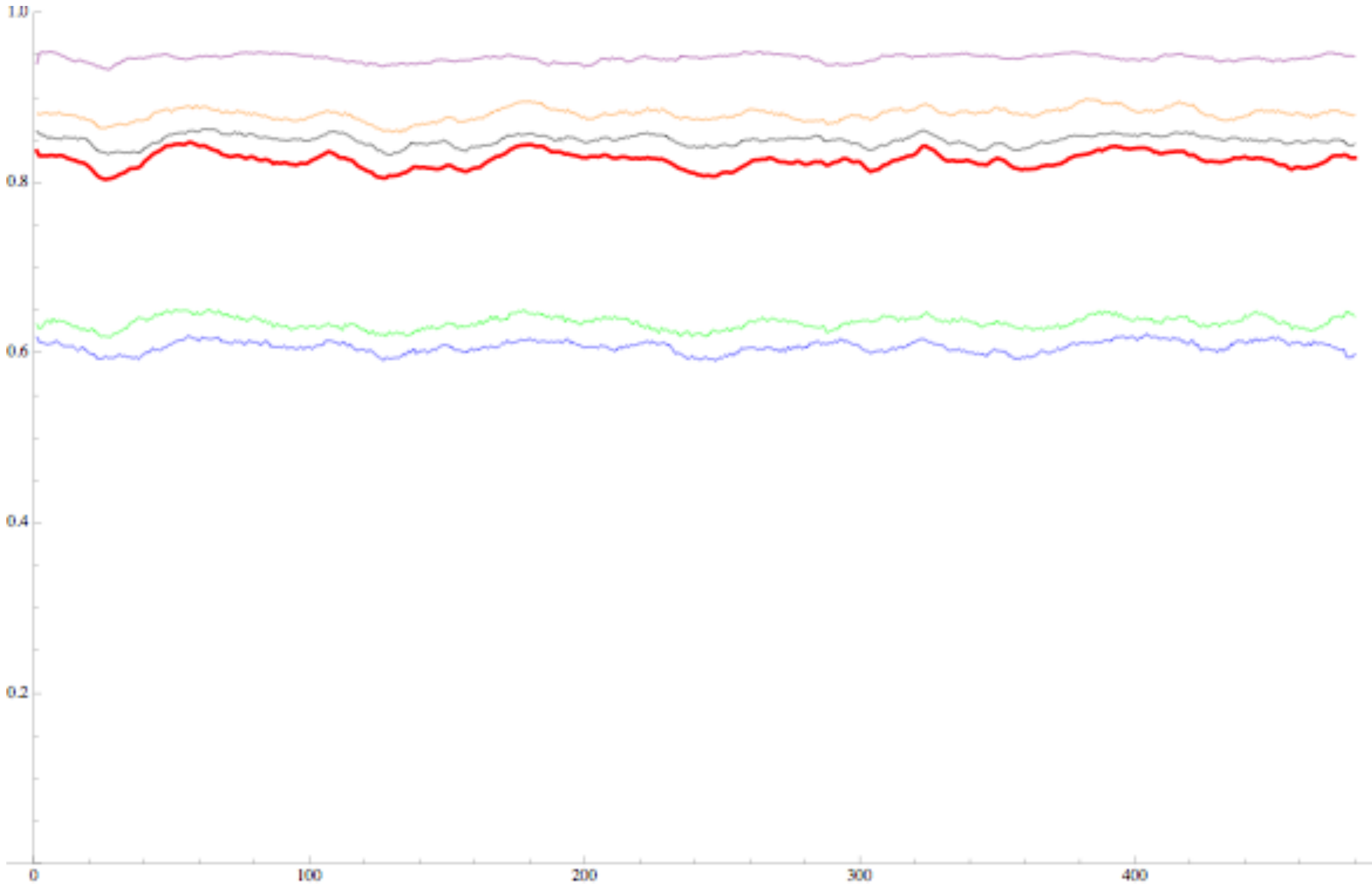
Rule 54

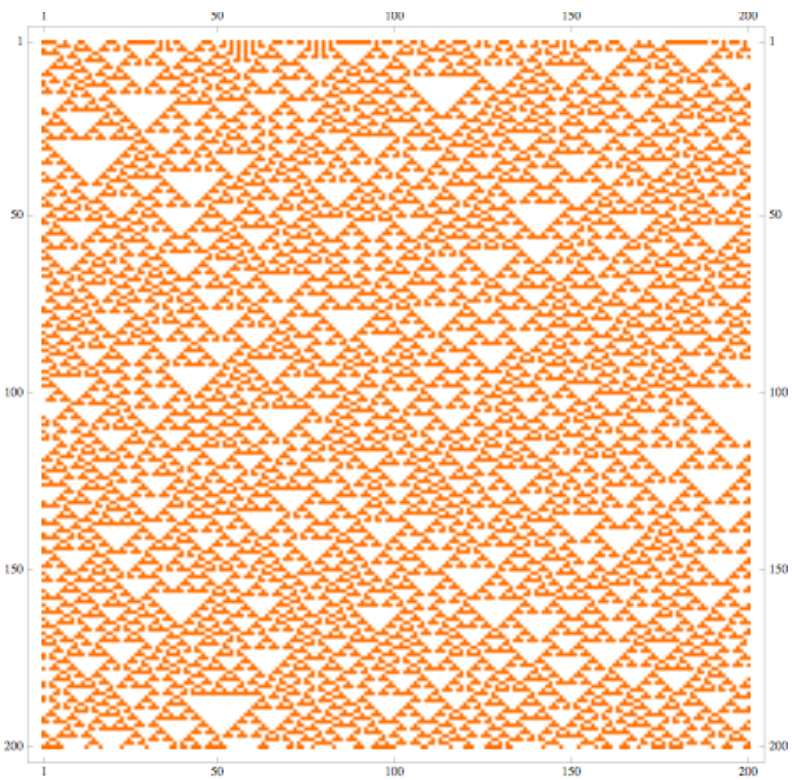


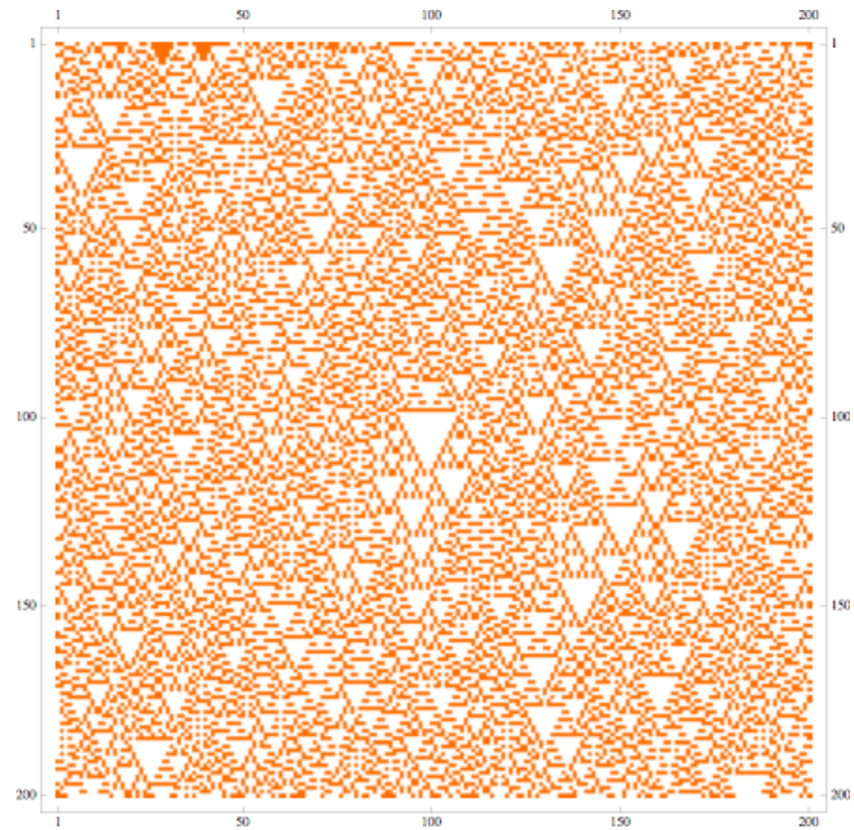

1
50
100
150
200


1
50
100
150
200

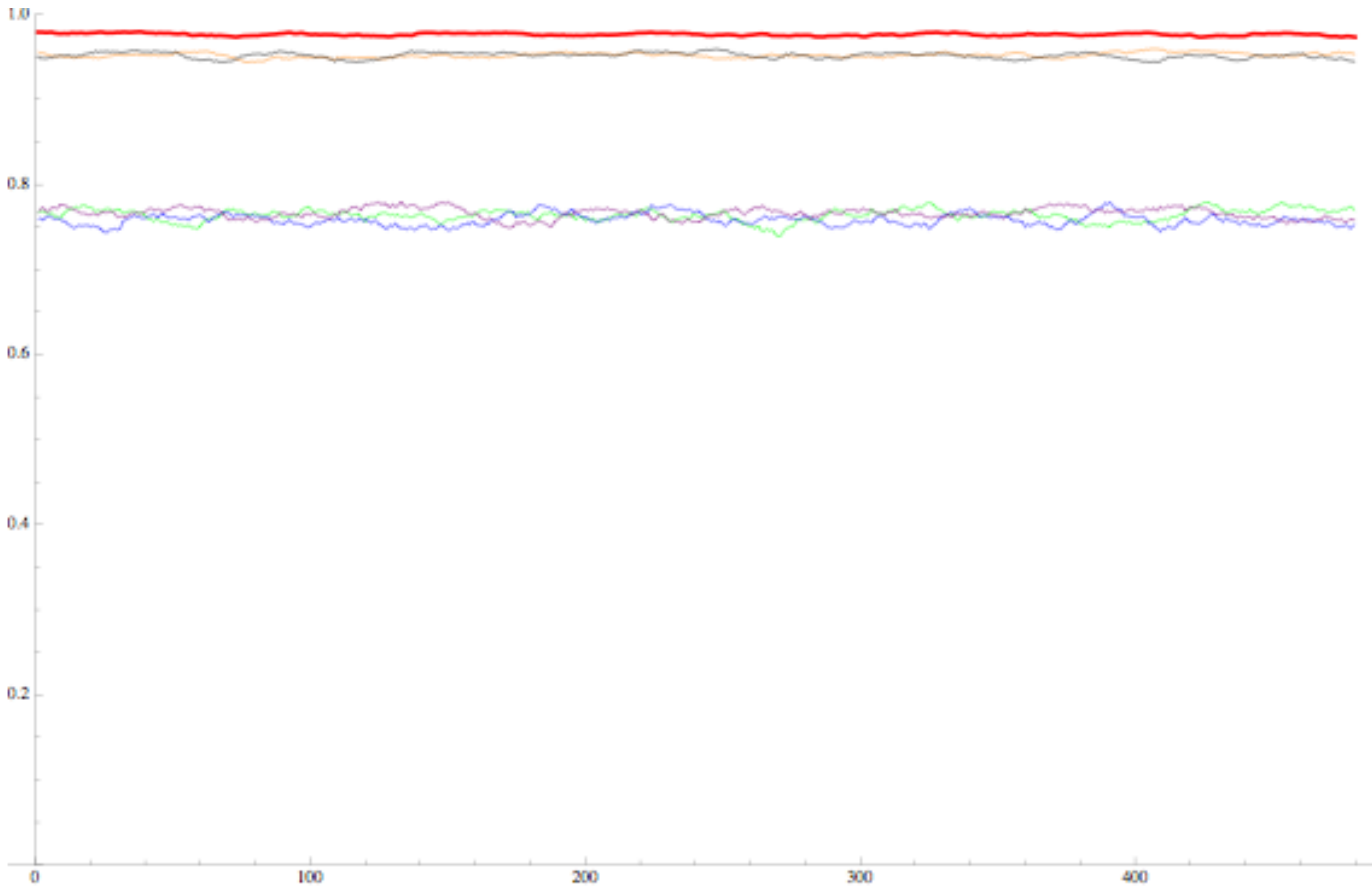
Rule 22

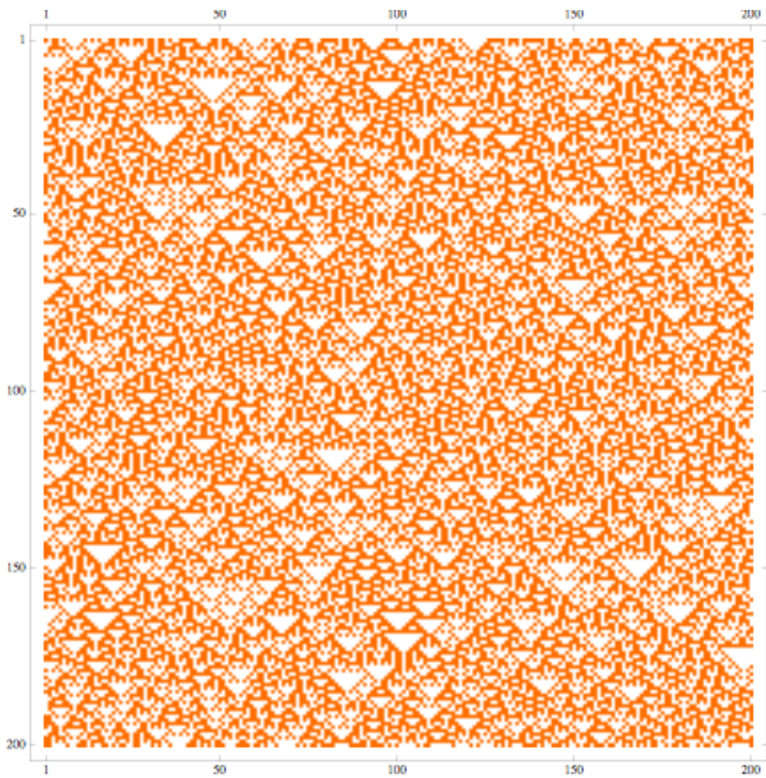


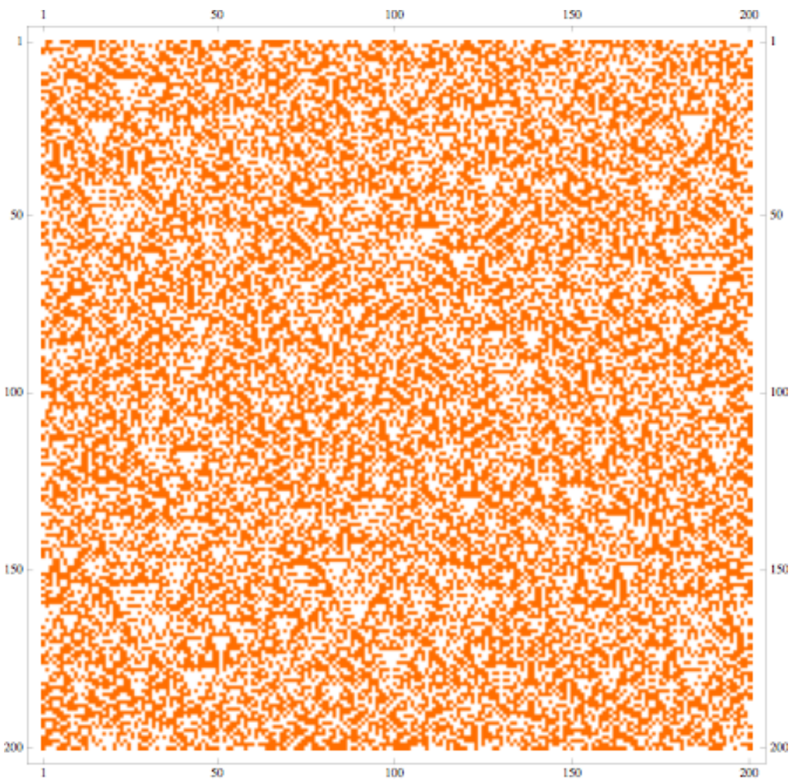

1 50 100 150 200
1 50 100 150 200


1 50 100 150 200
1 50 100 150 200

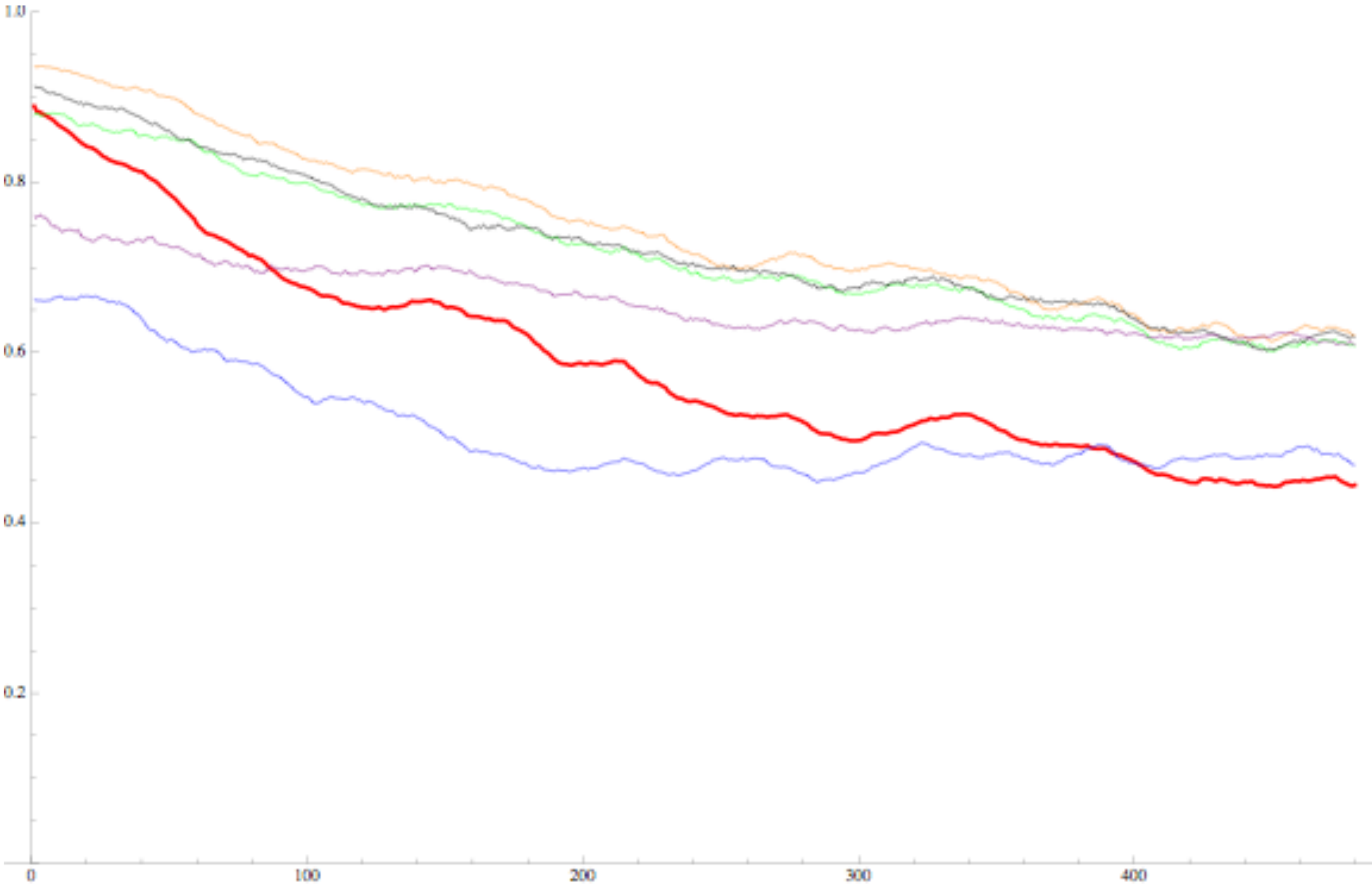
Rule 90

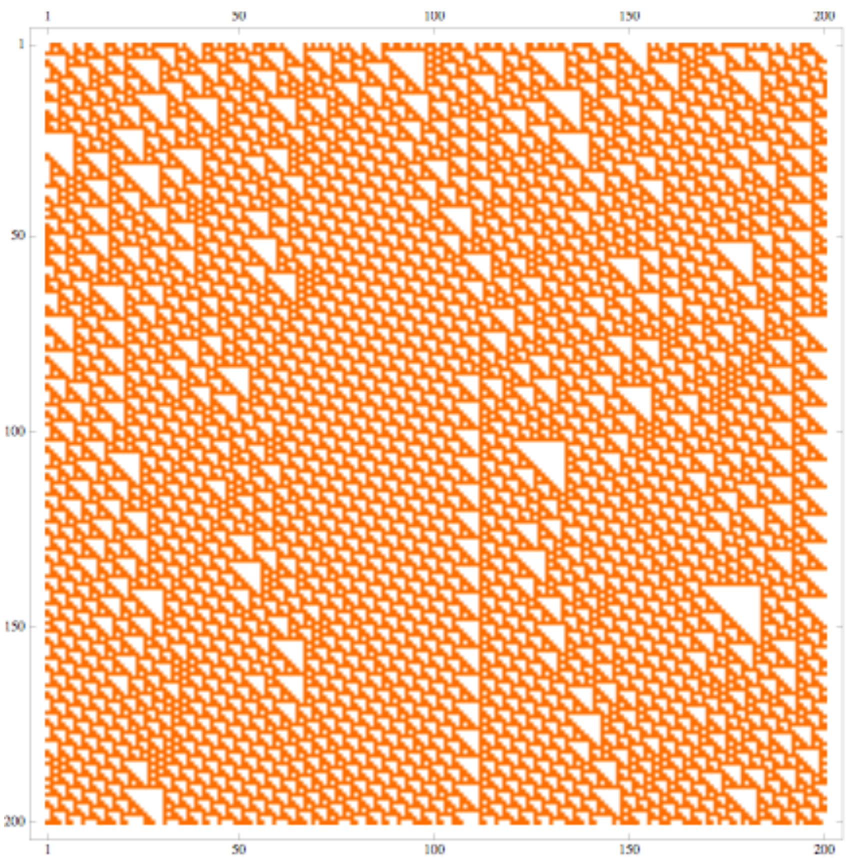


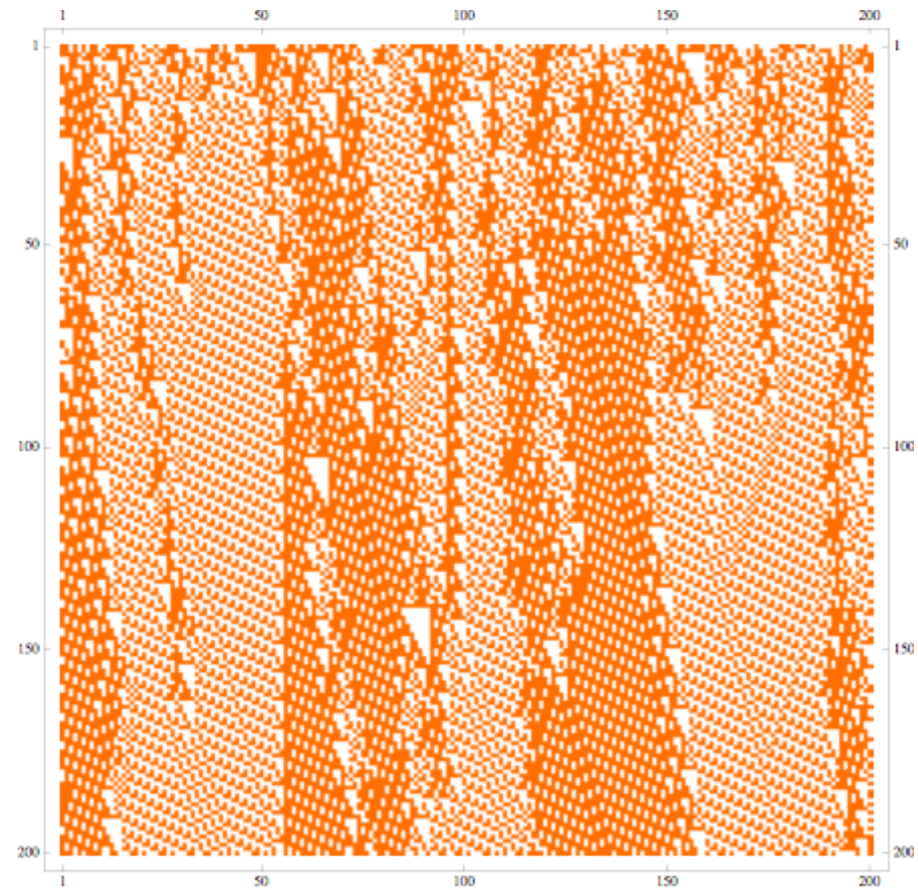

1
50
100
150
200


1
50
100
150
200

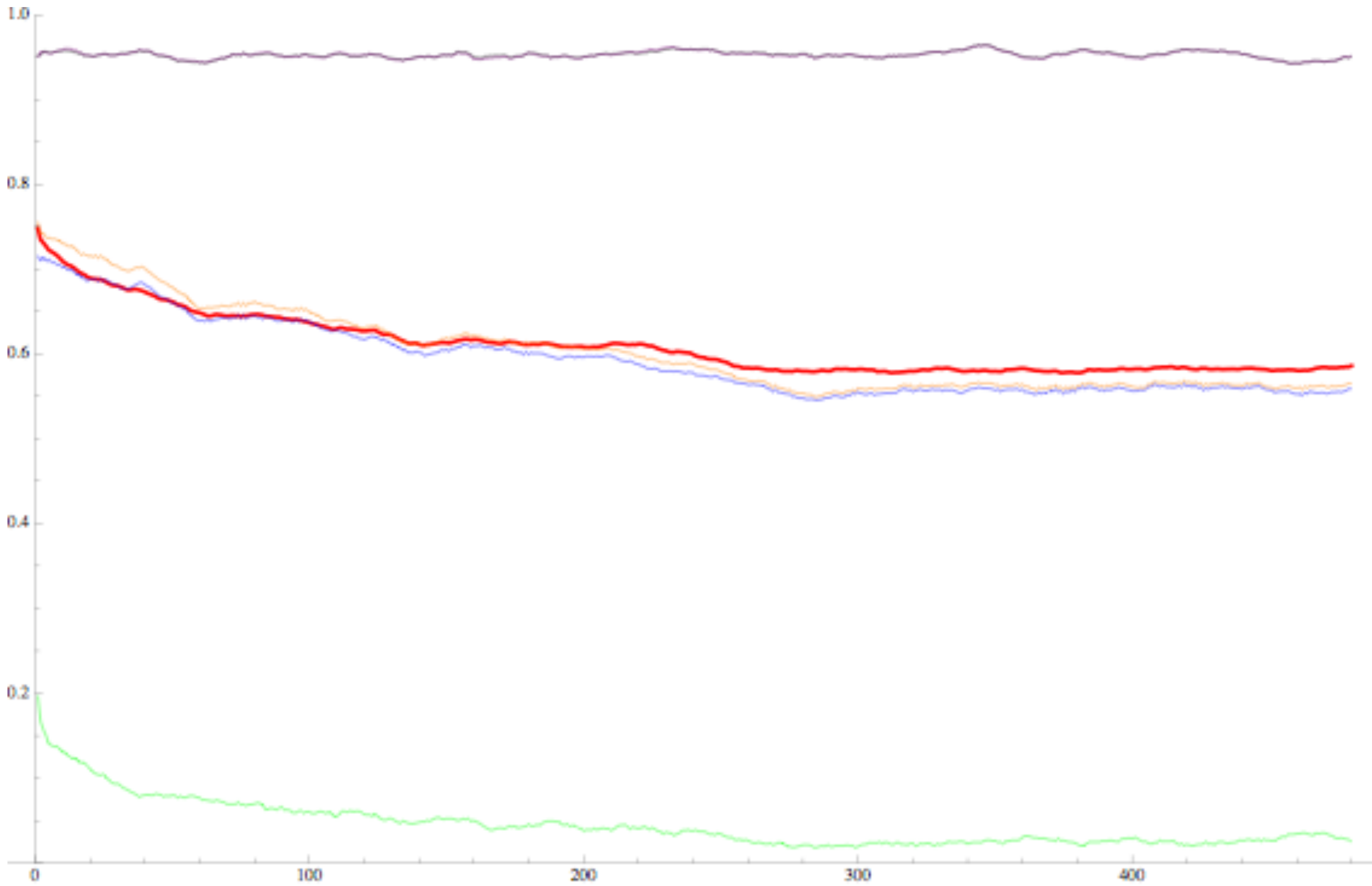
Rule 110



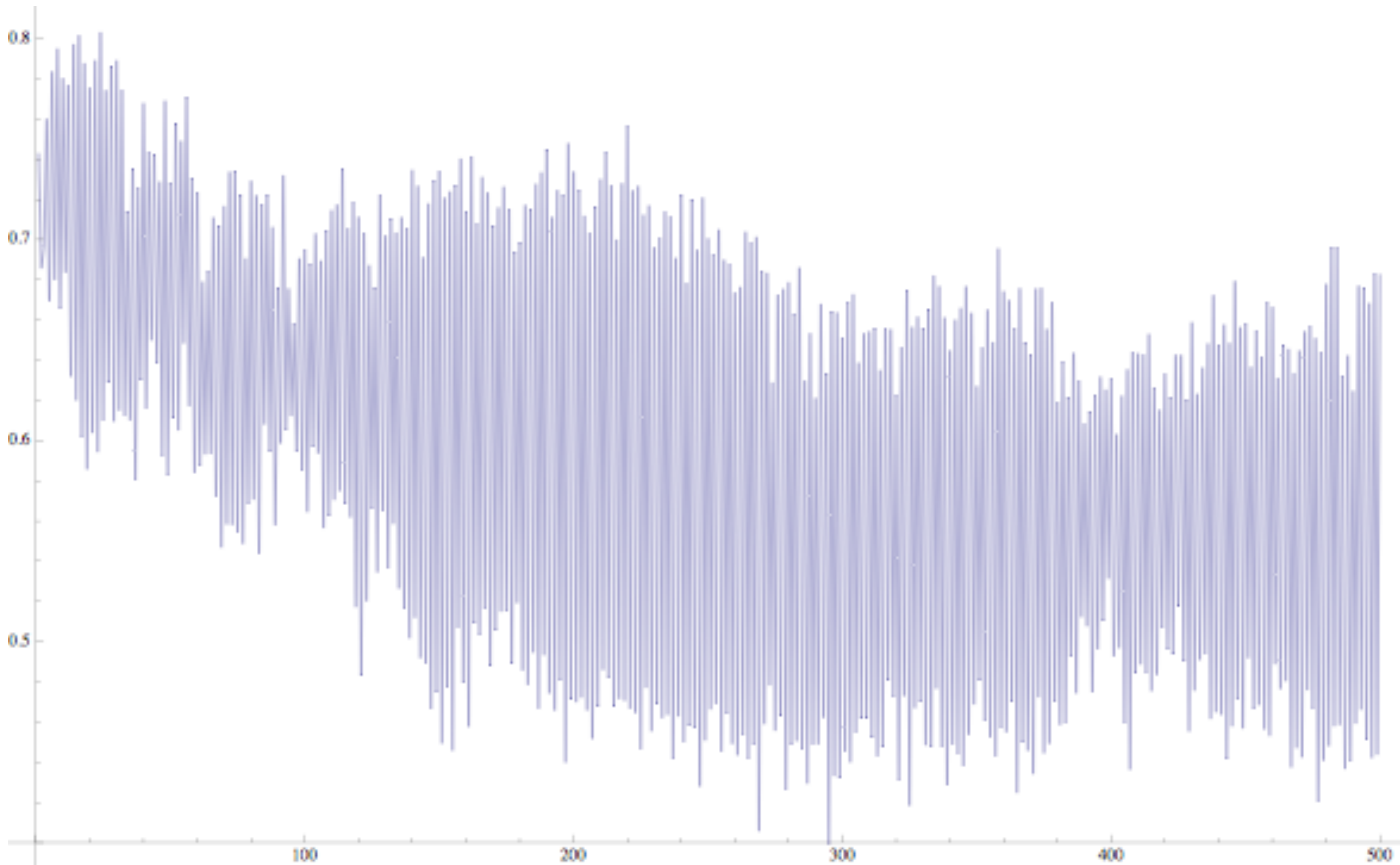

1
50
100
150
200


1
50
100
150
200

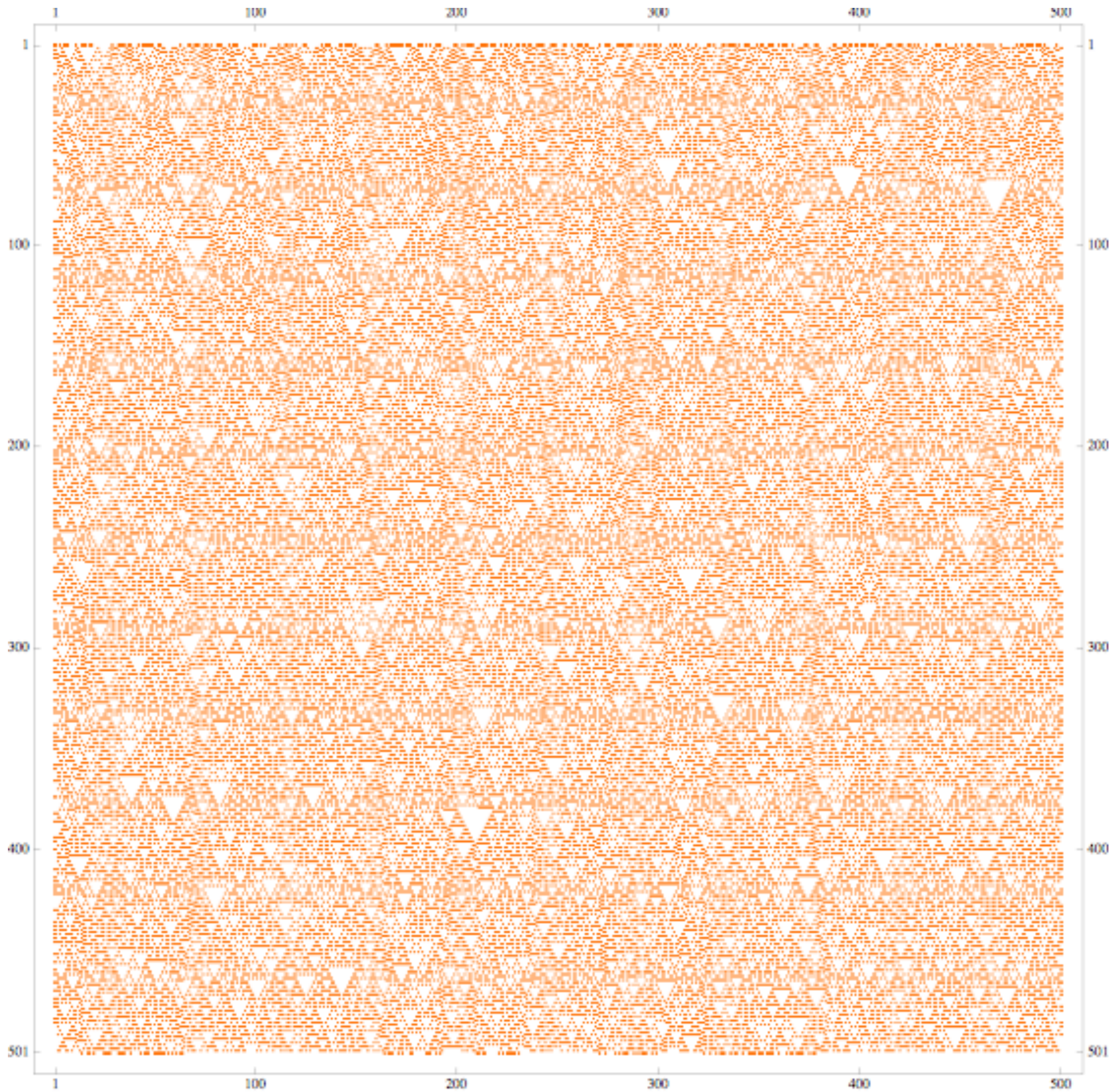
Rule 18



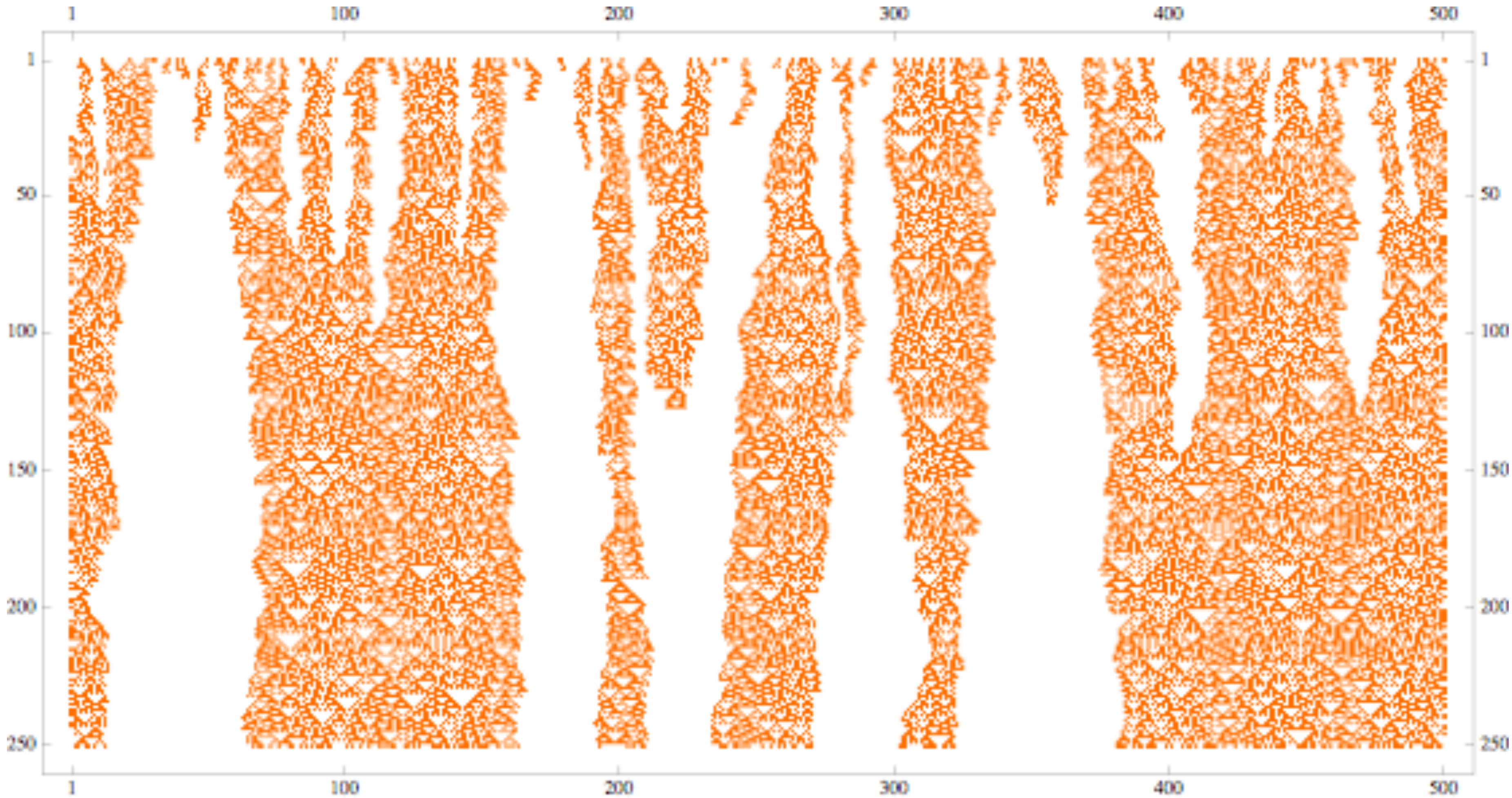
Rule 18 Course-Graining 0011



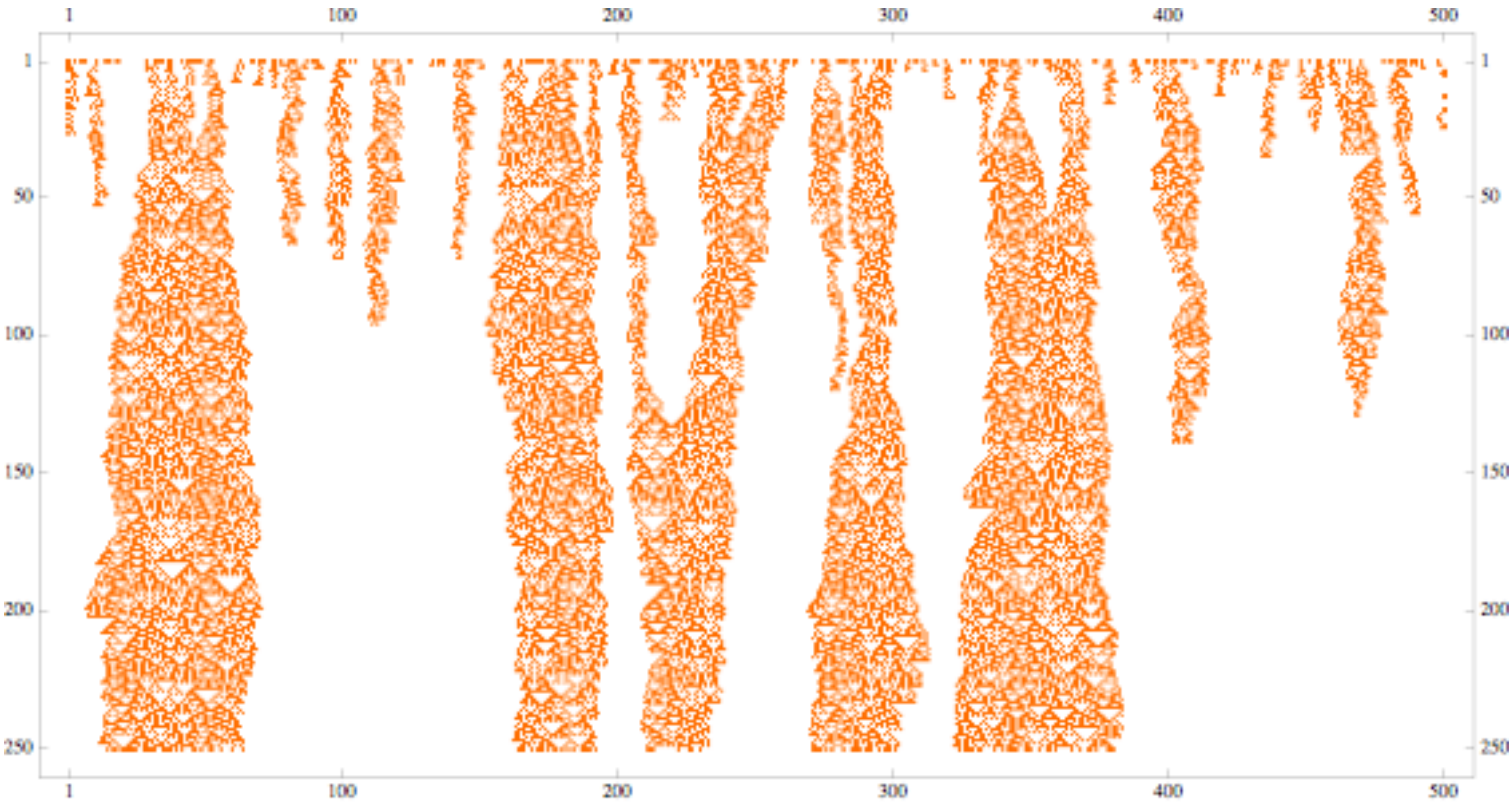
Rule 18 Course-Graining 0011

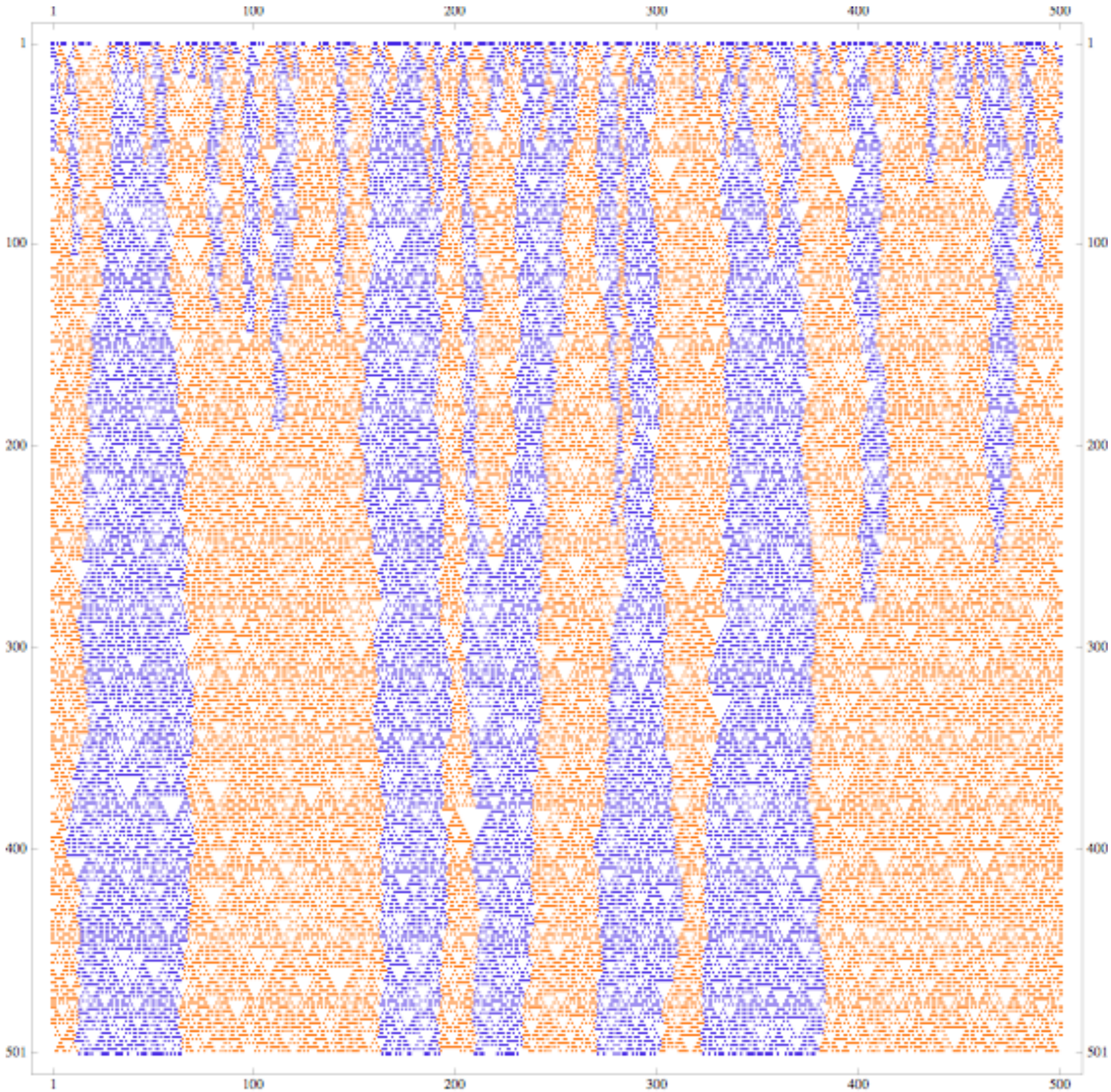


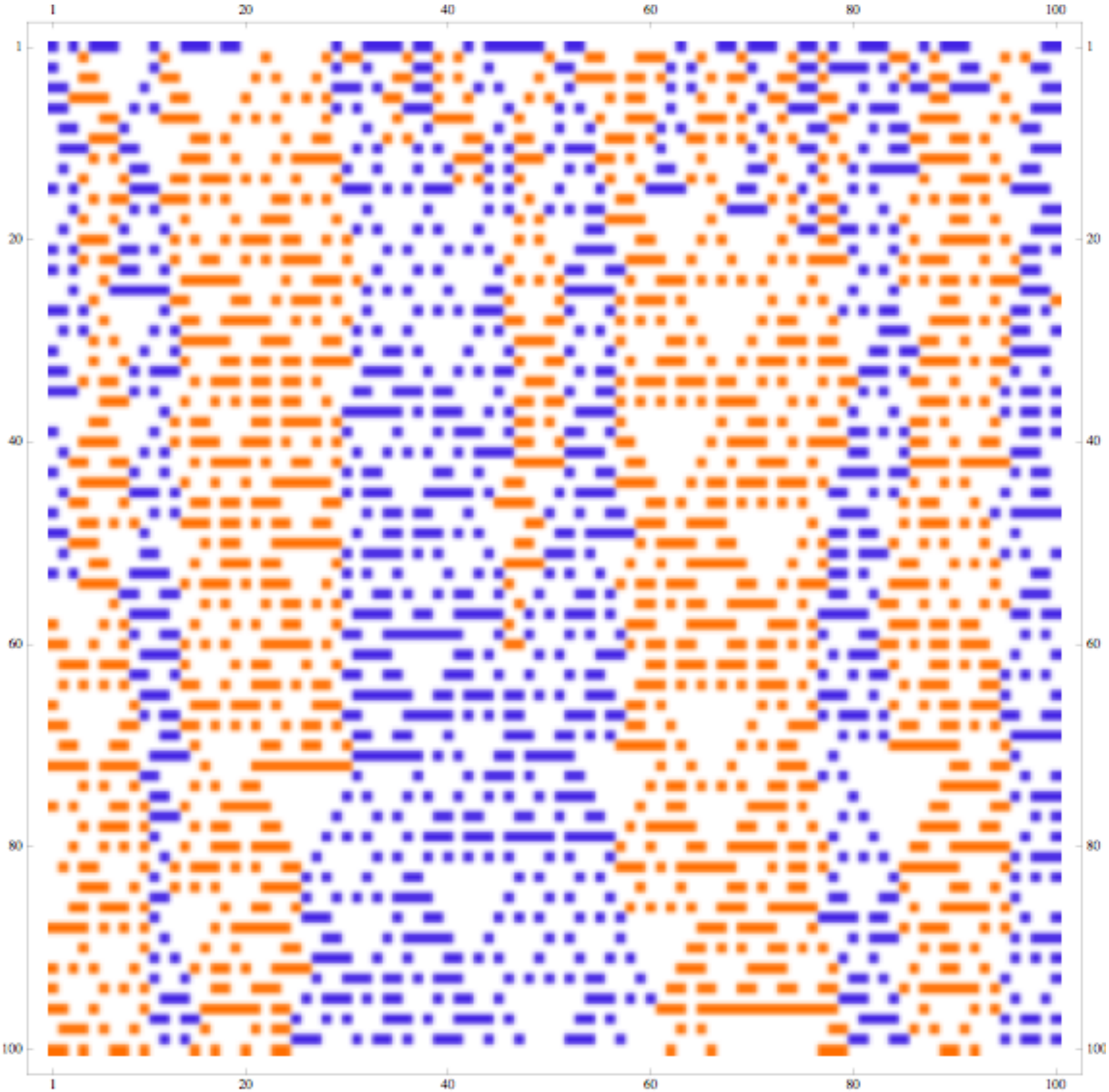
Rule 18 Course-Graining 0011 Evens



Rule 18 Course-Graining 0011 Odds




1
100
200
300
400
500
1
100
200
300
400
501


1
20
40
60
80
100

Moving Forward

-Why does rule 18 express different behavior when course-grained? (Is it a fluke?)

-What sort of course-grainings exist which don’t impose an arbitrary segmentation of the lattice?