Towards a Theory of Information Flow in the Finitary Process Soup

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Goals

- Analyze model of evolutionary self-organization in terms of information flow.
- Measure channel capacity of elementary ϵ-Transducers.
- Develop functional partitioning based on this and language properties.

Outline

ϵ-machines

ϵ-machines Defined

- $T = \{\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{T}}\}$
  - $\boldsymbol{\mathcal{S}}$ is a set of *causal states*
  - $\boldsymbol{\mathcal{T}}$ is the set of *transitions* between them: $T_{ij}^{(s)},\ s \in \mathcal{A}$

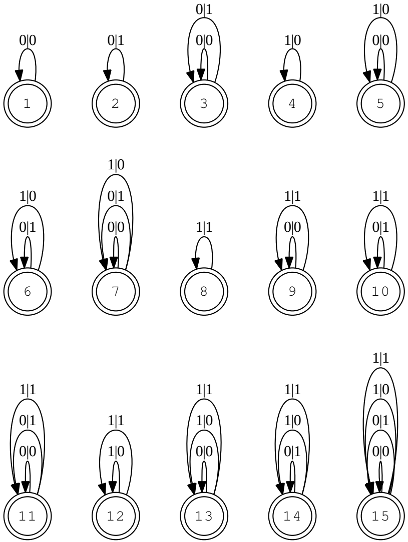
ϵ-machine Properties

- All of their recurrent states form a single *strongly connected* component.
- Transitions are *deterministic*.
- $\boldsymbol{\mathcal{S}}$ is *minimal*: an ϵ-machine is the smallest causal representation of the transformation it implements.

Transducers

- We interpret the symbols labeling the transitions in the alphabet $\mathcal{A}$ as consisting of two parts: an *input symbol* that determines which transition to take from a state and an *output symbol* which is emitted on taking that transition.
- Transducers implement functions:
  - Character to character
  - Input string to output string
  - Map sets to sets (languages)

Set of Single State Machines



Transducer Composition

- Create new mapping, input language of one machine becomes input language of another
- Not commutative
- Possible exponential growth in number of states.

Interaction Network

Interaction Matrix $\mathcal{G}^{(k)}$

$$\mathcal{G}_{ij}^{(k)} = \begin{cases} 1 & \text{if } T_k = T_j \circ T_i \\ 0 & \text{otherwise} \end{cases}$$

Single State Transition Matrix

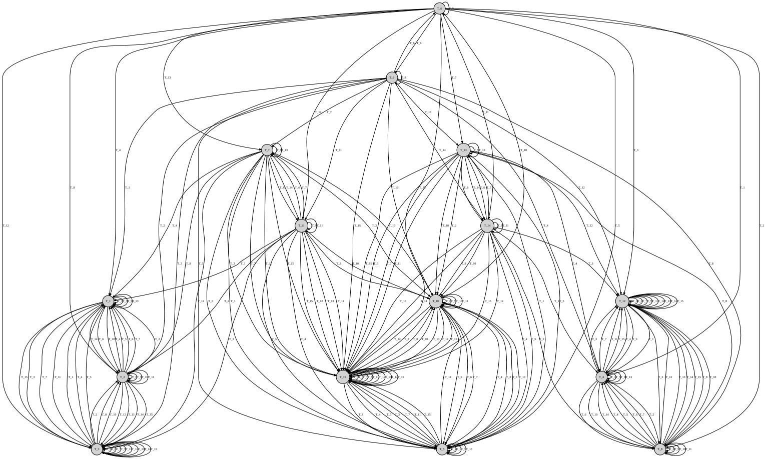
$$\begin{pmatrix}
T_1 & T_2 & T_3 & T_0 & T_1 & T_2 & T_3 & T_0 & T_1 & T_2 & T_3 & T_0 & T_1 & T_2 & T_3 \\
T_0 & T_0 & T_0 & T_1 & T_1 & T_1 & T_1 & T_2 & T_2 & T_2 & T_2 & T_3 & T_3 & T_3 & T_3 \\
T_1 & T_2 & T_3 & T_1 & T_1 & T_3 & T_3 & T_2 & T_3 & T_2 & T_3 & T_3 & T_3 & T_3 & T_3 \\
T_4 & T_8 & T_{12} & T_0 & T_4 & T_8 & T_{12} & T_0 & T_4 & T_8 & T_{12} & T_0 & T_4 & T_8 & T_{12} \\
T_5 & T_{10} & T_{15} & T_0 & T_5 & T_{10} & T_{15} & T_0 & T_5 & T_{10} & T_{15} & T_0 & T_5 & T_{10} & T_{15} \\
T_4 & T_8 & T_{12} & T_1 & T_5 & T_9 & T_{13} & T_2 & T_6 & T_{10} & T_{14} & T_3 & T_7 & T_{11} & T_{15} \\
T_5 & T_{10} & T_{15} & T_1 & T_5 & T_{11} & T_{15} & T_2 & T_7 & T_{10} & T_{15} & T_3 & T_7 & T_{11} & T_{15} \\
T_0 & T_0 & T_0 & T_4 & T_4 & T_4 & T_4 & T_8 & T_8 & T_8 & T_8 & T_{12} & T_{12} & T_{12} & T_{12} \\
T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 & T_9 & T_{10} & T_{11} & T_{12} & T_{13} & T_{14} & T_{15} \\
T_0 & T_0 & T_0 & T_5 & T_5 & T_5 & T_5 & T_{10} & T_{10} & T_{10} & T_{10} & T_{15} & T_{15} & T_{15} & T_{15} \\
T_1 & T_2 & T_3 & T_5 & T_5 & T_7 & T_7 & T_{10} & T_{11} & T_{10} & T_{11} & T_{15} & T_{15} & T_{15} & T_{15} \\
T_4 & T_8 & T_{12} & T_4 & T_4 & T_{12} & T_{12} & T_8 & T_{12} & T_8 & T_{12} & T_{12} & T_{12} & T_{12} & T_{12} \\
T_5 & T_{10} & T_{15} & T_4 & T_5 & T_{14} & T_{15} & T_8 & T_{13} & T_{10} & T_{15} & T_{12} & T_{13} & T_{14} & T_{15} \\
T_4 & T_8 & T_{12} & T_5 & T_5 & T_{13} & T_{13} & T_{10} & T_{14} & T_{10} & T_{14} & T_{15} & T_{15} & T_{15} & T_{15} \\
T_5 & T_{10} & T_{15} & T_5 & T_5 & T_{15} & T_{15} & T_{10} & T_{15} & T_{10} & T_{15} & T_{15} & T_{15} & T_{15} & T_{15}
\end{pmatrix}$$

Interaction Network

Interaction Graph $\mathcal{G}$

- Nodes correspond to ϵ-transducers
- If $T_C = T_B \circ T_A$, place directed edge connecting node T_A, to T_C, labeled with the transforming machine T_B.

Single State Interaction Network $\mathcal{G}$



Population Dynamics

- Population P
- N individuals

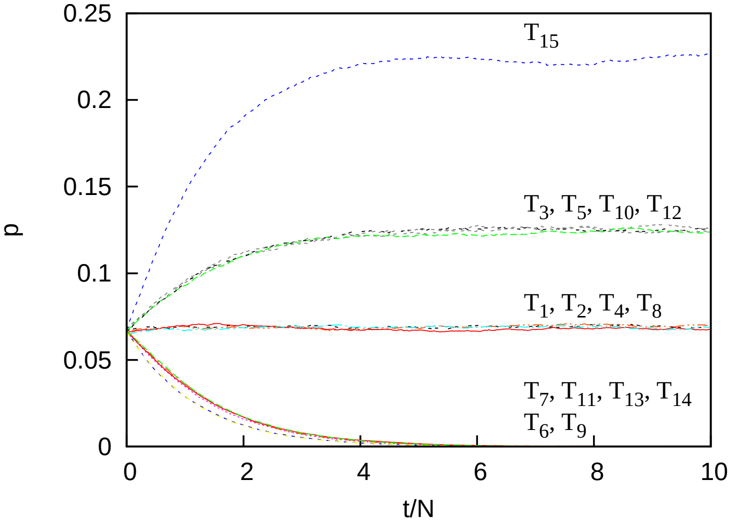
Population Dynamics

A single replication is determined through compositions and replacements in a two-step sequence:

1. Construct ϵ-machine T_C by forming the composition $T_C = T_B \circ T_A$ from T_A and T_B randomly selected from the population and minimizing.
2. Replace a randomly selected ϵ-machine, T_D, with T_C.

Note that there is no imposed notion of fitness nor spatial component.

Relaxation to Steady State (Single State Soup of Size 100,000)



Communication Channel

Discrete Channel [Cover and Thomas, 2006]

- $(\mathcal{X}, p(y|x), \mathcal{Y})$:
  - ($\mathcal{X}$ and $\mathcal{Y}$) are finite sets
  - $p(y|x)$ are probability mass functions

Channel Capacity

Cover and Thomas [Cover and Thomas, 2006] define the *channel capacity* of a discrete memoryless channel as:

$$C = \max_{p(x)} I(X;Y) \tag{1}$$

This capacity specifies the highest rate, in bits, at which information may be reliably transmitted through the channel, and Shannon's second theorem states that his rate is achievable in practice.

Mutual Information

Remember that **mutual information** is the reduction in uncertainty in one random variable due to knowledge of another.

$$I(X;Y) = H(Y) - H(Y|X) \tag{2}$$

Discrete Noiseless Channel

Discrete Noiseless Channel

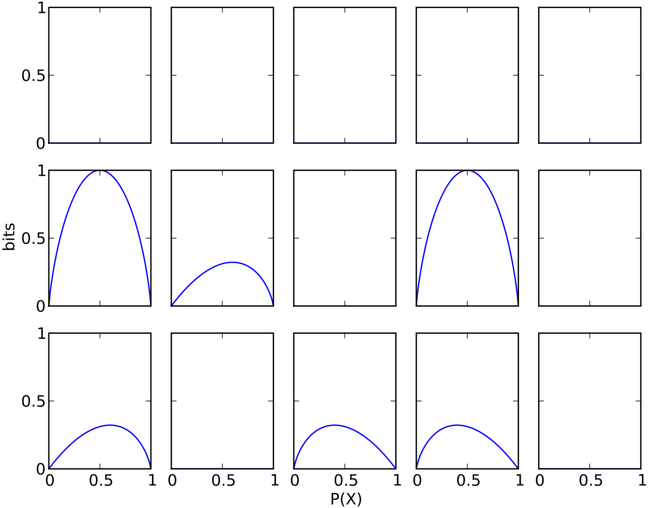
- One to one correspondence between input and output symbols

Languages

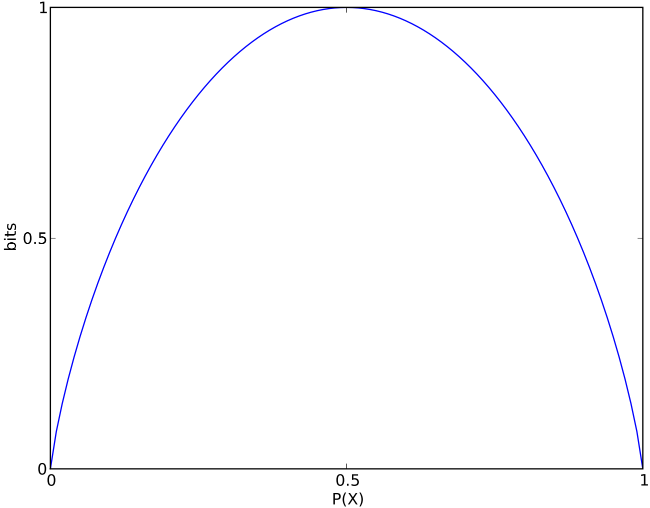
Partitioning based on language $\mathcal{L}$

- $\mathcal{L}_{in} = \Sigma^*$
- $\mathcal{L}_{out} = \Sigma^*$
- union of these – possibility for positive channel capacity

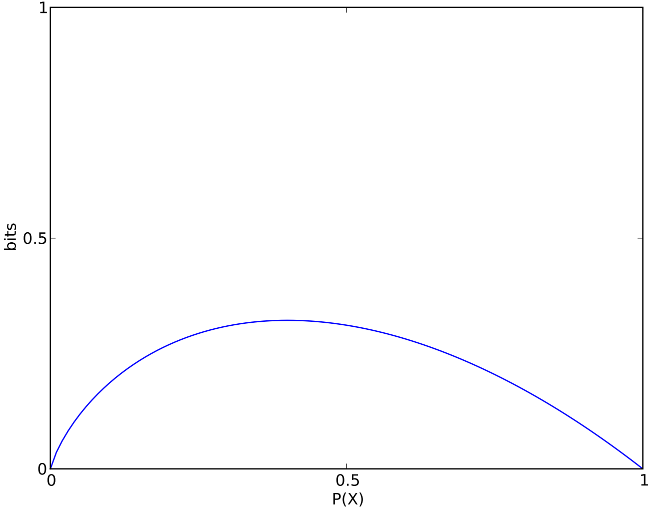
Single State Channel Capacities



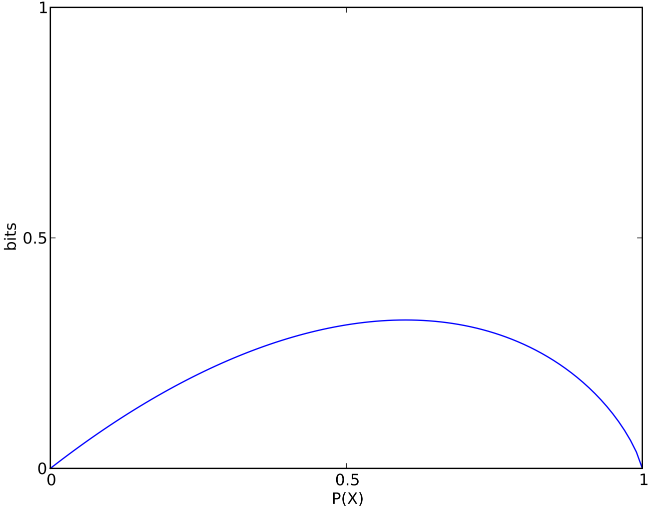
Machines of Maximal Channel Capacity



Partially Noisy Channels



Partially Noisy Channels



Single State Channel Capacities

Most are zero, except for...

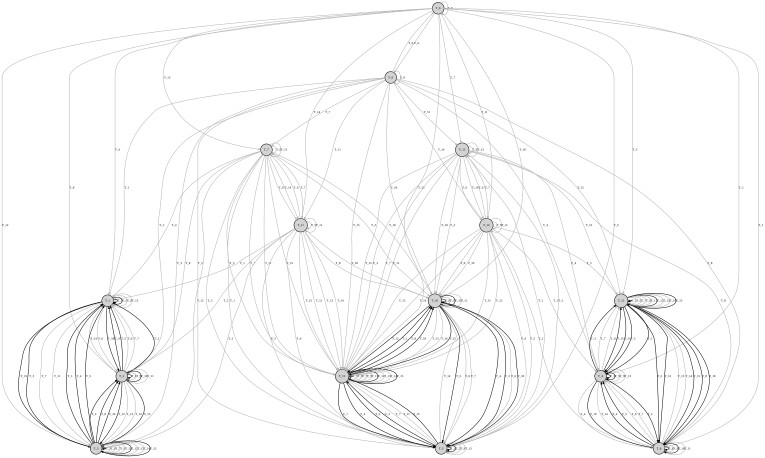
Machine Number	Channel Capacity	P(X=1)
6	1.0	0.5
7	0.32	0.6
9	1.0	0.5
11	0.32	0.6
13	0.32	0.4
14	0.32	0.4

Meta-machines

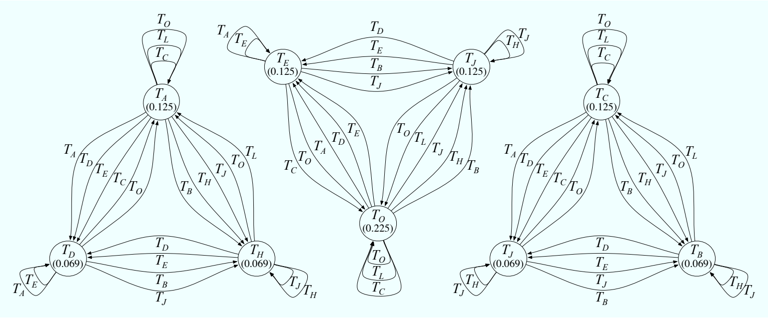
Meta-machine

- A set of machines that is closed and self-maintained under composition
- $\Omega \subseteq P$ is a meta-machine if and only if
  (i) $T_i \circ T_j \in \Omega$, for all $T_i, T_j \in \Omega$ and
  (ii) For all $T_k \in \Omega$, there exists $T_i, T_j \in \Omega$, such that $T_k = T_i \circ T_j$.
- Captures notion of invariant set: $\Omega = \mathcal{G} \circ \Omega$

Single State Metamachine



Single State Metamachine



[Crutchfield, 2006]

Summary

- Channel capacity is a function of machine structure.
- High channel capacity does not necessarily lead to persistence.
- All of the transducers in the single state meta-machine have zero channel capacity.
- Composition never increases channel capacity.

For Further Reading

T. M. Cover and J. A. Thomas
Elements of Information Theory
Wiley-Interscience, 2006

J. P. Crutchfield and O. Görnerup
Objects That Make Objects: The Population Dynamics of Structural Complexity
Journal of the Royal Society Interface, 3 (2006) 345-349.