# Computational Mechanics of the 2 Dimensional BTW Model

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# Goals of the Project

1. The 1D techniques we studied in class have been extended to 2D systems.
2. Understand the issues involved in 2 dimensional computational mechanics.
3. Ultimately, study the 2+1 dimensional Causal Dynamical Triangulation (CDT) Model.

# The 2+1 CDT Model

- Triangulated 2+1 dimensional manifold
- 2 dimensional spatial slices, $S^2$ topology.
- Gravitational path integral, Monte Carlo sampling.

# The 2 Dimensional Bak-Tang-Wiesenfeld Model

- A sandpile model that exhibits self-organized behavior.
- $L \times L$ square grid. Grains added to random sites.
- Exceeding threshold value (= 3) causes site to relax.
- A cascade of relaxing sites — "avalanche".

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 2 | 3 | 2 | 0 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 3 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 3 | 1 | 3 | 0 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 1 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 3 | 2 | 0 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 3 | 3 | 3 | 3 | 2 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 3 | 1 | 4 | 0 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 1 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 3 | 2 | 0 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 3 | 3 | 3 | 4 | 2 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 3 | 2 | 0 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 3 | 2 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 3 | 3 | 4 | 0 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 3 | 2 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 0 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 2 | 3 | 3 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 3 | 4 | 0 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 0 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 2 | 4 | 3 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 4 | 0 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 4 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 2 | 1 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 3 | 0 | 4 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 4 | 1 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 2 | 4 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 4 | 0 | 4 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 0 | 2 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 3 | 4 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 3 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
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| 4 | 0 | 4 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 0 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 4 | 0 | 4 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 4 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
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| 3 | 1 | 3 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 0 | 1 | 4 | 1 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 1 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 4 | 0 | 4 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 4 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
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| 0 | 2 | 0 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 1 | 3 | 2 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 4 | 0 | 4 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 2 | 4 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
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| 3 | 1 | 4 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 0 | 2 | 0 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 2 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 0 | 1 | 4 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 3 | 4 | 0 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 0 | 2 | 0 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 0 | 2 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 3 | 4 | 1 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 3 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 4 | 3 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 0 | 2 | 0 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 0 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 4 | 0 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 4 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 0 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 0 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 4 | 0 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 2 | 4 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
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| 0 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 1 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 0 | 1 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 3 | 4 | 2 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 1 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
| 2 | 2 | 1 | 0 | 3 | 3 | 0 | 1 | 3 | 2 | 0 | 3 |

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|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 0 | 4 | 1 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 0 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 1 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 0 | 2 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 4 | 0 | 3 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 2 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
| 2 | 2 | 1 | 0 | 3 | 3 | 0 | 1 | 3 | 2 | 0 | 3 |

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| 0 | 2 | 1 | 3 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 1 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 0 | 2 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 4 | 0 | 3 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 3 | 2 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
| 2 | 2 | 1 | 0 | 3 | 3 | 0 | 1 | 3 | 2 | 0 | 3 |

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| 0 | 2 | 1 | 3 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 1 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 0 | 1 | 3 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 4 | 2 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 1 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
| 2 | 2 | 1 | 0 | 3 | 3 | 0 | 1 | 3 | 2 | 0 | 3 |

# The 2 Dimensional Bak-Tang-Wiesenfeld Model

- A sandpile model that exhibits self-organized behavior.
- $L \times L$ square grid. Grains added to random sites.
- Exceeding threshold value (= 3) causes site to relax.
- A cascade of relaxing sites — “avalanche”.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 1 | 0 | 2 | 1 | 3 | 2 | 2 | 1 | 2 | 0 |
| 0 | 2 | 1 | 3 | 0 | 3 | 2 | 1 | 1 | 0 | 2 | 1 |
| 2 | 3 | 3 | 1 | 3 | 2 | 2 | 1 | 2 | 2 | 1 | 1 |
| 1 | 3 | 0 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 0 | 1 | 1 | 3 | 1 | 0 | 2 | 3 |
| 1 | 1 | 3 | 3 | 0 | 3 | 3 | 1 | 3 | 1 | 1 | 0 |
| 0 | 3 | 1 | 3 | 1 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| 2 | 1 | 1 | 0 | 2 | 3 | 3 | 2 | 2 | 2 | 1 | 0 |
| 1 | 0 | 2 | 3 | 0 | 2 | 1 | 2 | 0 | 1 | 0 | 2 |
| 1 | 2 | 2 | 3 | 0 | 2 | 2 | 3 | 3 | 1 | 2 | 1 |
| 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 1 | 0 | 3 |
| 2 | 2 | 1 | 0 | 3 | 3 | 0 | 1 | 3 | 2 | 0 | 3 |

# Entropy Measures of 2D BTW Model

## 1D Blocks

$$
\begin{aligned}
H(L) &= -\sum_{s^L \in \mathcal{A}} Pr(s^L) \log_2 Pr(s^L) \\
h_\mu &= \lim_{L \to \infty} \frac{H(L)}{L}
\end{aligned}
$$

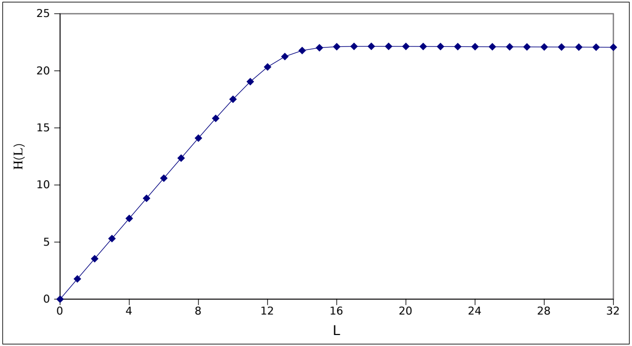



**Figure:** H(L) versus L for 1D Blocks

# Entropy Measures of 2D BTW Model

## 2D Blocks

$$
\begin{aligned}
\mathcal{B}_{i,j}^{M,N} &\equiv \begin{array}{ccccc}
s_{i,j} & s_{i,j+1} & s_{i,j+2} & \cdots & s_{i,j+M-1} \\
s_{i+1,j} & s_{i+1,j+1} & s_{i+1,j+2} & \cdots & s_{i+1,j+M-1} \\
s_{i+2,j} & s_{i+2,j+1} & s_{i+2,j+2} & \cdots & s_{i+2,j+M-1} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
s_{i+N-1,j} & s_{i+N-1,j+1} & s_{i+N-1,j+2} & \cdots & s_{i+N-1,j+M-1}
\end{array} \\
H(M,N) &= -\sum_{\mathcal{B}} Pr(\mathcal{B}^{\mathcal{M},\mathcal{N}}) \log_2 Pr(\mathcal{B}^{\mathcal{M},\mathcal{N}}) \\
h_\mu &= \lim_{M,N\to\infty} \frac{H(M,N)}{MN}
\end{aligned}
$$

# Entropy Measures of 2D BTW Model

## 2D Blocks

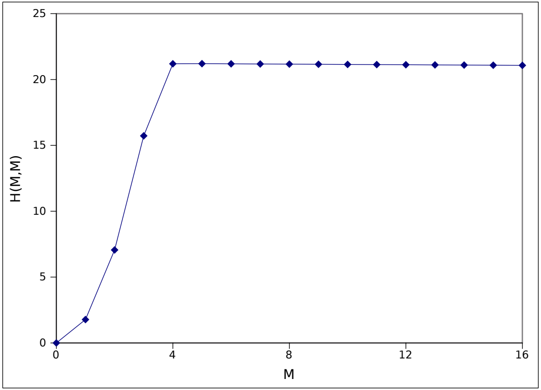



**Figure:** H(M,M) versus M for 2D Blocks

# Entropy Measure of 2D BTW Model

## Conditioned Block Entropy

In 1D we have:

$$h_\mu(L) = H[S_L | S_{L-1} S_{L-2} \cdots S_1]$$

What about 2D?

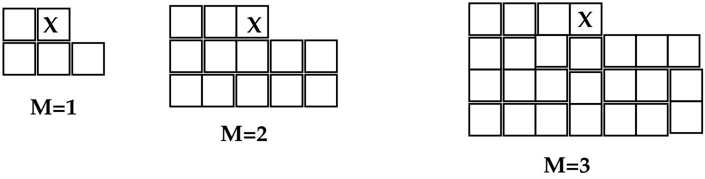
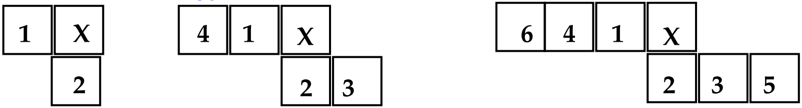



**Figure:** Conditioned Block Entropy

| | |
|---|---|
| 1 | 1.78 |
| 2 | $0.02 - 0.5$ |
| 3 | 0 |
| 4 | 0 |

# Entropy Measure of 2D BTW Model

## Conditioned Block Entropy




M=1 M=2 M=3

| | |
|---|---|
| 1 | 1.74398 |
| 2 | 1.73623 |
| 3 | 1.73225 |
| 4 | 1.7097 |
| 5 | 1.50407 |
| 6 | 0.813701 |
| 7 | 0.186773 |
| 8 | 0.0226799 |
| 9 | 0.00228094 |
| 10 | 0.000238476 |
| 11 | $1.91761 \times 10^{-5}$ |
| 12 | $7.7352 \times 10^{-7}$ |

# Conclusions and Future Work

1. 2D Computational Mechanics is hard!
2. Ambiguity about the shapes and ordering of the 2D blocks.
3. String length increases exponentially.
4. Poor choice of model? — binary alphabet models would be better.
5. Next step — investigate the $\uparrow\downarrow$ Ising Model on the honeycomb/hexagonal/triangular lattice.
6. What about CDT? All of the above issues plus...
   A non-fixed lattice!!!

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