# Binary Coding in ε-machine Reconstruction

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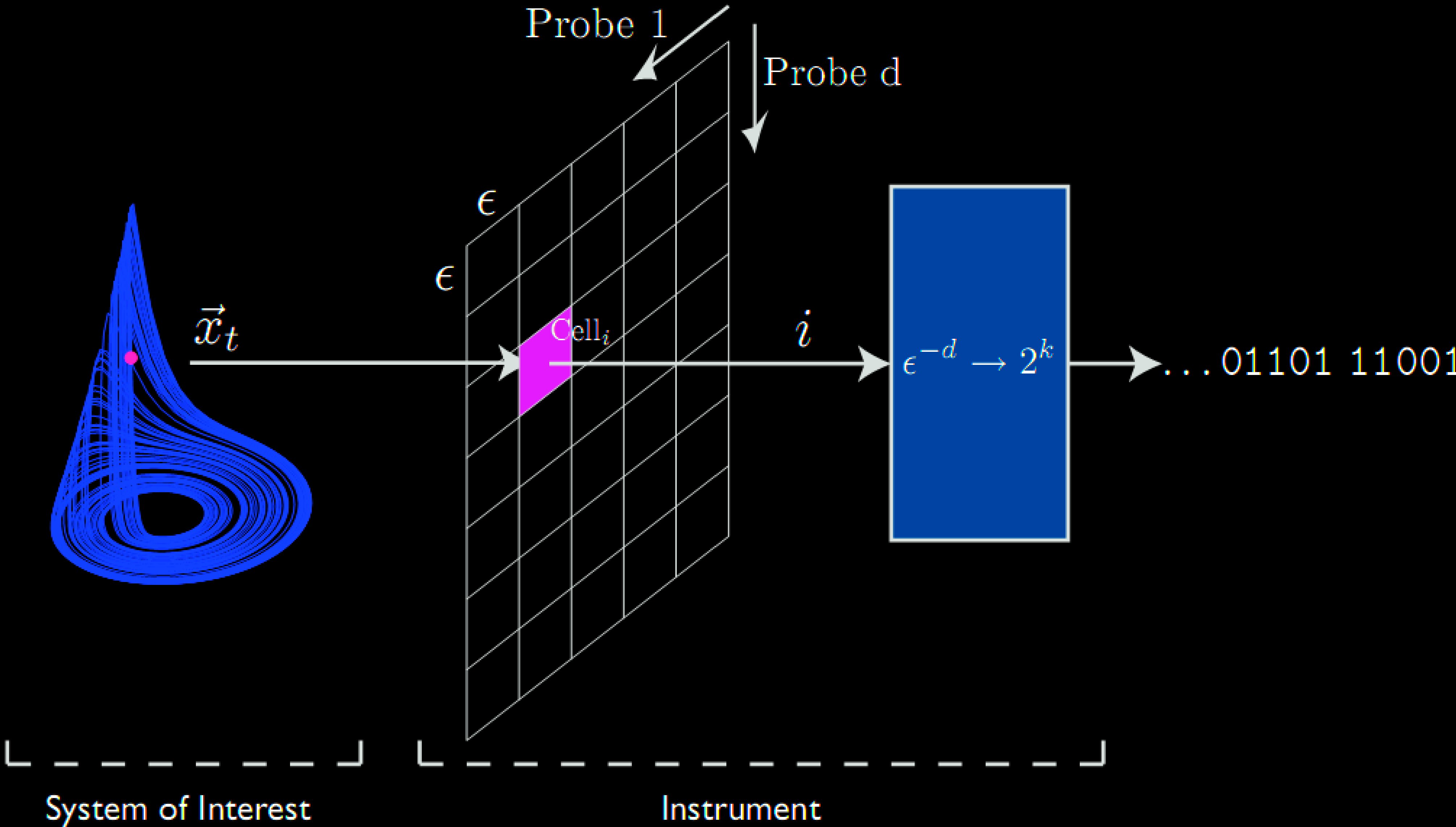
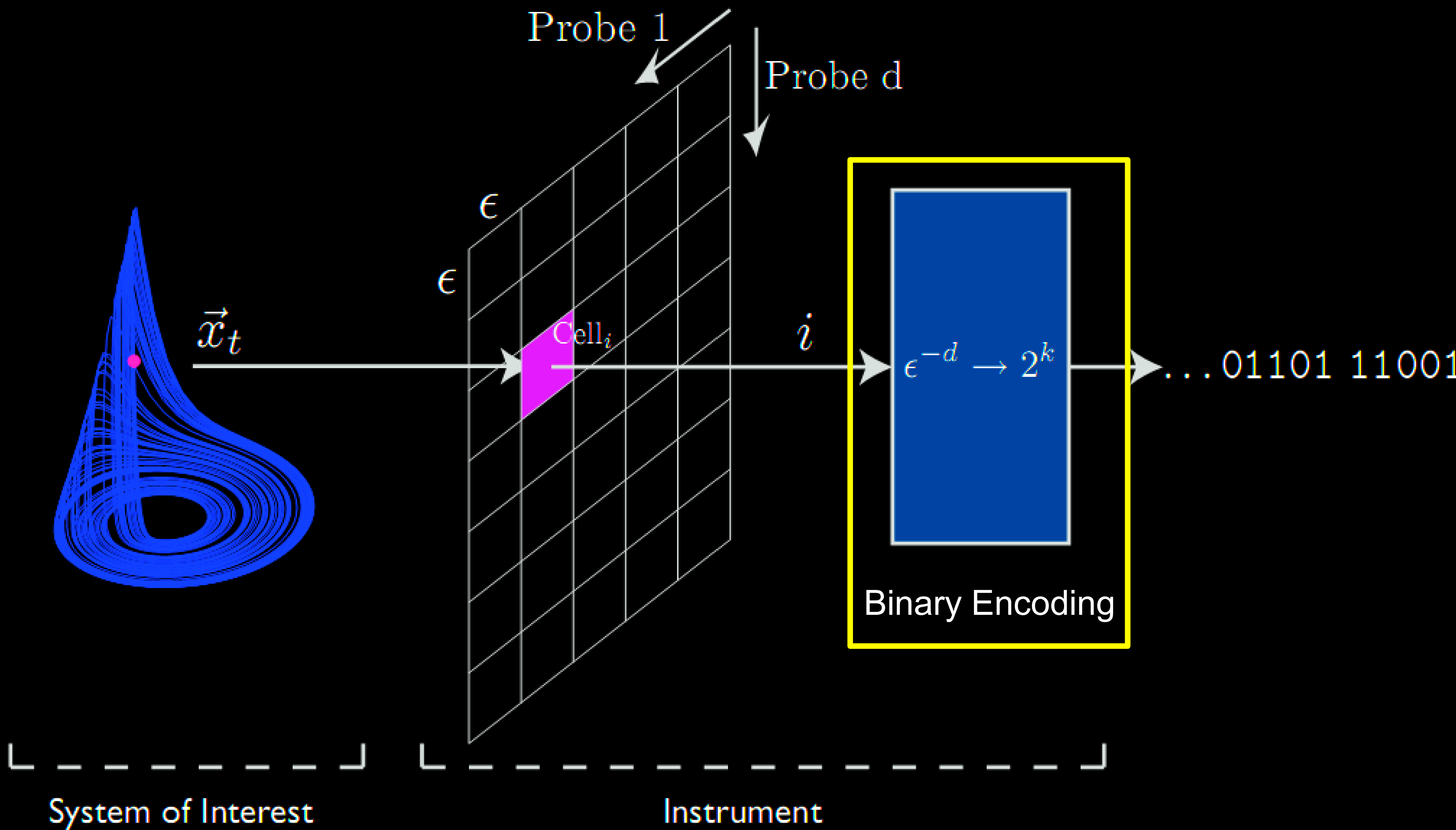
Final Project, Phys 256B

# From Determinism to Stochasticity ...




## Measurement Channel

# From Determinism to Stochasticity ...

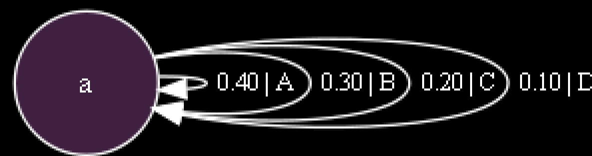



## Measurement Channel

# Questions

- How does binary encoding change the model?
  - Information quantities
  - Graph structure
- Can we map between the machines?
  - Closed form for composed machine
  - Invertibility of encoding, inversion synchronization
- Do all nontrivial coding methods give the same results?

# Experimental Setup:



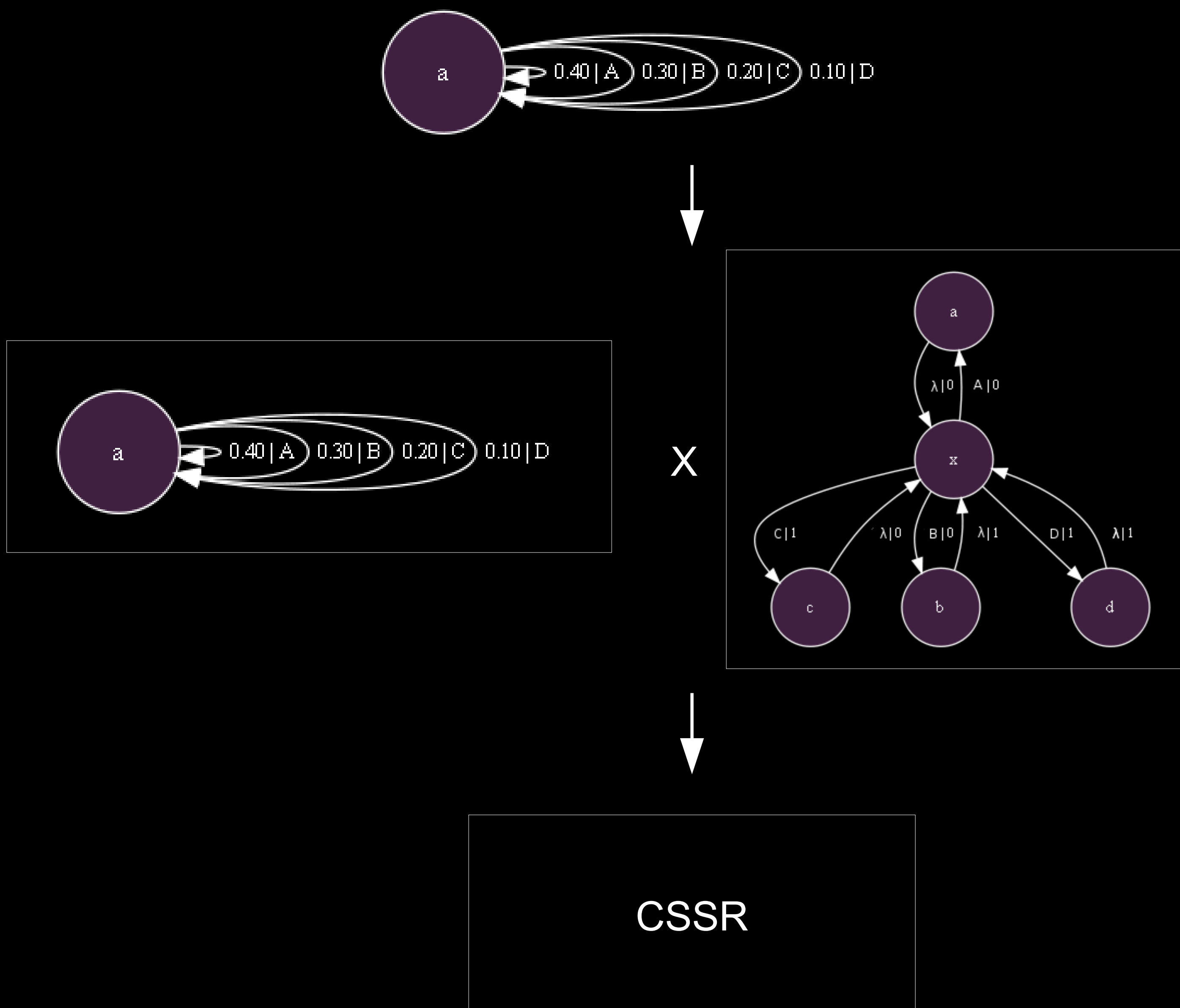

↓

A → 00
B → 01
C → 10
D → 11

↓

CSSR

# Experimental Setup:



# Thoughts on $h_\mu$

- Might expect $h_\mu^{bin} = h_\mu/2$
- Calculation process (H(2)):

| A | B | B | D | A | C | D | B | C | A | A | D | B | C | D | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 01 | 01 | 11 | 00 | 10 | 11 | 01 | 10 | 00 | 00 | 11 | 01 | 10 | 11 | 10 |

← n symbols
← 2n symbols

"in phase" count "out of phase" count

Example:

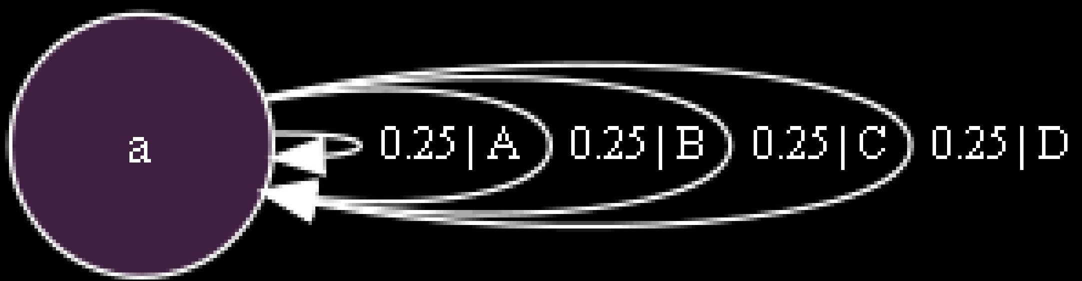
count(11) = count(D) + count(BD) + count(BC) + count(DD) + count(DC)

P(11) = count(11) / 2n

H(2) = H( P(xx) )

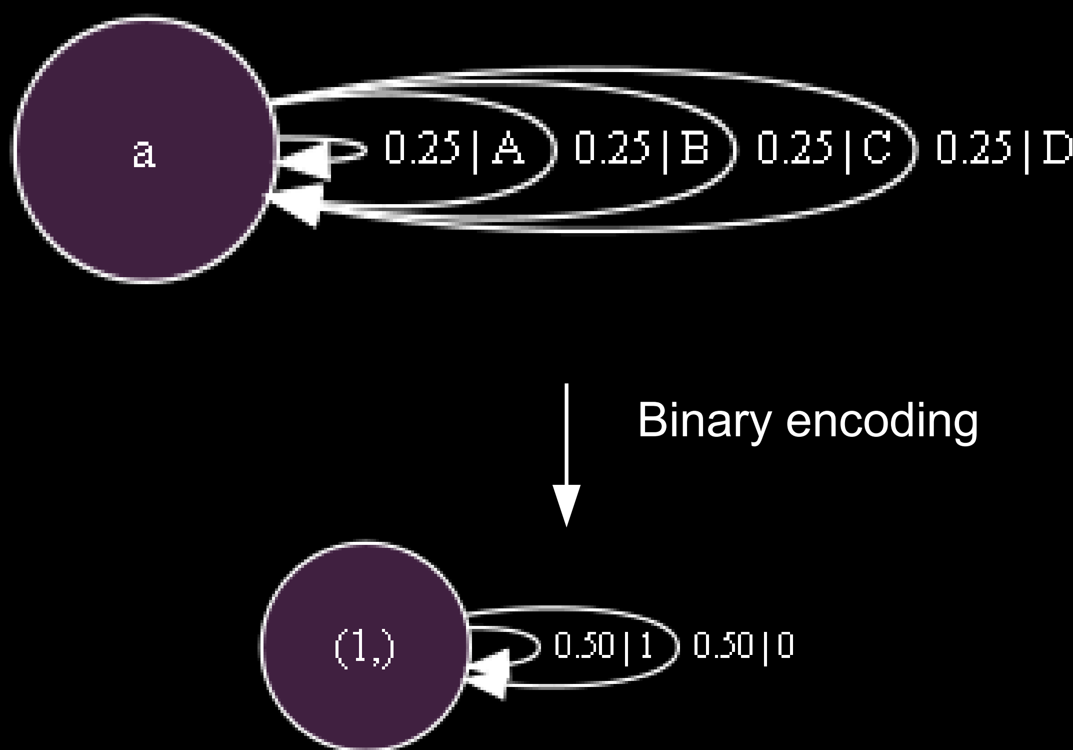
$h_\mu(2) = H(2) / 2$

# Fair 4-die



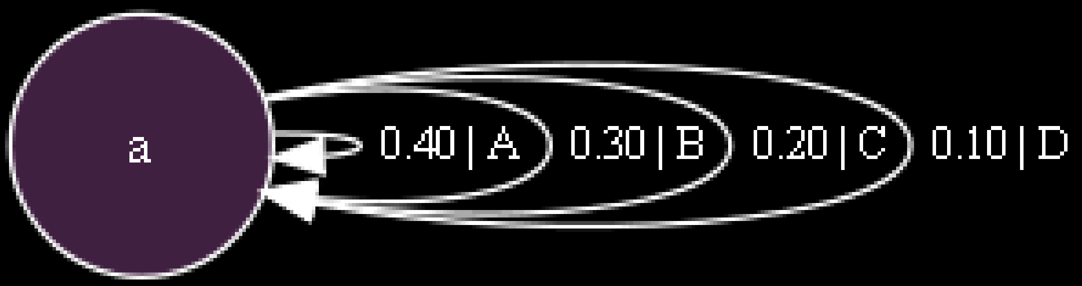

$h_\mu = 2$ $C_\mu = 0$

# Fair 4-die




$h_\mu = 1$ (Fair coin) $C_\mu = 0$

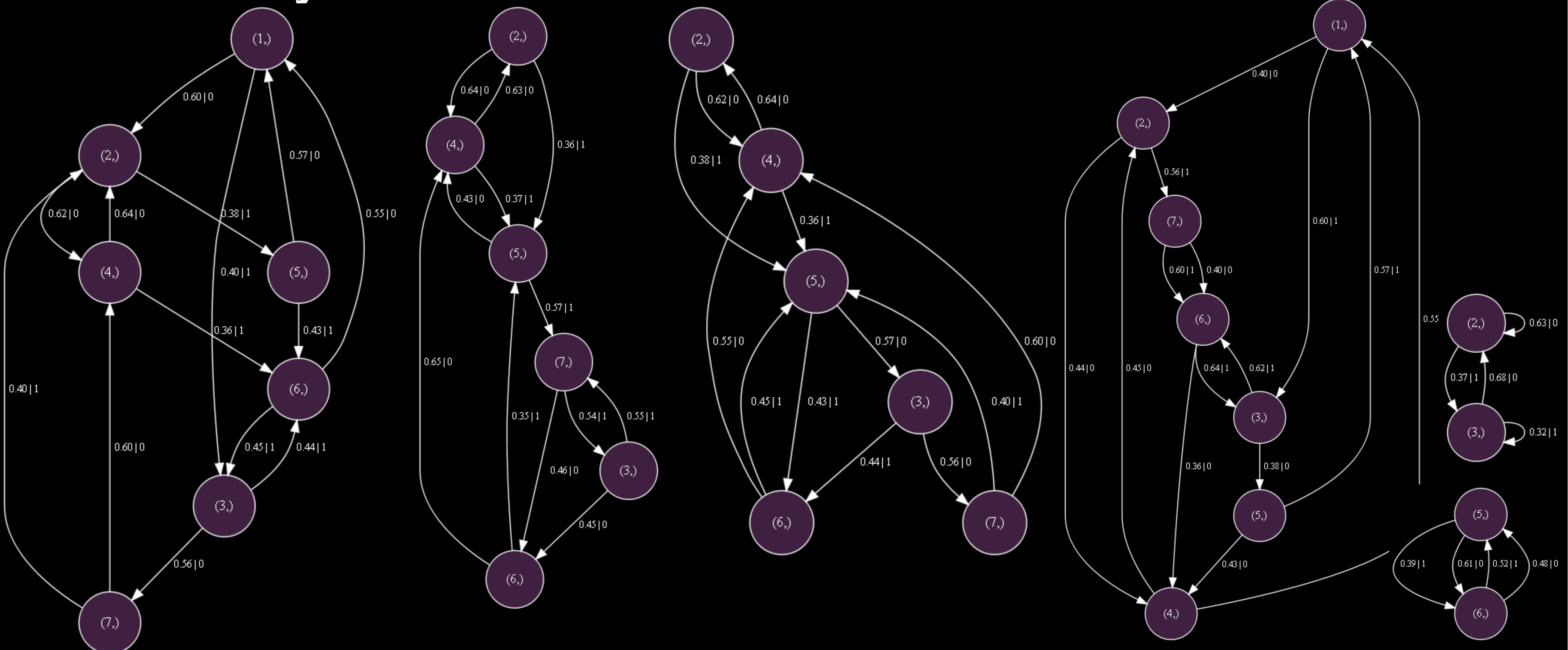
# "Linear" 4-die




$h_\mu = 1.846$

$C_\mu = 0$

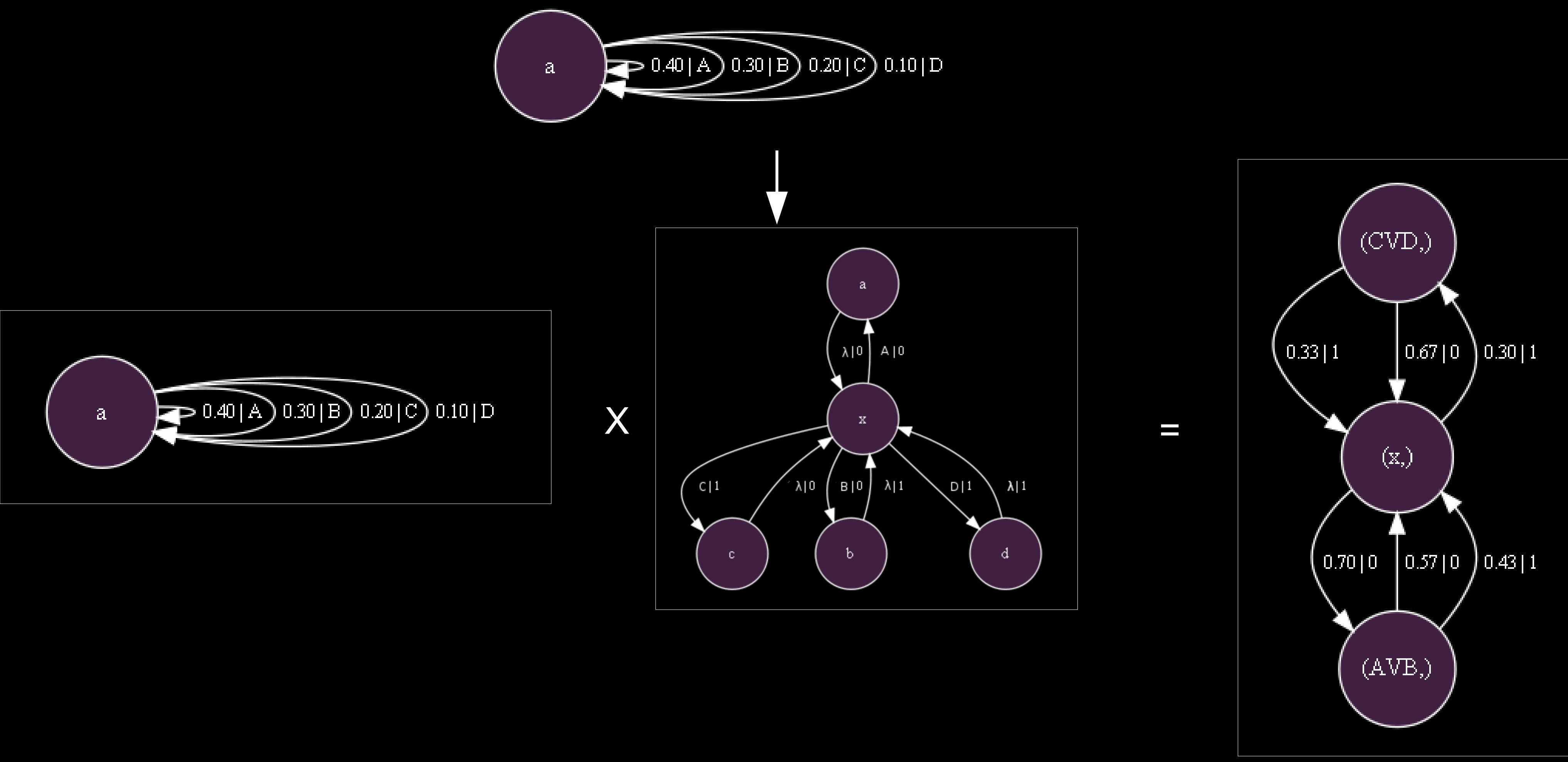
# Binary Encoded Machines from "Linear" 4-die




etc.

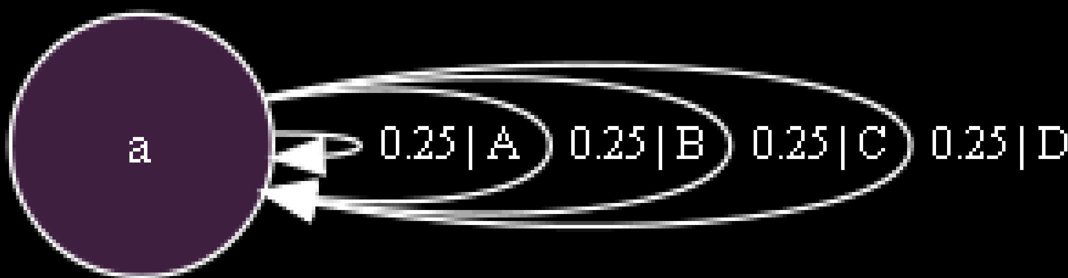
“TOO LITTLE DATA, SO I MAKE BIG” -Dr. Sbaitso

# Plan B:



a
0.40 | A
0.30 | B
0.20 | C
0.10 | D
a
0.40 | A
0.30 | B
0.20 | C
0.10 | D
X
a
λ | 0
A | 0
x
C | 1
λ | 0
B | 0
λ | 1
D | 1
λ | 1
c
b
d
=
(CVD,)
0.33 | 1
0.67 | 0
0.30 | 1
(x,)
0.70 | 0
0.57 | 0
0.43 | 1
(AVB,)

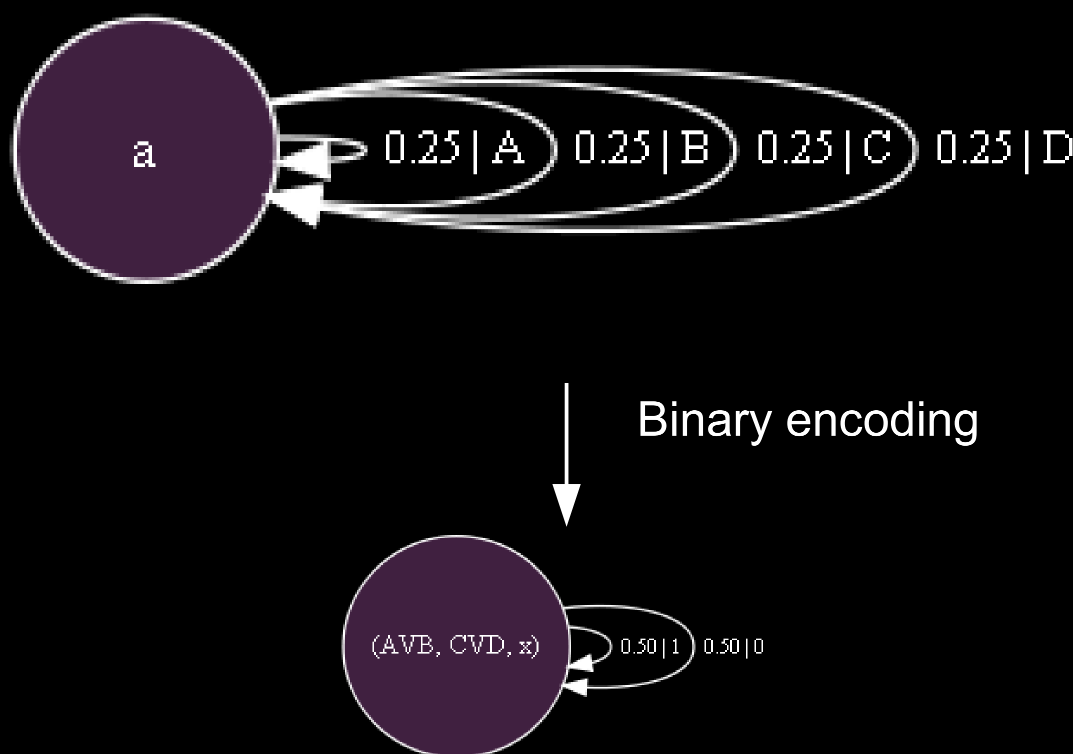
# Fair 4-die




$h_\mu = 2$

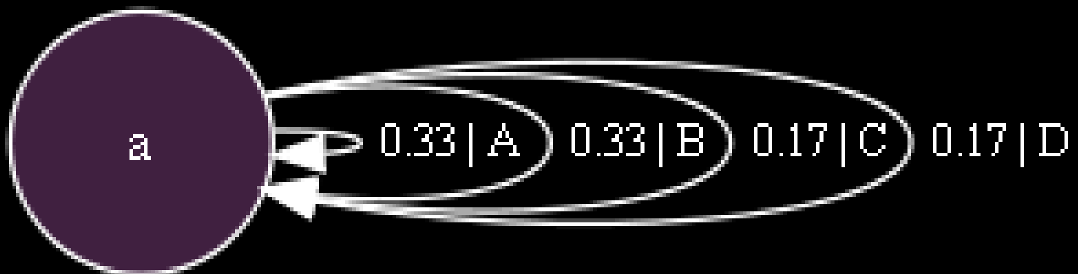
$C_\mu = 0$

# Fair 4-die




$h_\mu = 1$ (Fair coin) $C_\mu = 0$

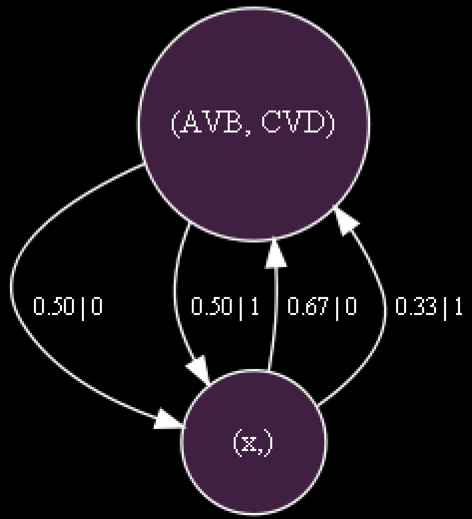
# "Step" 4-die




$h_\mu = 1.918$

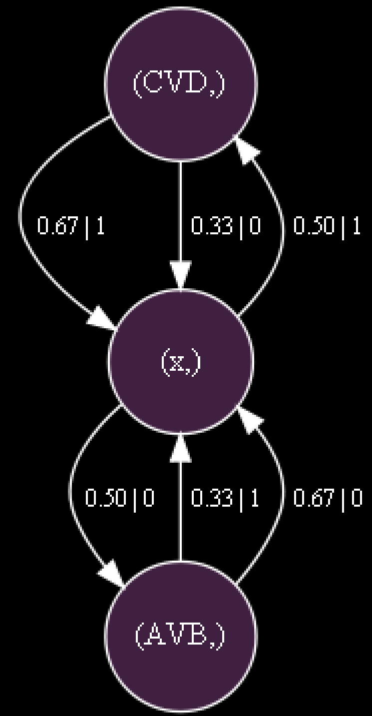
$C_\mu = 0$

# Binary Encoded Machines from "Step" 4-die




$C_\mu = 1$

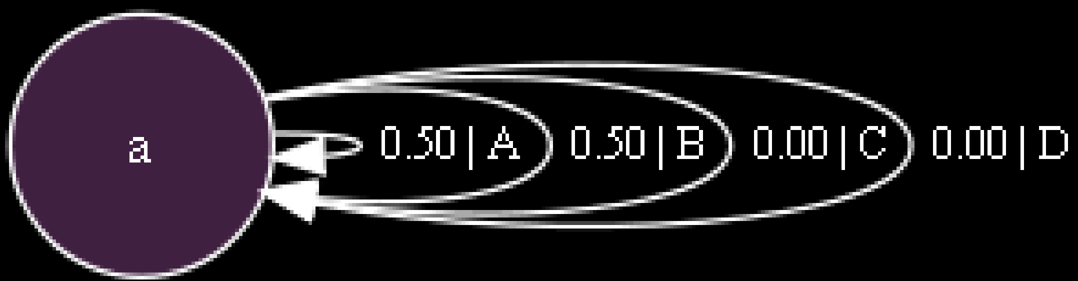
("topologically" exhaustive)




$C_\mu = 1.5$

$h_\mu^{bin} = .959 = h_\mu/2$

# "Coin" 4-die

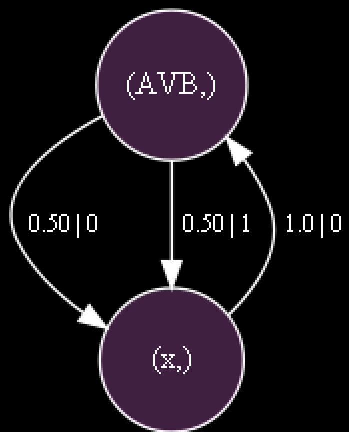



$h_\mu = 1$

$C_\mu = 0$

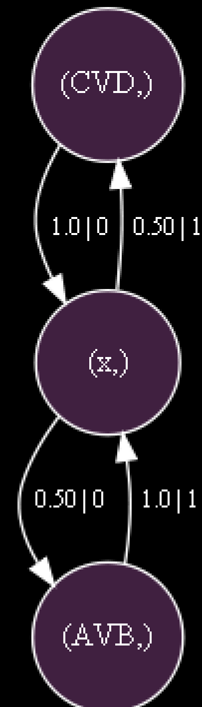
# Binary Encoded Machines from "Coin" 4-die

noisy period 2



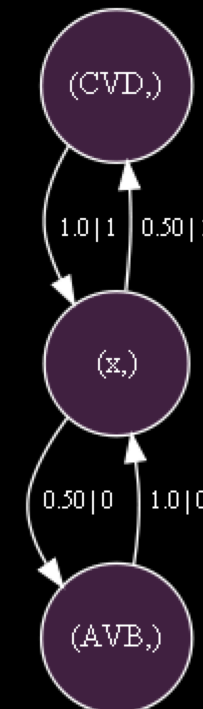

$C_\mu = 1$

random phase period 2




$C_\mu = 1.5$

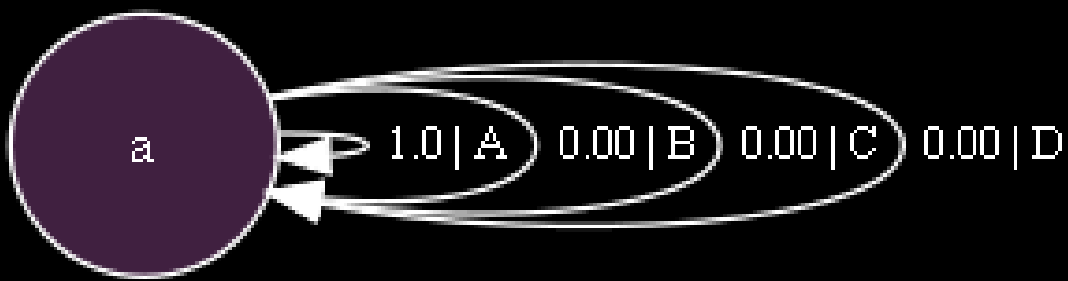
doubly even process




$C_\mu = 1.5$

$h_\mu^{bin} = .5 = h_\mu/2$

# "A" 4-die

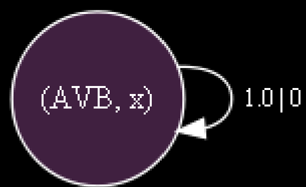



$h_\mu = 0$

$C_\mu = 0$

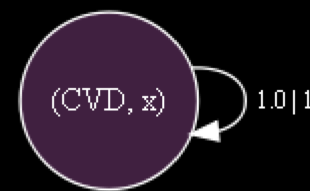
# Binary Encoded Machines from "A" 4-die

ones



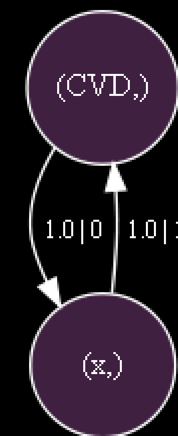

$C_\mu = 0$

zeros




$C_\mu = 0$

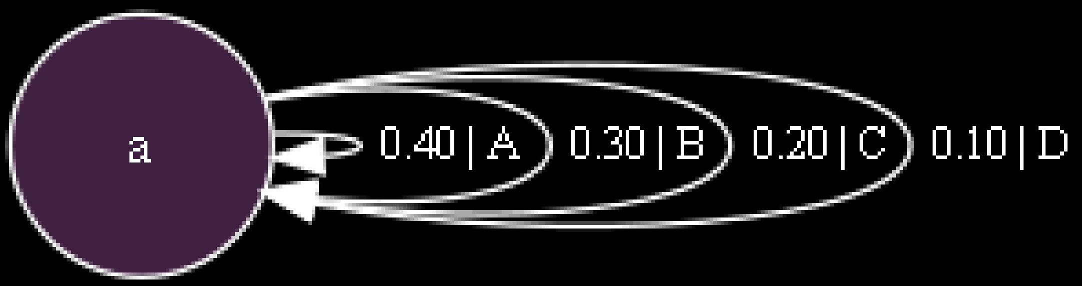
period-2




$C_\mu = 1$

$$h_\mu^{bin} = 0 = h_\mu/2$$

# "Linear" 4-die

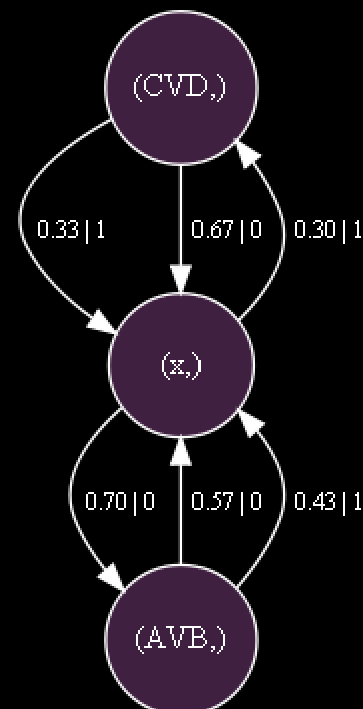



$h_\mu = 1.846$

$C_\mu = 0$

# Binary Encoded Machines from "Linear" 4-die

("topologically" general case)




$C_\mu$ = 1.441, 1.485, 1.5

$h_\mu^{bin} = .923 = h_\mu/2$

# Conclusions

- Graph structure clearly changes
- $h_\mu^{bin} = h_\mu/2$ for 4-dice
- Composed machine can be constructed analytically
- Coding method <u>does</u> affect binary machine / $C_\mu$

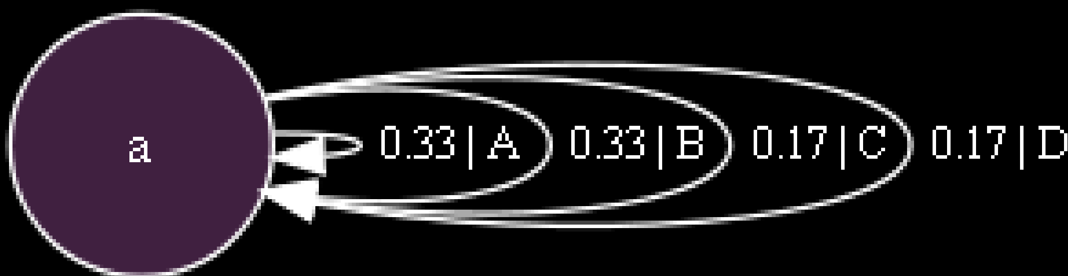
# Future Work

- Apply these methods to general 4-letter, $2^n$-letter, n-letter processes
- Functional relationship between $C_\mu^{bin}$ and $C_\mu$
- How do E and $\chi$ change?
- "Decoding" the binary machine / inversion synchronization
- Inhomogeneous coding (Huffman?)

Thanks!

# Plan A:
# Blooper Slides

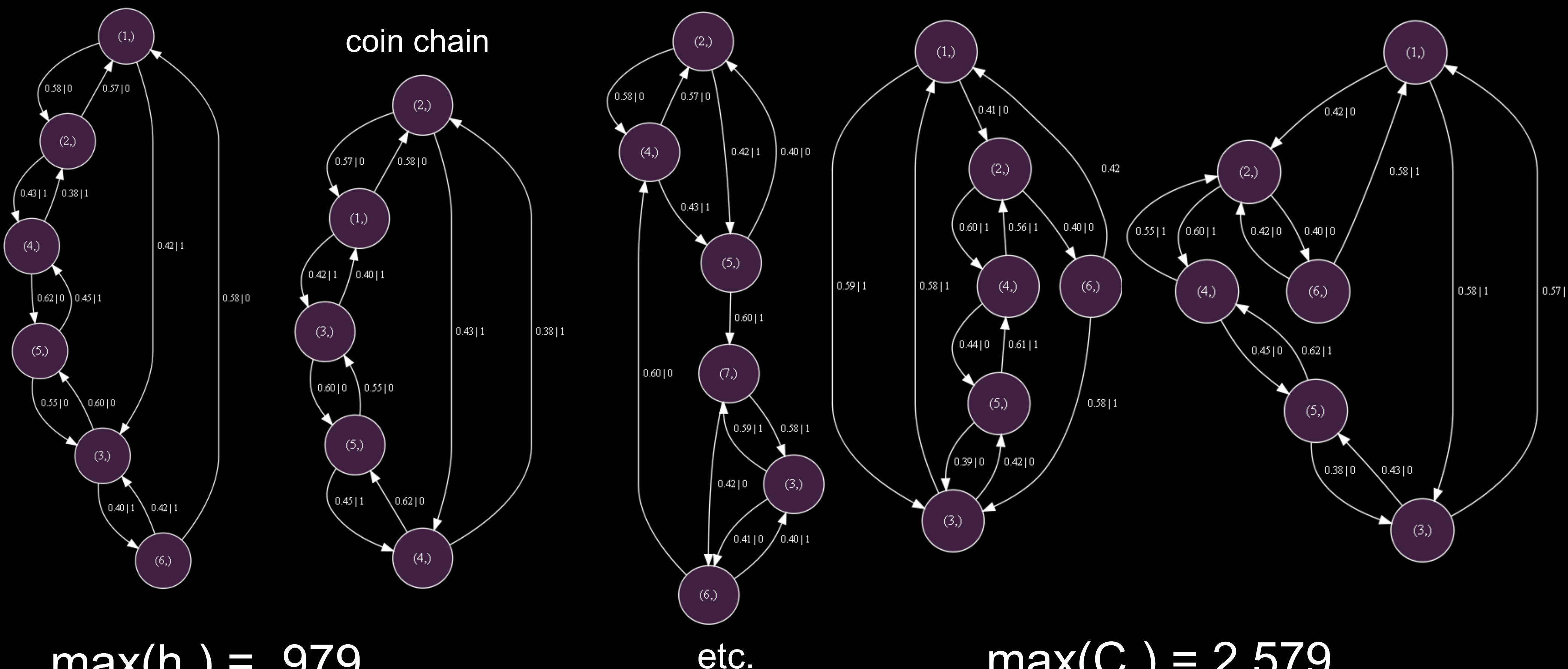
# "Step" 4-die




$h_\mu = 1.918$

$C_\mu = 0$

# Binary Encoded Machines from "Step" 4-die




$\max(h_\mu) = .979$

$\text{mean}(h_\mu) = .978$

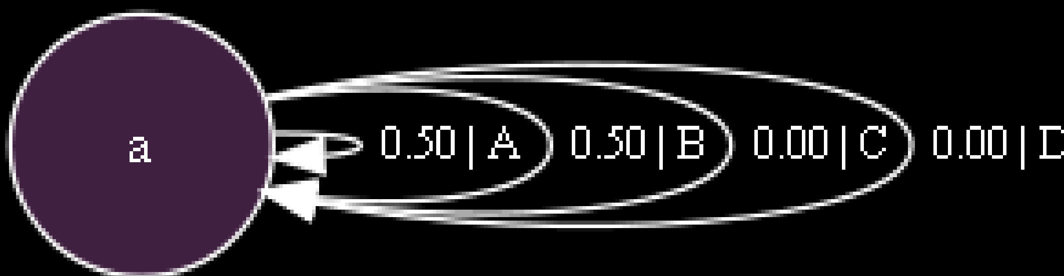
$\min(h_\mu) = .977$

$\max(C_\mu) = 2.579$

$\text{mean}(C_\mu) = 2.426$

$\min(C_\mu) = 2.258$

# "Coin" 4-die

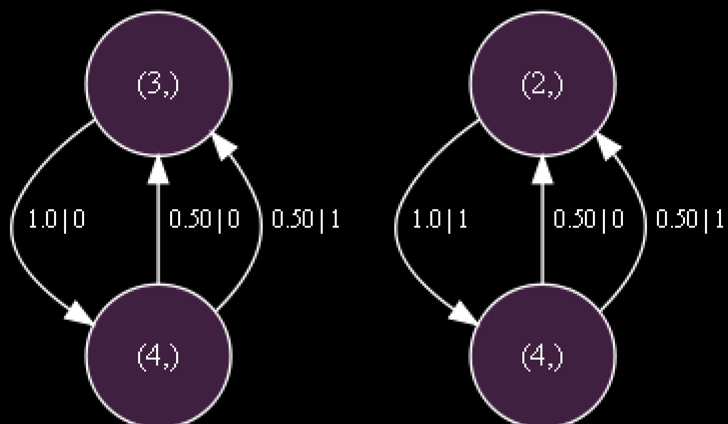



$h_\mu = 1$

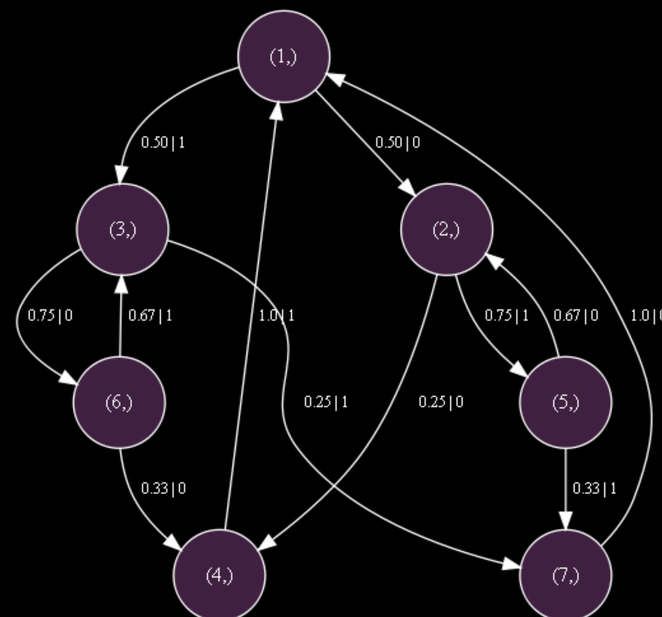
$C_\mu = 0$

# Binary Encoded Machines from "Coin" 4-die

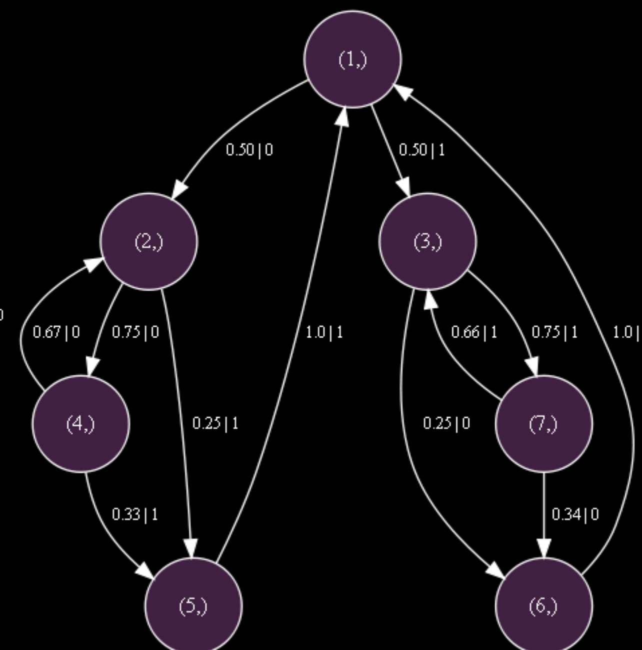
noisy period 2




random phase period 2




doubly even process




$\max(h_\mu) = .728$

$\text{mean}(h_\mu) = .579$

$\min(h_\mu) = .5$

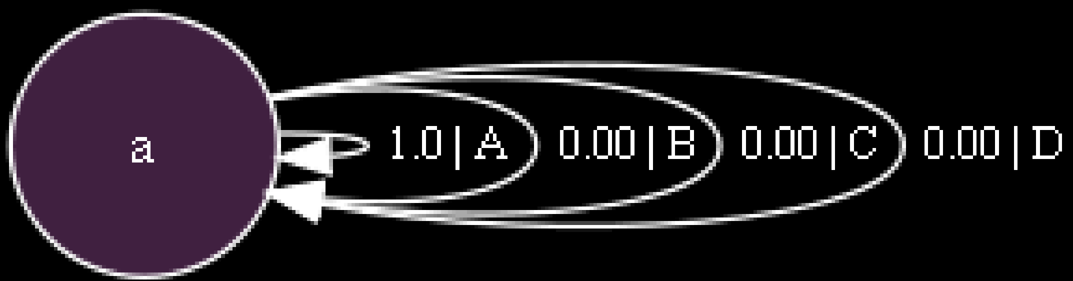
etc.

$\max(C_\mu) = 2.755$

$\text{mean}(C_\mu) = 1.585$

$\min(C_\mu) = 1$

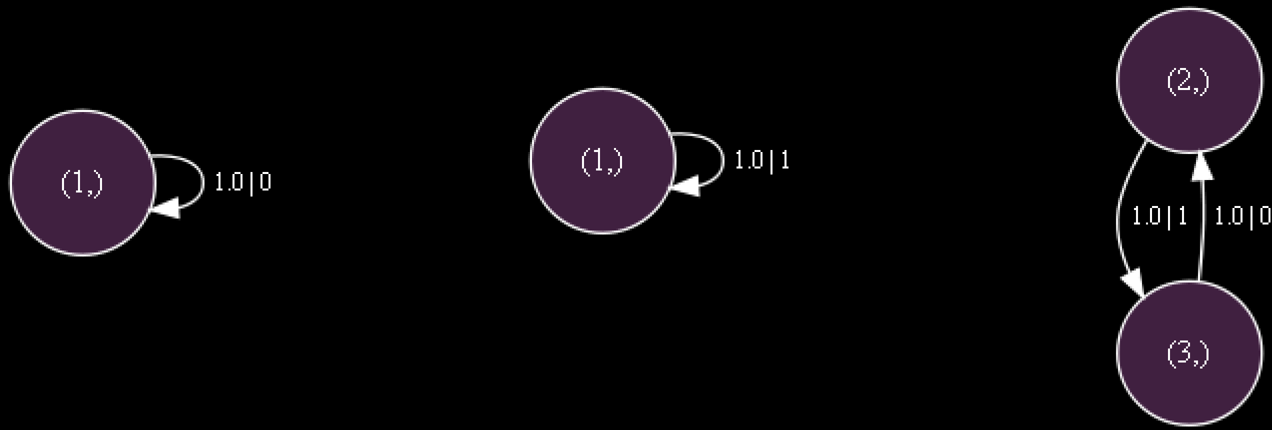
# "A" 4-die




$h_\mu = 0$

$C_\mu = 0$

# Binary Encoded Machines from "A" 4-die




(exhaustive)

$\max(h_\mu) = 0$

$\text{mean}(h_\mu) = 0$

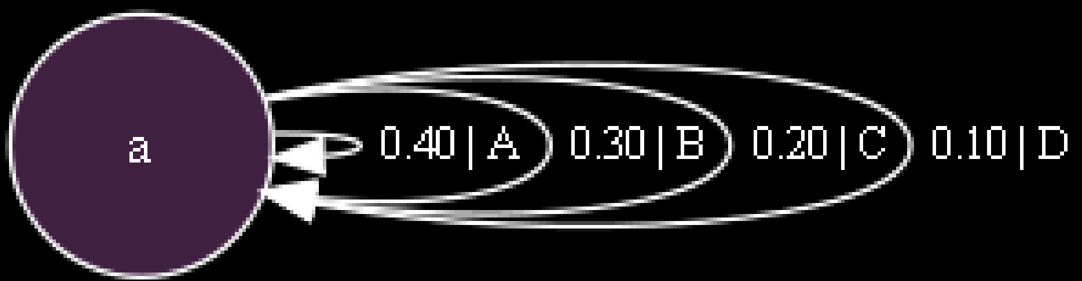
$\min(h_\mu) = 0$

$\max(C_\mu) = 1$

$\text{mean}(C_\mu) = .5$

$\min(C_\mu) = 0$

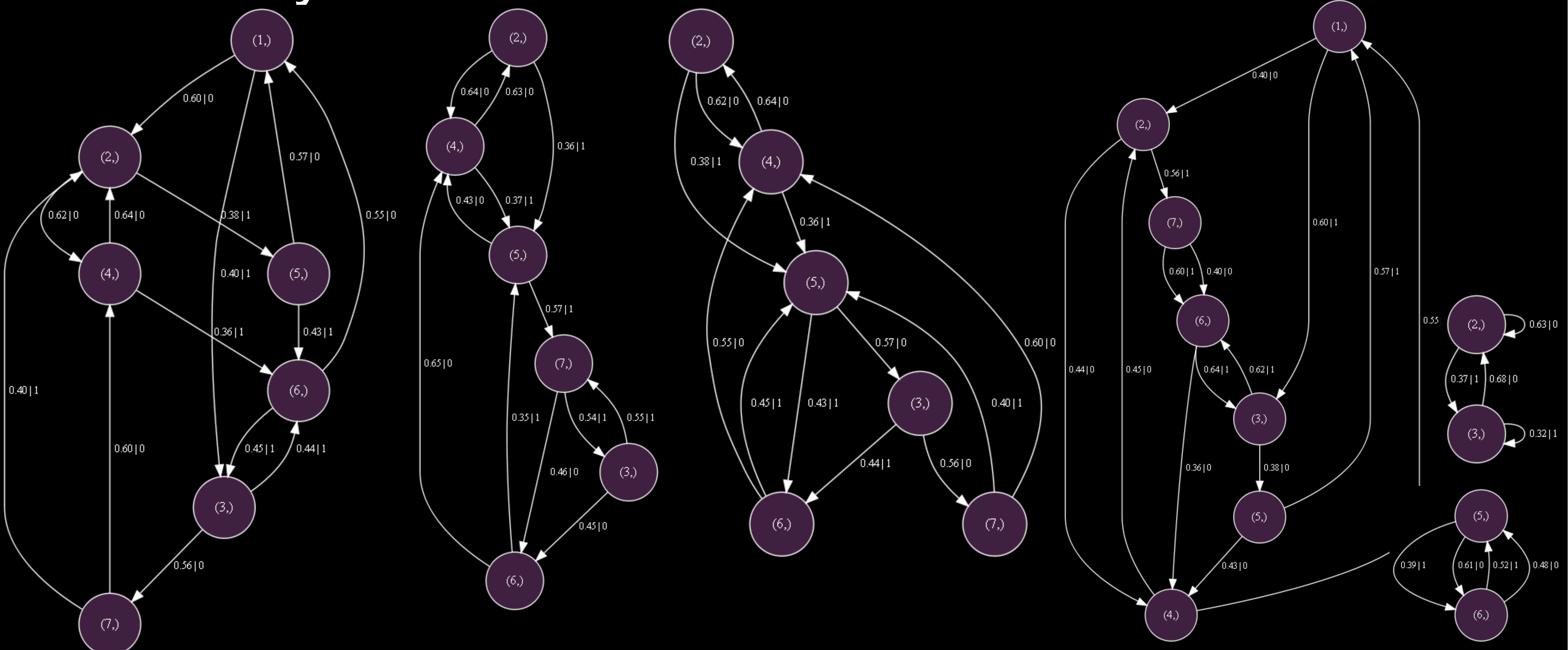
# "Linear" 4-die




$h_\mu = 1.846$

$C_\mu = 0$

# Binary Encoded Machines from "Linear" 4-die




max($h_\mu$) = .981

mean($h_\mu$) = .960

min($h_\mu$) = .930

etc.

max($C_\mu$) = 2.713

mean($C_\mu$) = 2.119

min($C_\mu$) = .934