Rikitake Model of Geomagnetic Reversal

I. Introduction & Background

To investigate the origin of the magnetic field of the earth and other celestial bodies, a natural idea is to consider there is a homogeneous dynamo in the earth's core. However, for a theory of homogeneous dynamo, it would be too complicated to obtain an exact solution when the couplings between electromagnetic fields and motions of electric conducting fluid are considered. To be specifically, we would need a simultaneous solver for the equations of the electromagnetism, hydrodynamics, and heat conduction in self-exciting dynamo.

Bullard[i],[ii],[iii] proposed a simplified idea for the possible analogies between the homogeneous dynamo with one- or multi-disk dynamos. This idea is quite interesting. As the method of electric circuit analysis, quantities of resistance, inductance and capacitance are easily obtained without investigating the distribution of electric currents or magnetic fields based on the fundamental equations of electricity and magnetism. Now we could replace the fluid motion by the rotation of a conducting disk, and the pondermotive force could be evaluated by the EM coupling between a disk and a coil. Rikitake [iv] furtherly proposed a model with coupled two disk dynamos.

II. Rikitake Model

The original differential equations derived by Rikitake are

$$L_1\frac{dI_1}{dt}+R_1 I_1=\Omega_1 M\, I_{2,}$$
$$L_2\frac{dI_2}{dt}+R_2 I_2=\Omega_2 N\, I_{1,}$$
$$C_1\frac{d\,\Omega_1}{dt}=G_1-M\, I_1 I_{2,}$$
$$C_2\frac{d\,\Omega_2}{dt}=G_2-N\, I_1 I_2$$

where L, R are the self-inductance and resistance of the coil, the electric currents, I, Ω, C, G are the electric currents, the angular velocity , momentum of inertia, and the driving force; M, N are the mutual inductance between the coils and the disks.

Now we consider a further simplification by

$$L_1=L_{2,}R_1=R_{2,}M=N\,,C_1=C_{2,}G_1=G_2$$

and set

$$I_1=\sqrt{\frac{G}{M}}x\,,I_2=\sqrt{\frac{G}{M}}y\,,\Omega_1=\sqrt{\frac{GL}{CM}}z\,,\Omega_2=\sqrt{\frac{GL}{CM}}(z-a),t=\sqrt{\frac{CL}{GM}}t'\,,v=R\sqrt{C/LMG}$$

where constant parameter a,v>0.

We obtain

$$\dot{x}=-vx+zy\,,$$
$$\dot{y}=-vy+(z-a)x\,,$$
$$\dot{z}=1-xy$$

But note x and y are corresponding to the electric currents, while z is corresponding to the angular velovity.

III. Dynamics of the Rikitake System

A. Convergence of the system

1. Volume contraction: dissipative?

The volume evolves as

$$\dot{V} = \int_v \nabla\cdot\vec{f}\, dV$$

where

$$\nabla\cdot\vec{f} = \frac{\partial}{\partial x}(-vx+zy) + \frac{\partial}{\partial y}(-vy+(z-a)x) + \frac{\partial}{\partial z}(1-xy) = -v-v = -2\mathrm{v} < 0$$

$$\rightarrow \dot{V} = -2\mathrm{vV}\text{, or } V(t) = V(0)e^{-2\mathrm{v}t}$$

Thus volumes in phase space shrink exponentially fast. Rikitake System is a dissipative system.

2. Liapunov exponent

As we could see from all figures that we plotted for exploring the parameter space, the Liaponov exponents are all positive, but almost all are smaller then 0.1, indicating there are chaos solutions and strange attractors are very likely to be exist.

B. Fixed Points

For fixed points (x*,y*,z*), we have

$$0 = -vx^* + z^* y^*$$
$$0 = -vy^* + (z^* - a)x^*$$
$$0 = 1 - x^* y^*$$

From the third Eq above, let x*=k, then $y^* = k^{-1}$,

$$0 = -vk + z^* k^{-1}$$
$$0 = -vk^{-1} + (z^* - a)k$$

we obtain $z^* = vk^2$, where $v(k^2 - k^{-2}) = a$.

If we require k to be positive, then

$$x^{*}=\pm k\,,\; y^{*}=\pm k^{-1}\,,\; z^{*}=vk^{2}$$

C. Stability of Fixed Points

The Jocobian

$$A=\begin{bmatrix} -v & z & y \\ z-a & -v & x \\ -y & -x & 0 \end{bmatrix}_{(x^{*},\,y^{*},\,z^{*})}$$

$$\begin{aligned} \tau &= tr\,A = -2\mathrm{v} < 0, \\ \Delta = det\,A &= -2\mathrm{x}^{*} y^{*} z^{*} - v\left(x^{*2}+y^{*2}\right) + a x^{*} y^{*} \\ &= -2\mathrm{v}\mathrm{k}^{2} - v\left(k^{2}+k^{-2}\right) + a \\ &= -2\mathrm{v}\left(k^{2}+k^{-2}\right) < 0 \end{aligned}$$

due to v>0, k>0

$$\begin{aligned} \tau^{2}-4\Delta &= (-2\mathrm{v})^{2} - 4\left(-2\mathrm{v}\left(k^{2}+k^{-2}\right)\right) \\ &= 4\mathrm{v}^{2} + 8\mathrm{v}\left(k^{2}+k^{-2}\right) > 0 \end{aligned}$$

Thus we know the fixed points are saddle points referring to Fig. 5.2.8 @Strogatz [v]. Thus for v>0, there is no bifurcations.

D. Symmetry

We see if replacing (x ↔ -x, y ↔ -y, z ↔ z), the Rikitake equations still hold.

Hence, if (x(t), y(t), z(t)) is a solution, so is (-x(t), -y(t), z(t)). In other words, all solutions are either symmetric themselves or have a symmetric partner.

E. Exploring the Parameter Space

We will investigate the dynamics of the Rikitake system with different parameter values, initial conditions, and indicate the long-term behavior.

Since the symmetry of parity of (x, y) we analyzed in Part D above, we know that we only need to consider the non-negative values of x(0) after exploring the whole real space of y(0) and z(0). I choose the parameter values and initial conditions below to explore the parameter space to 10^4 order (10^{-1}~10^3).

```
for v0=[0.1, 0.3, 0.9, 1, 1.2, 2, 10, 20, 50, 100, 160, 200, 400]
  for a0=[0.1, 0.5, 1, 2, 10, 100, 1000]
    for z0=[0, 0.1, -0.1, 1, -1, 2, -2, 10, -10, 50, -50]
      for y0=[0, 0.1, -0.1, 1, -1, 2, -2, 10, -10, 50, -50]
        for x0=[0, 0.1, 1, 2, 10, 50]
```

Thus we will produce 13x7x11x11x6=66066 data events totally. For every events, we plot at lease 7 figures: (x,y,z), x(t), y(t), z(t), z(x), z(y), y(x), thus producing 66066x7=462462 figures. The orbit diagram and Liapunov exponents will be plotted for certain interested cases. This is a big job, but I think it is meaningful and quite intuitive to explore the possible parameter space. In the next section, I will pick up few of them to illustrate the chaotic behavior of Rikitake system, and show the polarization reversal as well.

Now let's do some theoretical prediction for the large and small values of parameters. For the Rikitake system:

$$\dot{x} = -vx + zy\,,$$
$$\dot{y} = -vy + (z-a)x\,,$$
$$\dot{z} = 1 - xy$$

1. When v>>1, v>>x(0), v>>y(0), v>>z(0), z>>a

   then Rikitake system could be simplified as

$$\dot{x} = -vx\,,$$
$$\dot{y} = -vy\,,$$
$$\dot{z} = 1 - xy$$

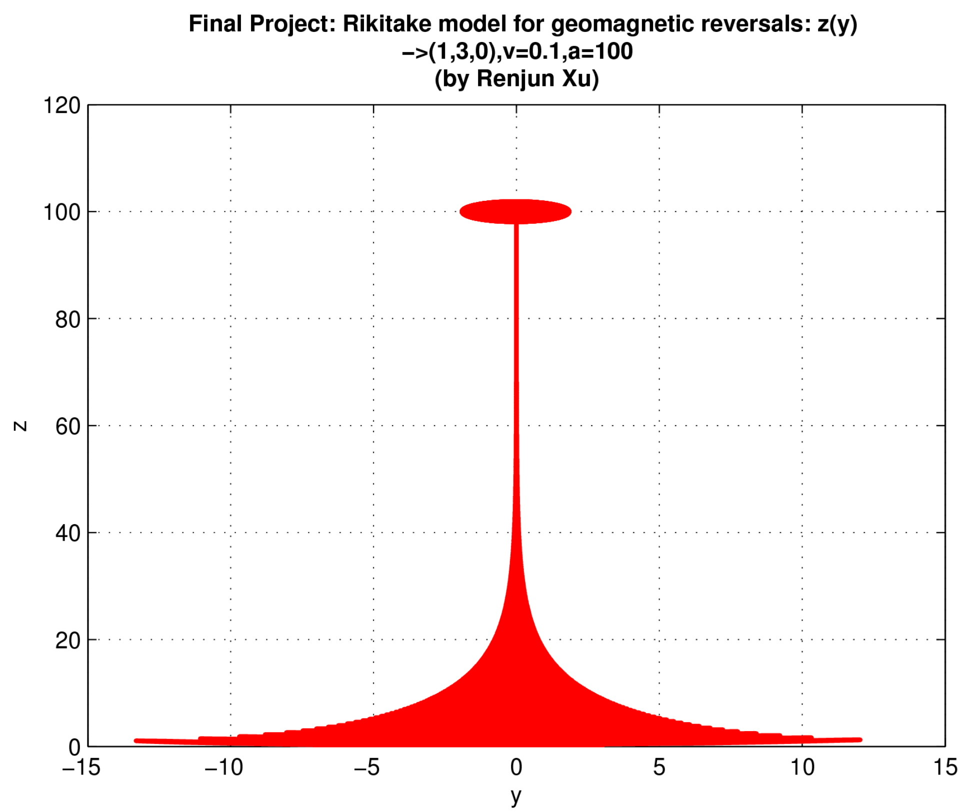
   Apparently, x and y will decay exponentially fast to zero.

   For z, when x(0) y(0) < 1, then z will increase initially based on the third

Eq. In addition, since x and y decay fast, the product of x(t)y(t) will soon be smaller than 1. Then z increases slowly. But since z grows and x and y decrease, the second quadratic term with z involved will make contribution after long-run.

2. When a is much bigger than others, v and the initial coordinates,

   Referring the figure below, at initial time, y will decrease fast since x is positive; after $y \to 0$, z will increase from the contraction of the third Eq; but since x & y and v both relatively small, thus at this stage only z grows fast, which is a bit like the case mentioned above. Later, when z is bigger enough, say the angular momentum is extremely fast, it will induce the electric currents.



IV. Figures, Data Output and Results

Here I just put a few interesting results for illustration.

For more and complete results, please login to our department student server by:

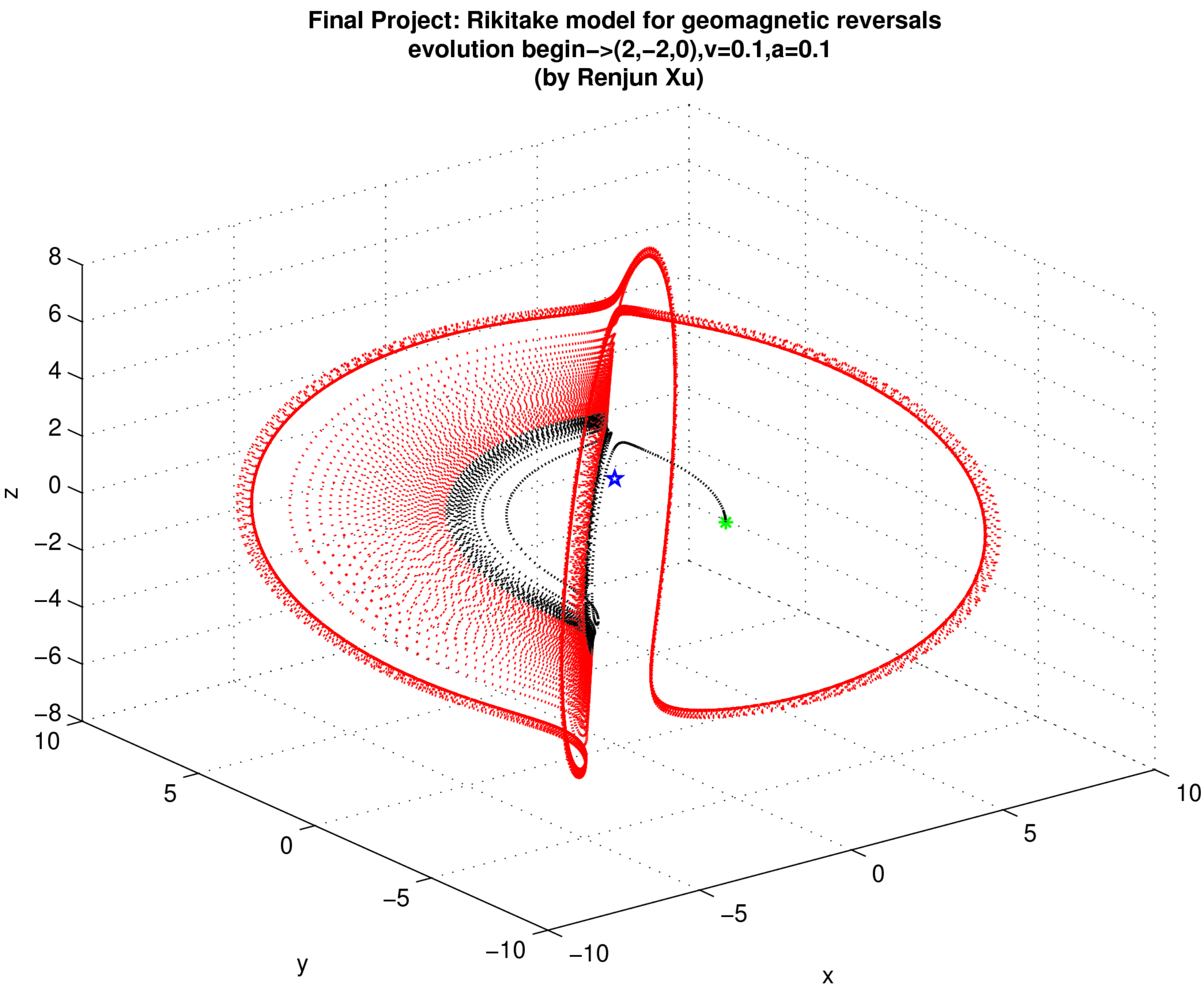
ssh student.physics.ucdavis.edu

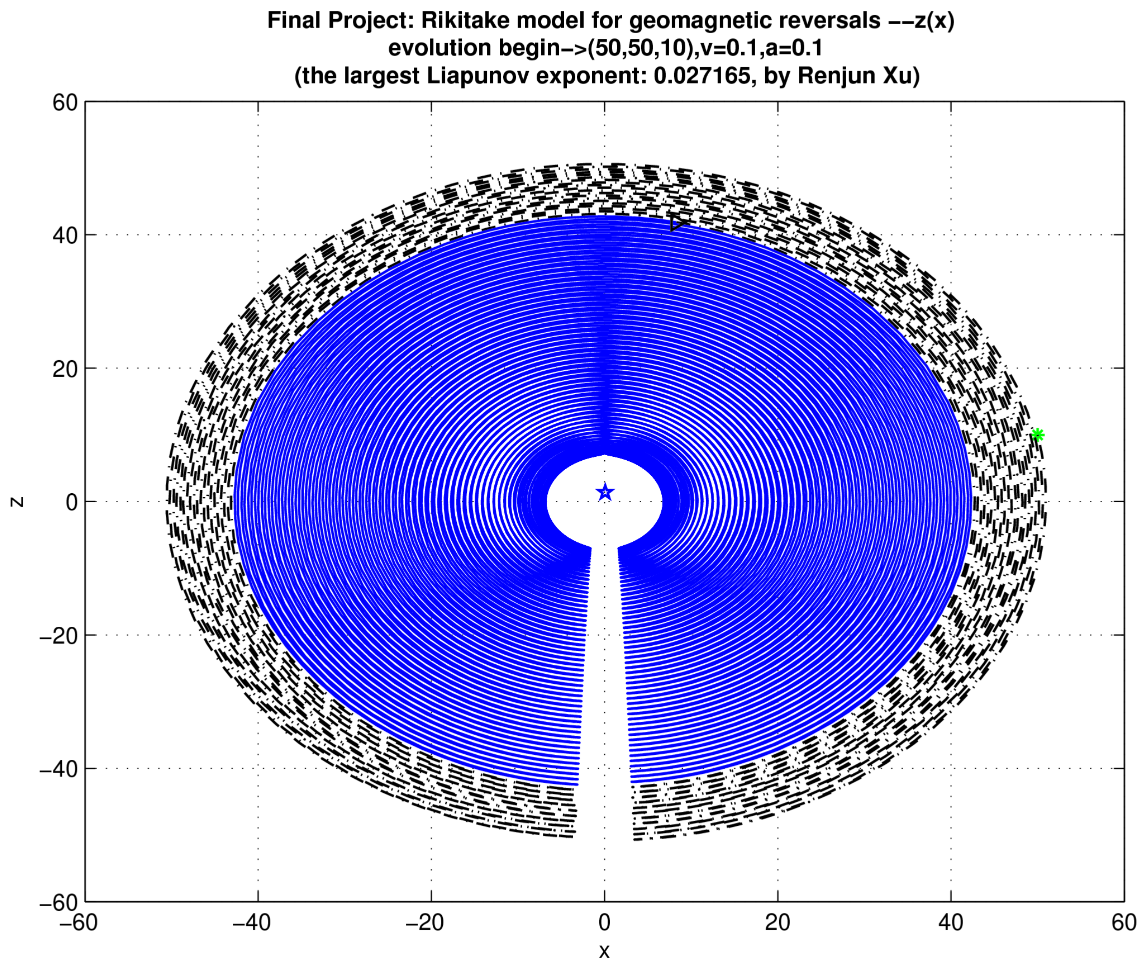
cd /tmp/chaos/FinalProject_Xu

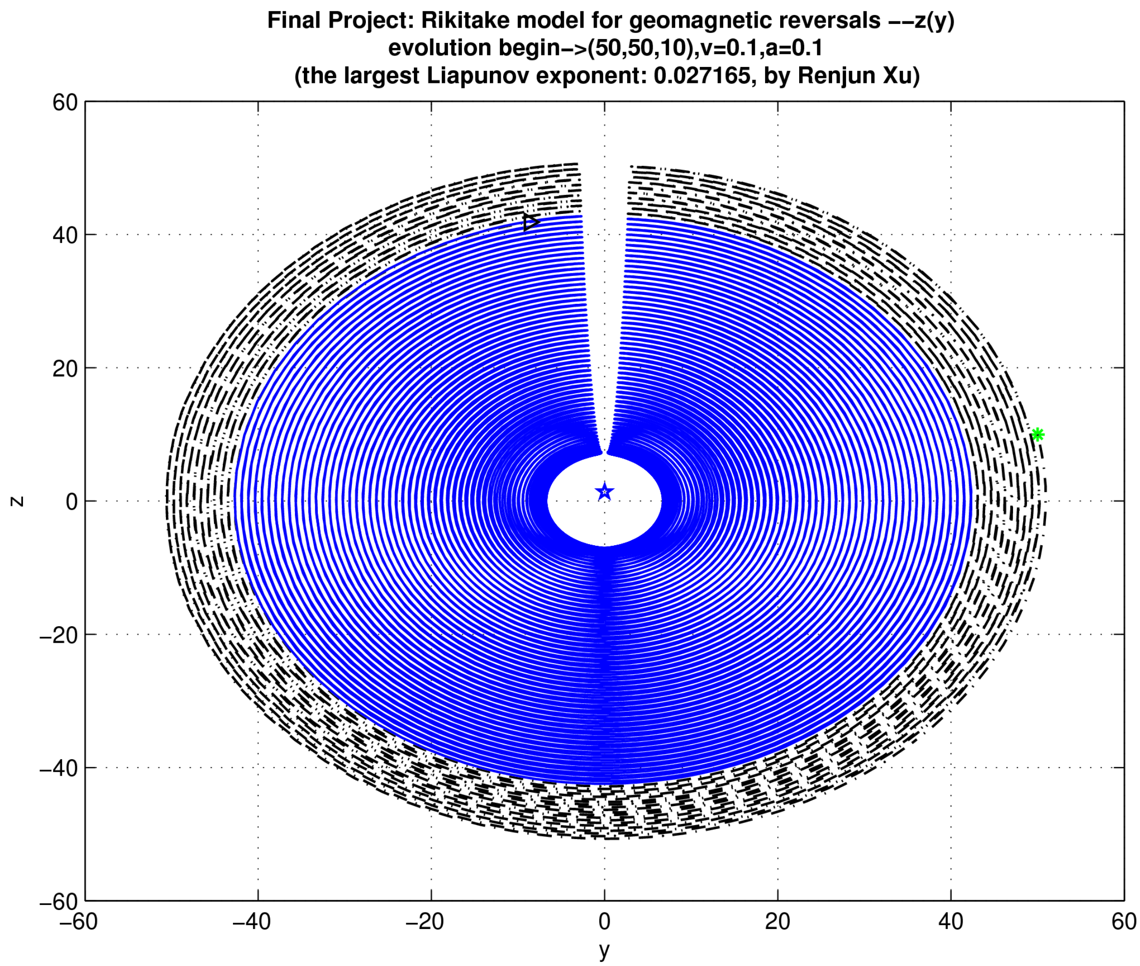
I directly linked my final folder there, and have changed the permission. But if you have any problem, please feel free to contact me. However, since the total size of those thousands of figures is over 1GB, I may cannot keep it for long on our server. My codes are free, I would be quite glad if someone find them useful.

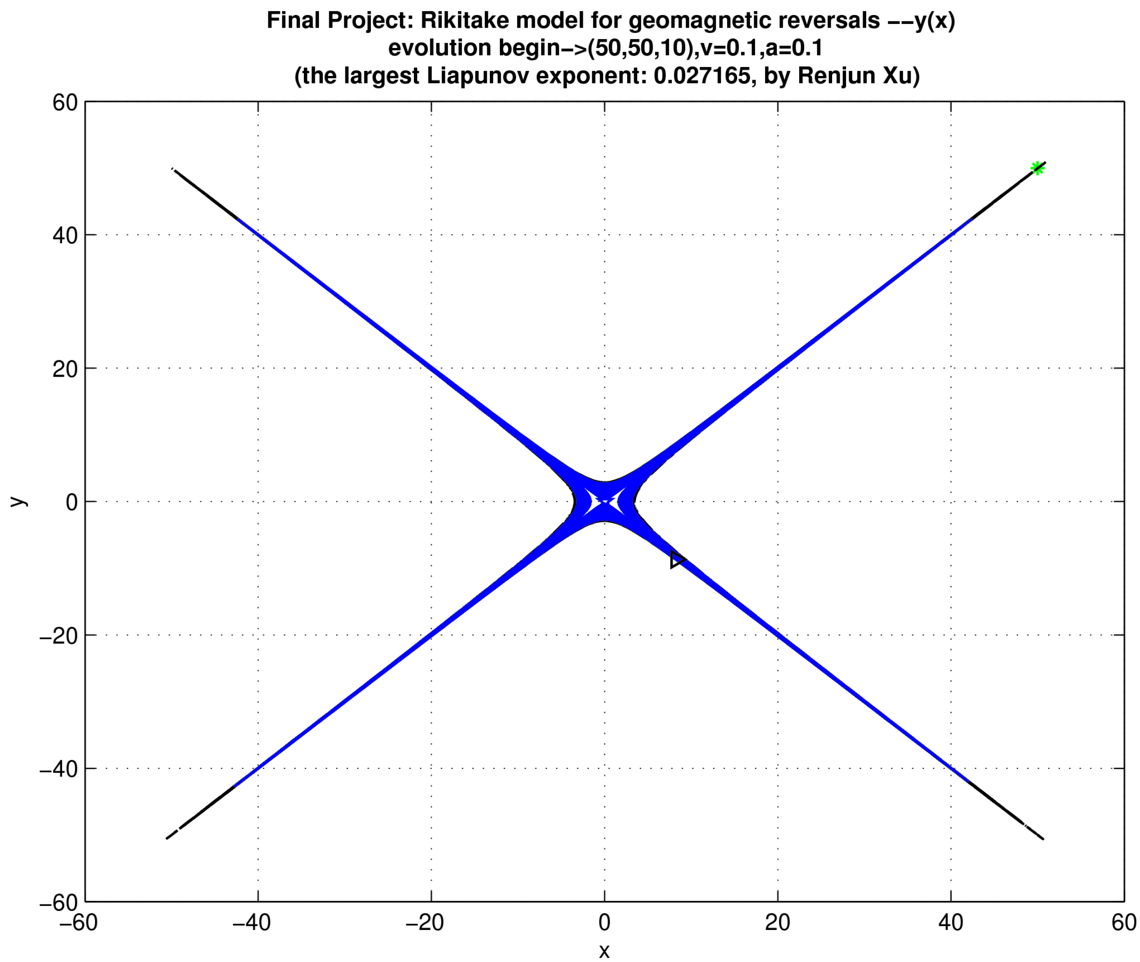
The initial conditions and the parameter values are directly indicated in the figures. It seems that though I output all figures in eps format, the quality lost when I imported to this word document. So please login to our student server to view high-quality produced Rikitake model shows.

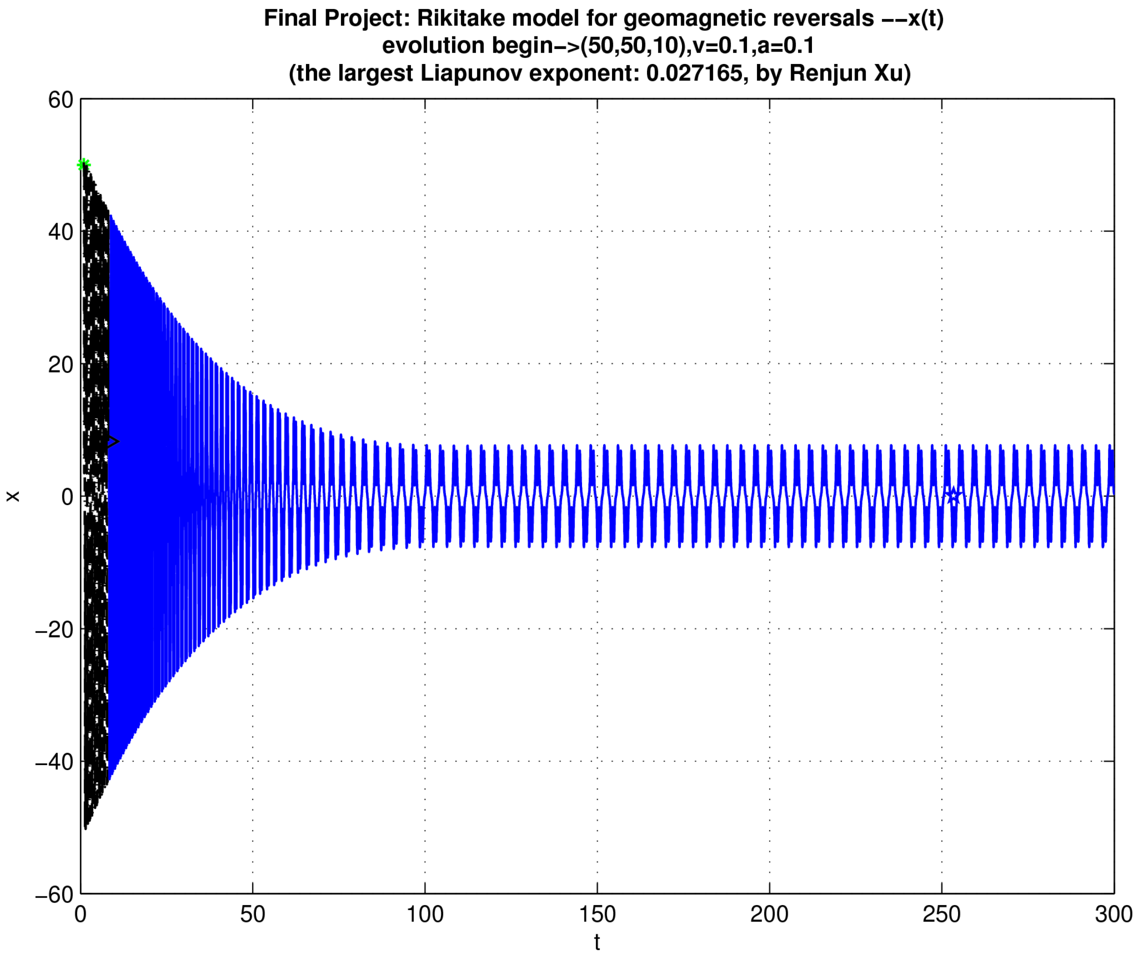
Thank you!

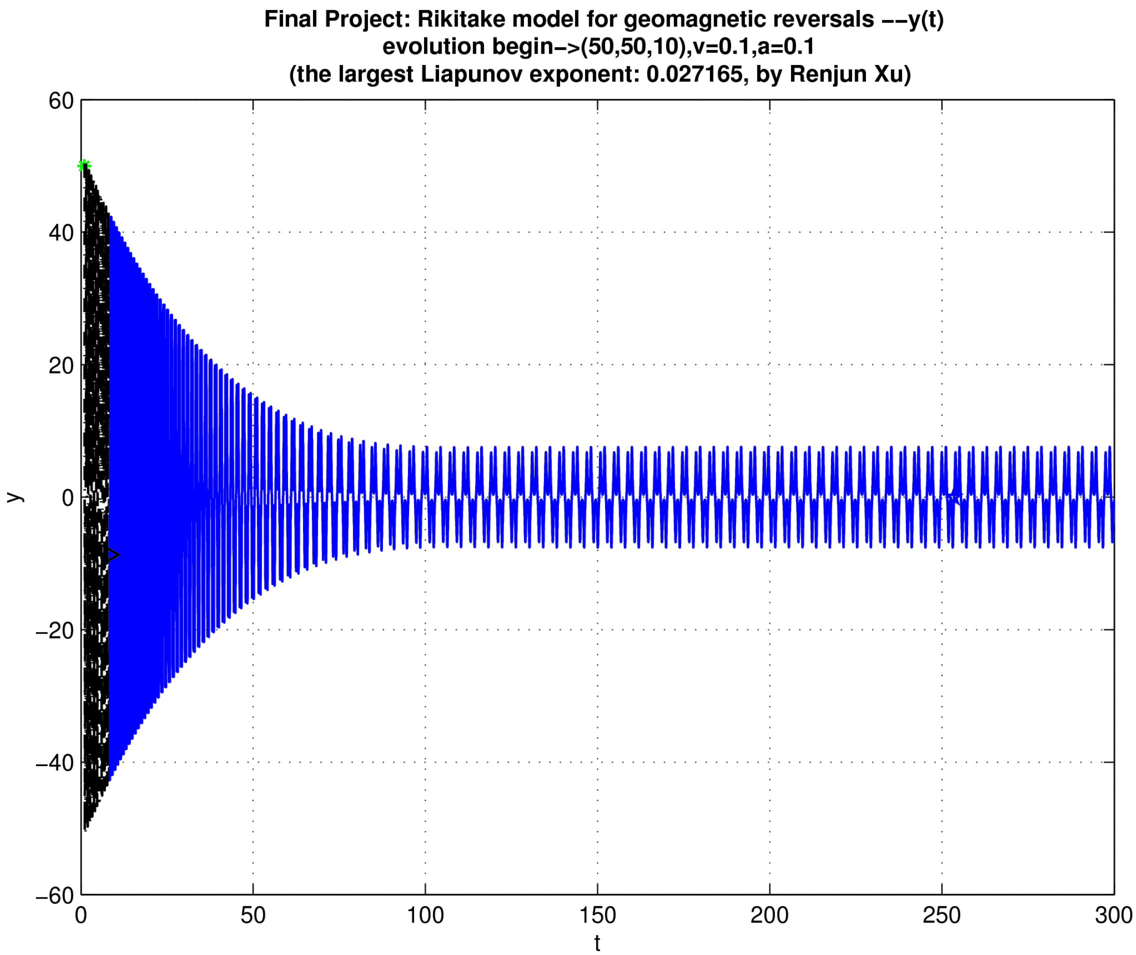

Final Project: Rikitake model for geomagnetic reversals
evolution begin−>(2,−2,0),v=0.1,a=0.1
(by Renjun Xu)
8
6
4
2
0
−2
−4
−6
−8
z
10
5
0
−5
−10
y
−10
−5
0
5
10
x

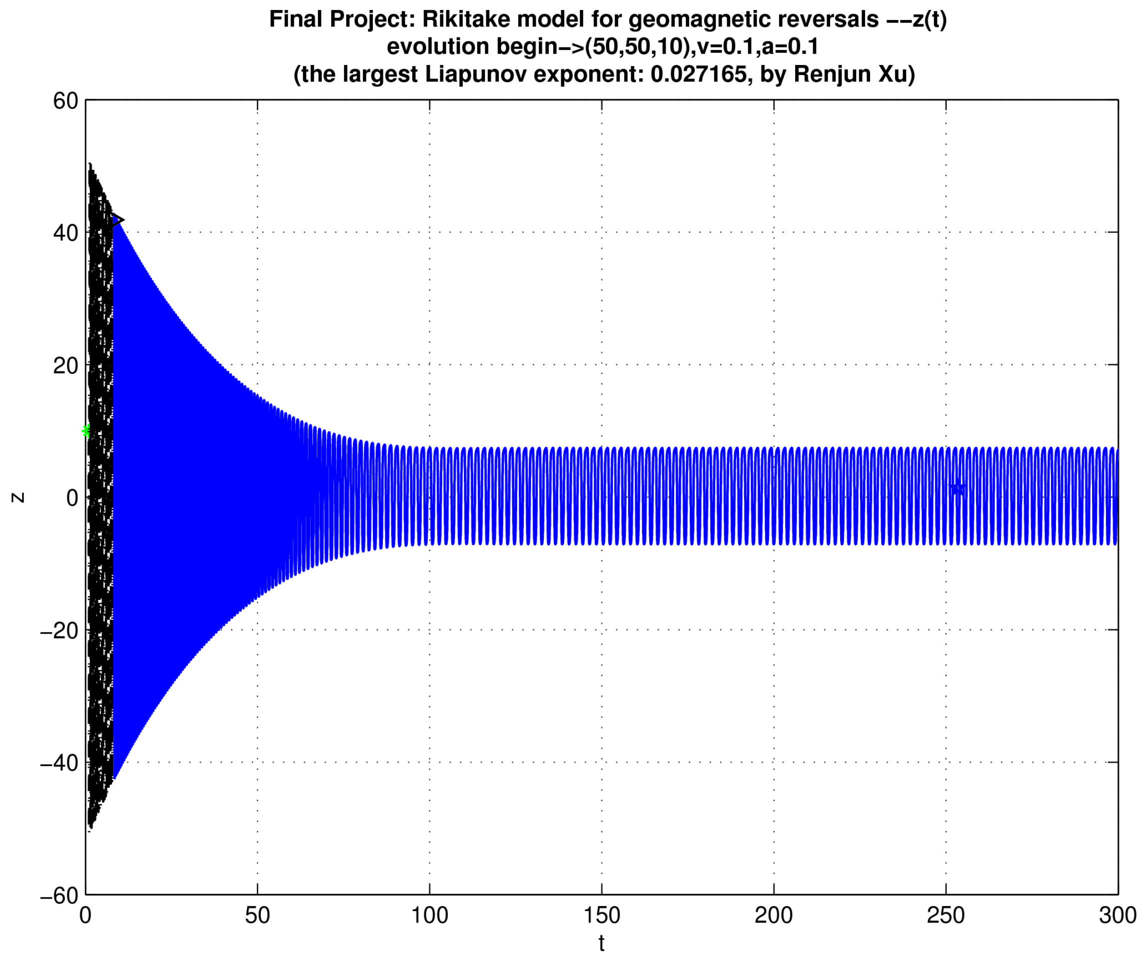

Final Project: Rikitake model for geomagnetic reversals −−z(x)
evolution begin−>(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
z
−60
−40
−20
0
20
40
60
x

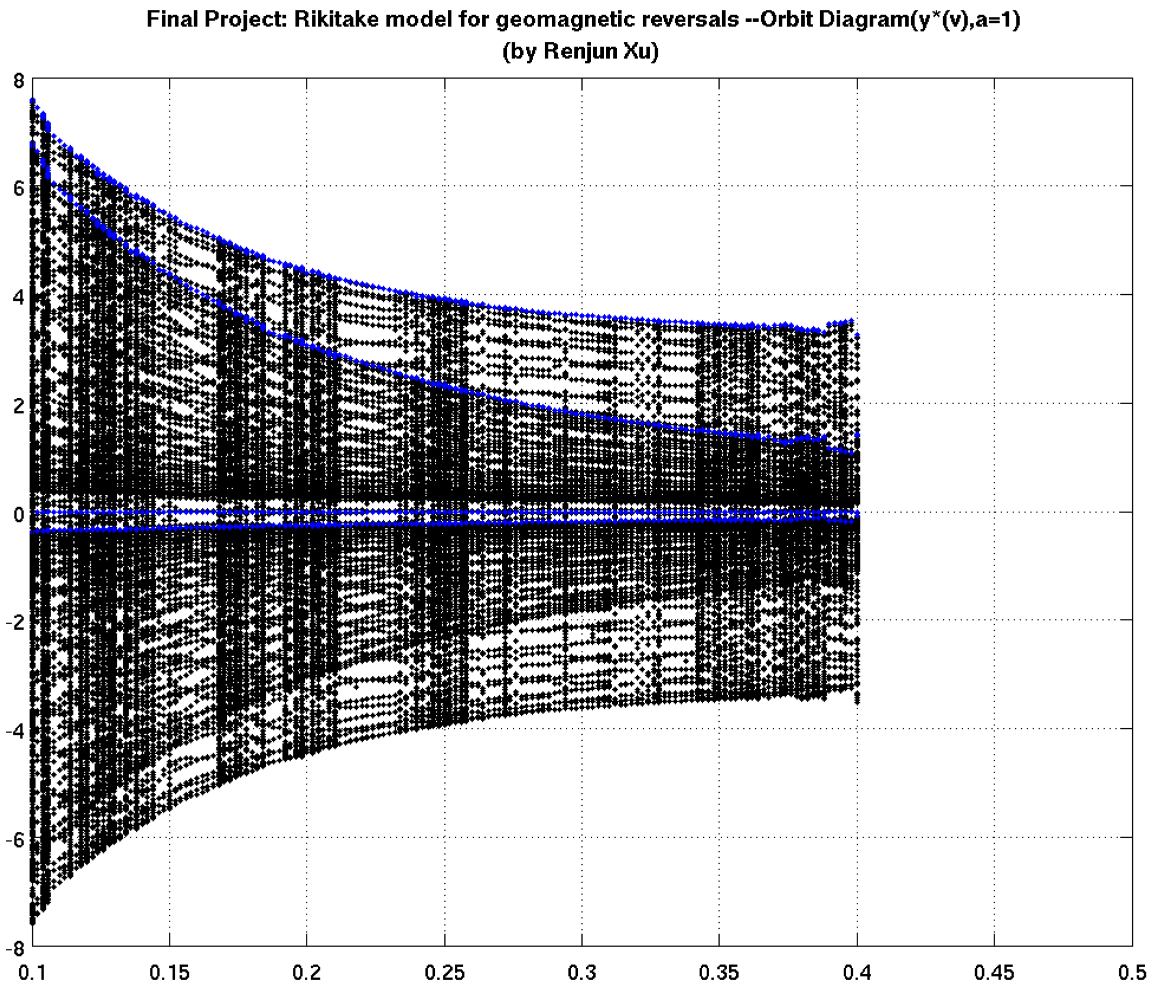

Final Project: Rikitake model for geomagnetic reversals −−z(y)
evolution begin−>(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
z
−60
−40
−20
0
20
40
60
y

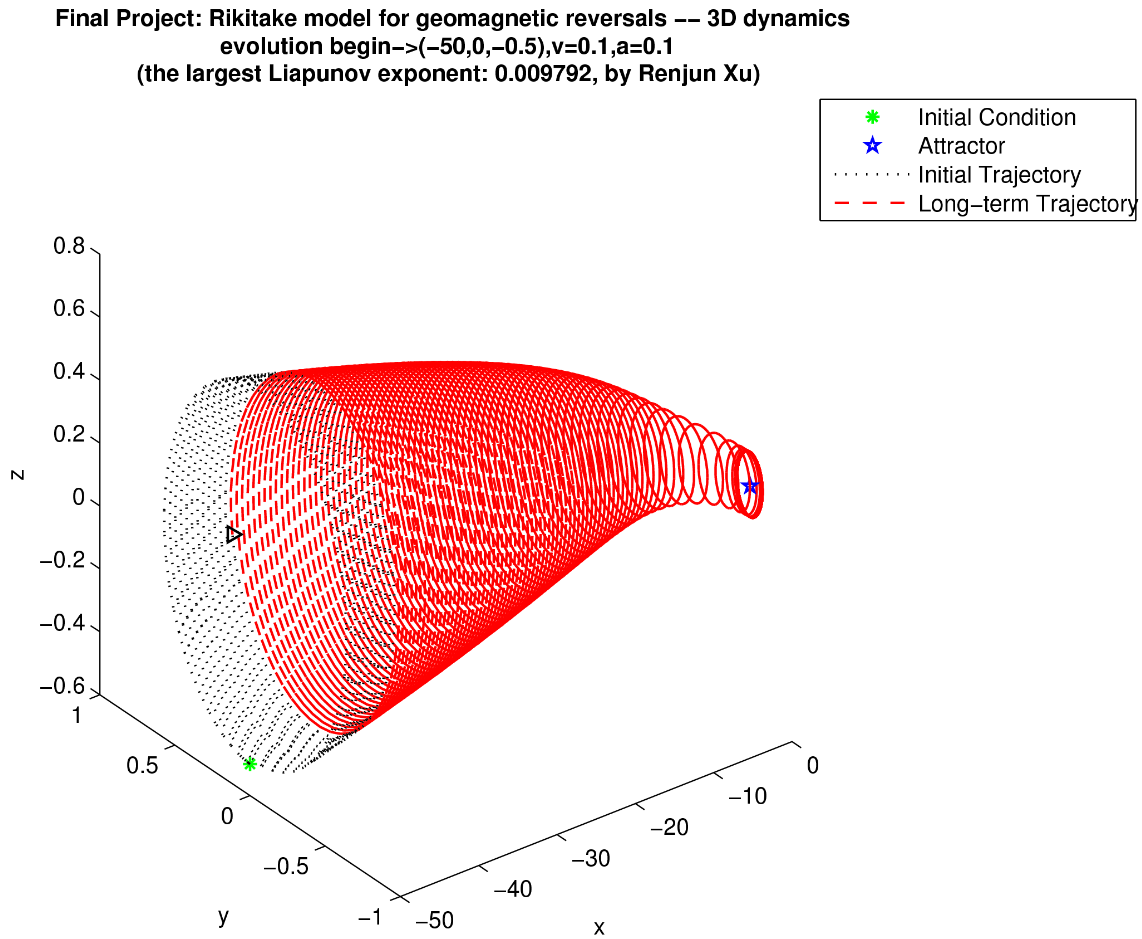

Final Project: Rikitake model for geomagnetic reversals --y(x)
evolution begin->(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
−60
−40
−20
0
20
40
60
x
y

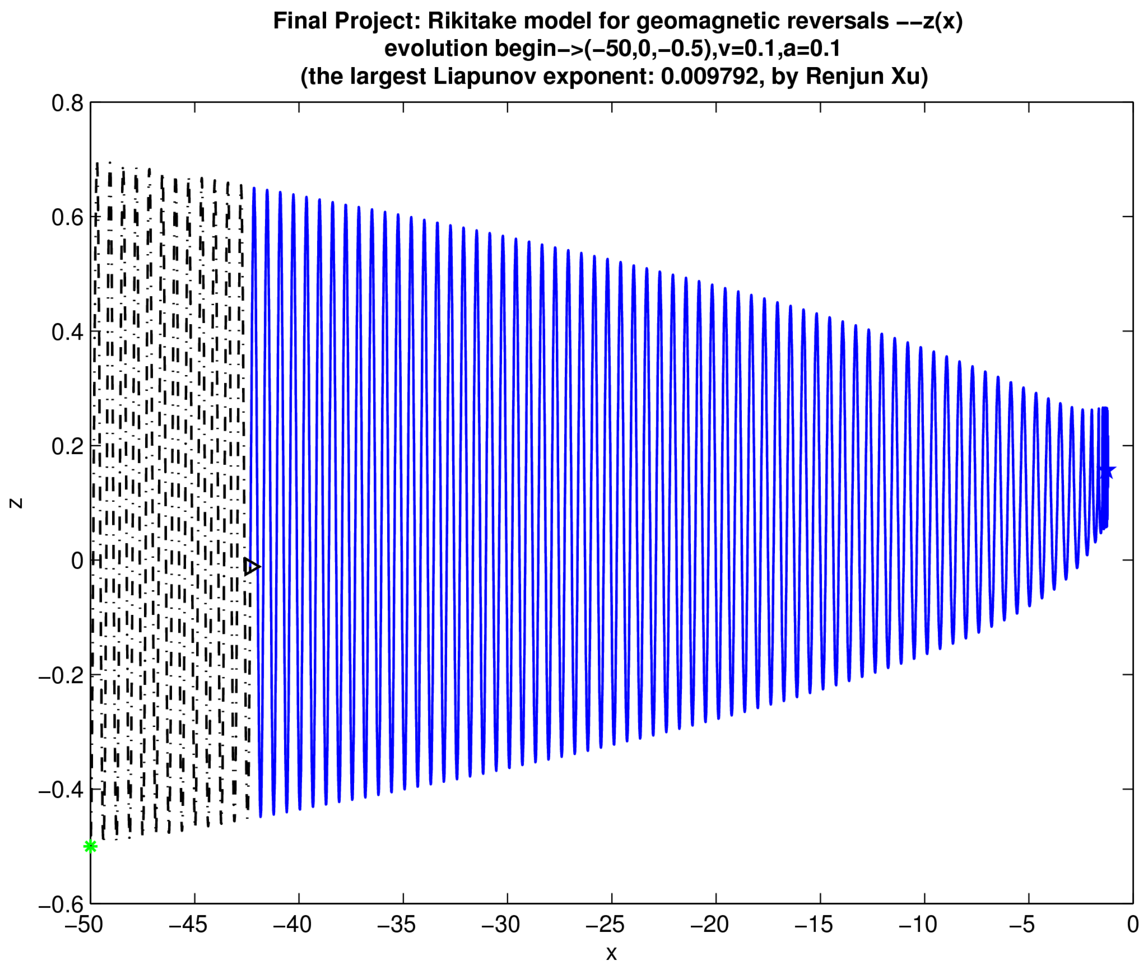

Final Project: Rikitake model for geomagnetic reversals --x(t)
evolution begin->(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
x
0
50
100
150
200
250
300
t

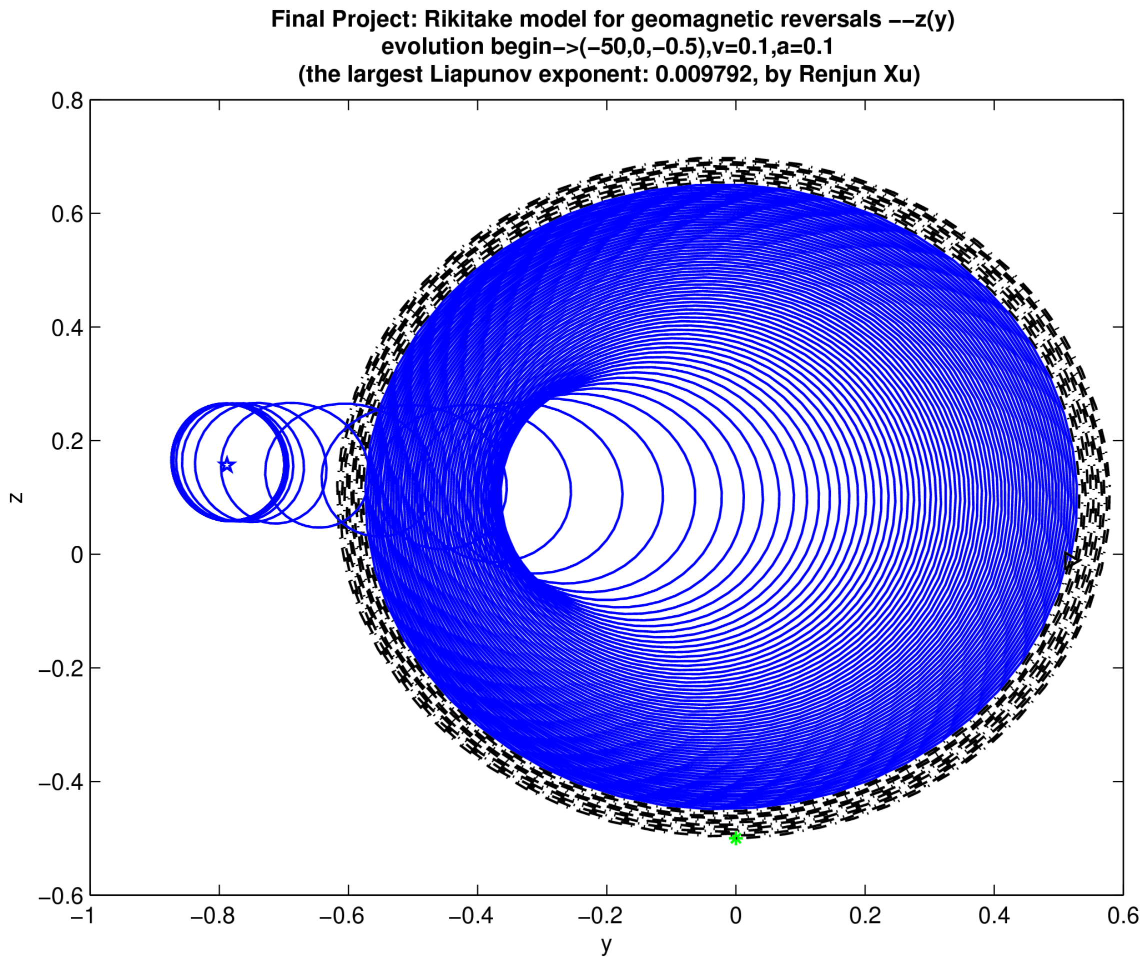

Final Project: Rikitake model for geomagnetic reversals --y(t)
evolution begin->(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
y
0
50
100
150
200
250
300
t

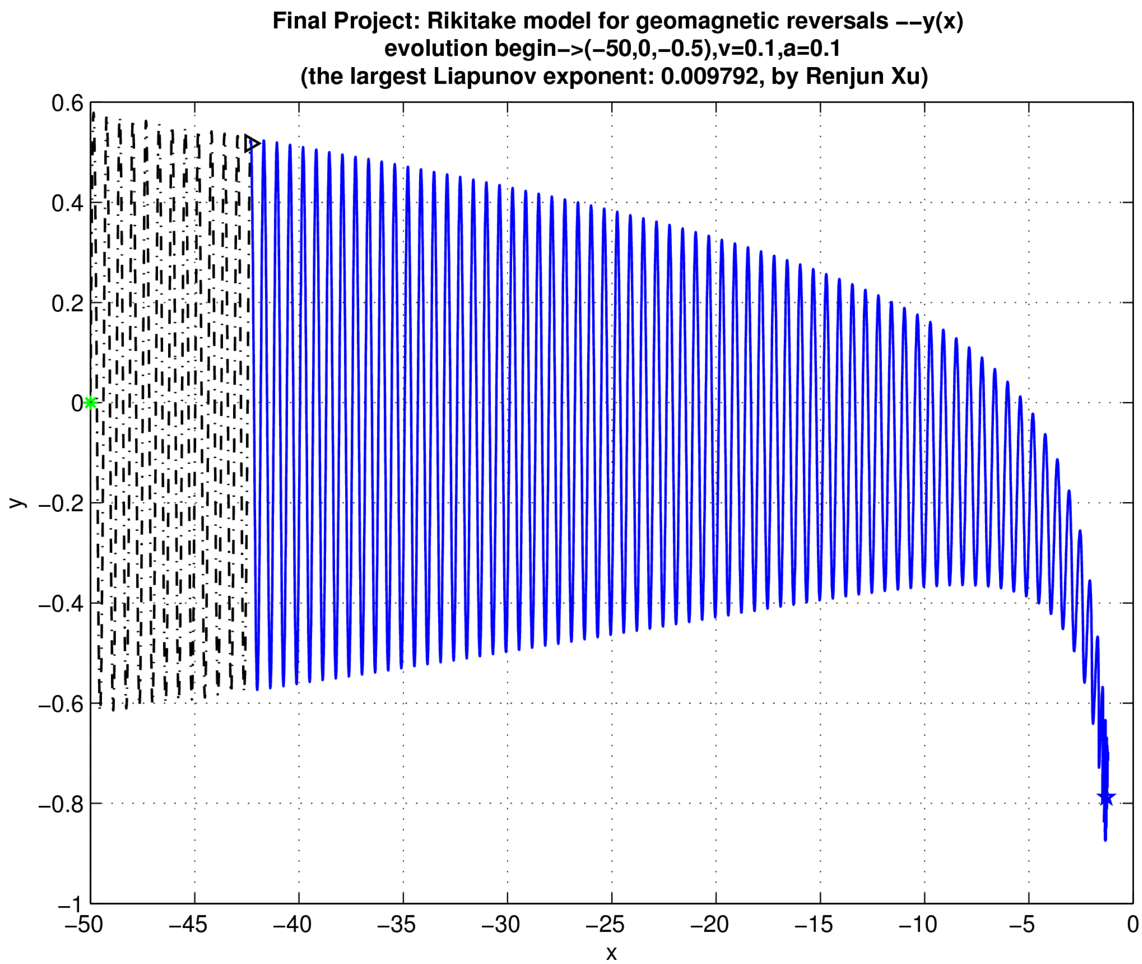

Final Project: Rikitake model for geomagnetic reversals --z(t)
evolution begin->(50,50,10),v=0.1,a=0.1
(the largest Liapunov exponent: 0.027165, by Renjun Xu)
60
40
20
0
−20
−40
−60
z
0
50
100
150
200
250
300
t

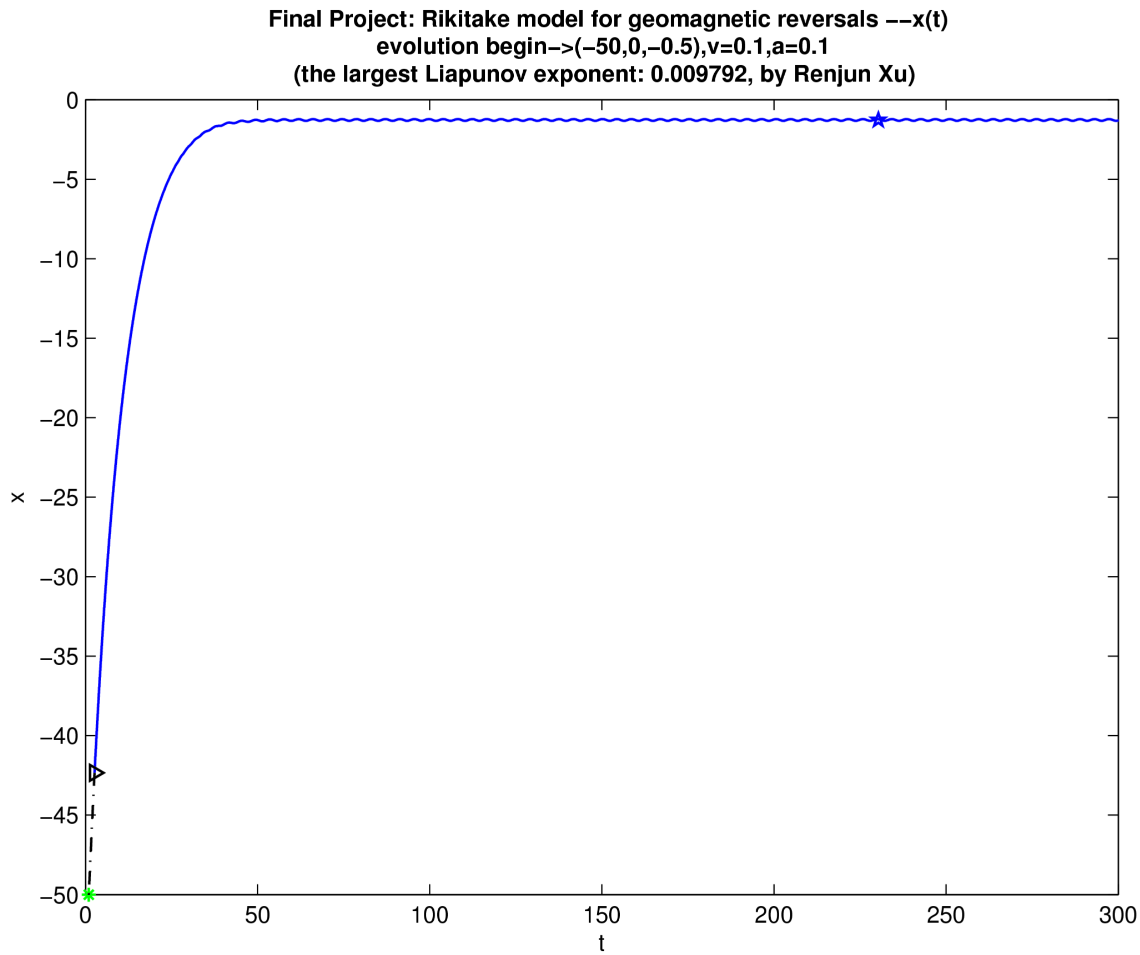

Final Project: Rikitake model for geomagnetic reversals --Orbit Diagram(y*(v),a=1)
(by Renjun Xu)
8
6
4
2
0
-2
-4
-6
-8
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

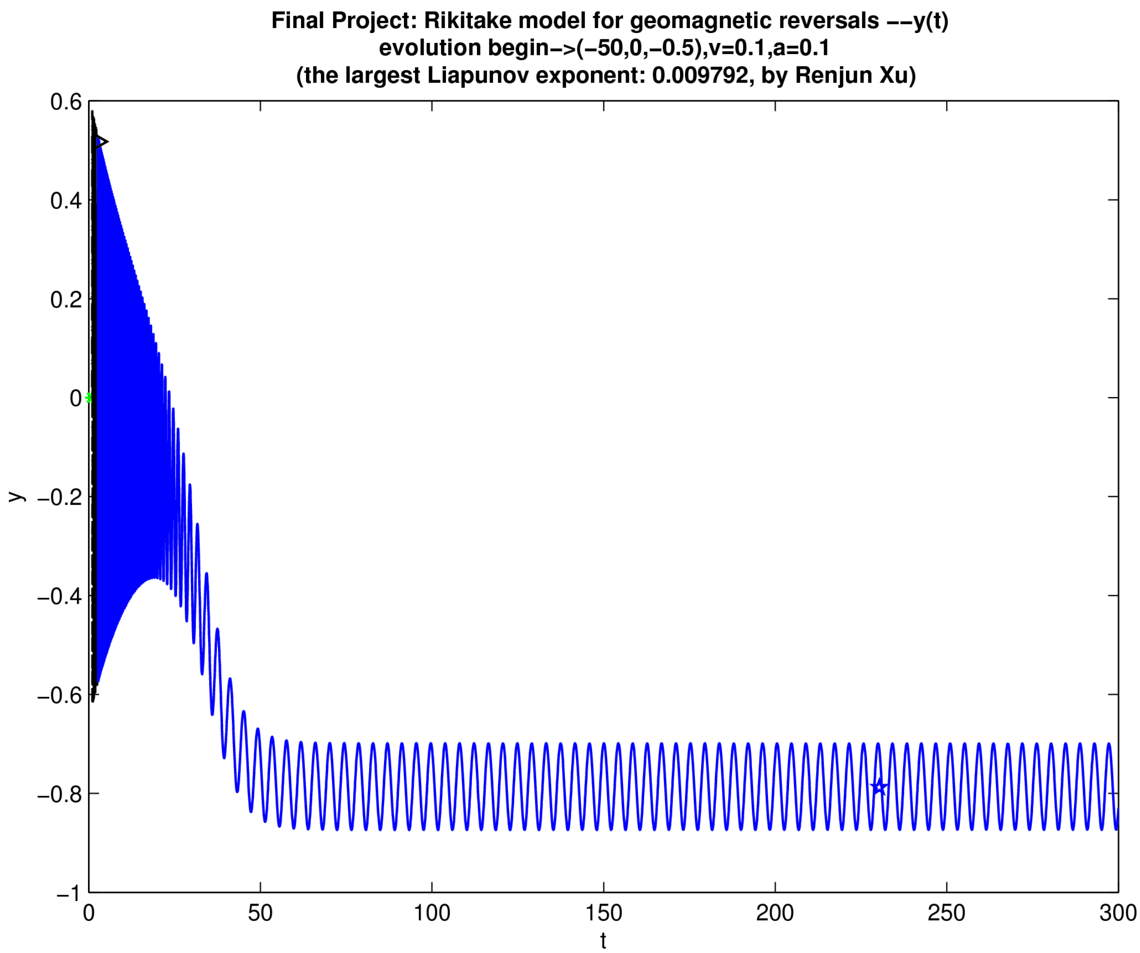

Final Project: Rikitake model for geomagnetic reversals -- 3D dynamics
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
Initial Condition
Attractor
Initial Trajectory
Long-term Trajectory
z
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
y
1
0.5
0
-0.5
-1
x
-50
-40
-30
-20
-10
0


Final Project: Rikitake model for geomagnetic reversals --z(x)
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
z
-50
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
x

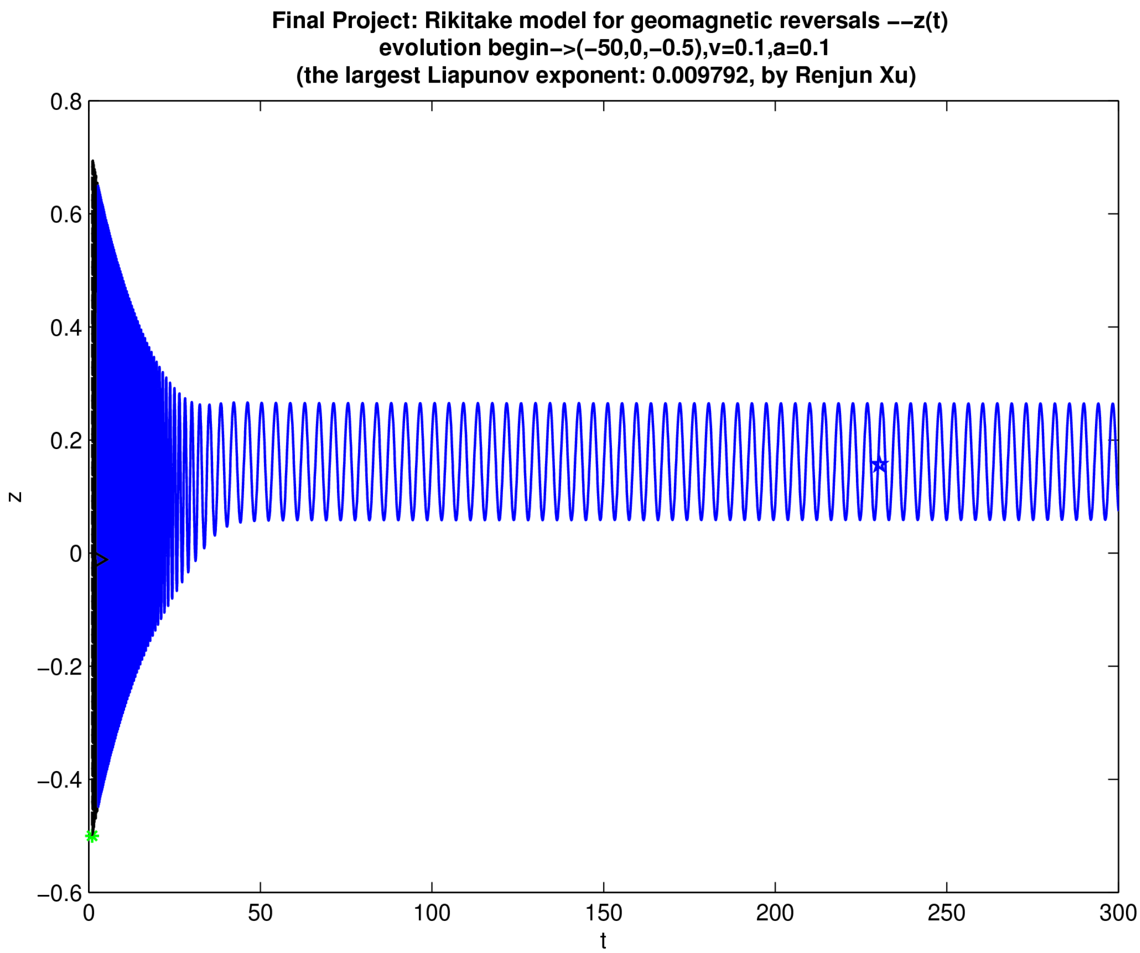

Final Project: Rikitake model for geomagnetic reversals −−z(y)
evolution begin−>(−50,0,−0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
z
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
y

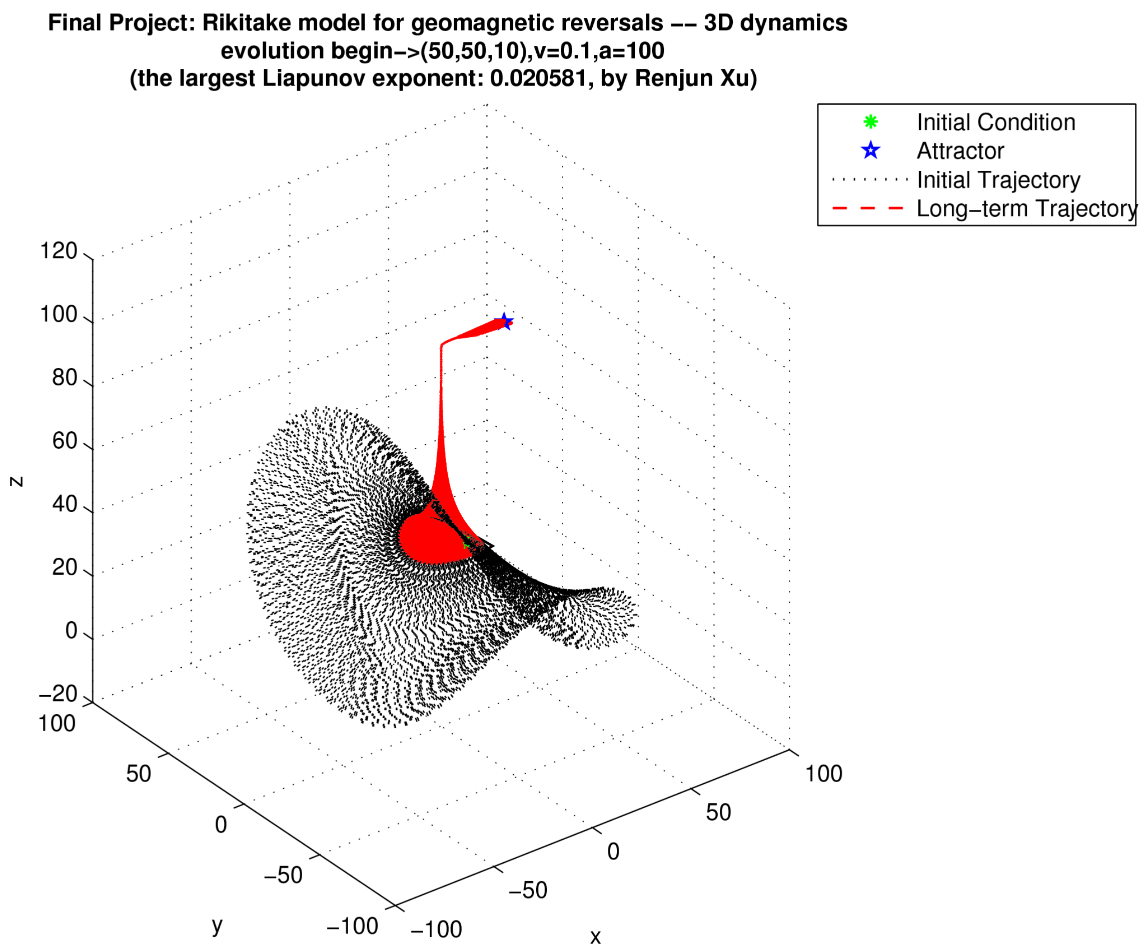

Final Project: Rikitake model for geomagnetic reversals --y(x)
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
-50
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
x

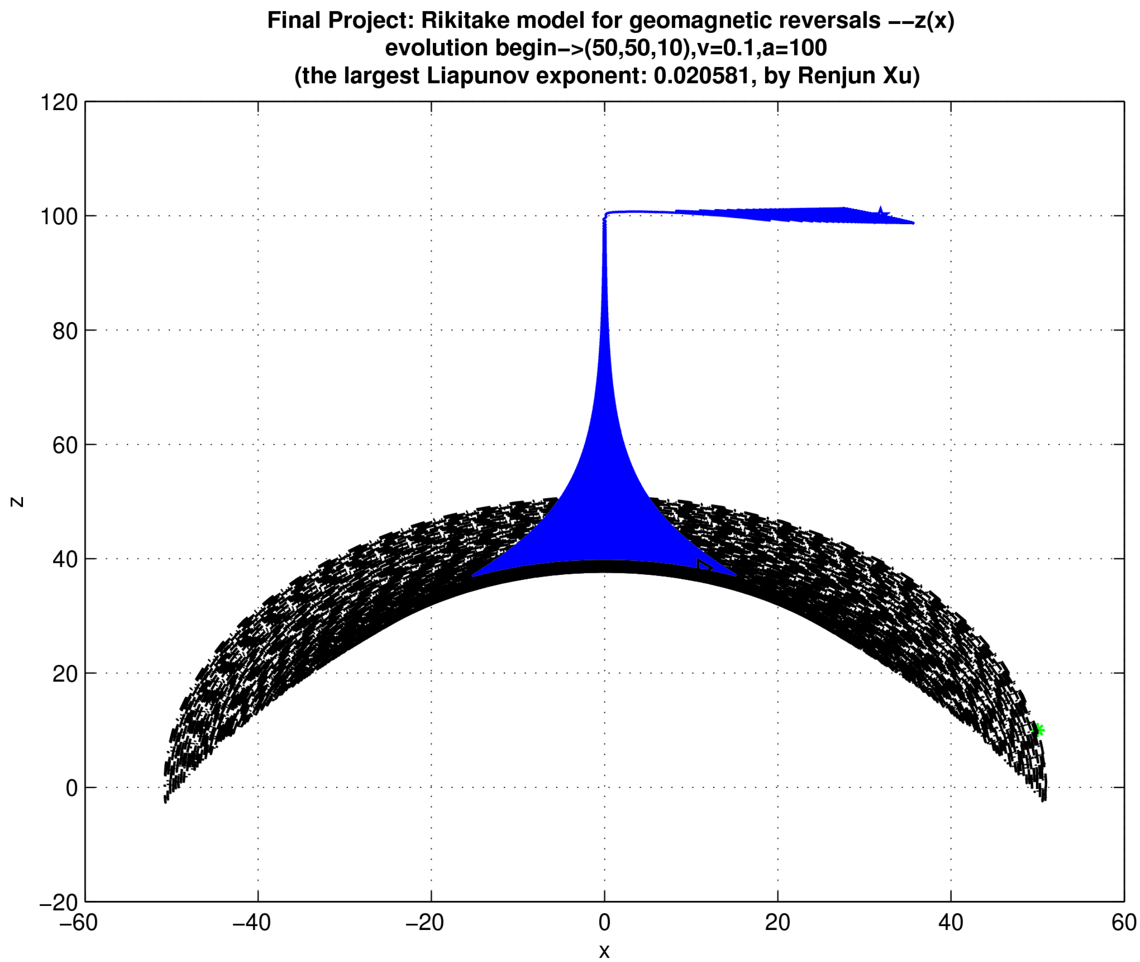

Final Project: Rikitake model for geomagnetic reversals --x(t)
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
x
0
50
100
150
200
250
300
t

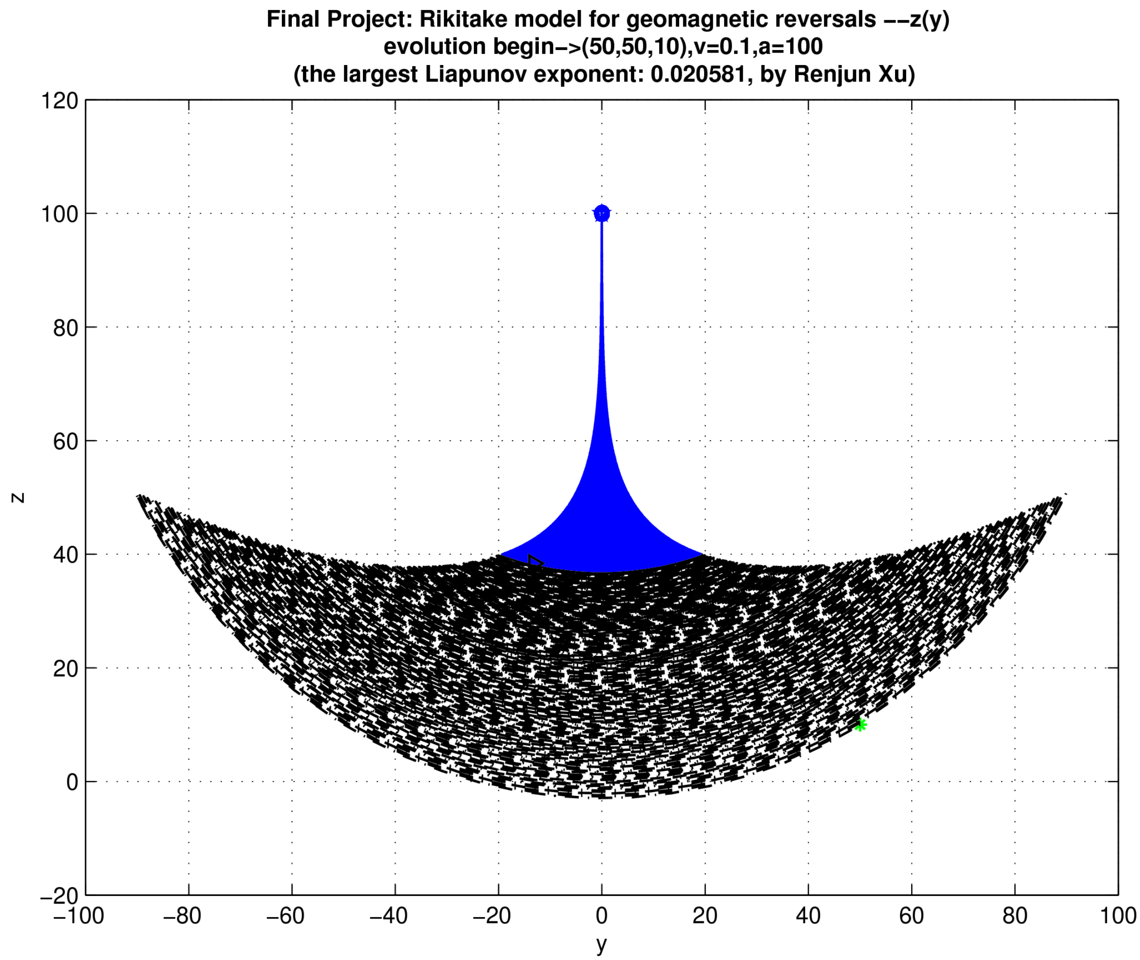


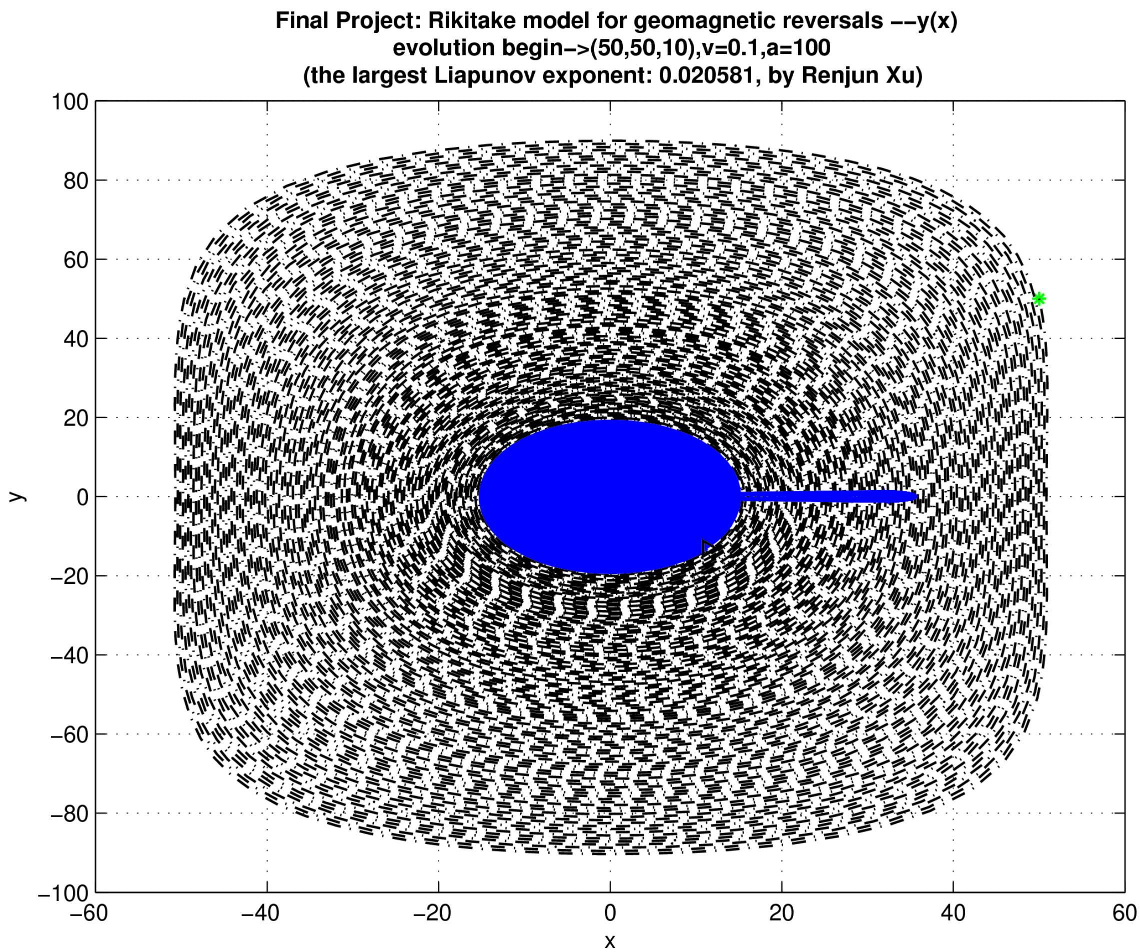
Final Project: Rikitake model for geomagnetic reversals --y(t)
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
0
50
100
150
200
250
300
t

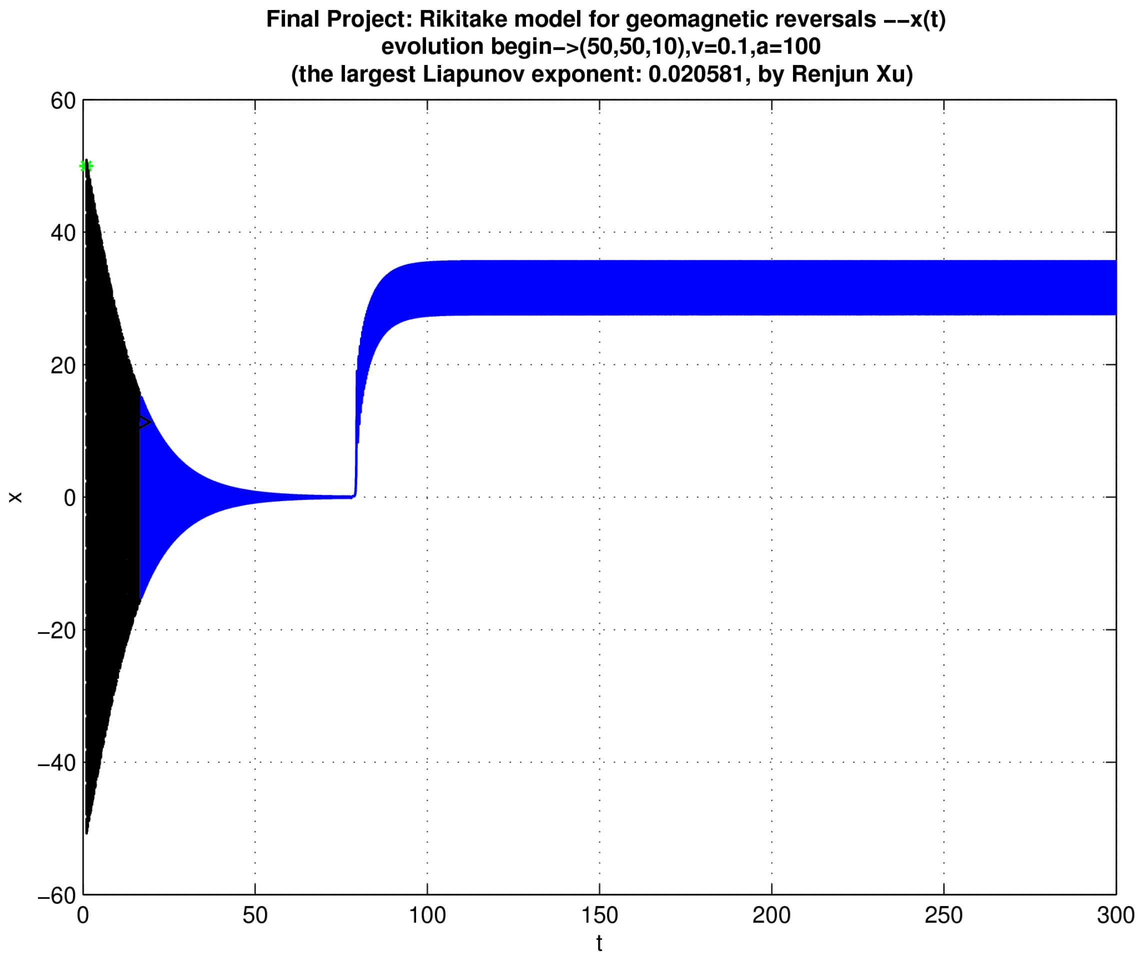

Final Project: Rikitake model for geomagnetic reversals --z(t)
evolution begin->(-50,0,-0.5),v=0.1,a=0.1
(the largest Liapunov exponent: 0.009792, by Renjun Xu)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
z
0
50
100
150
200
250
300
t

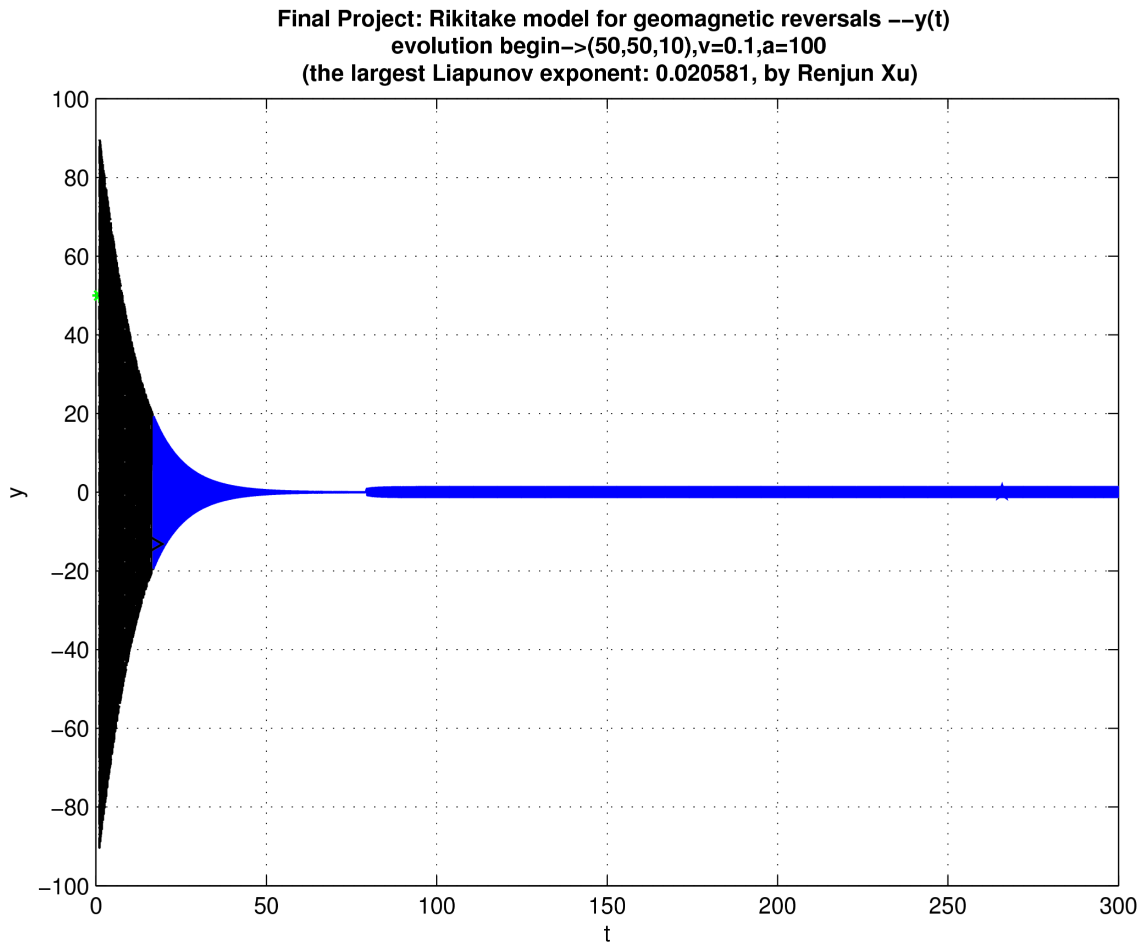

Final Project: Rikitake model for geomagnetic reversals –– 3D dynamics
evolution begin–>(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
Initial Condition
Attractor
Initial Trajectory
Long–term Trajectory
z
120
100
80
60
40
20
0
–20
y
100
50
0
–50
–100
x
–100
–50
0
50
100

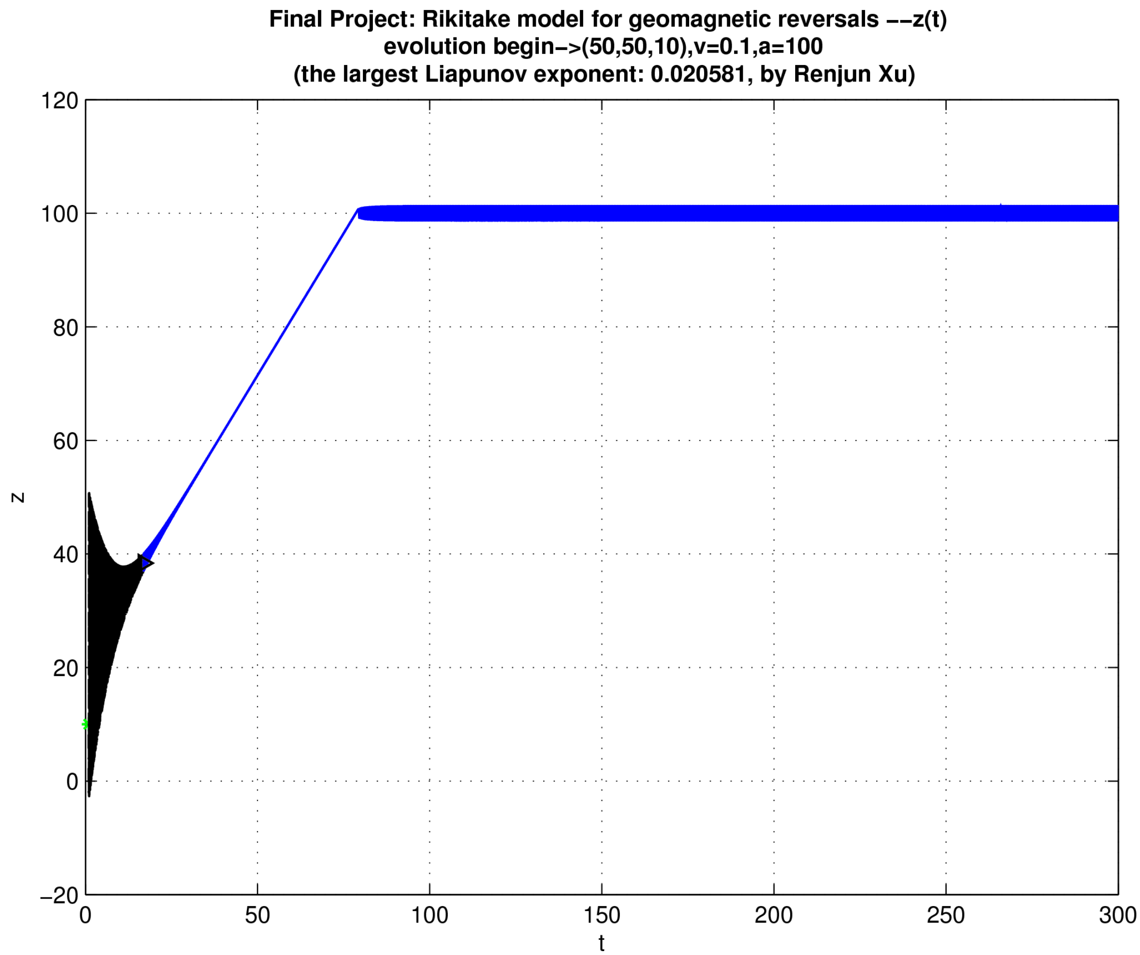

Final Project: Rikitake model for geomagnetic reversals −−z(x)
evolution begin−>(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
120
100
80
60
40
20
0
−20
z
−60
−40
−20
0
20
40
60
x


Final Project: Rikitake model for geomagnetic reversals −−z(y)
evolution begin−>(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
120
100
80
60
40
20
0
−20
z
−100
−80
−60
−40
−20
0
20
40
60
80
100
y


Final Project: Rikitake model for geomagnetic reversals --y(x)
evolution begin->(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
100
80
60
40
20
0
−20
−40
−60
−80
−100
y
−60
−40
−20
0
20
40
60
x


Final Project: Rikitake model for geomagnetic reversals --x(t)
evolution begin->(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
60
40
20
0
−20
−40
−60
x
0
50
100
150
200
250
300
t


Final Project: Rikitake model for geomagnetic reversals --y(t)
evolution begin->(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
100
80
60
40
20
0
−20
−40
−60
−80
−100
y
0
50
100
150
200
250
300
t


Final Project: Rikitake model for geomagnetic reversals --z(t)
evolution begin->(50,50,10),v=0.1,a=100
(the largest Liapunov exponent: 0.020581, by Renjun Xu)
120
100
80
60
40
20
0
−20
z
0
50
100
150
200
250
300
t

V. Code

My codes for this final project are written in Matlab.

```
%%%%%%%%%%%%%%%%%%%%%%Rikitake.m%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Final Project @ Chaos
%Author:Renjun Xu
%Date: 03/18/2009

clc; close all; clear all;
%Rikitake model for geomagnetic reversals
%dx/dt=-vx+zy;
%dy/dt=-vy+(z-a)x;
%dz/dt=1-xy;
dt=.001;

t1=1; %1e-18;
t2=300;
t=linspace(t1,t2);
par.t0 = t1;
delta_x0 = 1e-3;
delta_y0 = 1e-3;
delta_z0 = 1e-3;

for v0=[0.1,0.3,0.9,1,1.2,2,10,20,50,100,160,200,400]
   for a0=[0.1,0.5,1,2,10,100,1000]
      for z0=[0,0.1,-0.1,1,-1,2,-2,10,-10,50,-50]
         for y0=[0,0.1,-0.1,1,-1,2,-2,10,-10,50,-50]
            for x0=[0,0.1,1,2,10,50]
   par.v=v0;
   par.a=a0;

   par.x0 = x0;
   par.y0 = y0;
   par.z0 = z0;

%par.x0 = 50*randn;
%par.y0 = 50*randn;
%par.z0 = 50*randn;

options = odeset('RelTol',1e-4,'AbsTol',[1e-5 1e-5 1e-5],'Refine',4);
%[T,Y] = ode45(@(t,y)dr_dt(t,y,par),[t1 t2],[par.x0 par.y0 par.z0]);
[T,Y] = ode45(@(t,y)dr_dt(t,y,par),[t1 t2],[par.x0 par.y0 par.z0],options);
length_T1=floor(0.1*length(T));
length_T=floor(0.8*length(T));
figure(1);
plot3(Y(1:1,1),Y(1:1,2),Y(1:1,3),'*g',mean(Y(length_T:end,1)),mean(Y(length_T:end,2)),me
an(Y(length_T:end,3)),'pb',Y(1:length_T1,1),Y(1:length_T1,2),Y(1:length_T1,3),':k',Y(leng
th_T1:end,1),Y(length_T1:end,2),Y(length_T1:end,3),'--
r',Y(length_T1:length_T1,1),Y(length_T1:length_T1,2),Y(length_T1:length_T1,3),'>k','Lin
eWidth',1)
legend('Initial Condition','Attractor','Initial Trajectory','Long-term Trajectory')
xlabel('x');
ylabel('y');
```

```
zlabel('z');
%annotation('arrow',Y(length_T1:3:length_T1+3,1),Y(length_T1:3:length_T1+3,2),Y(length_T1:3:length_T1+3,3));
%plot(T(10:end),Y(10:end,3),'-b','LineWidth',3)
%xlim([t1,t2])
%xlabel('t');
%ylabel('$$ \delta_H $$','Interpreter','latex');
%title({'\bf Final Project:  Rikitake model for geomagnetic reversals';'(by Renjun Xu)'})
%title({'\bf Final Project: Rikitake model for geomagnetic reversals',['evolution begin->(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2str(par.a)],'(by Renjun Xu)'})

%title(['x_0=',num2str(par.x0)])
%legend('log(a(t))')

%[T3,Res]=lyapunov(3,@riktake,@ode45,0,0.5,200,[0 1 0],10);
%plot(T3,Res);
%title('Dynamics of Lyapunov exponents');
%xlabel('Time');
%ylabel('Lyapunov exponents');


[T2,Y2] = ode45(@(t,y)dr_dt(t,y,par),[t1 t2],[par.x0+delta_x0 par.y0+delta_y0 par.z0+delta_z0]);
%length_T2=floor(0.8*length(T));
delta_0=sqrt(delta_x0^2+delta_y0^2+delta_z0^2);
delta_t2=sqrt((Y2(end,1)-Y(end,1)).^2+(Y2(end,2)-Y(end,2)).^2+(Y2(end,3)-Y(end,3)).^2);
%delta_t=sqrt((Y2(:,1)-Y(:,1)).^2+(Y2(:,2)-Y(:,2)).^2+(Y2(:,3)-Y(:,3)).^2);
%lambda_Liapunov=log(delta_t/delta_0)/T;
lambda_Liapunov=log(delta_t2/delta_0)/t2;
fprintf('The largest Lyapunov Exponent=%10.6f\n',lambda_Liapunov);
title({'\bf Final Project: Rikitake model for geomagnetic reversals -- 3D dynamics',
['evolution begin->(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun Xu)']})
grid on

filename=[num2str(v0),'_a=',num2str(a0),'_Lyap=',num2str(lambda_Liapunov),'_(',num2str(x0),',',num2str(y0),',',num2str(z0)];
%fid = fopen(['Riktake_',filename,'.txt'], 'w');
%fprintf(fid, '%10.6f\n', lambda_Liapunov);
%fclose(fid)

figure(2)
plot(T(1),Y(1,1),'*g',mean(T(length_T:end)),mean(Y(length_T:end,1)),'pb',T(1:length_T1),
Y(1:length_T1,1),'-.k',T(length_T1:end),Y(length_T1:end,1),'-b',T(length_T1:length_T1),Y(length_T1:length_T1,1),'>k','LineWidth',1)
xlabel('t');
ylabel('x');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --x(t)',['evolution begin-
```

```
>(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2
str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun
Xu)']})
grid on

figure(3)
plot(T(1),Y(1,2),'*g',mean(T(length_T:end)),mean(Y(length_T:end,2)),'pb',T(1:length_T1),
Y(1:length_T1,2),'-.k',T(length_T1:end),Y(length_T1:end,2),'-
b',T(length_T1:length_T1),Y(length_T1:length_T1,2),'>k','LineWidth',1)
xlabel('t');
ylabel('y');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --y(t)',['evolution begin-
>(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2
str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun
Xu)']})
grid on

figure(4)
plot(T(1),Y(1,3),'*g',mean(T(length_T:end)),mean(Y(length_T:end,3)),'pb',T(1:length_T1),
Y(1:length_T1,3),'-.k',T(length_T1:end),Y(length_T1:end,3),'-
b',T(length_T1:length_T1),Y(length_T1:length_T1,3),'>k','LineWidth',1)
xlabel('t');
ylabel('z');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --z(t)',['evolution begin-
>(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2
str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun
Xu)']})
grid on

figure(5)
plot(Y(1,1),Y(1,3),'*g',mean(Y(length_T:end,1)),mean(Y(length_T:end,3)),'pb',Y(1:length_
T1,1),Y(1:length_T1,3),'-.k',Y(length_T1:end,1),Y(length_T1:end,3),'-
b',Y(length_T1:length_T1,1),Y(length_T1:length_T1,3),'>k','LineWidth',1)
xlabel('x');
ylabel('z');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --z(x)',['evolution begin-
>(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2
str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun
Xu)']})
grid on

figure(6)
plot(Y(1,2),Y(1,3),'*g',mean(Y(length_T:end,2)),mean(Y(length_T:end,3)),'pb',Y(1:length_
T1,2),Y(1:length_T1,3),'-.k',Y(length_T1:end,2),Y(length_T1:end,3),'-
b',Y(length_T1:length_T1,2),Y(length_T1:length_T1,3),'>k','LineWidth',1)
xlabel('y');
ylabel('z');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --z(y)',['evolution begin-
>(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2
str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun
Xu)']})
```

```
grid on

figure(7)
plot(Y(1,1),Y(1,2),'*g',mean(Y(length_T:end,1)),mean(Y(length_T:end,2)),'pb',Y(1:length_T1,1),Y(1:length_T1,2),'-.k',Y(length_T1:end,1),Y(length_T1:end,2),'-b',Y(length_T1:length_T1,1),Y(length_T1:length_T1,2),'>k','LineWidth',1)
xlabel('x');
ylabel('y');
title({'\bf Final Project: Rikitake model for geomagnetic reversals --y(x)',['evolution begin->(',num2str(par.x0),',',num2str(par.y0),',',num2str(par.z0),'),v=',num2str(par.v),',a=',num2str(par.a)],['(the largest Liapunov exponent: ',num2str(lambda_Liapunov), ', by Renjun Xu)']})
grid on


filename_3d = ['Riktake_3d_v=',filename];
filename_x = ['Riktake_x(t)_v=',filename];
filename_y = ['Riktake_y(t)_v=',filename];
filename_z = ['Riktake_z(t)_v=',filename];
filename_zx = ['Riktake_z(x)_v=',filename];
filename_zy = ['Riktake_z(y)_v=',filename];
filename_yx = ['Riktake_y(x)_v=',filename];

print('-f1','-depsc2',[filename_3d,'.eps']);
print('-f2','-depsc2',[filename_x,'.eps']);
print('-f3','-depsc2',[filename_y,'.eps']);
print('-f4','-depsc2',[filename_z,'.eps']);
print('-f5','-depsc2',[filename_zx,'.eps']);
print('-f6','-depsc2',[filename_zy,'.eps']);
print('-f7','-depsc2',[filename_yx,'.eps']);

            end
         end
      end
   end
end
```

```
%%%%%%%%%%%%%%Rikitake_orbit.m%%%%%%%%%%%%%%%%%%%%%%
%Final Project @ Chaos
%Author:Renjun Xu
%Date: 03/18/2009

clc; close all; clear all;
%Rikitake model for geomagnetic reversals
%dx/dt=-vx+zy;
%dy/dt=-vy+(z-a)x;
%dz/dt=1-xy;
dt=.001;
imax=0;

t1=1; %1e-18;
t2=250;
t=linspace(t1,t2);
par.t0 = t1;
delta_x0 = 1e-3;
delta_y0 = 1e-3;
delta_z0 = 1e-3;

v0=(0.1:0.002:0.4);
%a0=(0.1:0.02:0.4);
a0=0.1;
x0=50;
y0=50;
z0=10;
%(50,50,10),(10,10,10),(2,2,10)
%for z0=10
%    for y0=50
%        for x0=50
for i=1:length(v0)
    par.v=v0(i);
    par.a=a0;

    par.x0 = x0;
    par.y0 = y0;
    par.z0 = z0;

%par.x0 = 50*randn;
%par.y0 = 50*randn;
%par.z0 = 50*randn;

%options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-4],'Refine',4);
[T,Y] = ode45(@(t,y)dr_dt(t,y,par),[t1 t2],[par.x0 par.y0 par.z0]);
%[T,Y] = ode45(@(t,y)dr_dt(t,y,par),[t1 t2],[par.x0 par.y0 par.z0],options);
length_T1=floor(0.1*length(T));
length_T=floor(0.9*length(T));

YMax=zeros(1,(length(T)-length_T));
%TMax=zeros(1,(length(T)-length_T));
```

```
for k=length_T:length(T)-1
   if ((Y(k,2)>Y(k-1,2)) && (Y(k,2)>Y(k+1,2)))
      imax=imax+1;
      YMax(imax)=Y(k,2);
   end
end

plot(par.v,Y(length_T:end,2),'.k',par.v,YMax(1:imax),'.b','LineWidth',2)
hold on
end

hold off
title({'\bf Final Project: Rikitake model for geomagnetic reversals --Orbit
Diagram(y*(v),a=1)';'(by Renjun Xu)'})
grid on

filename_orb=['.(',num2str(x0),',',num2str(y0),',',num2str(z0),')'];
filename_orbit = ['Riktake_orbit',filename_orb];
print('-f1','-depsc2',[filename_orbit,'.eps']);
```

```
%functions
function dy=dr_dt(t,y,par)
dy = zeros(3,1);    % a column vector, y3=a
dy(1) = -par.v*y(1)+y(3).*y(2);
dy(2) = -par.v*y(2)+(y(3)-par.a).*y(1);
dy(3) = 1-y(1).*y(2);



%functions
function f=riktake(t,X)
% Values of parameters
v = 28;
a = 8/3;

x=X(1); y=X(2); z=X(3);

Y= [X(4), X(7), X(10);
   X(5), X(8), X(11);
   X(6), X(9), X(12)];

f=zeros(9,1);

%Lorenz equation
f(1)=-v*x+z*y;
f(2)=-v*y+(z-a)*x;
f(3)=1-x*y;

%Linearized system

 Jac=[-v,   z,  y;
      z-a, -v,  x;
       -y, -x,  0];

%Variational equation
f(4:12)=Jac*Y;

%Output data must be a column vector
```

```
function [Texp,Lexp]=lyapunov(n,rhs_ext_fcn,fcn_integrator,tstart,stept,tend,ystart,ioutp);

n1=n; n2=n1*(n1+1);

nit = round((tend-tstart)/stept);

y=zeros(n2,1); cum=zeros(n1,1); y0=y;
gsc=cum; znorm=cum;

y(1:n)=ystart(:);

for i=1:n1 y((n1+1)*i)=1.0; end;

t=tstart;


for ITERLYAP=1:nit

 [T,Y] = feval(fcn_integrator,rhs_ext_fcn,[t t+stept],y);

 t=t+stept;
 y=Y(size(Y,1),:);

 for i=1:n1
    for j=1:n1 y0(n1*i+j)=y(n1*j+i); end;
 end;


 znorm(1)=0.0;
 for j=1:n1 znorm(1)=znorm(1)+y0(n1*j+1)^2; end;

 znorm(1)=sqrt(znorm(1));

 for j=1:n1 y0(n1*j+1)=y0(n1*j+1)/znorm(1); end;

 for j=2:n1
    for k=1:(j-1)
       gsc(k)=0.0;
       for l=1:n1 gsc(k)=gsc(k)+y0(n1*l+j)*y0(n1*l+k); end;
    end;

    for k=1:n1
       for l=1:(j-1)
          y0(n1*k+j)=y0(n1*k+j)-gsc(l)*y0(n1*k+l);
       end;
    end;

    znorm(j)=0.0;
    for k=1:n1 znorm(j)=znorm(j)+y0(n1*k+j)^2; end;
    znorm(j)=sqrt(znorm(j));
```

```
    for k=1:n1 y0(n1*k+j)=y0(n1*k+j)/znorm(j); end;
  end;


  for k=1:n1 cum(k)=cum(k)+log(znorm(k)); end;

   for k=1:n1
     lp(k)=cum(k)/(t-tstart);
  end;

  if ITERLYAP==1
    Lexp=lp;
    Texp=t;
  else
    Lexp=[Lexp; lp];
    Texp=[Texp; t];
  end;

  if (mod(ITERLYAP,ioutp)==0)
    fprintf('t=%6.4f',t);
    for k=1:n1 fprintf(' %10.6f',lp(k)); end;
    fprintf('\n');
  end;

  for i=1:n1
    for j=1:n1
       y(n1*j+i)=y0(n1*i+j);
    end;
  end;

end;
```

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iii BULLABD, E. C. and GELLMAST, *H. Phil. Trans. A,* **247** (1954), **213**.

iv Rikitake, T., *Proc. Camb. Phil. Soc.* **54** (1958), 89.

v S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering* (Addison-Wesley, Reading, MA, 1994)