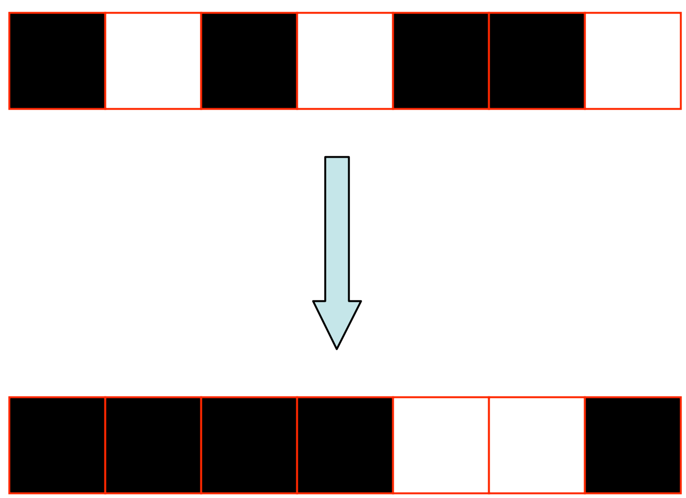
Computational Mechanics of ECAs, and Machine Metrics

Elementary Cellular Automata

- 1d lattice with N cells (periodic BC)
- Cells are binary valued {1,0} -- B or W
- Deterministic update rule, Φ, applied to all cells simultaneously to determine cell values at next time step.
- nearest neighbor interactions only

Example - Rule 54

000	001	010	011	100	101	110	111
0	1	1	0	1	1	0	0



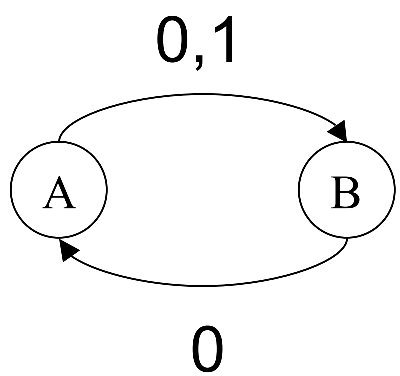
Typical Behavior of ECAs

- Emergence of “Domains” -- spatially homogeneous regions that spread through lattice as time progresses.
- Largely independent of lattice size N, for N big.
- Depends (sensitively) on update rule Φ.

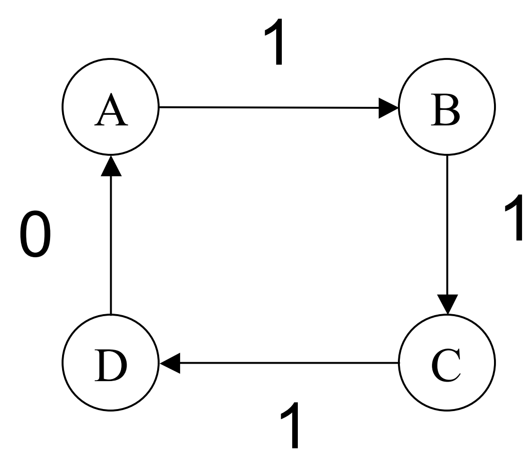
Characterizing ECA Behavior

Domains can be characterized by ε-Machines.

Rule 18 (0W)*



Rule 54 (1110)*



Formally Defining Domains

- Since each ECA Domain can be characterized by a DFA (ε-machine), domains are regular languages.
- Def: a (spatial) domain or (spatial) domain language Λ is a regular langauge s.t.

  (1) $\Phi(\Lambda) = \Lambda$ or $\Phi^p(\Lambda) = \Lambda$, for some p.
  (temporal invariance).

  (2) Process graph of Λ is strongly connected
  (spatial homogeneity).

Temporal Invariance?

- **Question**: Given a potential domain, Λ, with corresponding DFA, M, how do we determine temporal invariance? Can this even be done in general?
- **Answer**: Yes, but somewhat involved. Steps are:

  (1) Encode CA update rule as a Transducer, T.

  (2) Take composition T(M) = T’

  (3) Use T’ to construct M’ = $[T]_{out}$

  (4) Check if M’ = M

How to Determine Domains

- Visual Inspection in simple cases (#54)
- Epsilon Machine Reconstruction
- Fixed Point Equation

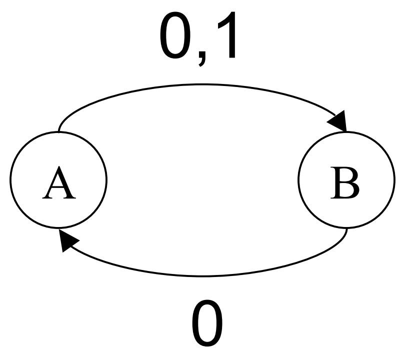
ε-Machine Reconstruction

Several Difficulties:

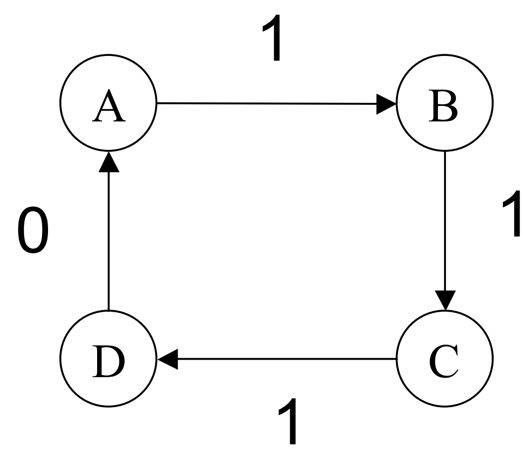
- ‘Experimental’ spatial data does not consist entirely of domain regions. Must sort out true transitions from anomalies.
- May be multiple domains
- Pattern may be spatio-temporal not simply spatial.

Rules that worked

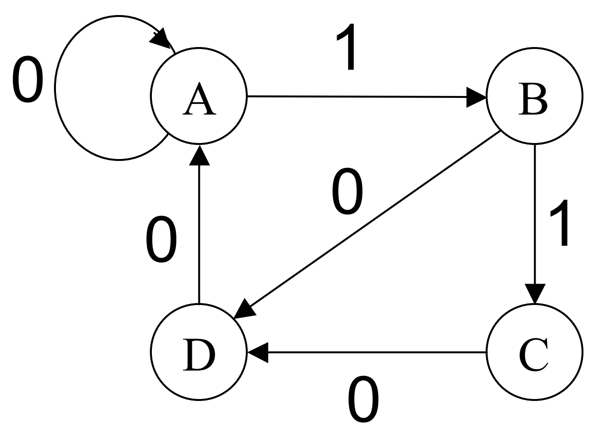
Rule 18 (0W)*



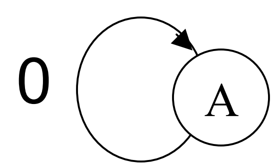
Rule 54 (1110)*



Rule 80 (00,0*,1/11…)

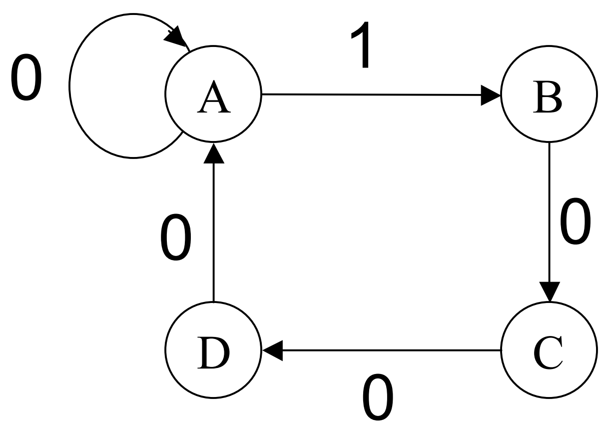


Rule 160 (0)*

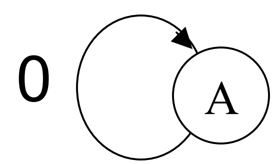


Rules that did NOT work

Rule 144 (1000,0*)



Rule 4, 107



No machines for 150, 180, 204 (and many others)

Results

- Good for entirely periodic spatial patterns, which are temporally fixed.
- Can reconstruct some spatial domains with indeterminancy e.g. Rule 18 = (0W)* , Rule 80.
- Can reconstruct some period 2 domains e.g. Rule 54.
- In general, difficulties for domains with lots of ‘noise’, non-block processes, low transition probabilities, and spatio-temporal processes.

Questions from Demos

- How to analyze patterns in space-time?
- Minimal invariant sets - domains within domains e.g. 000… in rule 18.
- What does it mean for a domain to be stable or attracting?
- Particles and transient dynamics?

Unit Perturbation DFAs

- The unit perturbation language L' of L is L' = { w' s.t. ∃ w in L s.t. $d(w',w) \leq 1$}
- Note: L regular $\Rightarrow$ L' regular

  L process $\not\Rightarrow$ L' process

Attractors

- A regular language L is a fixed point attractor for a CA, Φ, if

  (1) $\Phi(L) = L$

  (2) $\Phi^n(L') \subset L'$, for all n

  (3) For 'almost every' w in L' , $\Phi^n(w)$ is in L, for some n