THE ARMAGEDDON EQUATIONS

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Abstract

In this chaotic world, the threats posed by the small arms trade, which feeds asymmetric warfare, and the proliferation of weapons of mass destruction make arms control more critical by the day. However, there has been little research as to the impact of arms control methods in complex situations such as states with roiling insurgencies. In this paper, I study the impact of economic and military sanctions in such a state using a competition model. It was found that sanctions, while effective at disarming the state, resulted in much greater instability and the likely triumph of the insurgency. On the other hand, military and economic support were found effective at quashing an insurgency, and presumably disarmament could then be safely attempted. However, in one case, assistance was found to cause greater instability—in that stable or unstable outcomes could result, given any wavering in support.

INTRODUCTION

Synopsis

First I give some background on the work—both the motivation for studying the topic and a little history. I detail some underlying assumptions, and give an explanation as to why I built the model in a certain way. I then describe the model's construction and the limitations of certain designs. After adding the effect of sanctions to the simulation, I explain the results, then give final recommendations and some caveats to those conclusions.

Motivation for the Research

I have been interested in warfare research for some time, which is perhaps natural considering my family has long been involved in the military. However, I hope to better serve as a scientist than as a soldier.

My research interests, however, veer less toward the traditional path of development of weaponry than finding better ways to get rid of weapons. It is my belief that the way war is now waged, and how it will be conducted, is not conducive to the use of highly destructive weapons, and thus their presence poses more a danger than their existence is supposed to prevent.

This stems from my expectation that conflicts on the order of the World Wars will not be fought in the future (and if they are, no model could possibly predict the outcome). These days, large-scale conflict is far more expensive, and more costly to clean up—not to mention the fact that the rest of the world is always watching. Besides, why directly attack an enemy when one may slowly bleed it with the distraction of low-intensity fights, or wars by proxy? Consider Iraq in 2007; it was found that the “most lethal weapon against American troops” used by the insurgents at the time was an IED of Iranian manufacture [1]. And many non-states, notably powerful drug cartels, have great incentives for keeping small countries unstable in order to ply their trade. Succinctly stated, “The age of upheaval starts now” [2].

If this is the way war will be fought—indirectly, asymmetrically, or just messily—then highly destructive weapons are relatively useless to a state under siege. They are, however, very useful in the hands of an insurgency or terrorist group with which to create havoc. Then there is great incentive to remove them, especially since a state must fight to capture “hearts and minds” and build bridges, not demolish them. Insurgencies are maintained with light weapons, so it is also important to disrupt the flow of small arms; it is rather difficult to maintain an insurgency without weaponry.

There are two concurrent goals in such a situation: to undermine the insurgency and prevent instability, but at the same time decrease the military capabilities of the state, and hopefully the insurgency as well. One can already see the conflict between these two motivations. On the one hand, insurgencies are won with good governance, good intelligence, and *very* carefully chosen battles; without public dissatisfaction to feed from, an insurgency cannot survive. However, disarming a state at a critical point in time might still be disastrous. The question now is whether such goals can be achieved (preferably concurrently).

A reduction in weaponry may be achieved in several ways, and such methods are the domain of arms control theory. A state may attempt to prevent the creation of new weapons (nonproliferation), destroy existing weapons (antiproliferation) or simply prevent another state from acquiring or producing them (counterproliferation). Usually this requires a delicate diplomatic dance and finding the correct mix of carrots and sticks to get the job done; for the latter, that might include economic embargoes or restrictive export controls. At worst, a military intervention might be necessary. For example, in 2007 Israel launched air strikes against a suspected Syrian nuclear reactor [3], camouflaged as a Byzantine fortress [4]. On the other hand, disrupting arms trade networks (usually the black market) is an alternate method; after all, preventing access to weapons is as effective as averting their creation. However, this requires extensive intelligence-gathering capabilities along with diplomacy.

All these methods are relatively straightforward—except in the case of when an insurgency is also present. For example, a populace might be irritated with economic sanctions, which under normal circumstances would be a minor concern; but when public scrutiny itself is wielded as a weapon against the state (as an insurgency does), outcomes are less certain. The dynamics now are much more interesting, and in light of recent events in Pakistan, research on the topic perhaps more relevant.

BACKGROUND

I should note that in my model, I do not identify weaponry by type, and sanctions are blanket applied. Naturally, targeting particular weapons would make the model more complex and difficult to construct and tune. And though counterproliferation is typically referred to within the context of weapons of mass destruction (WMD), I effectively exclude WMD from my model.

First, I would not expect a state to use WMD (except perhaps North Korea). Their use is taboo, but more importantly, militarily they are not very useful. Chemical weapons have a poor track record. Though used in World War I with horrifying results, chemical weapons are too unpredictable to be of strategic benefit; for example, if the wind blows the wrong way, a state's own troops might be exposed to the weapon. Biological weapons are even less useful; the weapons require special preparation (weaponization of the biological agent) and a state's own population could potentially become infected, which is counterproductive. As for nuclear weapons, they are good for doing only one thing: destroying entire cities. Rarely in history has such a tactic accomplished much strategically [5]; and these days, civilian casualties are to be avoided—the rest of the world is always watching.

The real impact of WMD is the emotional reaction that they cause, which itself is a weapon, making their use attractive in asymmetric warfare. However, use by insurgencies and terrorist groups is rare. It is significantly more difficult to launch a chemical or biological attack, and due to the variable nature of the weapons, the payoff can be very low. As for nuclear weapons, nearly every group will *not* cross that line. Only one group has ever expressed interest in acquiring a nuclear or radiological weapon: al Qaeda. At this point, most of their top leadership has been killed or captured, and the strength of the global jihad movement is debatable. The backlash from the Muslim world (the target audience) against such an attack would only weaken the movement.

As such, I assume these attacks are very unlikely to occur.

As a result of these considerations, in my model I simulate only the application of conventional weapons and security measures, which are already dangerous enough.

DYNAMICAL SYSTEM

Construction of the Model

The construction of the model easily took the most amount of time. A typical cycle emerged: I would write up a set of equations, find it lacking in detail or that it behaved unrealistically, and then discard it for another. This process would continue until I was satisfied with the model. Unfortunately, my early results yielded rather dismal outcomes, and I dubbed the model the "Armageddon Equations" as it seemed appropriate at the time.

Initially I wanted to model the interactions between a state and a black market for weapons, where the black market fed off the state's military strength. I eventually realized that the problem was more economic in nature and could be better handled with a multi-agent model, the development of which could take significantly more time; also, arms control methods, which were the focus of the project, are directed towards the state, not the black market, and questions regarding the efficacy of sanctions would be ill-posed in that case.

Thus I decided to model a state with an insurgency, the dynamics of which are interesting. There are plenty of examples: Pakistan, Afghanistan, Sri Lanka, Colombia, etc. Pakistan is probably the most dangerous. The government barely controls the country; it has a roiling insurgency within its borders; and worst of all, Pakistan has weapons of mass destruction which could easily fall into the hands of a terrorist group or wind up on the black market.

A competition model seemed a natural approach; after all, a government competes with an insurgency for the control of the state. With Pakistan in mind, one of the earliest versions of the model (Mark II) was born:

$$G_S(n+1)=u_{gsge}G_E(n)+u_{gsgs}G_S(n)+u_{gsp}P(n)+u_{gsm}M(n)+u_{gst}T(n)-u_{gscs}e^{G_S(n)B_S(n)}-u_{gsce}e^{u_{cex}G_E(n)B_E(n)}$$
$$G_E(n+1)=u_{gege}G_E(n)+u_{gep}P(n)+u_{get}T(n)-u_{gece}e^{u_{cex}G_E(n)B_E(n)}$$
$$B_S(n+1)=u_{bsbe}B_E(n)+u_{bsbs}B_S(n)-u_{bst}T(n)-u_{bscs}e^{G_S(n)B_S(n)}-u_{bsce}e^{u_{cex}G_E(n)B_E(n)}-u_{bsp}P(n)-u_{bsm}M$$
$$B_E(n+1)=-u_{bege}G_E(n)+u_{bebe}B_E(n)-u_{bet}T(n)-u_{becs}e^{G_S(n)B_S(n)}-u_{bece}e^{u_{cex}G_E(n)B_E(n)}$$
$$P(n+1)=u_{pt}T(n)-u_{pm}M(n)$$
$$M(n+1)=u_{mgs}G_S(n)+u_{mp}P(n)-u_{mt}T(n)$$
$$T(n+1)=u_{tgs}G_S(n)+u_{tbs}B_S(n)+u_{tp}P(n)+u_{tm}M(n)-u_{cs}e^{u_{csx}G_S(n)B_S(n)}-u_{ce}e^{u_{cex}G_E(n)B_E(n)}$$

G_S and G_E represent the strength and support for the state (government) and its economic power or resources, respectively; similarly, B_S and B_E describe the insurgency. P represents the support of the people for either the government (positive values) or the insurgency (negative values). M is the military and police force, wielded by the government against the insurgency. Finally, T is the relative stability of the state; intuitively, positive values indicate that the state is stable, and negative values indicate the opposite.

Unfortunately, I quickly discovered one of the limitations of the model: signed quantities can produce very strange results.

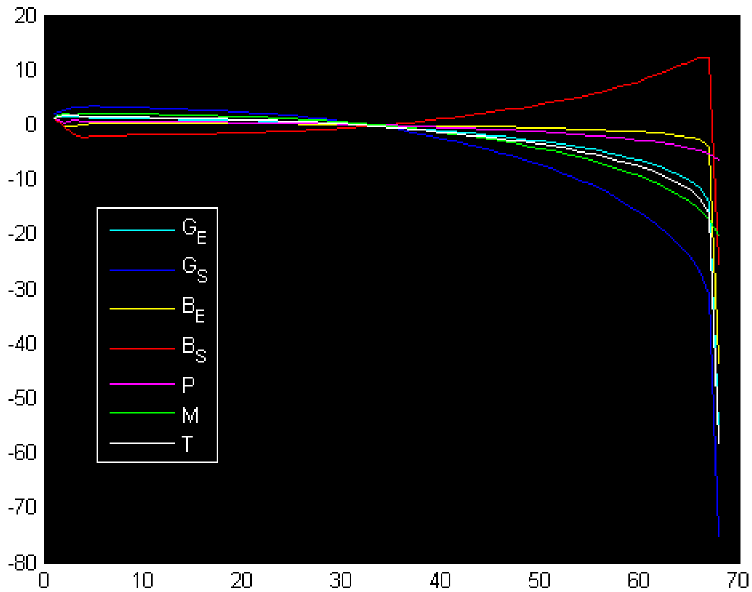

Figure 1. Armageddon occurs after 68 iterations. This illustrates the dangers of using signed quantities in the model.

Thus I made most of the quantities strictly positive so that relative strengths would be measured only as a magnitude, recalling the LPA model [6] that I had studied as an undergraduate. Exponentiation provided a neat solution; the exponent determines the relative change in quantity, either growth or decay. The model behaved as expected until I simulated the effects of arms control.

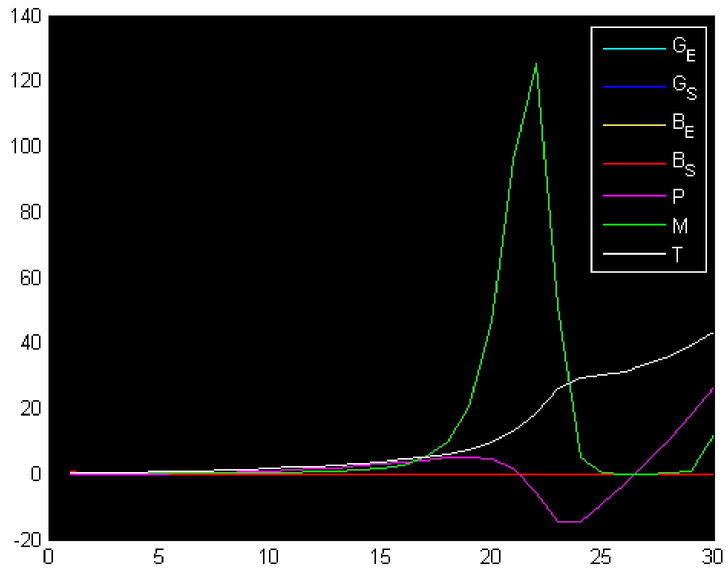

Figure 2. Overzealous economic sanctions lead to a military coup. This indicates that the military and state terms must be coupled.

In this case, it appears that the state has transformed into a military dictatorship, which was not an outcome I had planned to simulate (though in reality, juntas are common). To solve this problem, I coupled G_S and M. The equations are very symmetric in the sense that the government and insurgency are mirrors of each other, each benefiting from their separate economies, and each declining from the effects of competition. Similarly, the government uses the military against the insurgency and to maintain order, and the insurgency uses the stability of the state itself against the government. The fate of the military and the maintenance of order is thus tied to that of the state, and without instability, the insurgency cannot survive. Thus the competition between the two entities is tied to the stability of the state. In the simulation, stable outcomes are correlated with the government as the victor; unstable outcomes result in the triumph of the insurgency.

The final version of the Armageddon Equations (Mark VI) are thus:

$$
\begin{aligned}
G_S(n+1) &= G_S(n)\, e^{u_{gsge} G_E(n) + u_{gsp} P(n) + u_{gsm} M(n) + u_{gst} T(n) - u_{gscs} B_S(n)} \\
G_E(n+1) &= G_E(n)\, e^{u_{gegs} G_S(n) + u_{get} T(n) - u_{gem} M(n) - u_{gece} B_E(n)} \\
B_S(n+1) &= B_S(n)\, e^{u_{bsbe} B_E(n) - u_{bst} T(n) - u_{bsp} P(n) - u_{bsm} M(n) - u_{bscs} G_S(n)} \\
B_E(n+1) &= B_E(n)\, e^{u_{bebs} B_S(n) - u_{bet} T(n) - u_{becs} G_S(n) - u_{bece} G_E(n)} \\
P(n+1) &= P(n) + u_{pt} T(n) + u_{pgs} G_S(n) + u_{pge} G_E(n) - u_{pbs} B_S(n) - u_{pbe} B_E(n) - u_{pm} M(n) \\
M(n+1) &= M(n)\, e^{-u_{mg} G_E(n) \ln(G_S(n)) + u_{mcse} B_S(n) G_S(n) G_E(n) + u_{mp} P(n) - u_{mtgs} T(n) G_S(n) G_E(n) - u_{mb} B_S(n) B_E(n)} \\
T(n+1) &= T(n) + u_{tgs} G_S(n) - u_{tbs} B_S(n) + u_{tge} G_E(n) + u_{tbe} B_E(n) + u_{tp} P(n) + u_{tg} G_S(n) M(n) \\
&\quad + u_{tb} B_E(n) \ln(B_S(n)) - u_{tcs} e^{u_{csx} G_S(n) B_S(n)} - u_{tce} e^{u_{cex} G_E(n) B_E(n)}
\end{aligned}
$$

A detailed explanation of the terms is in Appendix A.

While picking and choosing terms was difficult, scaling the parameters and initial values was a challenge all unto itself. I found it best to simply use integer values to represent the relative effects of the terms, and then "batch-scaling" the parameters with a scaling parameter.

In order to select the scaling parameters, I rendered a basin of attraction and looked for regions with interesting dynamics, assuming that areas of the basin not dominated by stable or unstable outcomes indicated parameter combinations that allowed for a "fair fight."

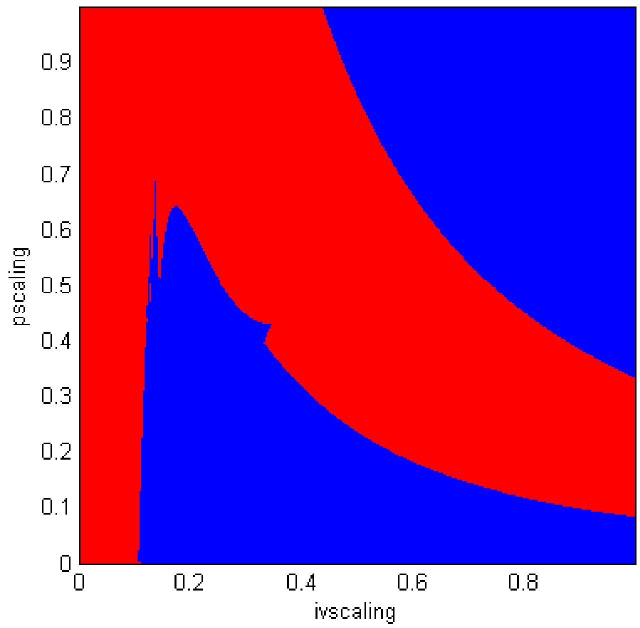

Figure 3. Two intertwined basins of attraction representing initial values that lead to stable outcomes (blue points) and unstable outcomes (red points), where the parameter scaling and initial value scaling terms are varied to test a range of scenarios. Along the boundaries of the basins, "fair fights" are assumed to be expected.

I wound up picking simpler values for the scaling parameters which coincided with a boundary of the basins of attraction (0.1 for both). A "fair fight" using the "tuned" parameters with scaling is shown next.

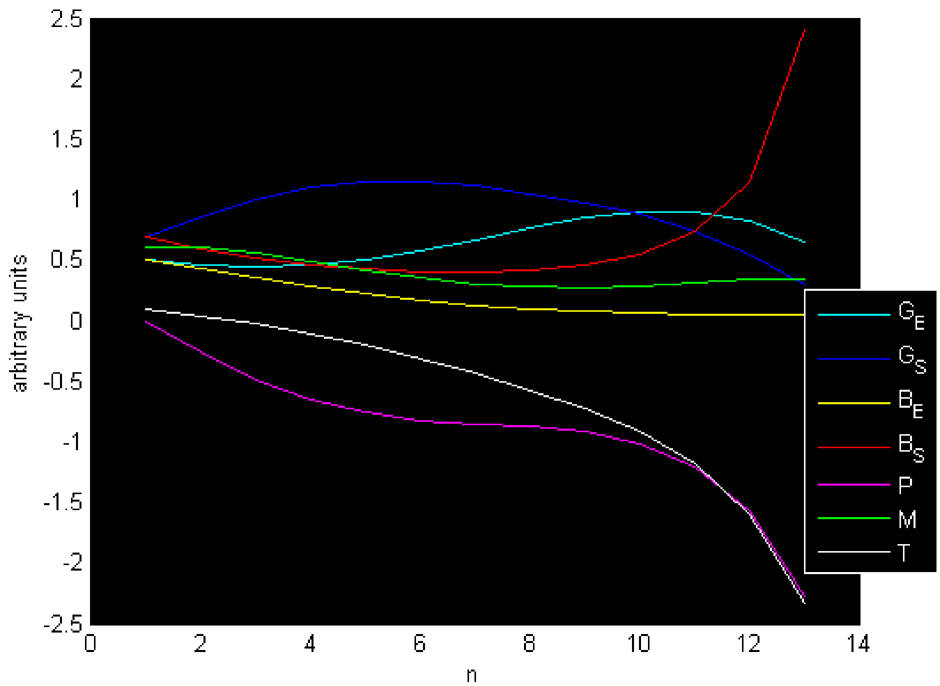

Figure 4. A “fair fight.” The government initially starts out strongly, but the presence of the military and the competition between the government and insurgency anger the populace and create instability. The government attempts to regain control by boosting its military, but this angers the populace even more and drains the economy, causing even greater instability. The insurgency triumphs.

Methods and Results

Once I had a working model, I was able to test the effects of arms control methods. I assumed that diplomatic carrots and sticks would come in the form of economic and military sanctions (or assistance, depending on the scaling) and added appropriate terms:

$$
\begin{aligned}
G_S(n+1) &= G_S(n)\,e^{u_{gsge}G_E(n)+u_{gsp}P(n)+u_{gsm}M(n)+u_{gst}T(n)-u_{gscs}B_S(n)} \\
G_E(n+1) &= G_E(n)\,e^{u_{gegs}G_S(n)+u_{get}T(n)-u_{gem}M(n)-u_{gece}B_E(n)-u_{gek}G_E(n)} \\
B_S(n+1) &= B_S(n)\,e^{u_{bsbe}B_E(n)-u_{bst}T(n)-u_{bsp}P(n)-u_{bsm}M(n)-u_{bscs}G_S(n)} \\
B_E(n+1) &= B_E(n)\,e^{u_{bebs}B_S(n)-u_{bet}T(n)-u_{becs}G_S(n)-u_{bece}G_E(n)-u_{bek}B_E(n)} \\
P(n+1) &= P(n)+u_{pt}T(n)+u_{pgs}G_S(n)+u_{pge}G_E(n)-u_{pbs}B_S(n)-u_{pbe}B_E(n)-u_{pm}M(n) \\
M(n+1) &= M(n)\,e^{-u_{mg}G_E(n)\ln(G_S(n))+u_{mcse}B_S(n)G_S(n)G_E(n)+u_{mp}P(n)-u_{mtgs}T(n)G_S(n)G_E(n)-u_{mb}B_S(n)B_E(n)-u_{mk}M(n)} \\
T(n+1) &= T(n)+u_{tgs}G_S(n)-u_{tbs}B_S(n)+u_{tge}G_E(n)+u_{tbe}B_E(n)+u_{tp}P(n)+u_{tg}G_S(n)M(n) \\
&\quad +u_{tb}B_E(n)\ln(B_S(n))-u_{tcs}e^{u_{csx}G_S(n)B_S(n)}-u_{tce}e^{u_{cex}G_E(n)B_E(n)}
\end{aligned}
$$

Again, I used the “scaling trick” to test a variety of outcomes. I set the sanctions parameters u_{gek}, u_{bek}, and u_{mk} to intuitive integer values and then scaled them twice: first with the default parameter scaling term (0.1) and then with the appropriate sanctions scaling term (separate terms were used for economic and military sanctions parameters). I varied the sanctions scaling terms to render another basin of attraction.

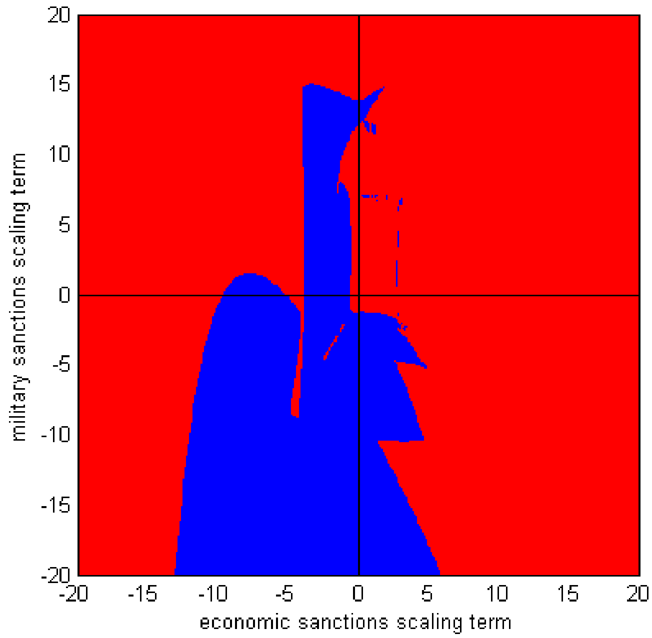

Figure 5. Two intertwined basins of attraction. The red basin includes initial values that lead to unstable outcomes (red points), and the blue basin are those that lead to stable outcomes (blue points). Positive scaling terms result in sanctions being applied. Negative scaling terms indicate support or assistance. Sanctions tend to lead to unstable outcomes while support or assistance tends to lead to the triumph of the government over the insurgency.

The first quadrant represents "sticks only" sanctions—both economic and military sanctions were applied. The second and fourth quadrants represent a mixture of carrots and sticks, and the third quadrant shows the effects of a "carrots only" policy.

Unfortunately, according to the model, military and economic sanctions, though they might disarm the state, are extremely destabilizing. Arming and assisting the state, immoral as such actions might be, allow the state to quell the insurgency. However, if sanctions are too generous, they might help the insurgency; economic benefits trickle down to the insurgency too, and if the state's military is too zealous, the people are driven toward the comparatively gentler insurgency.

Unfortunately, I was unable to find any cyclic behavior in the Armageddon Equations. As a result, I could not apply Markov methods to the data, as there was nothing resembling a continuous process. However, I was fortunate to stumble upon a very small riddled basin of attraction (likely just the lower boundary of the graph).

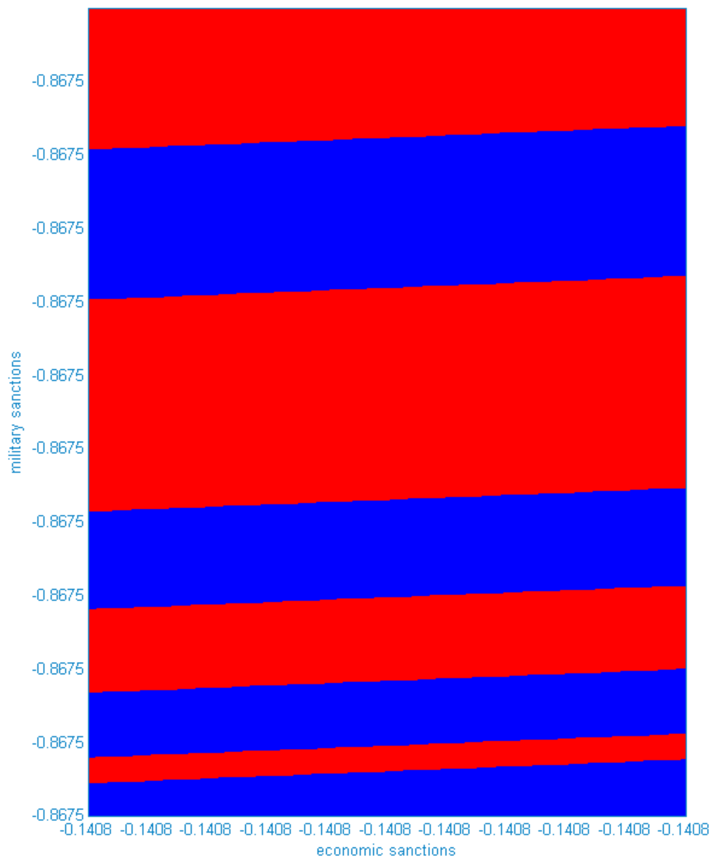

Figure 6. Similar to Figure 5, two intertwined basins represent stable (blue points) and unstable (red points) outcomes. However, there is riddling near the base of the graph. Within the riddled basin of attraction, any slight deviation from a starting point in one basin can result in winding up in the other basin of attraction.

Since the two basins are intertwined, the dense fractal banding present near the bottom of the graph implies that the basins are riddled there: given some initial condition in one basin, any slight perturbation can result in winding up in the other basin of attraction. In other words, if the state applying sanctions or assisting the target state (the one being simulated) wavered only slightly in their commitment, the insurgency could end up the victor. The implications of the model suggest that economic and military sanctions can be destabilizing in a far more dangerous way—given any variation in the application of sanctions (or support), it is impossible to predict the outcome of the result of the arms control methods.

Fortunately, I found that the basin was the result of an erroneously scaled sanctions term. I had intended to add a "sanctions irritation term" $u_{pek} P(n)$ to the popular mood, as shown below:

$$P(n+1)=P(n)+u_{pt}T(n)+u_{pgs}G_S(n)+u_{pge}G_E(n)-u_{pbs}B_S(n)-u_{pbe}B_E(n)-u_{pm}M(n)-u_{pek}P(n)$$

Unfortunately, with the mistake in scaling, the term acted as a highly effective insurgency recruitment term, assuming the insurgency had a brilliant propaganda campaign, or the state was badly mistreating its citizens:

$$P(n+1)=P(n)+u_{pt}T(n)+u_{pgs}G_S(n)+u_{pge}G_E(n)-u_{pbs}B_S(n)-u_{pbe}B_E(n)-u_{pm}M(n)-u_{pr}P(n)$$

I had wanted to avoid modeling propaganda, as it is complex and there are other models that handle the dynamic far more effectively; SIR models, typically used to model disease epidemics, have been successful in modeling insurgency recruitment.

Removal of the erroneous, but interesting term resulted in the disappearance of the fractal boundary in the basin of attraction. Apparently the recruitment effect was a delicate one. I was unable to find any other fractal boundaries or regions in the basin of attraction, so I created my dataset from the "interesting mistake."

I fixed the economic sanctions scaling term and then varied the military sanctions scaling term along the fractal boundary region. By taking finer and finer measurements (reducing the step size between points) I was able to generate longer datasets. I then mapped unstable outcomes in the basin (red points) to 0 and stable outcomes (blue points) to 1, yielding a binary string; if the step size is made infinitely small, the "measured process" is continuous. Using this dataset, I was able to reconstruct an epsilon machine.

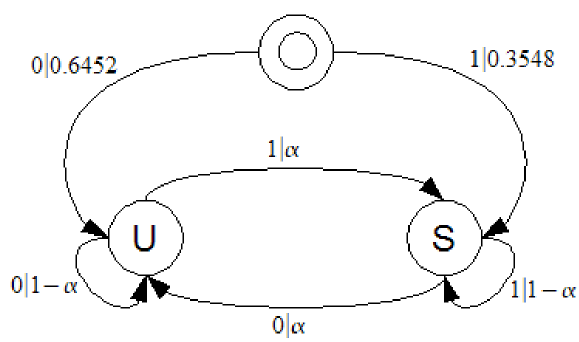

Figure 7. An epsilon machine constructed using data from the fractal basins of attraction shown in Figure 6; mapping red points to 1's and blue points to 0's results in in a long binary string.

The process effectively resembles a biased coin that, after being flipped once, sticks to the floor; there is a tiny chance α that the coin will reverse. The process synchronizes after a *single* measurement.

This is due to the nature of the dataset. While the banding of the basins is fractal, the bands are sufficiently wide, and the step size sufficiently small, that the number of transitions between basins are comparatively few. This is indicated by the very, very small chance of transitions between the U and S causal states.

Conclusions

Judging from Figure 5, the effects of economic and military sanctions, while they may be effective at disarming a state, they are dangerously destabilizing. Military sanctions may work, but only if applied in conjunction with some economic assistance. It appears that the best route to achieving stability in a state, and disarmament after, is economic and military *support*. In the long-term, according to this model it is better to arm a state (ignoring possible moral issues along the way) in order to later disarm it. And if the recruiting power of an insurgency is taken into account, it is possible that assistance may make the state even more unstable—in that the effect of the sanctions is to make either outcome just as likely. Truly, the path to Armageddon may be paved with trade incentives and military support.

However, the Armageddon Equations are a toy model. Though they were created to focus on the dangers of instability, which is very much a post-Cold War viewpoint, they are still plagued by a problem encountered in warfare research in any era: international security is unsimulatable. In other words, the most accurate model is the object itself.

Certainly, I could add more equations and terms for greater realism, but that makes the model more unwieldy. Other methods have been suggested [7], but I think that perhaps a multi-agent model would be the most effective. Already there is the Sentient World Simulation [8], which seems to be effective, judging from the amount of funding it is receiving from the Department of Defense. Results are likely classified for now.

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APPENDICES

Appendix A

Here I give a full explanation of every term in the model.

Scaling terms

Scaling term	Description	Typical value
pscaling	Blanket scaling term of the parameters	0.1
ivscaling	Blanket scaling term of the initial values	0.1
econ_sanc	Scaling term applied to economic sanctions parameters	[FIGURE] varied from [-20,20] [FIGURE] varied from [-0.1407795, -0.14078] [EMACH] fixed at -0.140779
mil_sanc	Scaling term applied to the military sanctions parameter	[FIGURE] varied from: [-20,20] [FIGURE] varied from [-0.867515, -0.867504] [EMACH] varied from [-0.867515, -0.867500]

Equation terms

Entity (left-hand-side)	Term (right-hand-side)	Typical value	Scaled by	Description
$G_S(n+1)$	$u_{gsge} G_E(n)$	3	pscaling	The government benefits from the economy (note that insurgencies do not pay taxes to the state on black market gains).
	$u_{gsp} P(n)$	4	pscaling	The government is supported (or not) by the people).
	$u_{gsm} M(n)$	2	pscaling	The government enforces its will via the military.
	$u_{gst} T(n)$	2	pscaling	The government benefits from stability, and suffers if there is instability.
	$u_{gscs} B_S(n)$	3	pscaling	The government suffers from the presence of the insurgency.
$G_E(n+1)$	$u_{gegs} G_S(n)$	3	pscaling	The economy benefits from the presence of the government.
	$u_{get} T(n)$	2	pscaling	The economy thrives in an ordered atmosphere.
	$u_{gem} M(n)$	2	pscaling	Militaries cost resources.
	$u_{gece} B_E(n)$	4	pscaling	The economy suffers from competition with the black market.
	$u_{gek} G_E(n)$	5	pscaling econ_sanc	The economy suffers if economic sanctions are imposed.
$B_S(n+1)$	$u_{bsbe} B_E(n)$	4	pscaling	The insurgency benefits from the

				black market.
	$u_{bst}T(n)$	3	pscaling	The insurgency grows stronger in an unstable atmosphere, and suffers declines if the state is ordered.
	$u_{bsp}P(n)$	3	pscaling	The insurgency is supported (or not) by the people.
	$u_{bsm}M(n)$	2	pscaling	The insurgency suffers from the presence of the military.
	$u_{bscs}G_S(n)$	3	pscaling	The insurgency competes with the government.
$B_E(n+1)$	$u_{bebs}B_S(n)$	3	pscaling	The black market benefits from the support of the insurgency.
	$u_{bet}T(n)$	2	pscaling	The black market benefits from instability and suffers from stability.
	$u_{becs}G_S(n)$	2	pscaling	The black market is supported by the insurgency.
	$u_{bece}G_E(n)$	4	pscaling	The black market suffers from competition with the economy.
	$u_{bek}B_E(n)$	5	pscaling econ_sanc	The black market also suffers from economic sanctions.
$P(n+1)$	$u_{pt}T(n)$	2	pscaling	The people support whoever *seems* to be in power.
	$u_{pgs}G_S(n)$	3	pscaling	The people support the government if it is strong enough.
	$u_{pge}G_E(n)$	2	pscaling	The people support the government if the economy is strong.
	$u_{pbs}B_S(n)$	3	pscaling	The people support the insurgency if it is large enough.
	$u_{pbe}B_E(n)$	5	pscaling	The people support the black market depending on how ubiquitous it is.
	$u_{pm}M(n)$	4	pscaling	The people dislike intrusions by the military—searches, checkpoints, abuses, etc.
	$u_{pr}P(n)$ (term not present in MK6)	4	pscaling	The people are actively recruited by the insurgency.
$M(n+1)$	$u_{mg}G_E(n)\ln(G_S(n))$	2	pscaling	The military is supported by the government, but declines if the government is weak.
	$u_{mcse}B_S(n)G_S(n)G_E(n)$	2	pscaling	The military is boosted in response to the threat of the insurgency, proportional to the strength of the government and economy.
	$u_{mp}P(n)$	3	pscaling	Support (or hindrance) from the people.

	$u_{mtgs} T(n) G_S(n) G_E(n)$	2	pscaling	If the state is stable, decrease the military's strength proportional to the strength of the government and economy. If not, increase proportional to the threat.
	$u_{mb} B_S(n) B_E(n)$	4	pscaling	The insurgency attacks the military proportional to its strength and resources.
	$u_{mk} M(n)$	5	pscaling mil_sanc	Sanctions drain the military.
$T(n+1)$	$u_{tgs} G_S(n)$	3	pscaling	The government brings stability.
	$u_{tbs} B_S(n)$	3	pscaling	The insurgency creates instability.
	$u_{tge} G_E(n)$	2	pscaling	A strong economy can help keep order, *regardless* of the source. If the government cannot provide services, the market will fill the need.
	$u_{tbe} B_E(n)$	2	pscaling	
	$u_{tp} P(n)$	3	pscaling	If the people do not support the government, instability results; and vice-versa.
	$u_{tg} G_S(n) M(n)$	3	pscaling	The military and police continually bring order, but is only as effective as the government that wields it.
	$u_{tb} B_E(n) \ln(B_S(n))$	4	pscaling	The insurgency uses instability as a weapon; when it is weak ($B_S(n)<1$), it will create instability proportional to its resources.
	$u_{tcs} e^{u_{csx} G_S(n) B_S(n)}$	1, 1	pscaling	Stability suffers from competition between the government and insurgency.
	$u_{tce} e^{u_{cex} G_E(n) B_E(n)}$	2, 1	pscaling	Instability results from competition between the economy and black market.

Initial Values

Entity	Typical value (always scaled by ivscaling)	Comments
$G_S(0)$	7	The government starts out relatively strong.
$G_E(0)$	5	The economy is healthy.
$B_S(0)$	6	These values are lower than the government's starting values, but the insurgency does not need as much to disrupt the government.
$B_E(0)$	4	The black market is active.
$P(0)$	0	The people start out apathetic.
$M(0)$	6	The military is quite strong; anything lower than 5 usually results in the insurgency winning.
$T(0)$	1	The state is somewhat stable.

Appendix B

[10]