Physics of Information
Physics $2^8$A
**Syllabus (Winter)**

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## Contents



# 1 First Lecture: Overview

**Readings** (available via course website):

- *Chaos*, JP Crutchfield, JD Farmer, NH Packard, RS Shaw, Scientific American **255** (1986) 46–57.

- *Odds*, Stanislaw Lem, New Yorker **54** (1978) 38–54.

**Topics**:

1. Introduction and motivations
2. Physics of Information 256A: Dynamics, Self-Organization, Measurement Theory, Information Theory
3. Physics of Computation 256B
4. Survey interests, background, and abilities
5. Course logistics
6. Exams
7. CMPy Labs

# 2 Self-Organization

**Reading**: *Nonlinear Dynamics and Chaos*, Strogatz (NDAC), and Course Lecture Notes

**Theme**: Forms of Randomness, Order, and Intrinsic Instability

1. Nonlinear Dynamics:
   (a) Qualitative dynamics
   (b) ODEs and maps
   (c) Bifurcations
   (d) Stability, instability, and chaos
   (e) Quantifying (in)stability
2. Pattern-forming systems:
   (a) Instability and stabilization of patterns
   (b) Cellular automata, map lattices, spin systems

## 2.1 Lecture 2: The Big Picture

**Reading**: *NDAC*, Chapters 1 and 2.

**Topics**:

1. Pendulum demo
2. Discuss *Chaos* and *Odds* readings and homework
3. Qualitative dynamics: A geometric view of behavior
4. State space
5. Flows
6. Attractors
7. Basins
8. Submanifolds
9. Concrete, but simple example: One-dimensional flows

**Homework**: Assign Week 0's homework today. Everyday unpredictability; see handout or website. Due in one week, but be prepared to discuss at next meeting.

## 2.2 Lecture 3: Example Dynamical Systems

**Reading**: *NDAC*, Sections 6.0-6.7, 7.0-7.3, and 9.0-9.4.

**Topics**:

1. Continuous-time ODEs
    (a) 2D flows: Fixed points (Sec. 6.0-6.4)
    (b) 2D flows: Limit cycles (Sec. 7.0-7.3)
    (c) 3D flows: Chaos in Lorenz (Sec. 9.0-9.4)
    (d) Simulation demo
2. From continuous to discrete time (Sec. 9.4)
    (a) Poincaré maps and sections
    (b) Lorenz ODE to cusp map
    (c) Rössler ODE to logistic map (pp. 376–379)
    (d) Discrete-time maps

## 2.3 Lecture 4: The Big, Big Picture I

**Reading**: *NDAC*, Chapters 3 and 8 and Sec. 10.0-10.4.

**Topics**:

1. Qualitative dynamics: Space of all dynamical systems
2. Example: Bifurcations of one-dimensional flows
    (a) Saddle node
    (b) Transcritical
    (c) Pitchfork
3. Catastrophe theory
    (a) Catastrophes: Fixed point to fixed point bifurcation
    (b) Example: Cusp Catastrophe
    (c) Catastrophe theory classification of fixed point bifurcations

**Homework**: Collect Week 0's, assign Week 1's today.

## 2.4 Lecture 5: The Big, Big Picture II

**Reading**: *NDAC*, Chapters 3 and 8 and Sec. 10.0-10.4.

**Topics**:

1. Bifurcations in discrete-time maps: Logistic map
2. Fixed point to limit cycle
3. Phenomenon and calculation
4. Limit cycle to limit cycle
5. Phenomenon and calculation

6. Routes to chaos: Period-doubling cascade
7. Phenomenon and calculation
8. Band-merging
9. Periodic windows and intermittency
10. Simulation demo

### 2.5 Lecture 6: Mechanism of Chaos: Stable Instability

**Reading**: *NDAC*, Sec. 12.0-12.3, 9.3, and 10.5.

**Topics**:

1. Chaotic mechanisms: Stretch and fold
2. Baker's map
3. Cat map (and stretch demo)
4. Henon map: stretch-fold and self-similarity
5. Roessler attractor branched manifold
6. Dot spreading: Roessler and Lorenz ODEs
7. Lyapunov characteristic exponents (LCEs)
8. Time to unpredictability
9. Dissipation rate
10. Attractor LCE classification
11. Chaos defined

**Homework**: Collect Week 1's, assign Week 2's today.

### 2.6 Lecture 7: Example Chaotic Maps (that you can analyze)

**Reading**: *NDAC*, Chapter 10.

**Topics**:

1. Shift map
2. LCEs for maps
3. Tent map
4. Logistic map
5. LCE view of period-doubling route to chaos
6. Period-doubling self-similarity
7. Renormalization group analysis of scaling

### 2.7 Lecture 8: Pattern Formation I

**Reading**: Lecture Notes.

**Topics**:

1. Review last lecture.

2. Spatially Extended Dynamical Systems
3. Synchronous Cellular Automata
4. Lattice Maps: Logistic Lattice and Dripping Handrail

**Homework**: Collect Week 2's, assign Week 3's today.

### 2.8 Lecture 9: Pattern Formation II

**Reading**: Lecture Notes.

**Topics**:

1. Review last lecture.
2. Asynchronous Cellular Automata
3. Spin Systems

## 3 From Determinism to Stochasticity

**Reading**: Lecture Notes.

**Theme**: Stochasticity and Measurement

1. Probability Theory of Dynamical Systems
2. Stochastic Processes
3. Measurement Theory

### 3.1 Lecture 10: Probability Theory of Dynamical Systems

**Reading**: Lecture Notes.

**Topics**:

1. Probability theory review
2. Dynamical evolution of distributions
3. Invariant measures
4. Examples

**Homework**: Collect Week 3's, assign Week 4's today.

### 3.2 Lecture 11: Stochastic Processes

**Reading**: Lecture Notes.

**Topics**:

1. Review last lecture.
2. Processes

3. Markov chains
4. Statistical equilibrium
5. Hidden Markov models
6. Examples: Fair coin, periodic, golden mean, even, and others

### 3.3 Lecture 12: Measurement Theory I

**Reading**: Lecture Notes.

**Topics**:

1. Review last lecture.
2. State-space partitioning
3. Orbit and sequence spaces
4. Good instruments and informative measurements

**Homework**: Collect Week 4's, assign Week 5's today.

### 3.4 Lecture 13: Measurement Theory II

**Reading**: Lecture Notes.

**Topics**:

1. Review last lecture.
2. Markov partitions in 1D
3. Generating partitions in 1D
4. Example: 1D maps
5. Generating partitions in 2D
6. Example: 2D Cat map

## 4 Information Processing

**Reading**: *Elements of Information Theory*, Cover and Thomas (EIT), and *Computational Mechanics Reader*, JPC (CMR)

**Theme**: Information, Uncertainty, and Memory

1. Entropies
2. Communication Channel (and coding theorems)
3. Mutual Information and Information metric
4. Excess Entropy
5. Transient Information
6. Connection to Dynamics: Entropy rate and LCEs

### 4.1 Lecture 14: Entropies

**Reading**: *EIT*, Chapters 1 and 2.

**Topics**:

1. Motivation: Information $\neq$ Energy
2. Information as uncertainty and surprise
3. Information sources: Ignorance of forces or initial conditions, deterministic chaos, and ...?
4. Axioms for a measure of information
5. Entropy function
6. Convexity
7. Joint and Conditional Entropy
8. Mutual information
9. Examples

**Homework**: Collect Week 5's, assign Week 6's today.

### 4.2 Lecture 15: Information in Processes I

**Reading**: *EIT*, Sec. 5-5.4 and 8-8.5 and Chapter 4.

**Topics**:

1. Communication channels
2. Coding theorems
3. Examples

### 4.3 Lecture 16: Information in Processes II

**Reading**: *EIT*, Sec. 5-5.4 and 8-8.5 and Chapter 4.

**Topics**:

1. Entropy rates for Markov chains
2. Entropies for times series
3. Connection to Dynamics: Entropy rate and LCEs

**Homework**: Collect Week 6's, assign Week 7's today.

### 4.4 Lecture 17: Memory in Processes I

**Reading**: *CMR* article RURO.

**Topics**:

1. Entropy convergence
2. Excess entropy
3. Examples

### 4.5 Lecture 18: Memory in Processes II

**Reading**: *CMR* article RURO.

**Topics**:

1. Generalized synchronization
2. Transient information
3. Examples

**Homework**: Collect Week 7's, assign Week 8's today.

### 4.6 Lecture 19: Rate Distortion Theory I

**Reading**: *EIT*, Chapter 10.

**Topics**:

1. Rate distortion theory

### 4.7 Lecture 20: Rate Distortion Theory II

**Reading**: *EIT*, Chapter 10.

**Topics**:

1. Rate distortion theory

**Homework**: Collect Week 8's.