QUANTUM MECHANICS AND QUANTUM INFORMATION

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John Templeton Foundation

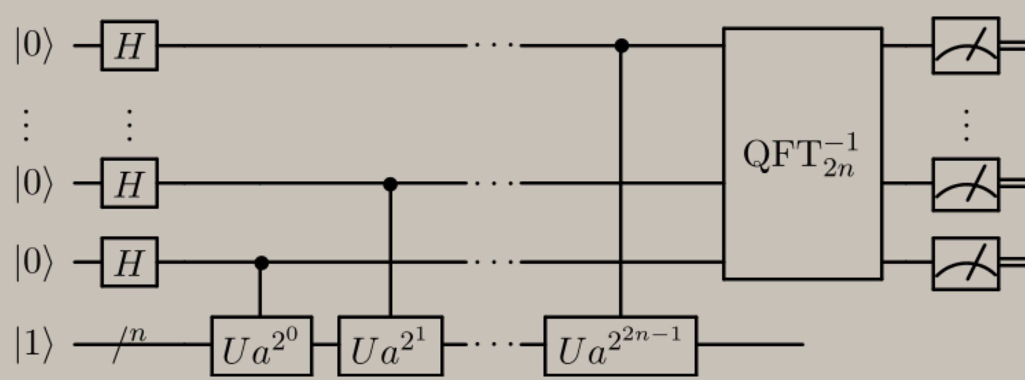
John Mahoney - Natural Computation and Self-Organization 2016

reference: QCQI, Nielsen and Chuang

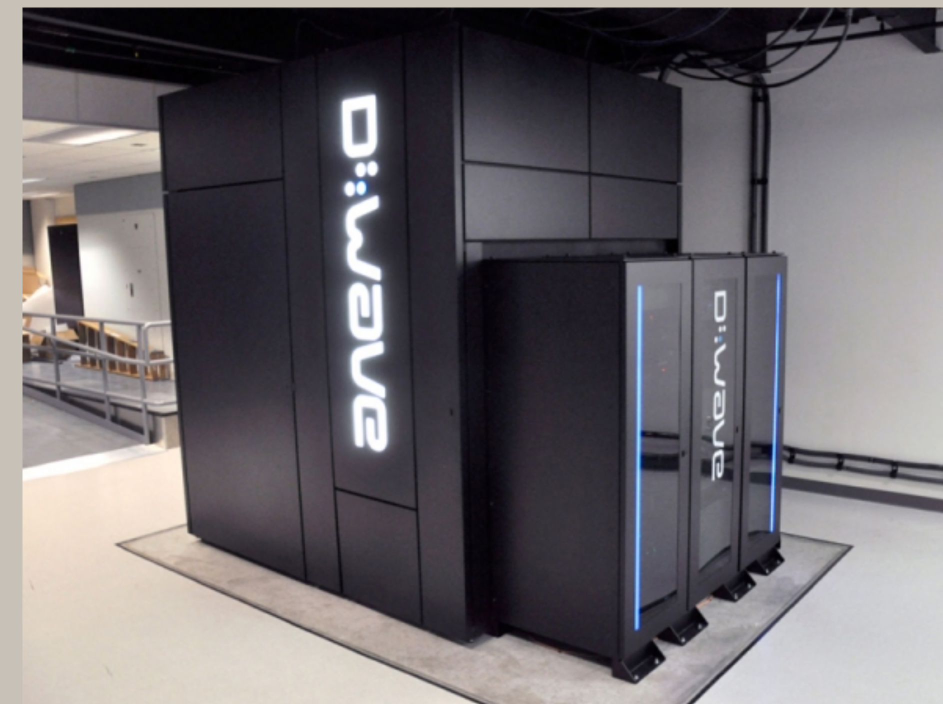
WHY DO WE NEED A QUANTUM THEORY?

- Atomic spectra
- Photo electric effect
- Stern Gerlach
- Double slit experiment

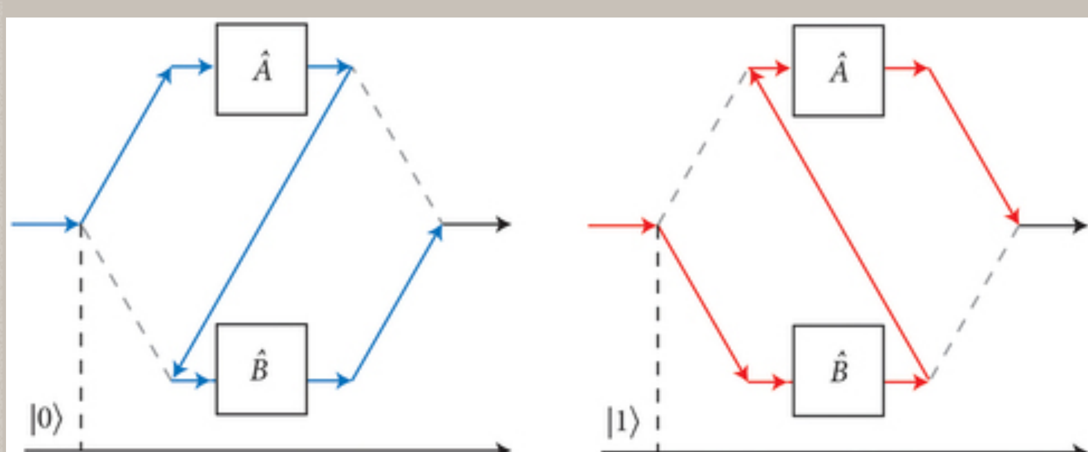
MORE RECENTLY



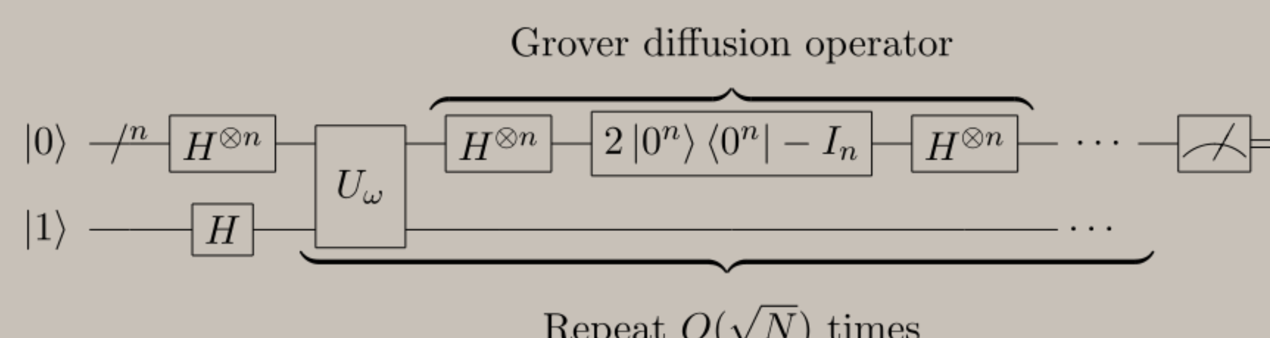
Shor's algorithm



commercial q-computers?



causal ambiguity



Grover's algorithm

GOALS

- What is a quantum state?
- What does a quantum measurement look like?
- What are quantum mixed states?
- Difference btwn mixture and superposition?
- What is quantum entropy? (von Neumann)
- What is entanglement? (and relation to entropy)

RESEARCH IDEAS

- Entanglement is thought to underly much of the power of quantum computation. What is the role of entanglement in these representations?
    - Are there “quantum” processes that require no repr entanglement? or classical that do?
- We think about representations of physical processes. Some quantum representations are more efficient. Are these “more natural”?

QUANTUM STATES

$$|\psi\rangle$$

lives in a complex vector space
Hilbert space

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

quantum amplitudes
orthonormal basis

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

..just linear algebra

QUANTUM STATES

Addition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$|\phi\rangle = \gamma\,|0\rangle + \delta\,|1\rangle$$

$$|\psi\rangle + |\phi\rangle = (\alpha + \gamma)\,|0\rangle + (\beta + \delta)\,|1\rangle$$

Scalar Multiplication

$$\lambda\,|\psi\rangle = \lambda\alpha\,|0\rangle + \lambda\beta\,|1\rangle$$

Inner Product

$$\langle\psi|\phi\rangle = \alpha^*\gamma\,|0\rangle + \beta^*\delta\,|1\rangle$$

QUANTUM STATES

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\alpha = e^{i\gamma} r$$

$$\beta = e^{i\gamma}(b + ci)$$

$$|\,|\psi\rangle\,| = \sqrt{\langle\psi|\psi\rangle} = 1$$

$$|e^{i\gamma}|^2(r^2 + b^2 + c^2) = 1$$

$$r^2 + b^2 + c^2 = 1$$

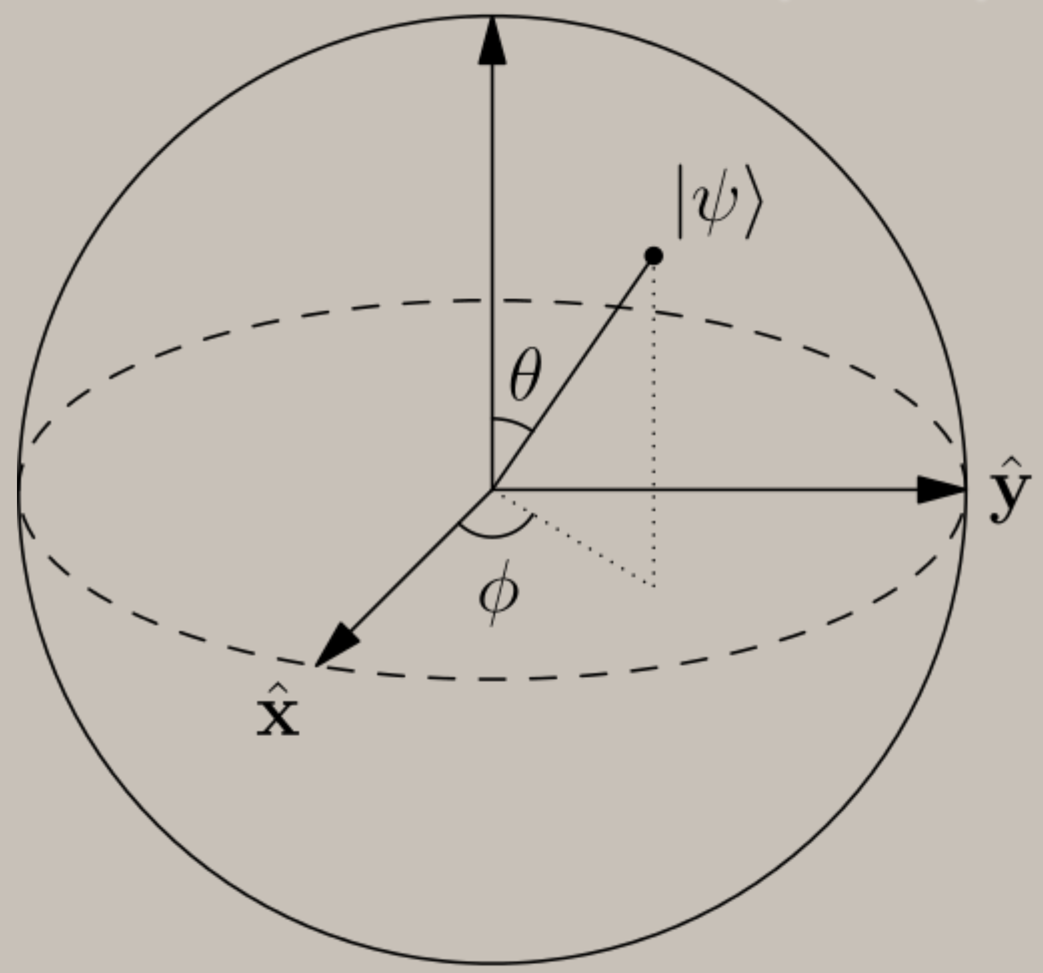
This is the surface of a sphere!

BLOCH SPHERE

$$|\psi\rangle = e^{i\gamma}(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle)$$

nonphysical global phase

$\hat{\mathbf{z}} = |0\rangle$ *"spin up"*



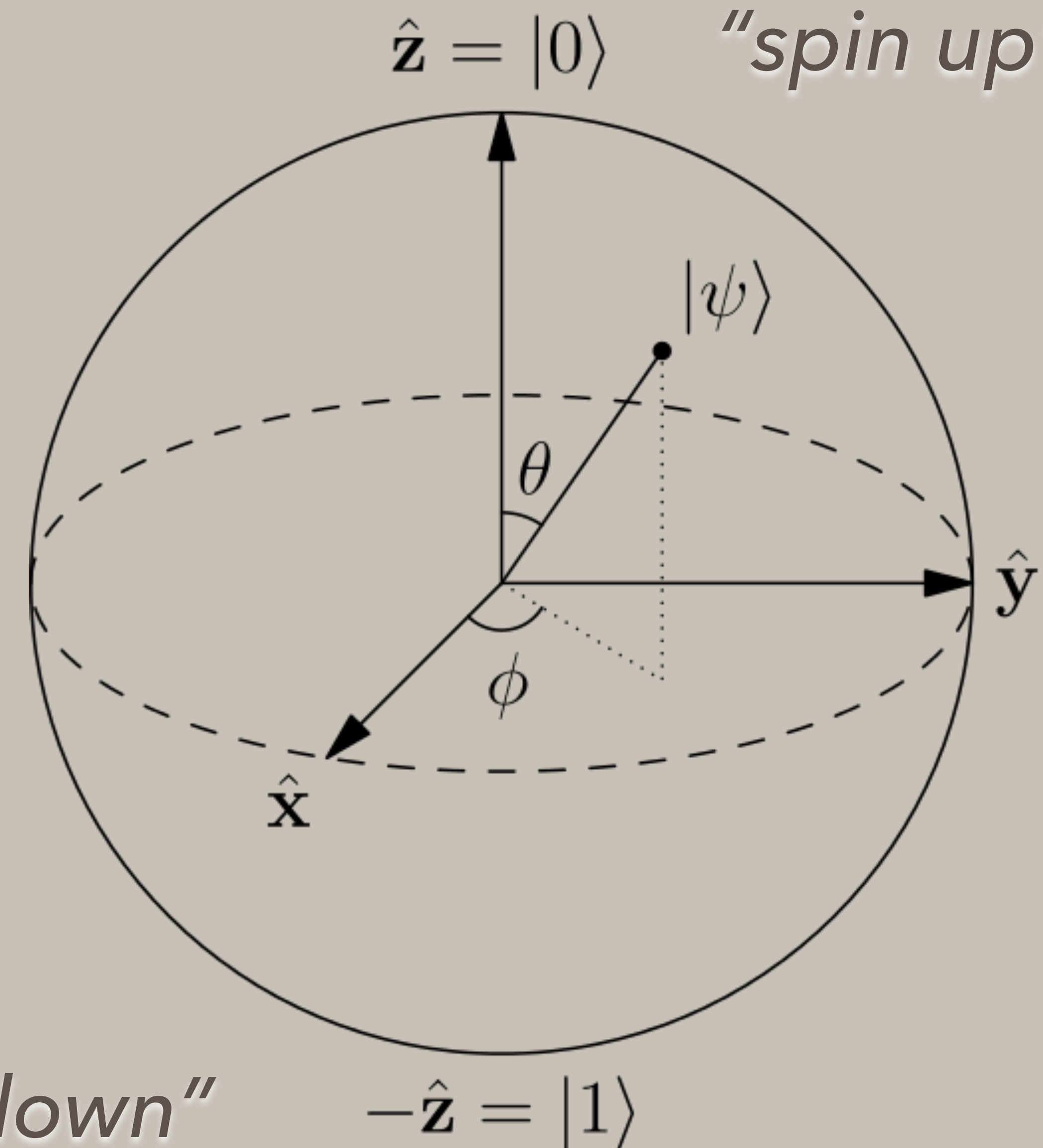
"spin down" $-\hat{\mathbf{z}} = |1\rangle$

BLOCH SPHERE

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

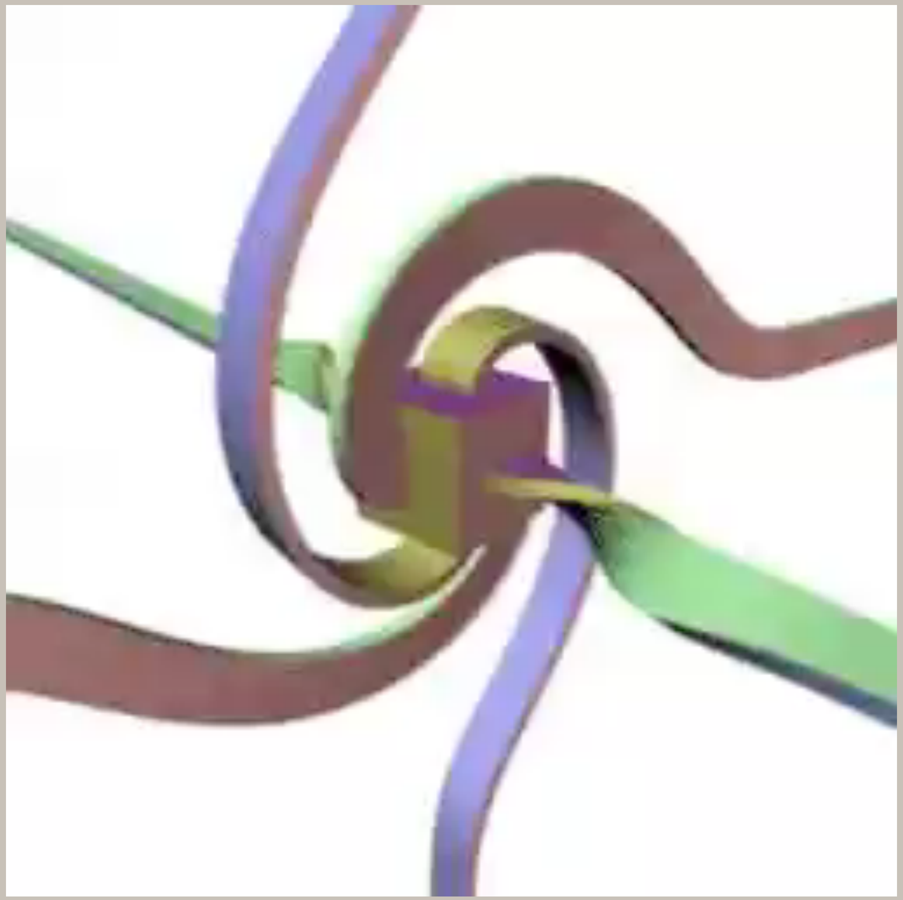
**projective Hilbert space*

"spin up"



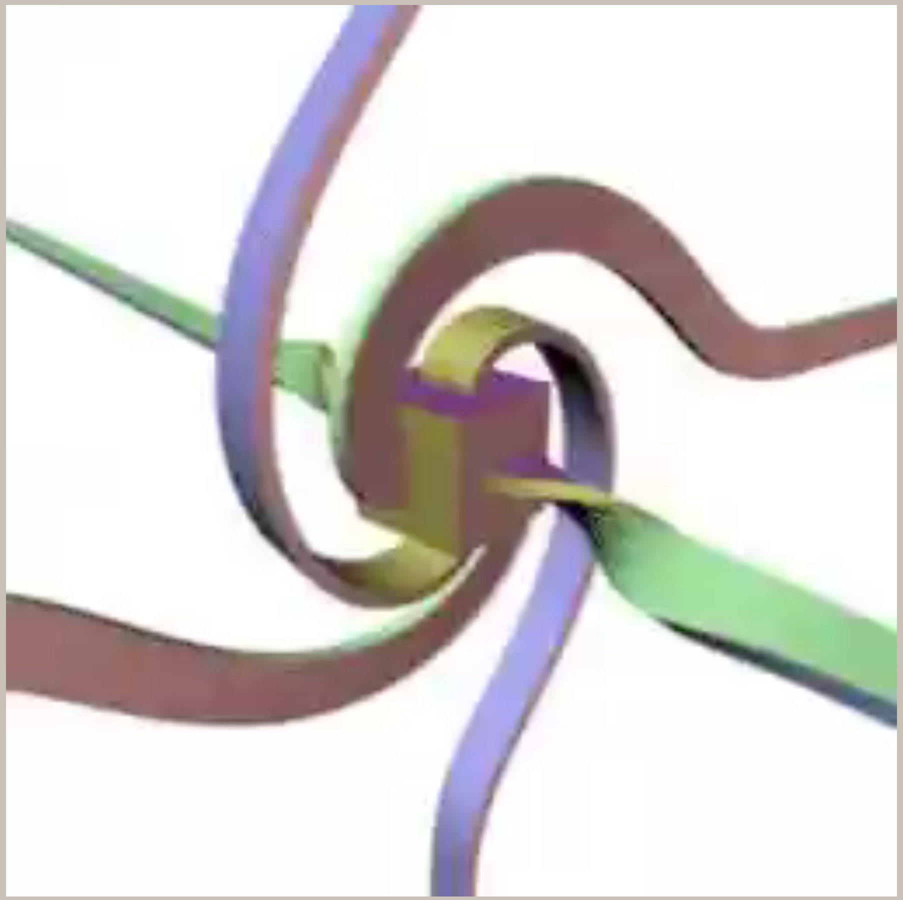
"spin down"

Dirac Belt Trick



https://en.wikipedia.org/wiki/Plate_trick

Dirac Belt Trick



[*https://en.wikipedia.org/wiki/Plate_trick*](https://en.wikipedia.org/wiki/Plate_trick)

EVOLUTION: CLOSED SYSTEM

Unitary evolution

$$|\psi'\rangle = U\,|\psi\rangle$$

overlaps maintained

$$\langle\psi|U^\dagger U|\phi\rangle = \langle\psi|I|\phi\rangle = \langle\psi|\phi\rangle$$

columns define ON-basis

$$U^\dagger U = 1$$

maps one ONB to another ONB

$$U = \sum_i |w_i\rangle\langle v_i|$$

…it's just a big rotation

EVOLUTION: CLOSED SYSTEM

Schrodinger

$$|\psi'\rangle = e^{-iHt/\hbar}|\psi\rangle$$

EVOLUTION: CLOSED SYSTEM

Schrodinger

$$|\psi'\rangle = e^{-iHt/\hbar}\,|\psi\rangle$$

$$U(t)\,|E_1\rangle = e^{-iE_1 t/\hbar}\,|E_1\rangle$$

$$U(t)\frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle) = e^{-iE_1 t/\hbar}\,|E_1\rangle + e^{-iE_2 t/\hbar}\,|E_2\rangle$$

$$= e^{-iE_1 t/\hbar}(|E_1\rangle + e^{-i\underline{(E_2 - E_1)}t/\hbar}\,|E_2\rangle)$$

MEASUREMENT: PROJECTIVE

Observables modeled by Hermitian operators

$$M = M^\dagger$$

Projectors are complete

$$\sum_m P_m = 1$$

Spectral decomposition

$$M = \sum_m m P_m$$

m - outcomes
P - projectors

How to compute probabilities

$$Pr(m) = \langle \psi | P_m | \psi \rangle$$

Born Rule

MEASUREMENT: PROJECTIVE

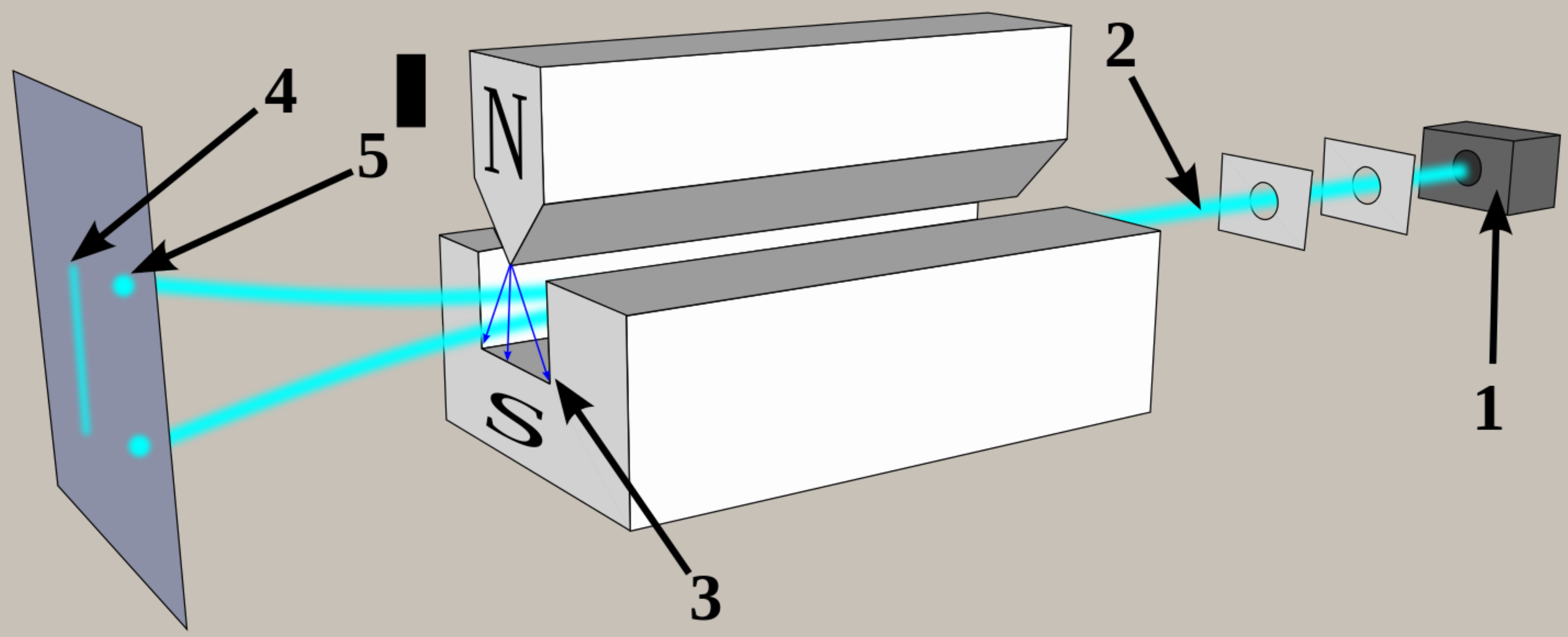
Post-measurement state

$$\begin{aligned} |\psi'\rangle &= \frac{P_m |\psi\rangle}{\sqrt{Pr(m)}} \\ &= \frac{P_m |\psi\rangle}{\sqrt{\langle\psi|P_m|\psi\rangle}} \\ &= \frac{P_m |\psi\rangle}{\sqrt{\langle\psi|P_m^\dagger P_m|\psi\rangle}} \\ &= \frac{P_m |\psi\rangle}{|P_m |\psi\rangle\,|} \end{aligned}$$

project and normalize

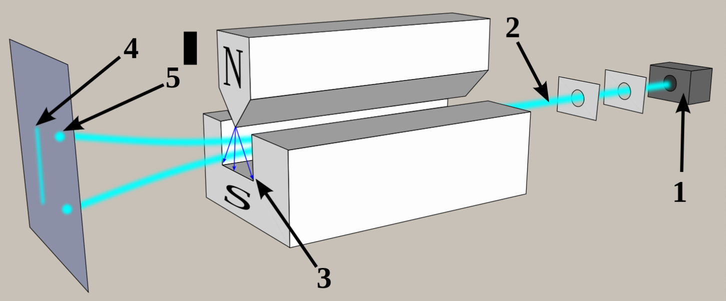
MEASUREMENT: PROJECTIVE

$$M = +1\,|0\rangle\langle 0| - 1\,|1\rangle\langle 1|$$



wikipedia-Stern-Gerlach

MEASUREMENT: PROJECTIVE



$$M = +1\,|0\rangle\langle 0| - 1\,|1\rangle\langle 1|$$

$$|\psi\rangle = |0\rangle$$

outcome +1

$$|\psi'\rangle \propto |0\rangle\langle 0|\,|0\rangle = |0\rangle$$

$$Pr(+1) = \langle 0|0\rangle = 1$$

$$|\psi'\rangle = |0\rangle/\sqrt{1} = |0\rangle$$

outcome -1

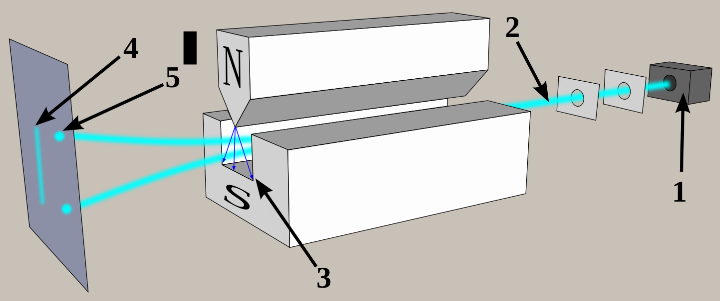
$$|\psi'\rangle \propto |1\rangle\langle 1|\,|0\rangle = 0$$

$$Pr(-1) = 0^2 = 0$$

$$|\psi'\rangle = 0/\sqrt{0} = \text{undef}$$

MEASUREMENT: PROJECTIVE

$$M = +1\,|0\rangle\langle 0| - 1\,|1\rangle\langle 1|$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

outcome +1

$$|\psi'\rangle \propto |0\rangle\langle 0|\,\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\,|0\rangle$$

$$Pr(+1) = |\frac{1}{\sqrt{2}}\,|0\rangle\,|^2 = \frac{1}{2}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\,|0\rangle\,/\sqrt{\frac{1}{2}} = |0\rangle$$

outcome -1

$$|\psi'\rangle \propto |1\rangle\langle 1|\,\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\,|1\rangle$$

$$Pr(-1) = |\frac{1}{\sqrt{2}}\,|1\rangle\,|^2 = \frac{1}{2}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\,|1\rangle\,/\sqrt{\frac{1}{2}} = |1\rangle$$

EXPECTATION VALUE

$$\begin{aligned} E(M) &= \sum_m m Pr(m) \\ &= \sum_m m \langle \psi | P_m | \psi \rangle \\ &= \langle \psi | \sum_m m P_m | \psi \rangle \\ &= \langle \psi | M | \psi \rangle \end{aligned}$$

This is the first time that the eigenvalues of M matter

MEASUREMENT: GENERAL

set of measurement operators $\{M_m\}$

probabilities

$$Pr(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

post-measurement state

$$|\psi'\rangle = \frac{M_m\,|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

MEASUREMENT: POVM

(positive operator-valued measure)

set of measurement operators $\{M_m\}$

(operator) positive elements

$$E_m = M_m^\dagger M_m$$

$$\sum_i E_m = 1$$

probabilities

$$Pr(m) = \langle\psi|E_m|\psi\rangle$$

DISTINGUISHABILITY

Given two non-orthogonal states can we distinguish them?

$$\langle \psi_1 | E_1 | \psi 1 \rangle = 1 \quad \langle \psi_2 | E_2 | \psi 2 \rangle = 1$$

By completeness and first relation,

$$\langle \psi_1 | E_2 | \psi 1 \rangle = 0$$

by positivity $\sqrt{E_2}\, |\psi_1\rangle = 0$

$$|\psi_2\rangle = \alpha\, |\psi_1\rangle + \beta\, |\phi\rangle$$

$$\langle \psi_2 | E_2 | \psi_2 \rangle = |\beta|^2 \langle \phi | E_2 | \phi \rangle \leq |\beta|^2 < 1$$

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$$|\psi_2\rangle = \alpha \, |\psi_1\rangle + \beta \, |\phi\rangle$$

contradiction

$$\langle \psi_2 | E_2 | \psi_2 \rangle = |\beta|^2 \langle \phi | E_2 | \phi \rangle \leq |\beta|^2 < 1$$

DISTINGUISHABLE?

$$|\psi_1\rangle = |0\rangle$$

$$|\psi_2\rangle = \frac{1}{2}(|0\rangle + |1\rangle)$$

$$E_1 = \frac{\sqrt{2}}{1+\sqrt{2}}\,|1\rangle\langle 1|$$

$$E_2 = \frac{\sqrt{2}}{1+\sqrt{2}}\,\frac{(|0\rangle - |1\rangle)(\langle 0| - \langle 1|)}{2}$$

$$E_3 = 1 - E_1 - E_2$$

DISTINGUISHABLE?

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$$E_3 = 1 - E_1 - E_2$$

perfect distinguishability!

…but not every time

MIXED STATES

Physical situation: input to SG is random choice: up, down.

How can we model this state with a quantum state?

SG output is random: +1, -1.

$$Pr(+1) = 1/2$$
$$Pr(-1) = 1/2$$

We know how to model this:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

MIXED STATES

What if we measured the random input with a different SG?

$$M = +1\frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) - 1\frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|)$$

SG output is random: +1, -1.

$$Pr(+1) = 1/2$$

$$Pr(-1) = 1/2$$

Apply this measurement to our model?..

$$Pr(+1) = 0$$

$$Pr(-1) = 1$$

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$$Pr(+1) = 1/2$$

$$Pr(-1) = 1/2$$

Apply this measurement to our model?..

$$Pr(+1) = 0$$

$$Pr(-1) = 1$$

our notion of state is insufficient

MIXED STATES

Density operator is a trace 1, positive operator.

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|$$

$$\rho = \frac{1}{2}\,|0\rangle\langle 0| + \frac{1}{2}\,|1\rangle\langle 1|$$

Our example:

probabilistic mixture of up and down

MIXED STATES: NON-UNIQUE

Our example:

probabilistic mixture of up and down

$$
\begin{aligned}
\rho &= \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\
&= \frac{1}{2}\,|0\rangle\langle 0| + \frac{1}{2}\,|1\rangle\langle 1|\\
&= \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + |1\rangle) + \frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - |1\rangle)
\end{aligned}
$$

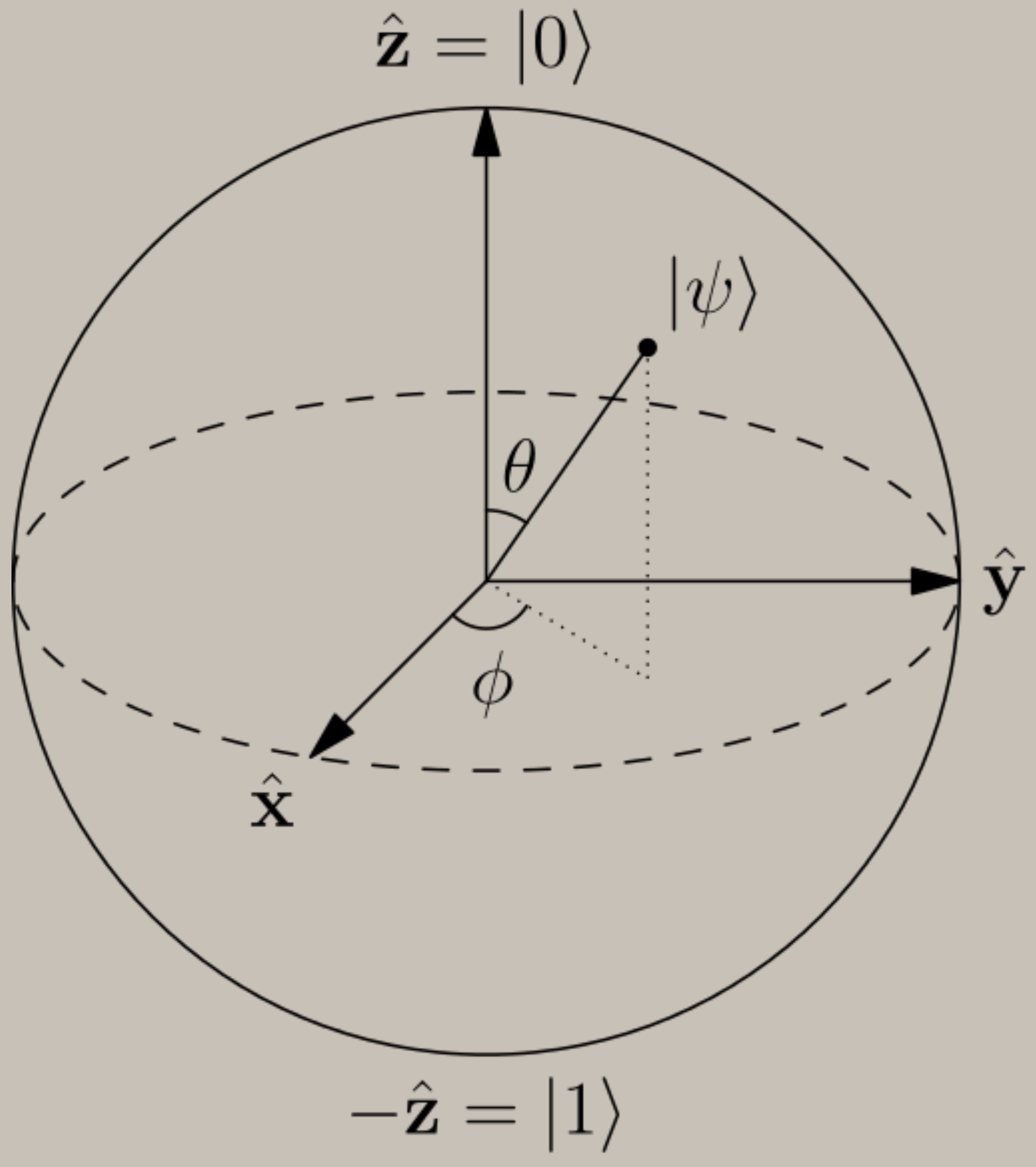
MIXED STATES: BLOCH BALL

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\vec{r} = [x, y, z] \quad \vec{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$$

$$\boxed{\rho = \frac{1}{2}(1 + \vec{r} \cdot \vec{\sigma})}$$

$$\rho_{\text{max mix}} = \frac{1}{N}1 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



MIXED STATES: NON-UNIQUE

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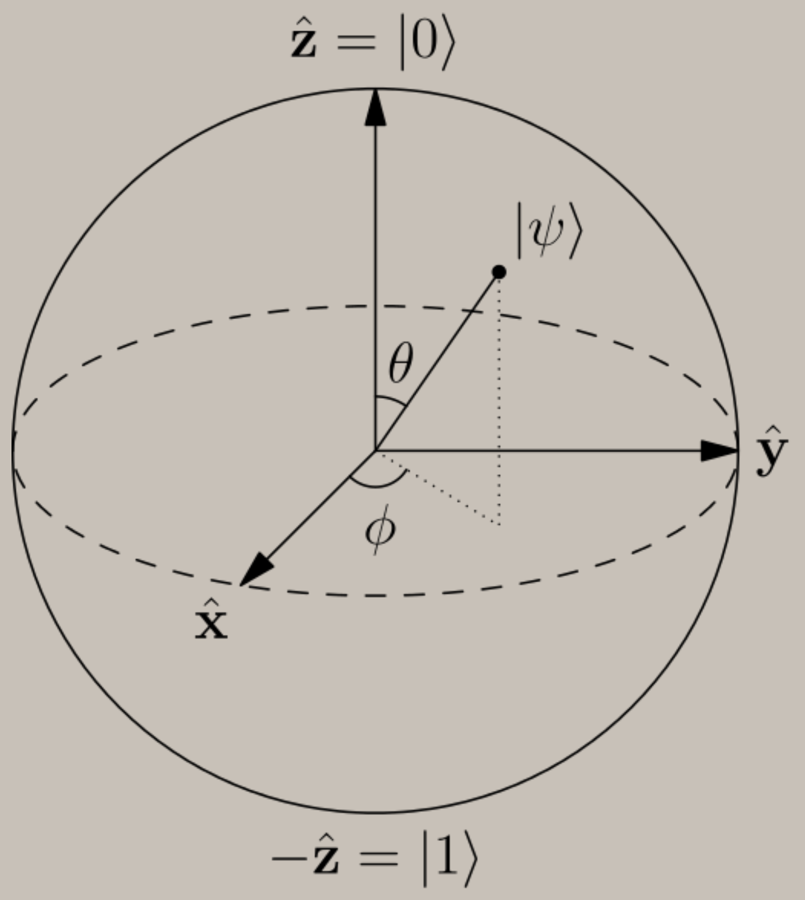

$$\rho = \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$= \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + |1\rangle) + \frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - |1\rangle)$$

MIXED STATES: EVOLUTION

$$\rho = \sum_i p_i \left|\psi_i\right\rangle\!\left\langle\psi_i\right| \xrightarrow{U} \sum_i p_i U \left|\psi_i\right\rangle\!\left\langle\psi_i\right| U^\dagger = U\rho U^\dagger$$

MIXED STATES: MEASUREMENT

set of measurement operators $\{M_m\}$

probabilities

$$Pr(m) = \mathrm{tr}\left(M_m^\dagger M_m \rho\right)$$

post-measurement state

$$\rho' = \frac{M_m \rho M_m^\dagger}{\mathrm{tr}\left(M_m^\dagger M_m \rho\right)}$$

MIXED STATES: ENTROPY

von Neumann entropy

$$
\begin{aligned}
S(\rho) &= \mathrm{tr}(\rho \log \rho) \\
&= \mathrm{tr}\big(UDU^\dagger \log UDU^\dagger\big) \\
&= \mathrm{tr}\big(UDU^\dagger U \log DU^\dagger\big) \\
&= \mathrm{tr}\big(U^\dagger UDU^\dagger U \log D\big) \\
&= \mathrm{tr}(D \log D) \\
&= \sum_i \lambda_i \log \lambda_i
\end{aligned}
$$

Minimal randomness in complete projective measurement

CODING

Shannon's noiseless coding theorem:

$\{X\}$ is an i.i.d. information source with (Shannon) entropy rate $H(X)$.

For $R > H(X)$, there exists a reliable compression scheme.

For $R < H(X)$ there is no reliable scheme.

Schumacher's noiseless coding theorem:

$\{\rho\}$ is an i.i.d. information source with (von Neumann) entropy rate $S(\rho)$.

For $R > S(\rho)$, there exists a reliable compression scheme.

For $R < S(\rho)$ there is no reliable scheme.

Coding theorem gives quantum entropy physical meaning.

NEXT TIME…

- Composite systems
- Entanglement
- Quantum analog of C_mu
- Operational meaning in terms of channel
- Candidate quantum encoding construction
- Relation to cryptic order
- Computational methods / illustration

COMPOSITE SYSTEMS

$$\left|\psi^{AB}\right\rangle = \left|\psi^{A}\right\rangle \otimes \left|\phi^{B}\right\rangle$$

one electron spin

another electron spin

$$\left|\psi^{A}\right\rangle \in \mathcal{H}^{A} \quad \left|\phi^{B}\right\rangle \in \mathcal{H}^{B}$$

COMPOSITE SYSTEMS

$$\left|\psi^{AB}\right\rangle = \left|\psi^{A}\right\rangle \otimes \left|\phi^{B}\right\rangle$$

electron spin

same electron position

$$\left|\psi^{A}\right\rangle \in \mathcal{H}^{A} \quad \left|\phi^{B}\right\rangle \in \mathcal{H}^{B}$$

COMPOSITE SYSTEMS

$$\{|i^A\rangle\} \quad \{|j^B\rangle\} \to \{|i^A\rangle \otimes |j^B\rangle\}$$

bases combine

$$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$$

example: two spins

$$|\psi^{AB}\rangle = |0^A\rangle \otimes |1^B\rangle = |01\rangle$$

shorthand

COMPOSITE SYSTEMS: SUPERPOSITIONS

A *B*

$$\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) =$$

$$\frac{1}{2}(|00\rangle - |11\rangle) =$$

COMPOSITE SYSTEMS: SUPERPOSITIONS

$$\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) = \underbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}^{A} \otimes \underbrace{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}^{B}$$

$$\frac{1}{2}(|00\rangle - |11\rangle) =$$

COMPOSITE SYSTEMS: SUPERPOSITIONS

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$$\begin{aligned}\frac{1}{2}(|00\rangle - |11\rangle) &= (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes (\gamma\,|0\rangle + \delta\,|1\rangle) \\ &= \alpha\gamma\,|00\rangle + \alpha\delta\,|01\rangle + \beta\gamma\,|10\rangle + \beta\delta\,|11\rangle\end{aligned}$$

COMPOSITE SYSTEMS: SUPERPOSITIONS

$$\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) = \underbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}^{A} \otimes \underbrace{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}^{B}$$

$$\frac{1}{2}(|00\rangle - |11\rangle) = (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes (\gamma\,|0\rangle + \delta\,|1\rangle)$$
$$= \alpha\gamma\,|00\rangle + \alpha\delta\,|01\rangle + \beta\gamma\,|10\rangle + \beta\delta\,|11\rangle$$

$$\alpha\gamma = +1 \quad \alpha\delta = 0$$
$$\beta\delta = -1 \quad \beta\gamma = 0$$

no factorization

Entangled

COMPOSITE SYSTEMS

simple : $\rho_1 = |00\rangle \otimes \langle 00|$

superposition : $\rho_2 = \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle) \otimes \frac{1}{\sqrt{2}}(\langle 00| - \langle 01|)$

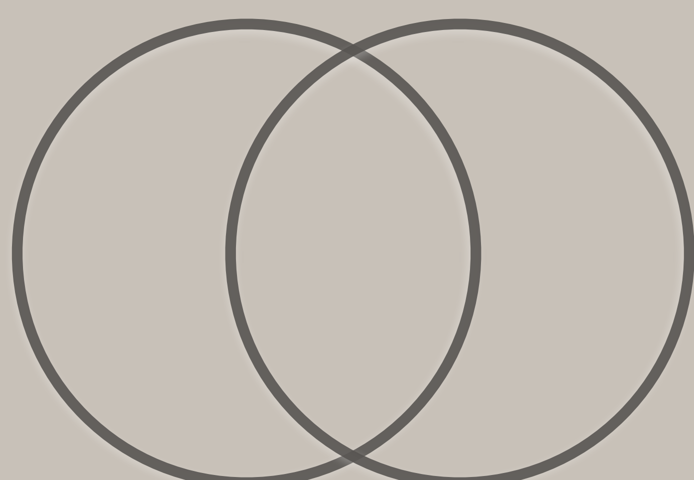
entangled : $\rho_3 = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \otimes \frac{1}{\sqrt{2}}(\langle 00| - \langle 11|)$

$$S(\rho_1) = \quad ?$$
$$S(\rho_2) = \quad ?$$
$$S(\rho_3) = \quad ?$$

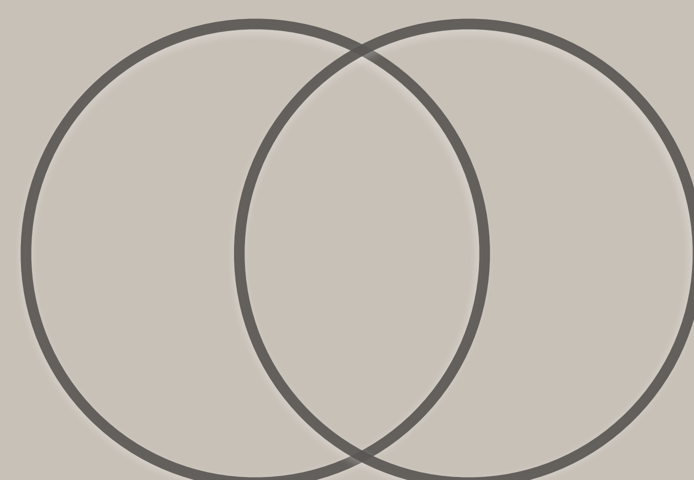
What is the von Neumann entropy of these three states?

COMPOSITE SYSTEMS

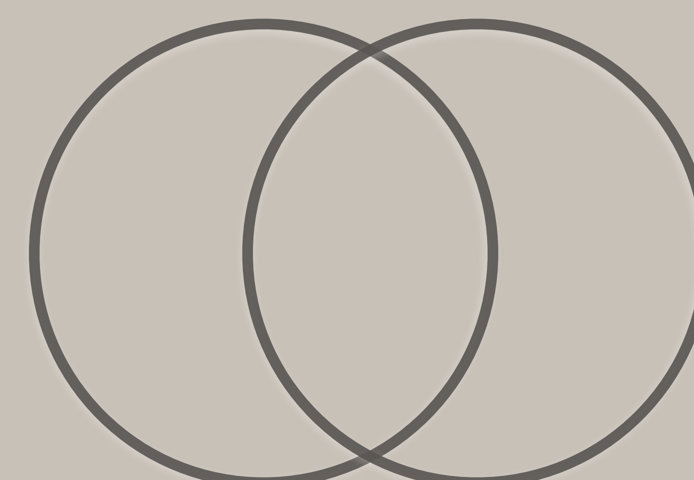
vNE involves probing global measurements.
What about local?



$$|00\rangle$$



$$\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle)$$



$$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

COMPOSITE SYSTEMS

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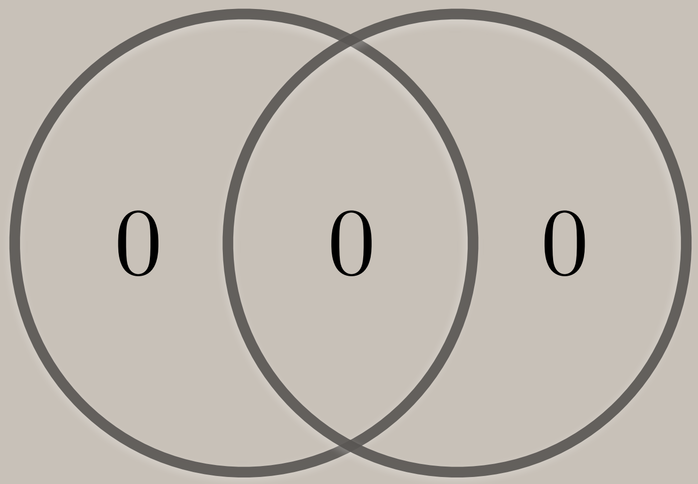
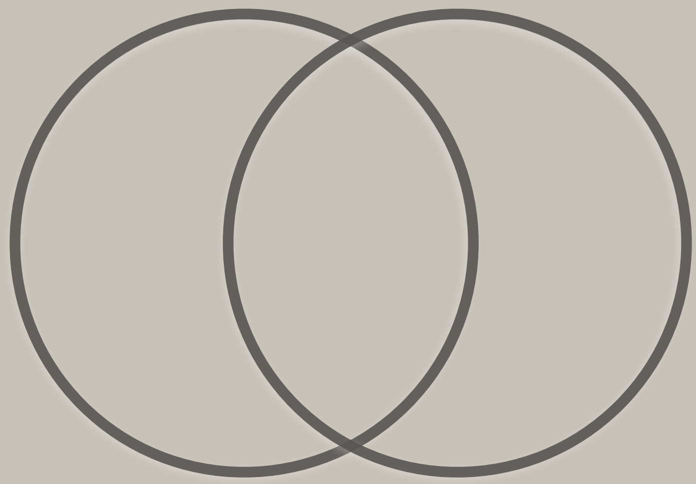
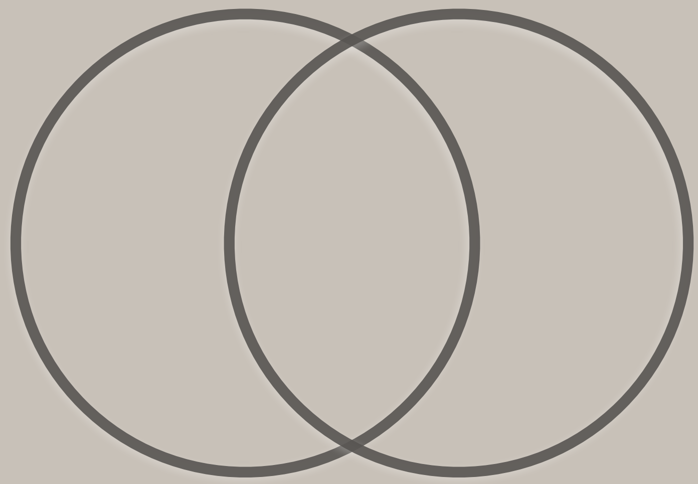
What about local?



$|00\rangle$



$\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle)$



$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

COMPOSITE SYSTEMS

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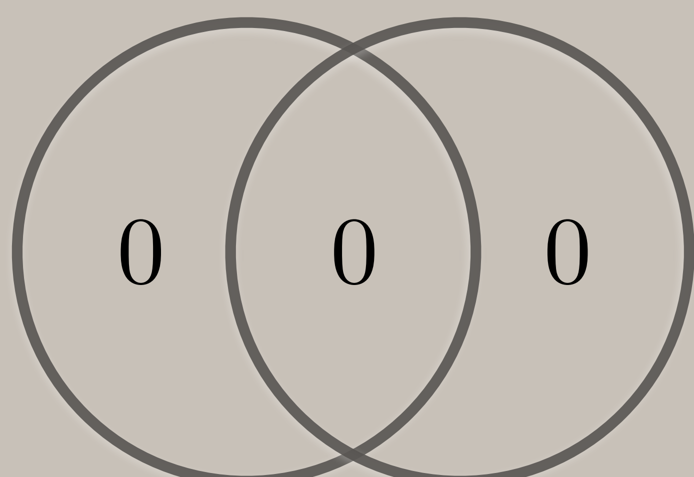
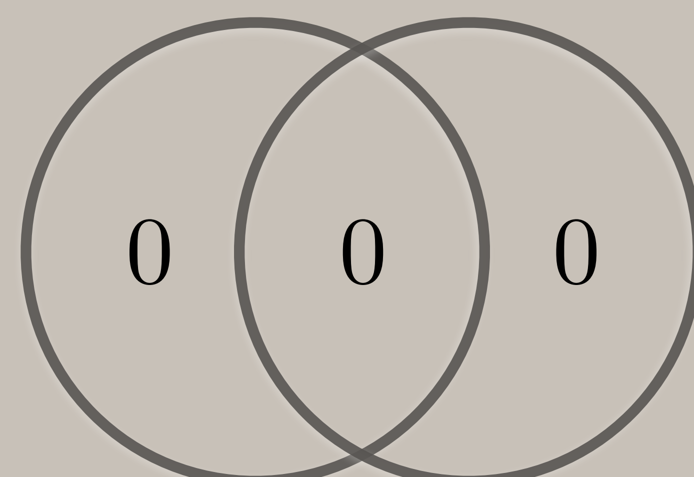
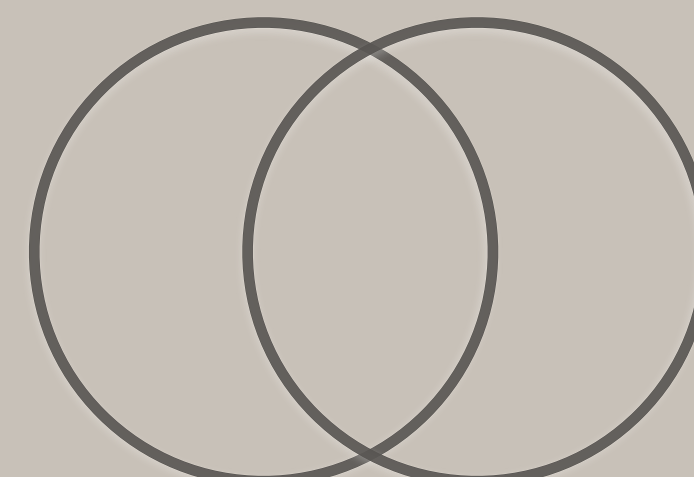


$$|00\rangle$$



$$\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle)$$

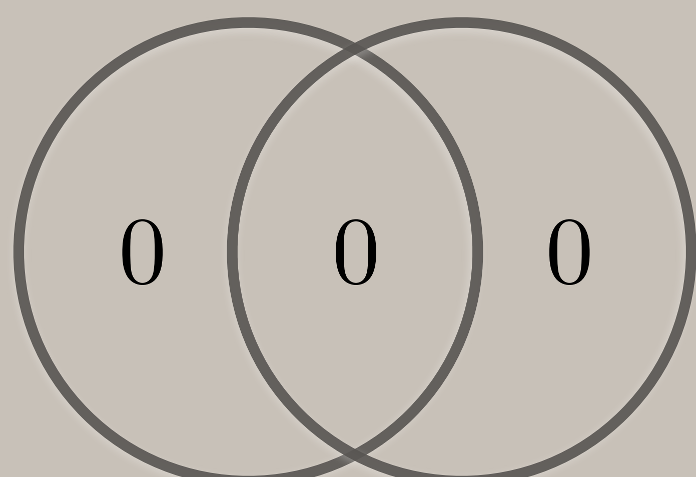


$$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

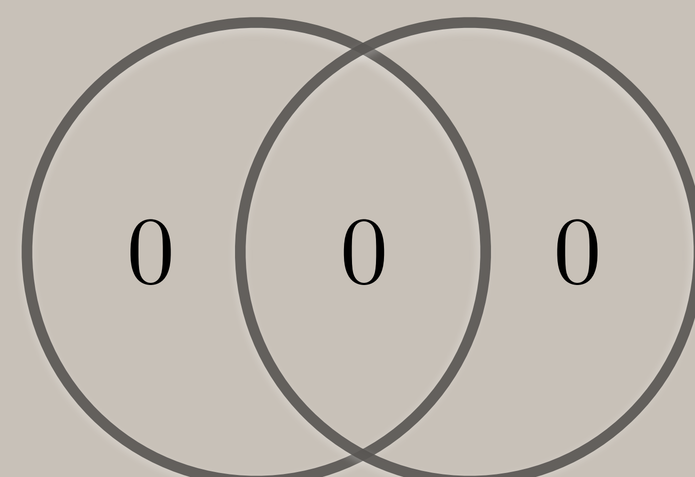
COMPOSITE SYSTEMS

vNE involves probing global measurements.
What about local?

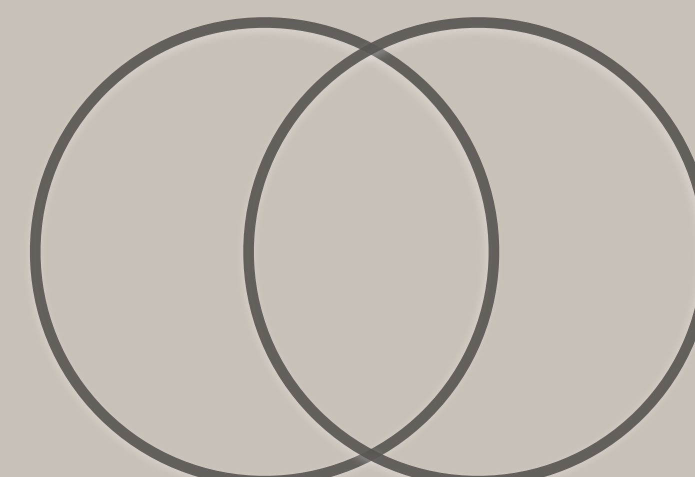
???



$$|00\rangle$$



$$\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle)$$



$$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

PARTIAL TRACE

"ignore one quantum subsystem"

$$\rho^{AB} \stackrel{?}{\rightarrow} \rho^{A}$$

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$$\rho^{AB} \xrightarrow{?} \rho^{A}$$

Given meas of A,

$$\{M^A\}$$

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$$\rho^{AB} \xrightarrow{?} \rho^{A}$$

Given meas of A, *corresponding meas of joint:*

$$\{M^A\} \longrightarrow M_m^{AB} = M_m^A \otimes 1^B$$

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We know this:

$$Pr(m) = \mathrm{tr}\big(M_m^A \otimes 1^B \rho^{AB}\big)$$

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"ignore one quantum subsystem"

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Given meas of A, *corresponding meas of joint:*

$$\{M^A\} \longrightarrow M_m^{AB} = M_m^A \otimes 1^B$$

We know this:

$$\rho^A \equiv \mathrm{tr}_B(\rho^{AB}) \longleftarrow Pr(m) = \mathrm{tr}\big(M_m^A \otimes 1^B \rho^{AB}\big)$$

PARTIAL TRACE

$$\begin{aligned}\rho^A &= \mathrm{tr}_B(\rho^{AB}) \\ &= \sum_i \left\langle i^B \right| \rho^{AB} \left| i^B \right\rangle\end{aligned}$$

$$\begin{aligned}\rho^B &= \mathrm{tr}_A(\rho^{AB}) \\ &= \sum_i \left\langle i^A \right| \rho^{AB} \left| i^A \right\rangle\end{aligned}$$

PARTIAL TRACE

$$\rho^A = \mathrm{tr}_B(\rho^{AB}) \\ = \sum_i \langle i^B | \rho^{AB} | i^B \rangle$$

$$\rho^B = \mathrm{tr}_A(\rho^{AB}) \\ = \sum_i \langle i^A | \rho^{AB} | i^A \rangle$$

$$\rho^{AB} = \frac{1}{2}(|00\rangle\langle 00| - |00\rangle\langle 11| - |11\rangle\langle 00| + |11\rangle\langle 11|)$$

PARTIAL TRACE

$$\rho^A = \mathrm{tr}_B(\rho^{AB}) = \sum_i \langle i^B| \rho^{AB} |i^B\rangle$$

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$$\rho^A = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$\rho^B = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$S(\rho^{AB}) = a + b + c = 0$$

$$S(\rho^A) = a + b = 0$$

$$S(\rho^B) = b + c = 0$$

PARTIAL TRACE

$$\rho^A = \mathrm{tr}_B(\rho^{AB}) \qquad\qquad \rho^B = \mathrm{tr}_A(\rho^{AB})$$

$$= \sum_i \langle i^B | \, \rho^{AB} \, | i^B \rangle \qquad\qquad = \sum_i \langle i^A | \, \rho^{AB} \, | i^A \rangle$$

$$\rho^{AB} = \frac{1}{2}(|00\rangle\langle 00| - |00\rangle\langle 11| - |11\rangle\langle 00| + |11\rangle\langle 11|)$$

$$\rho^A = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

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PARTIAL TRACE

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$$\rho^{AB} = \frac{1}{2}(|00\rangle\langle 00| - |00\rangle\langle 11| - |11\rangle\langle 00| + |11\rangle\langle 11|)$$

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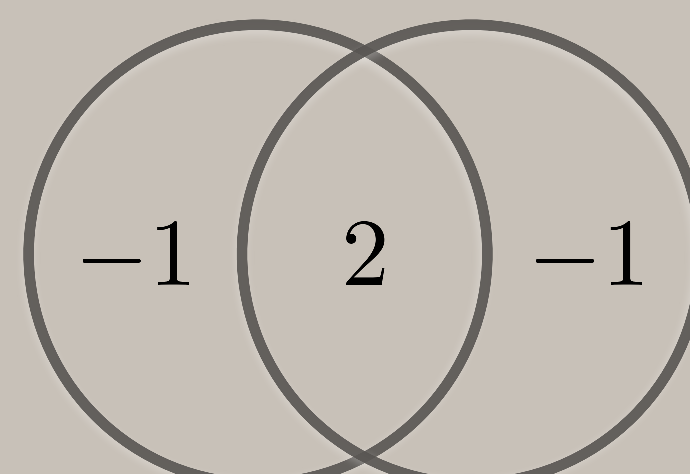

TELEPORTATION

Alice has some qubit state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

she wishes to send to Bob.
How much information must be transmitted?

What if they share an entangled pair?
(and a codebook for how to use it)

Entanglement is a resource

ENTANGLEMENT MEASURES

Entanglement entropy
Entanglement of formation
Entanglement of distillation

ENTANGLEMENT: EXTENSION TO MIXED STATES

state EoF def

convex roof

difficult optimization problem