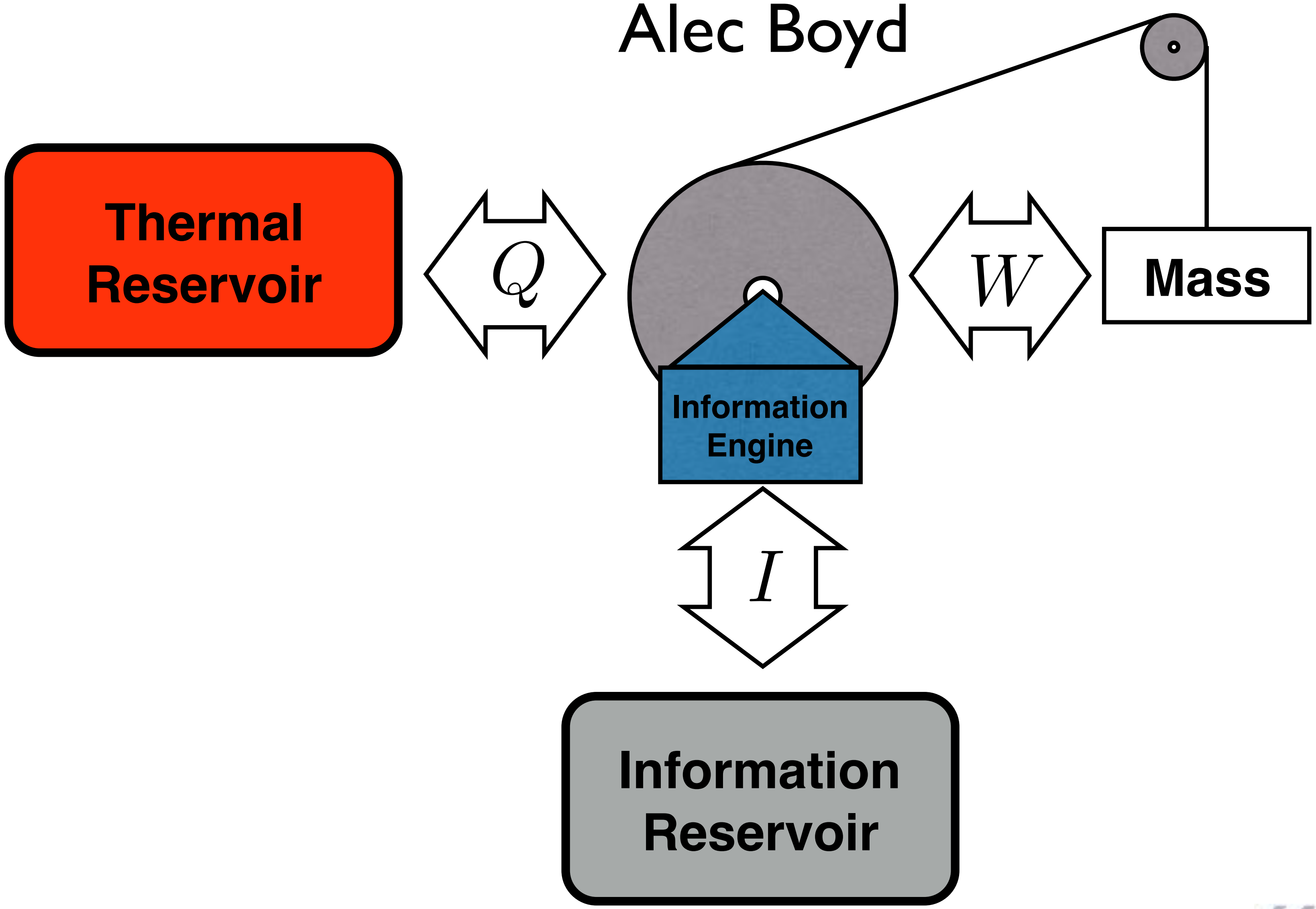
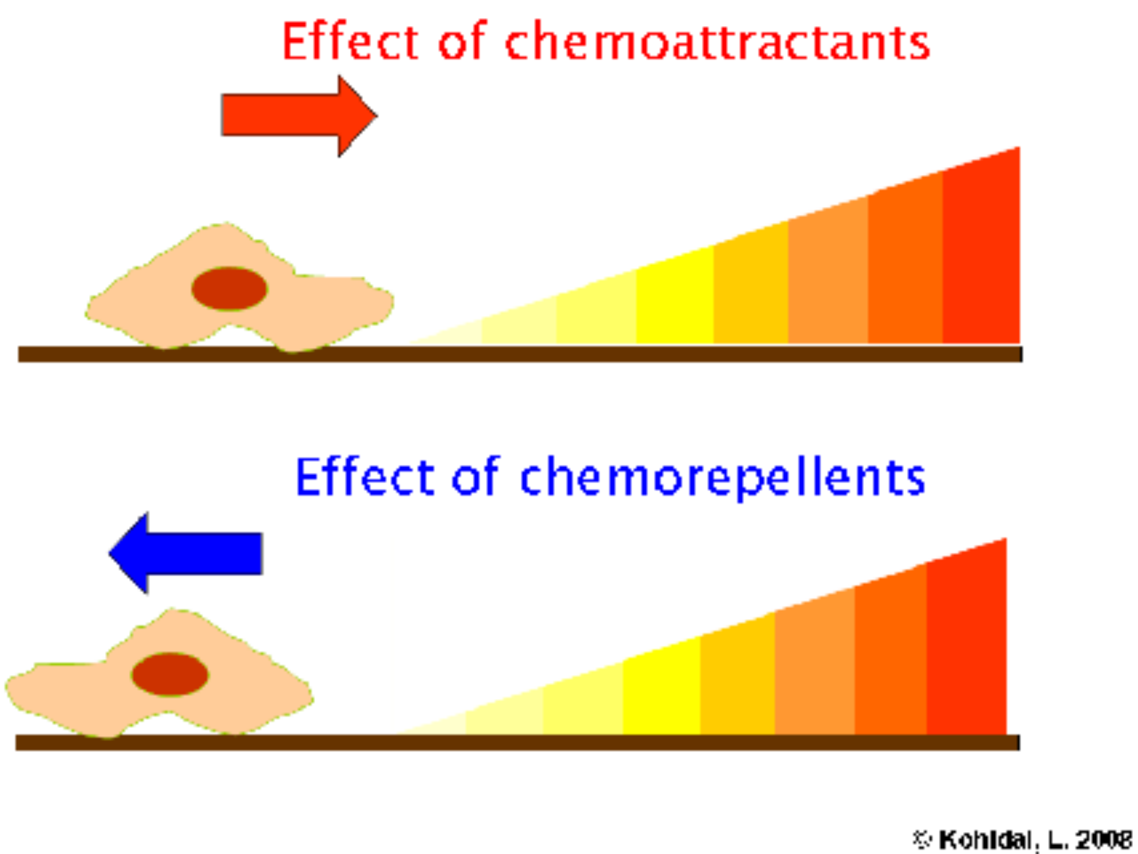
Thermodynamics of Information
Part II

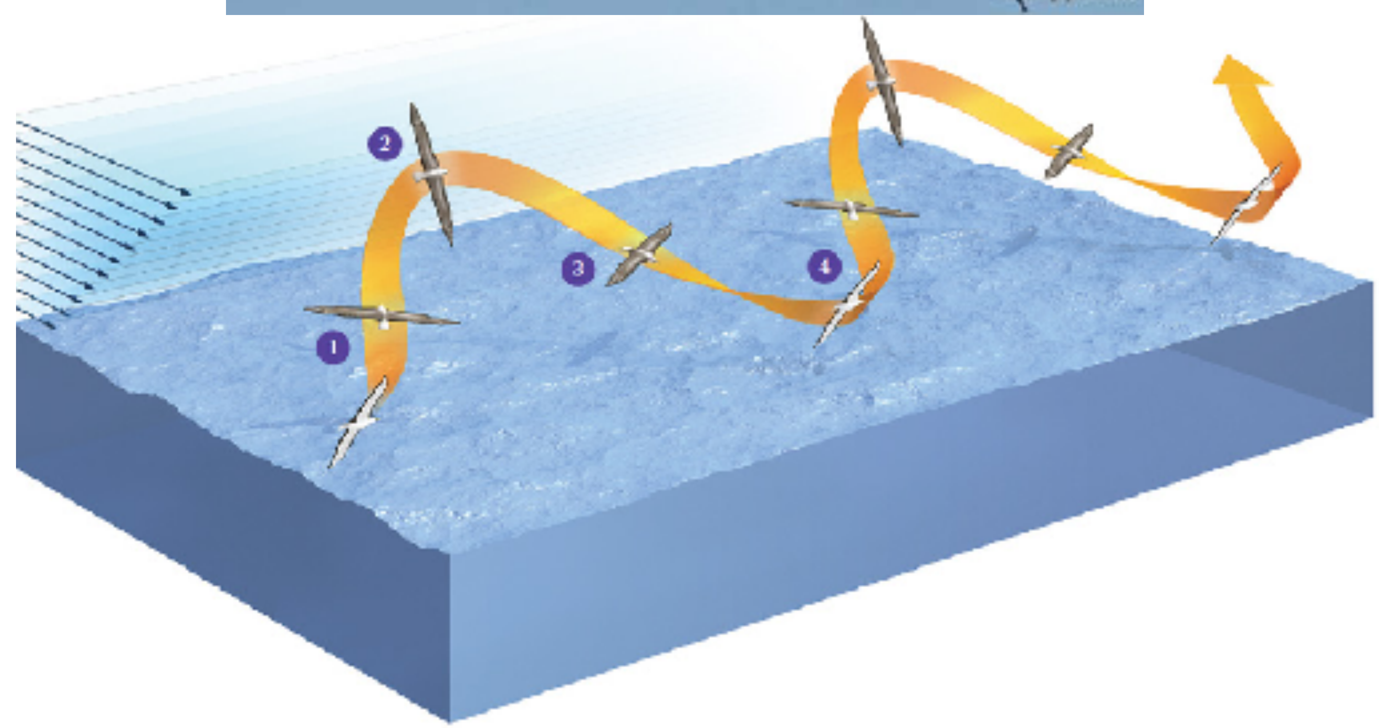
Alec Boyd



Benefit of Information Processing

Information processing allows systems to leverage ordered environments





http://spectrum.ieee.org/aerospace/robotic-exploration/the-nearly-effortless-flight-of-the-albatross

Benefit of Information Processing

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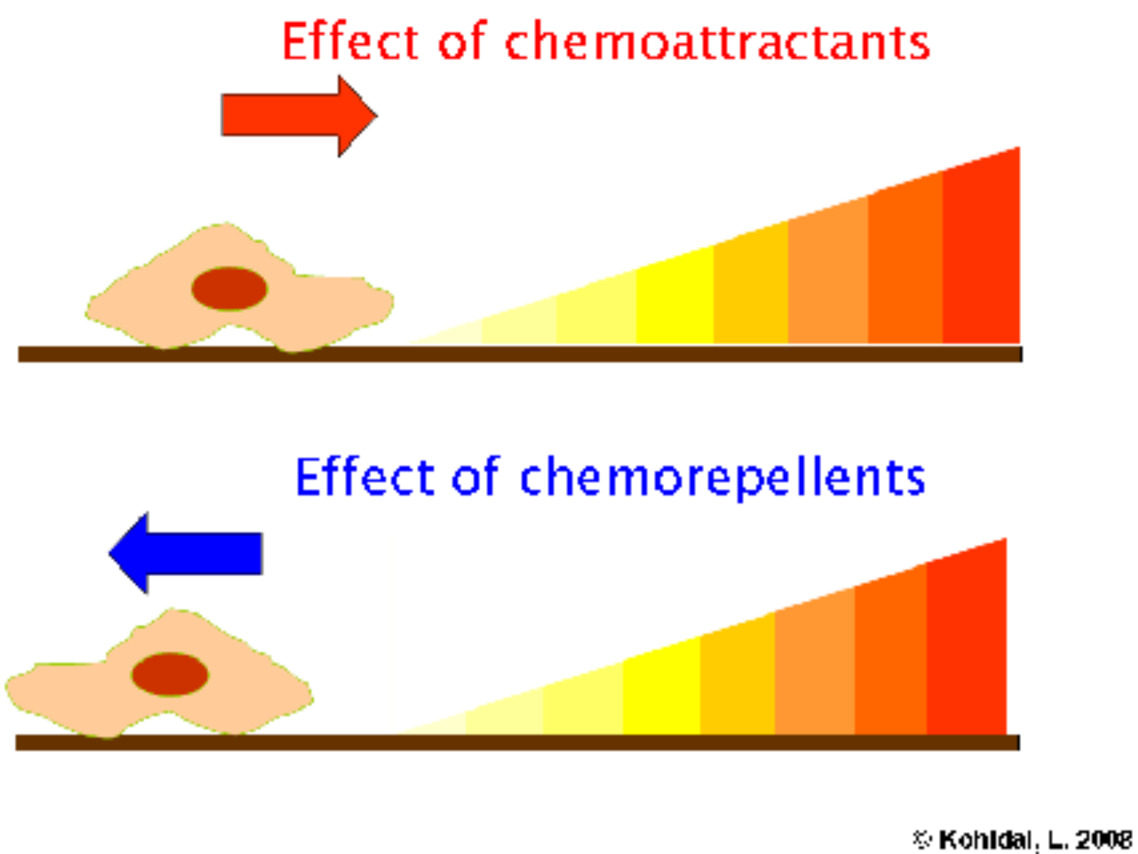
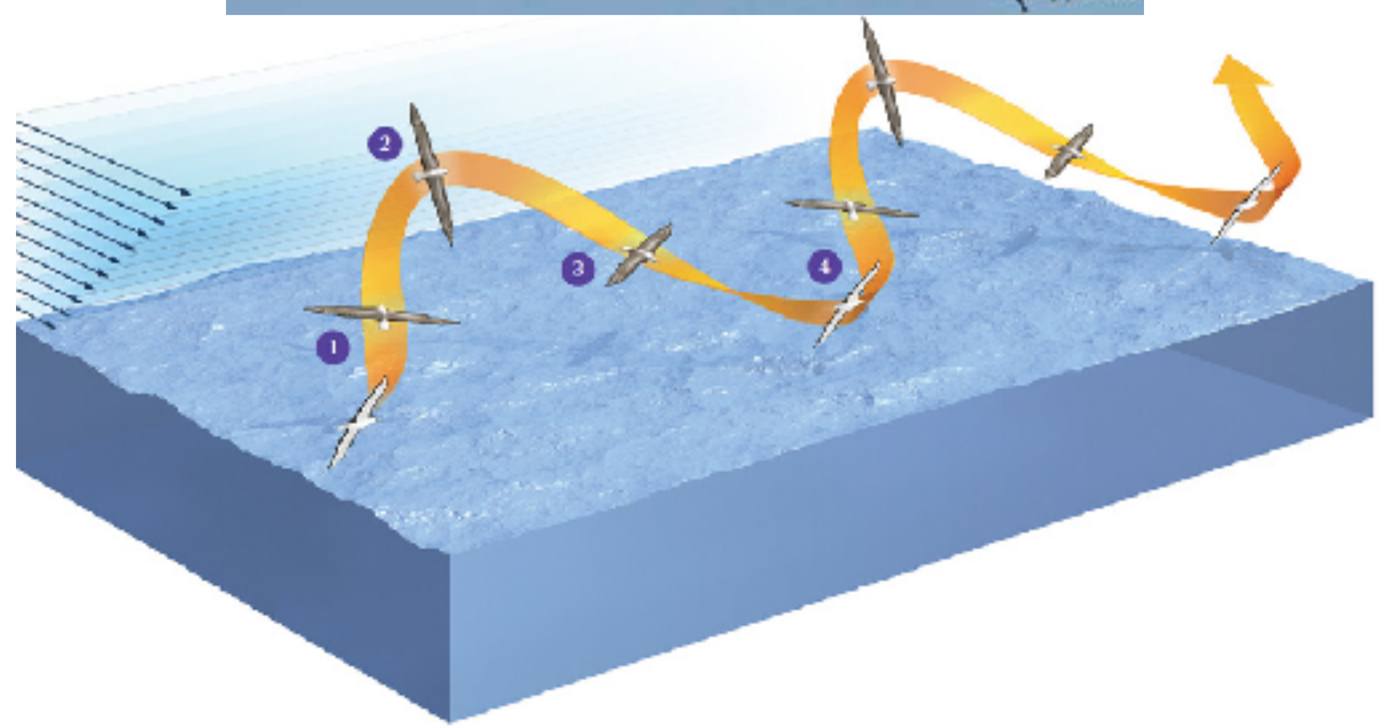
Simple



Complex



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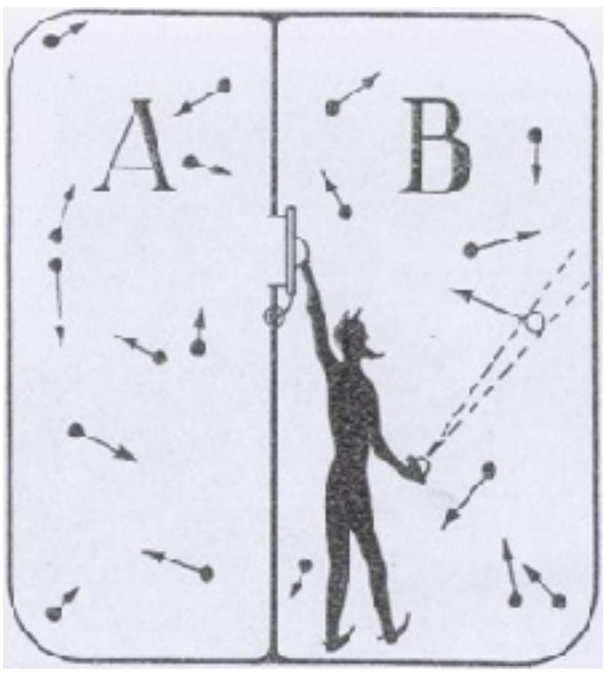
Information Bearing Degrees of Freedom

Maxwell's Demon



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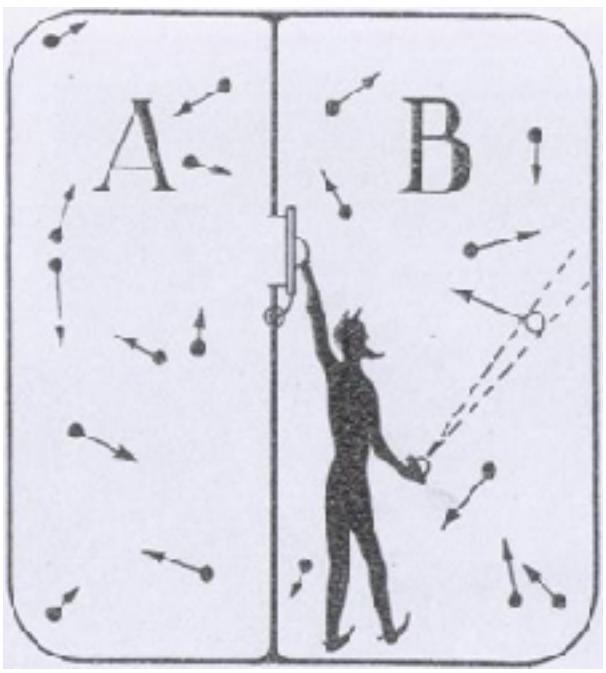

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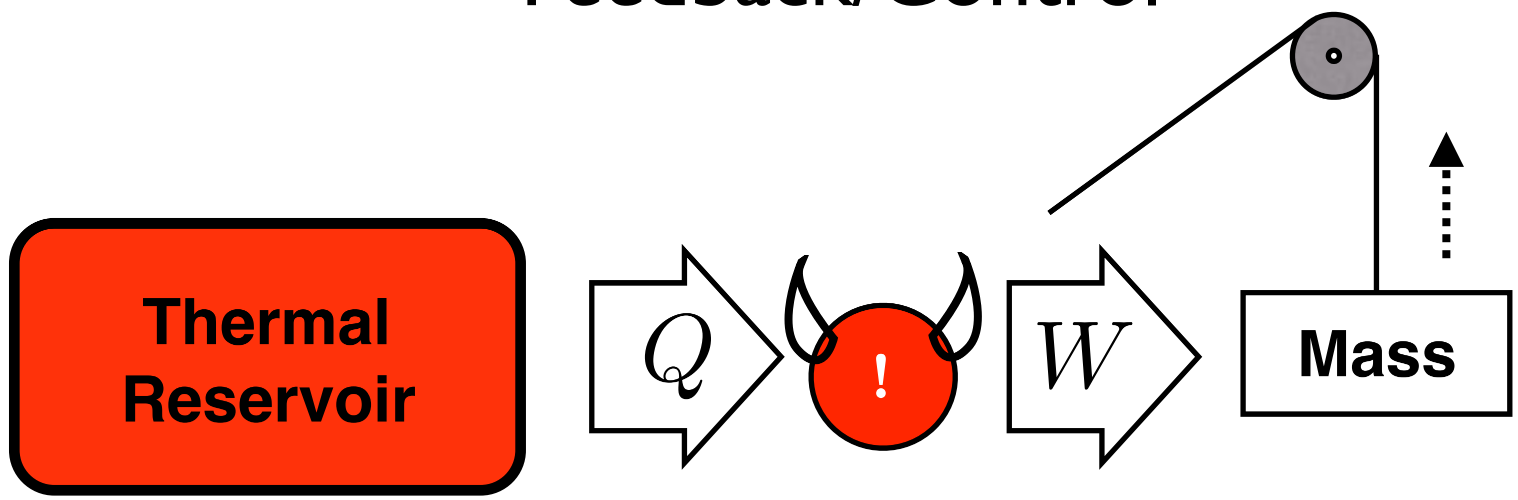
Feedback/Control



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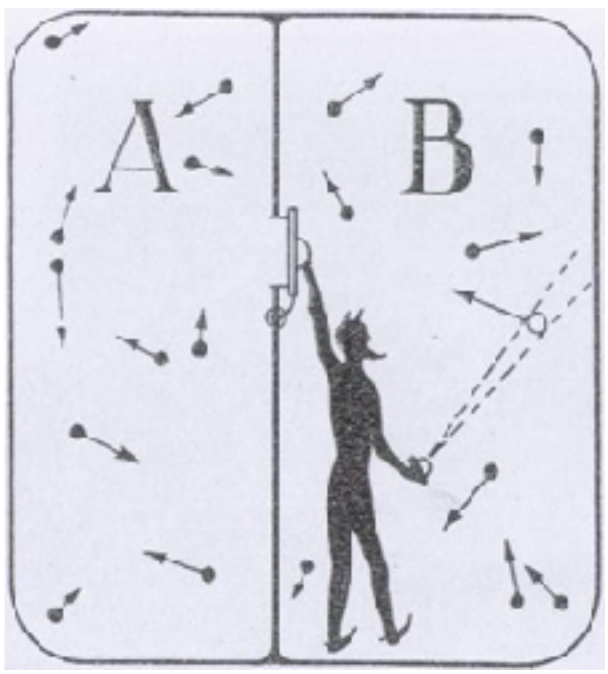

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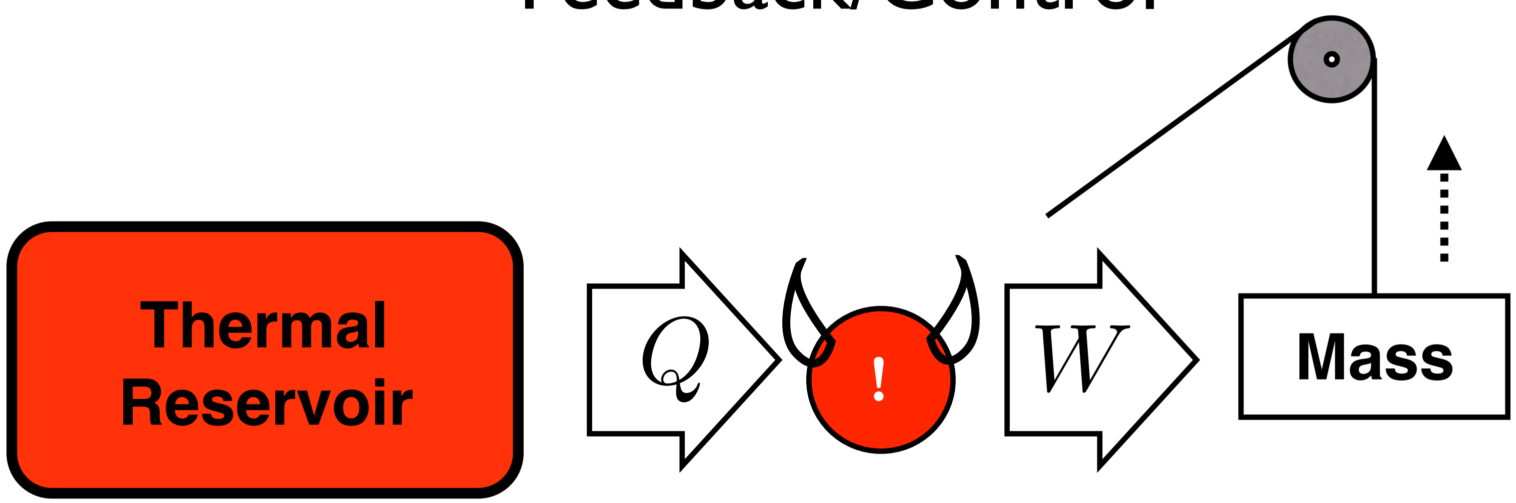
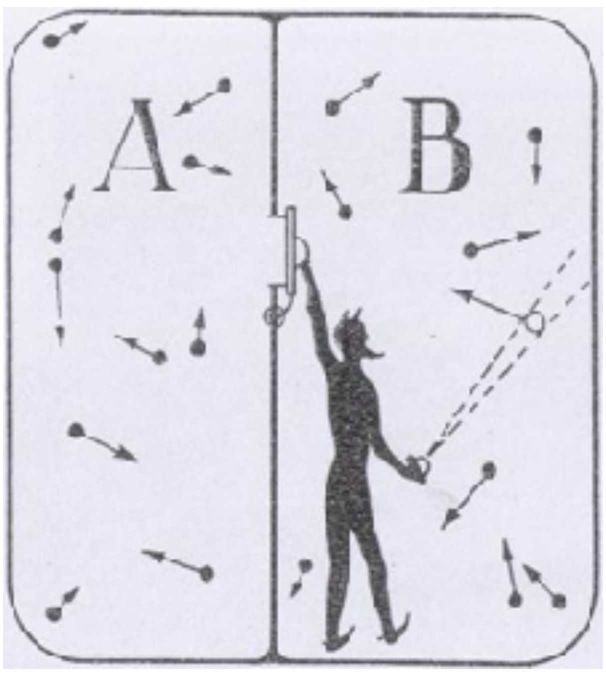


$$\Delta S_{\text{total}} < 0?$$

Information Bearing Degrees of Freedom

Maxwell's Demon



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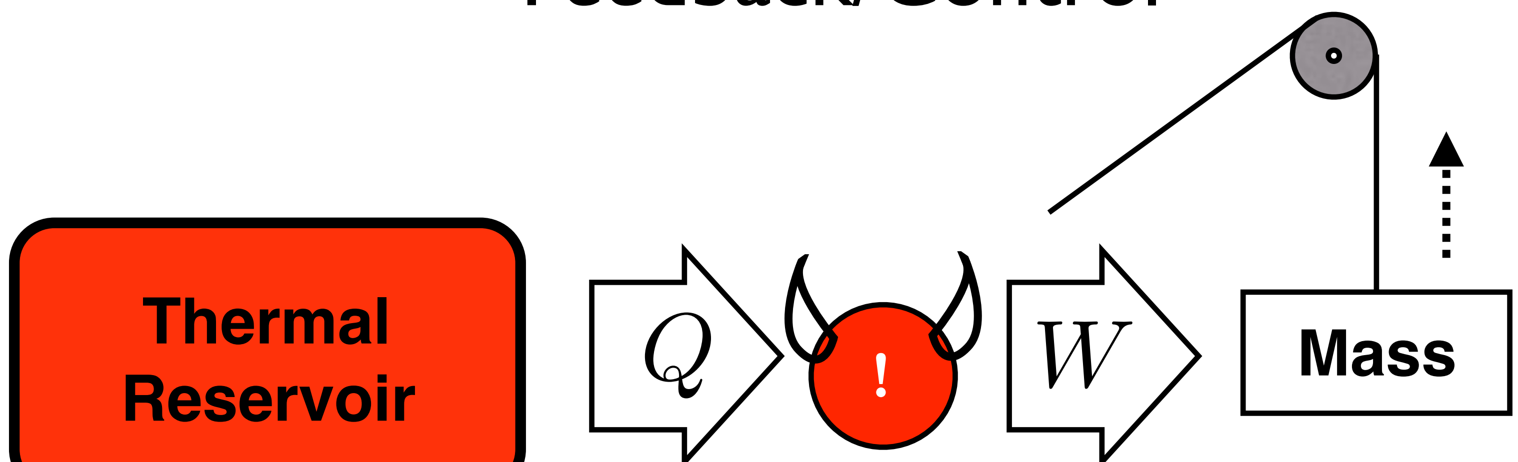
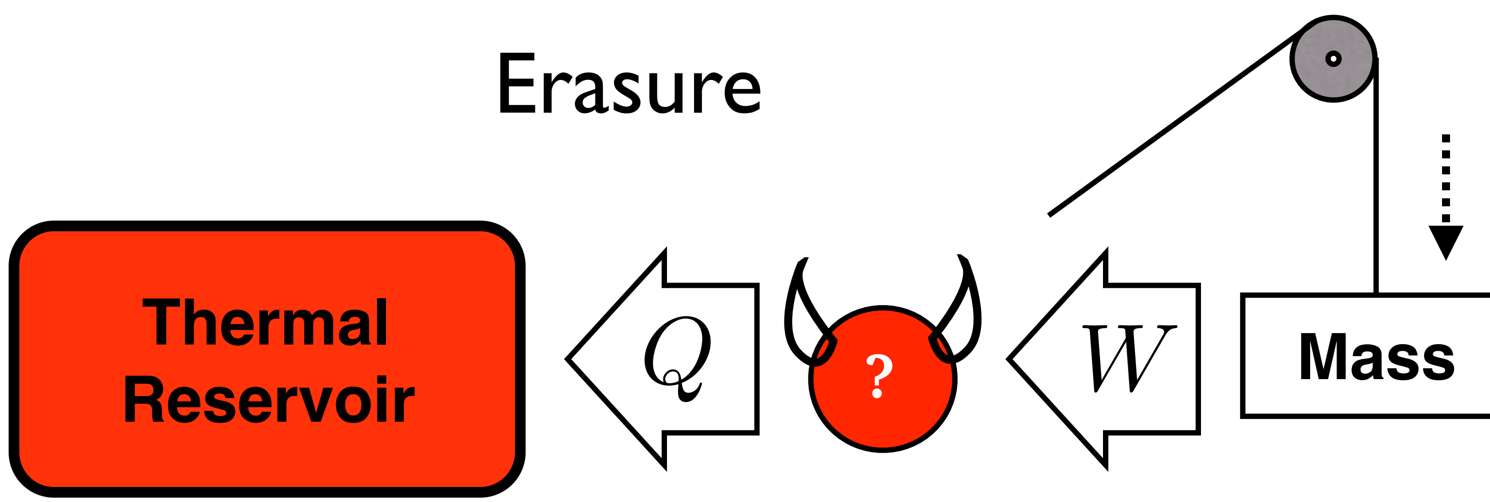


$$\cancel{\Delta S_{\text{total}} < 0?}$$

Erasure



Information Bearing Degrees of Freedom

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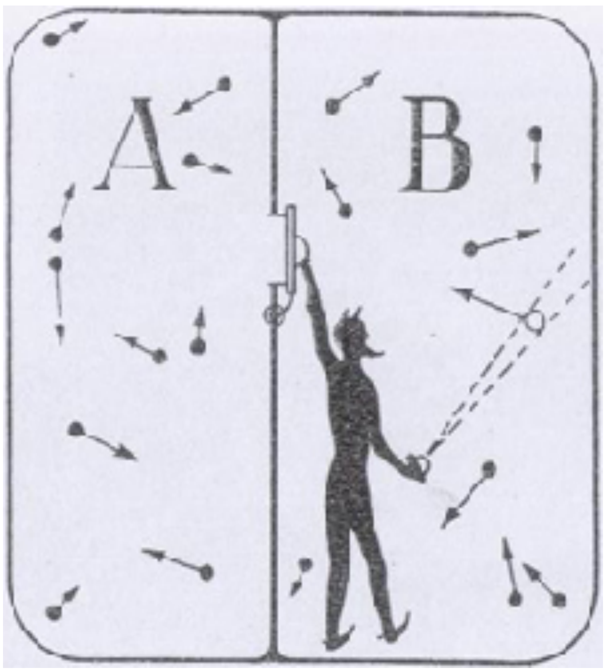

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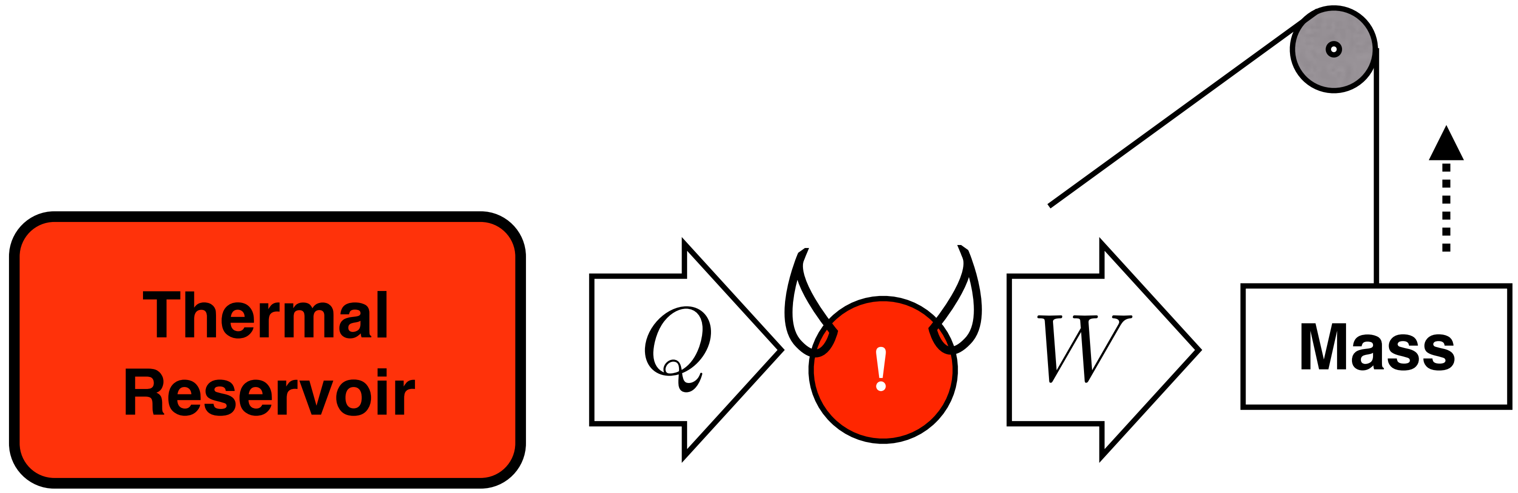

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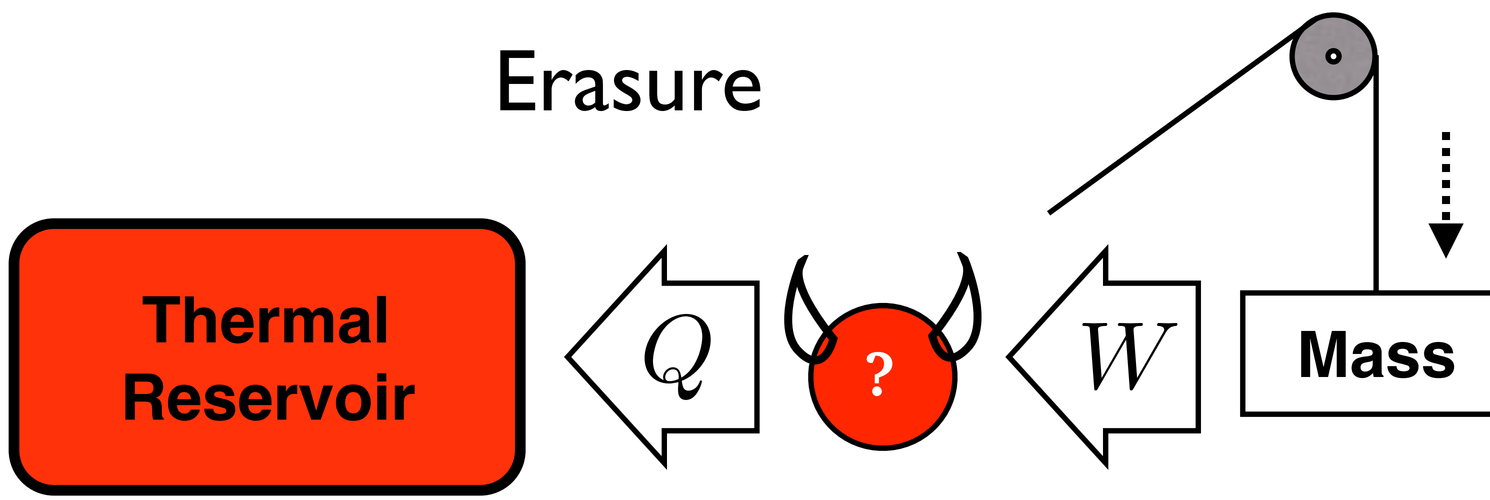
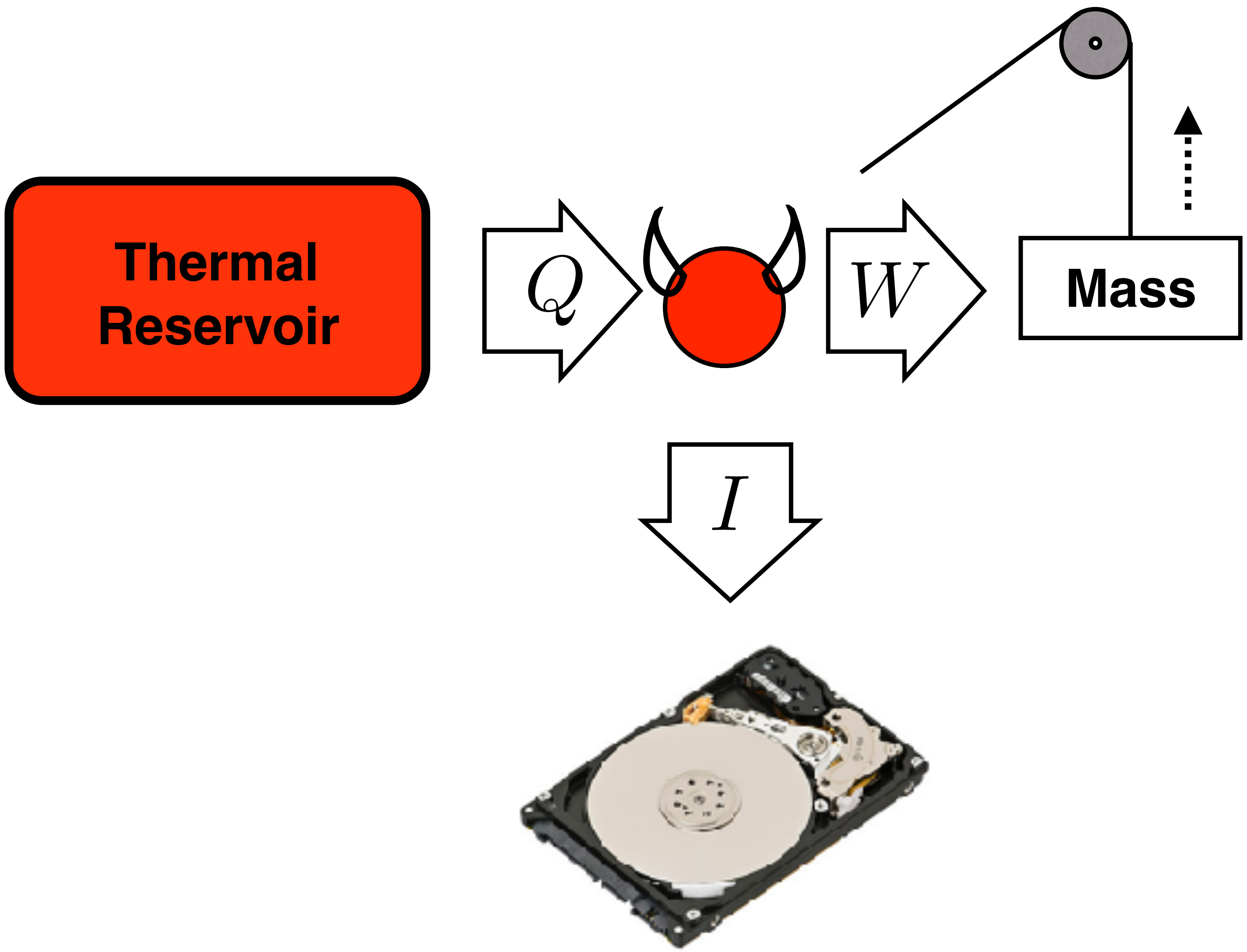
Erasure



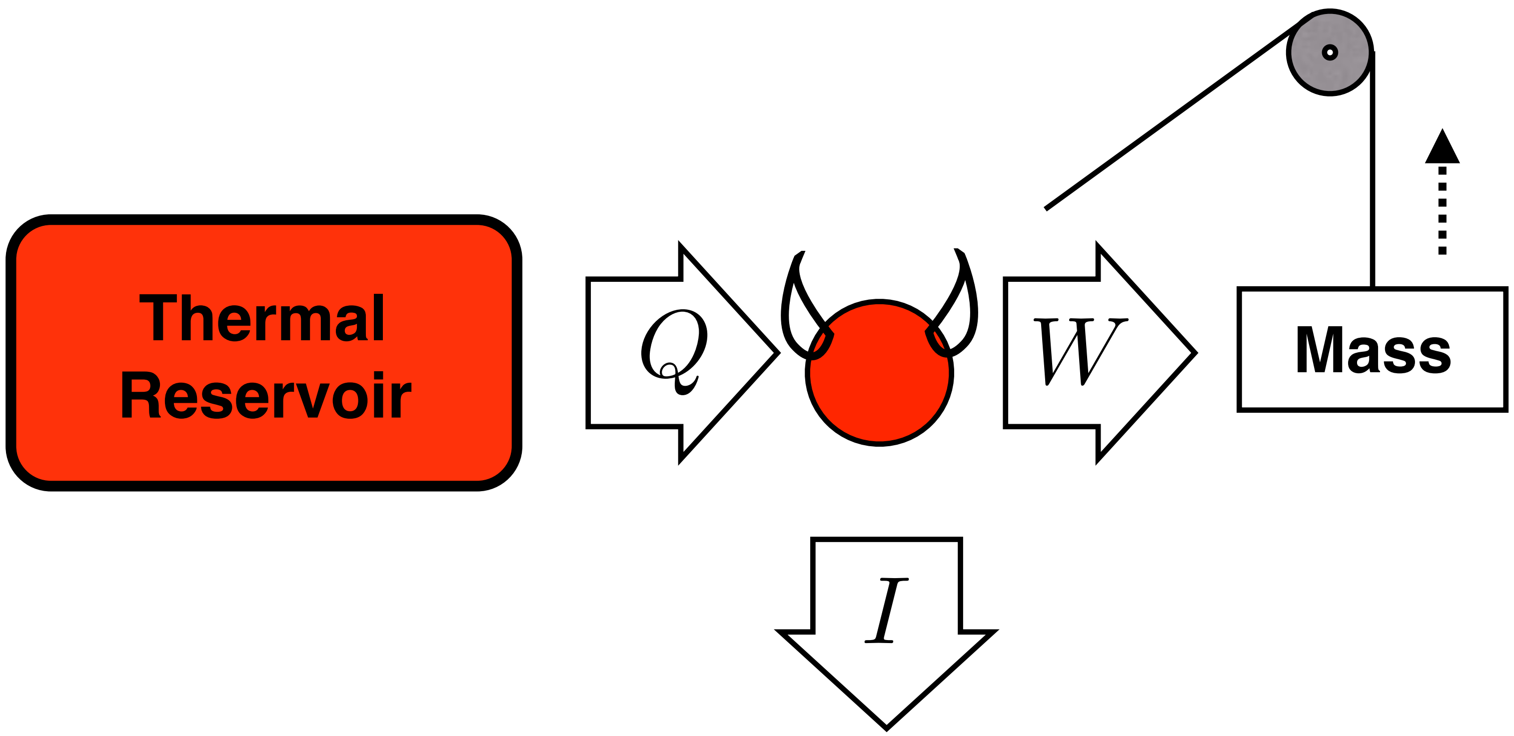
Landauer's Principle: $W_{\text{erase}} \geq k_B T \ln 2 \rightarrow \Delta S_{\text{total}} \geq 0$

Information is a Thermodynamic Fuel



Instead of erasing, write to a hard drive

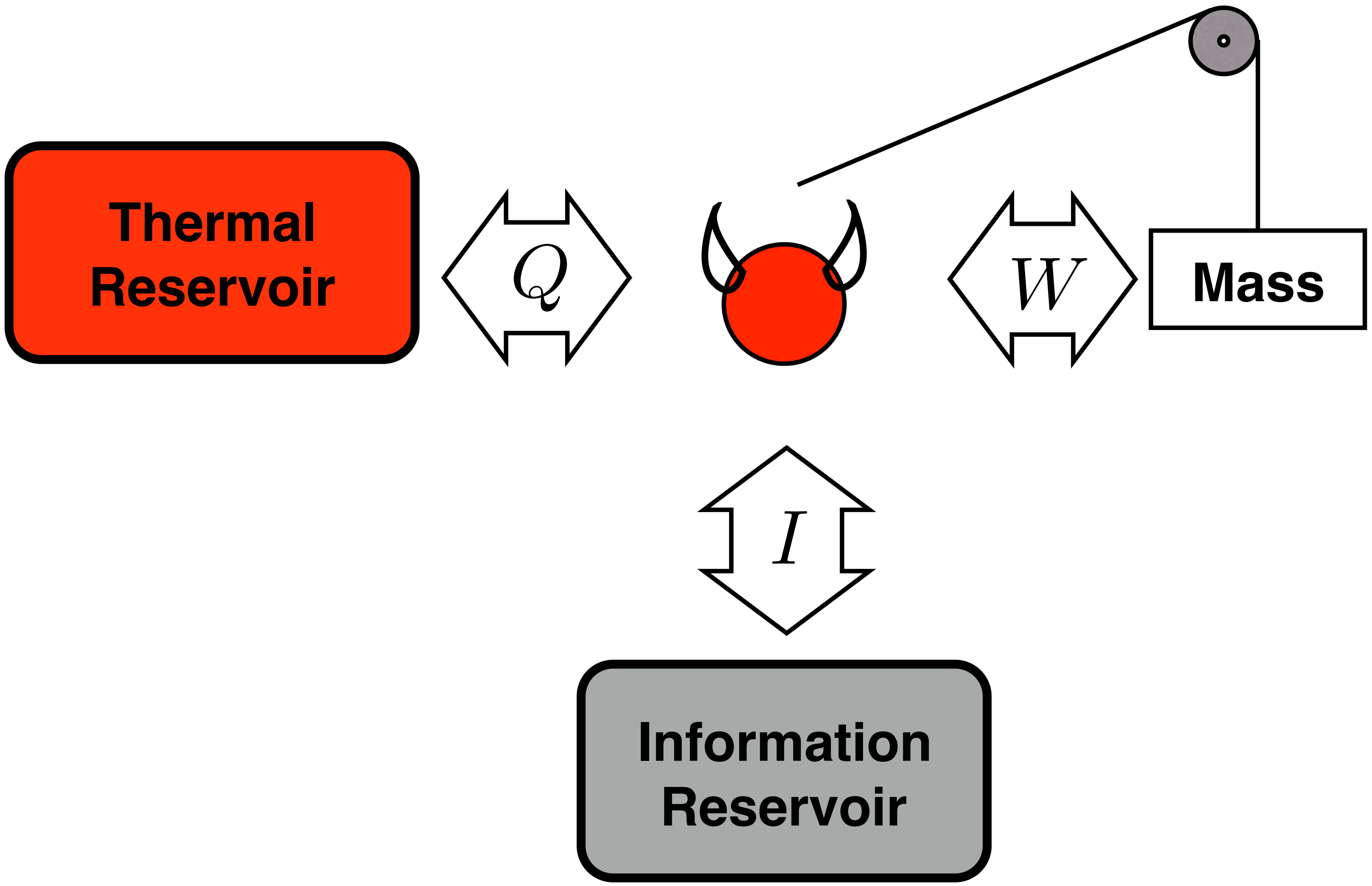
Information is a Thermodynamic Fuel



Instead of erasing, write to a hard drive

$$W > 0,\ Q < 0,\ \Delta S_{\text{hard drive}} > 0 \rightarrow \Delta S_{\text{total}} \geq 0$$

Information Engines

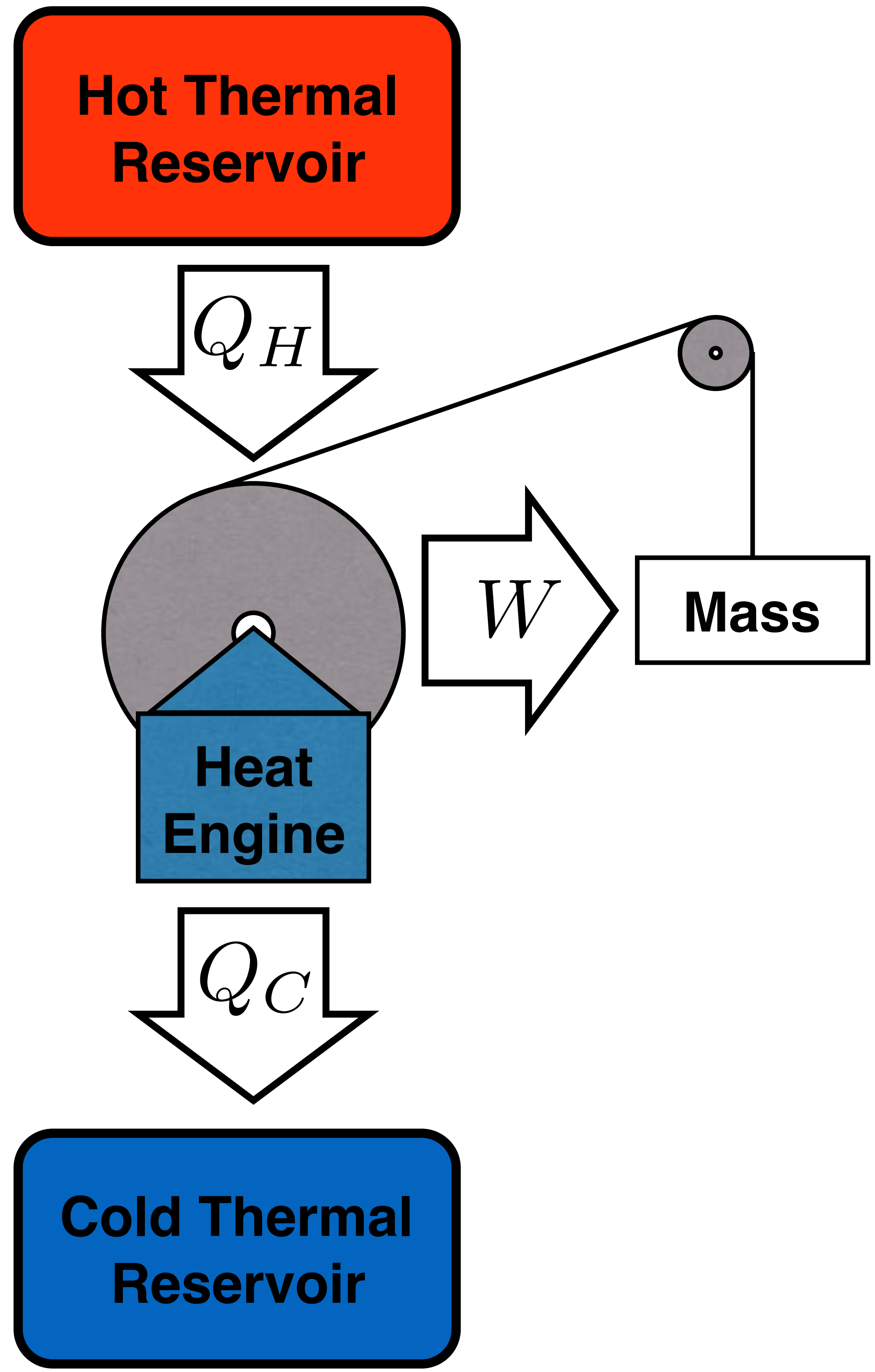


Simultaneously processes information and manipulates energy

Heat Engines

Energy flows between three reservoirs to produce work:

1) Hot Thermal
2) Cold Thermal
3) Work



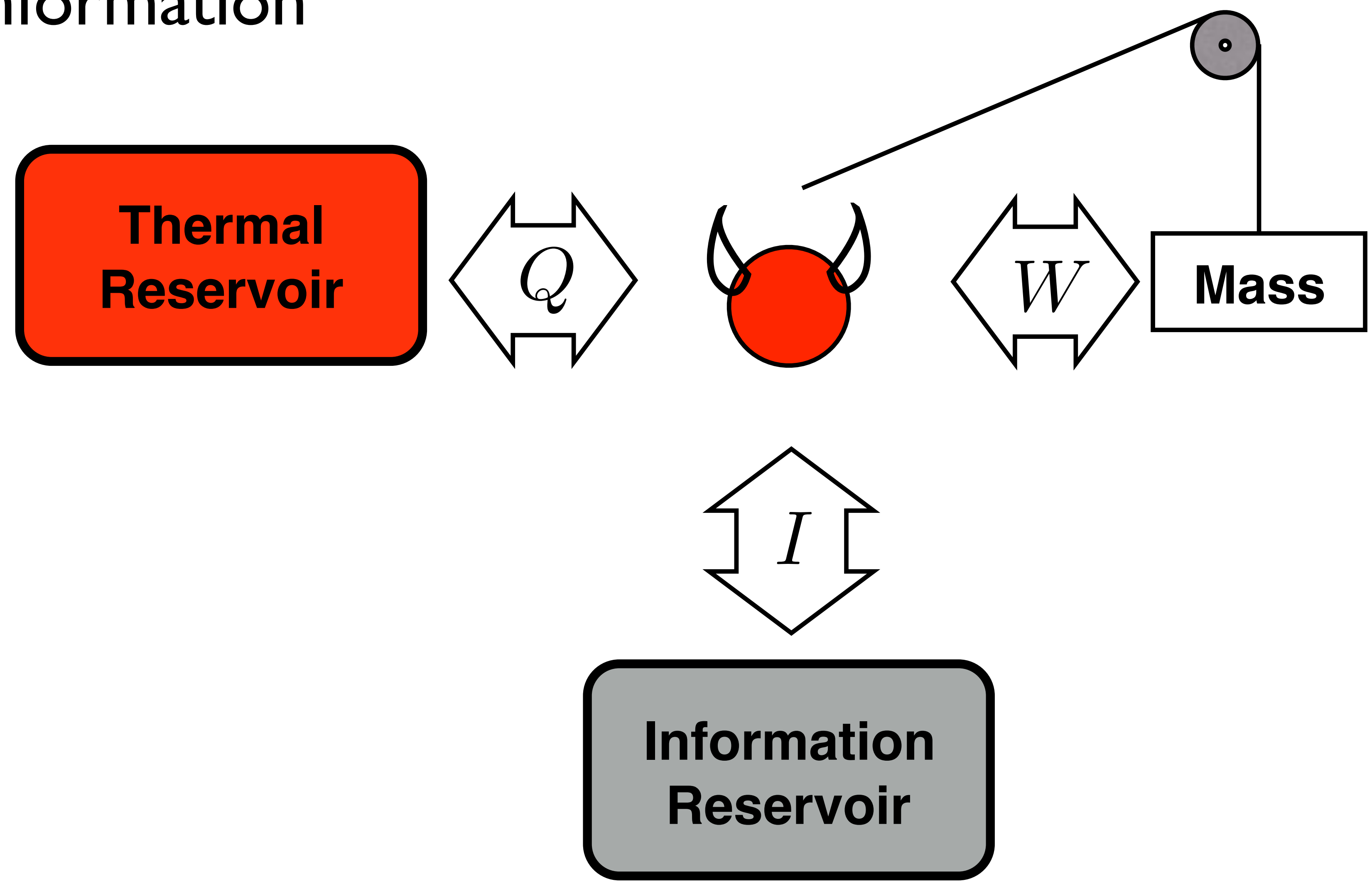
Carnot efficiency bound:

$$\Delta S \geq 0 \Rightarrow \eta = \frac{W}{Q_H} \leq 1 - \frac{T_C}{T_H}$$

Information Engines

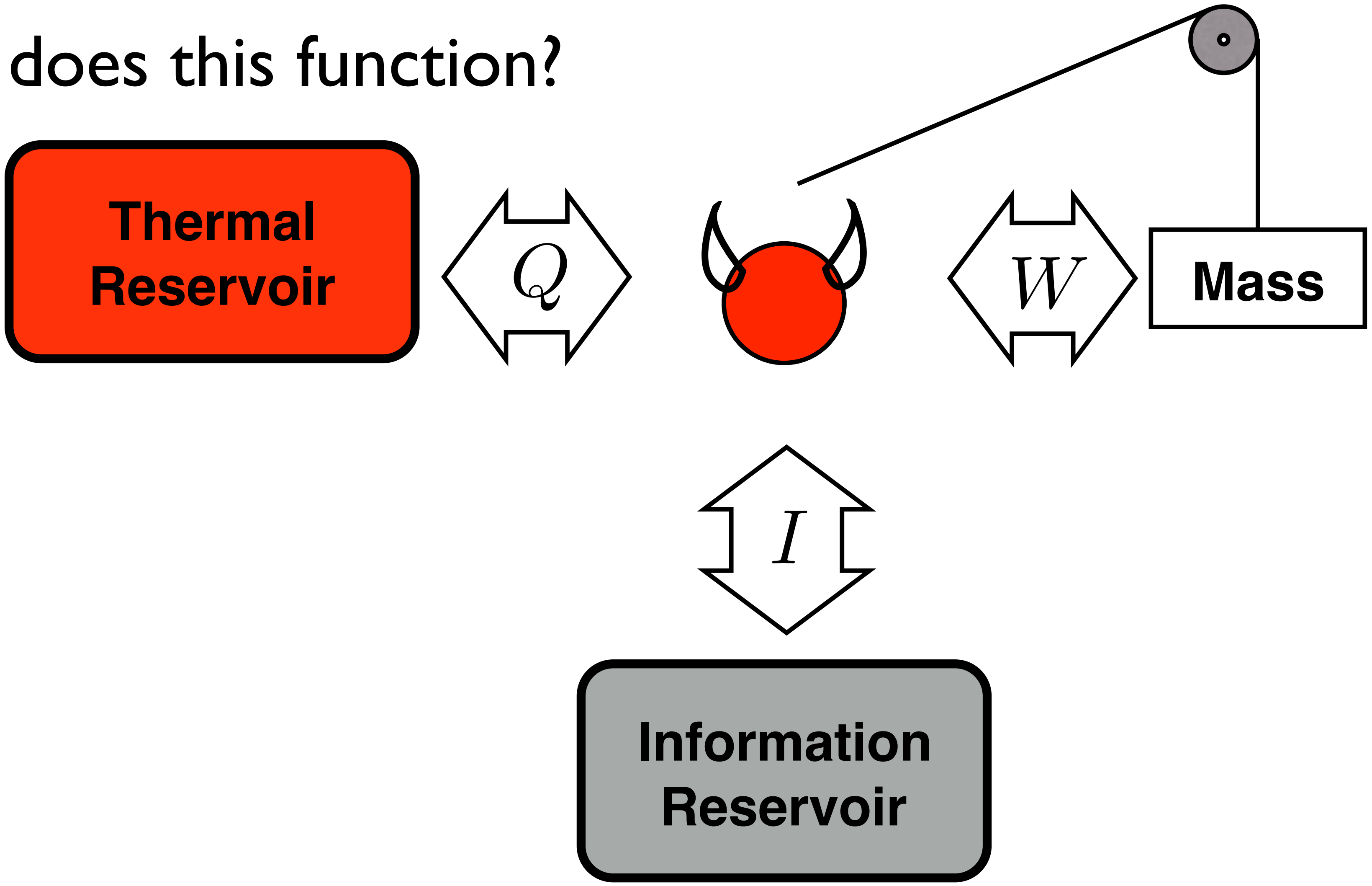
Energy and information flow between three reservoirs:

1) Thermal
2) Work
3) Information



Information Engines

How does this function?



Information Engines

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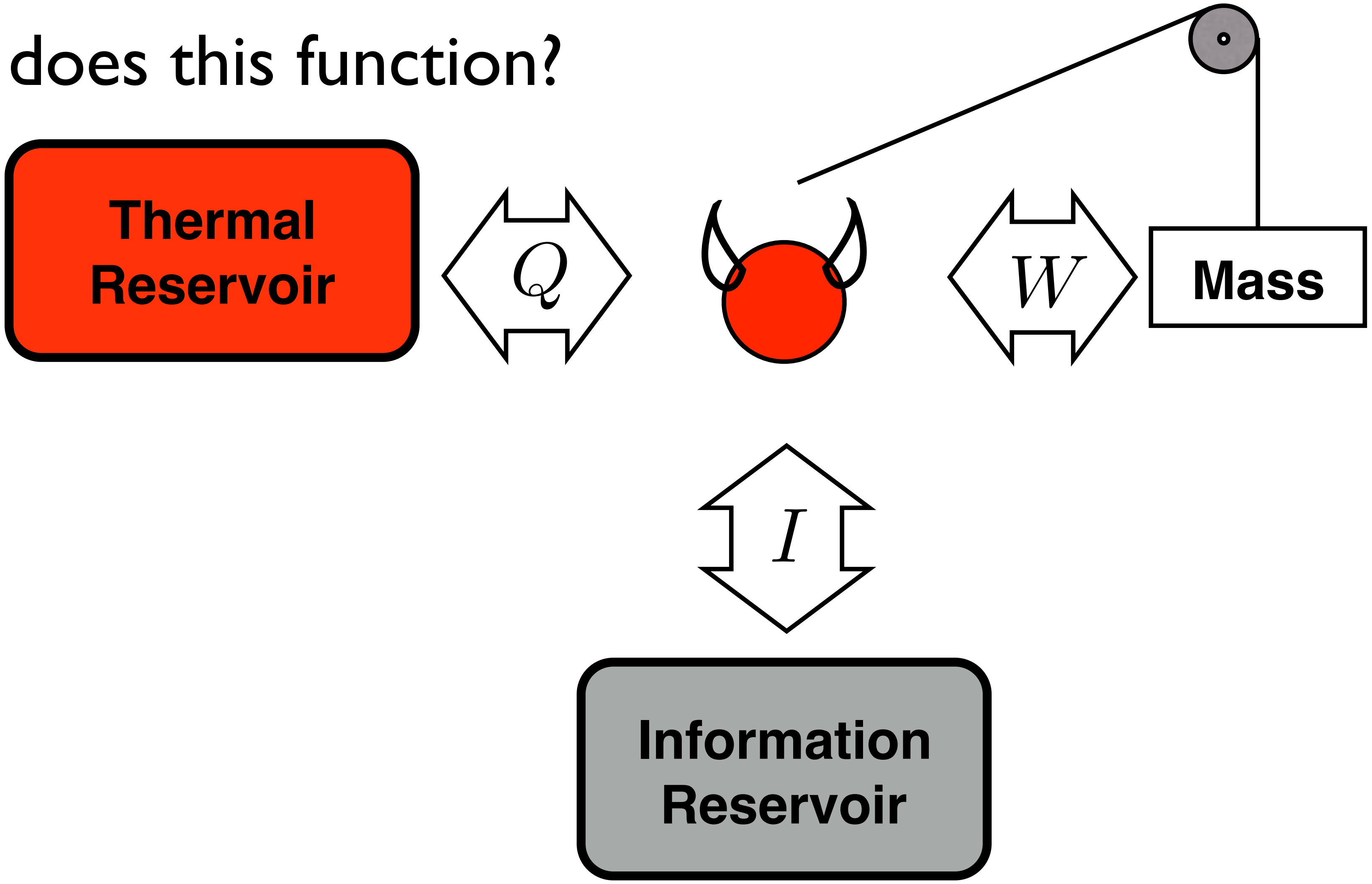

What is an information reservoir?

Information Reservoir

A bit string is an information reservoir

1	1	0	1	0	0	0	1	0	0	1	0	1	1	1	0	1	. . . .

Information Reservoir

A bit string is an information reservoir

1	1	0	1	0	0	0	1	0	0	1	0	1	1	1	0	1	....

if every 0 and 1 have the same energy, and so every configuration of the system has equal energy.

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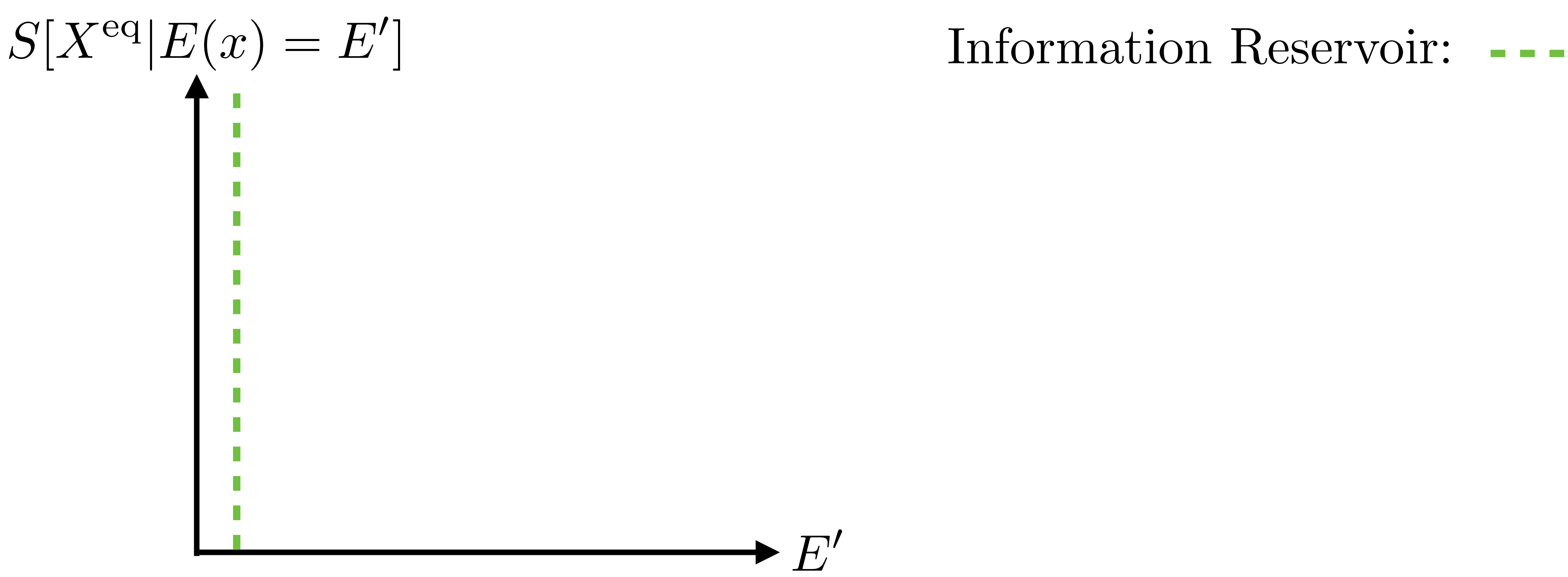

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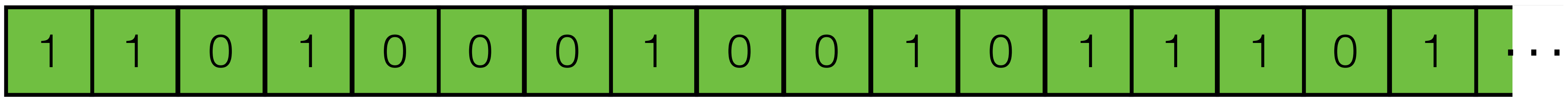

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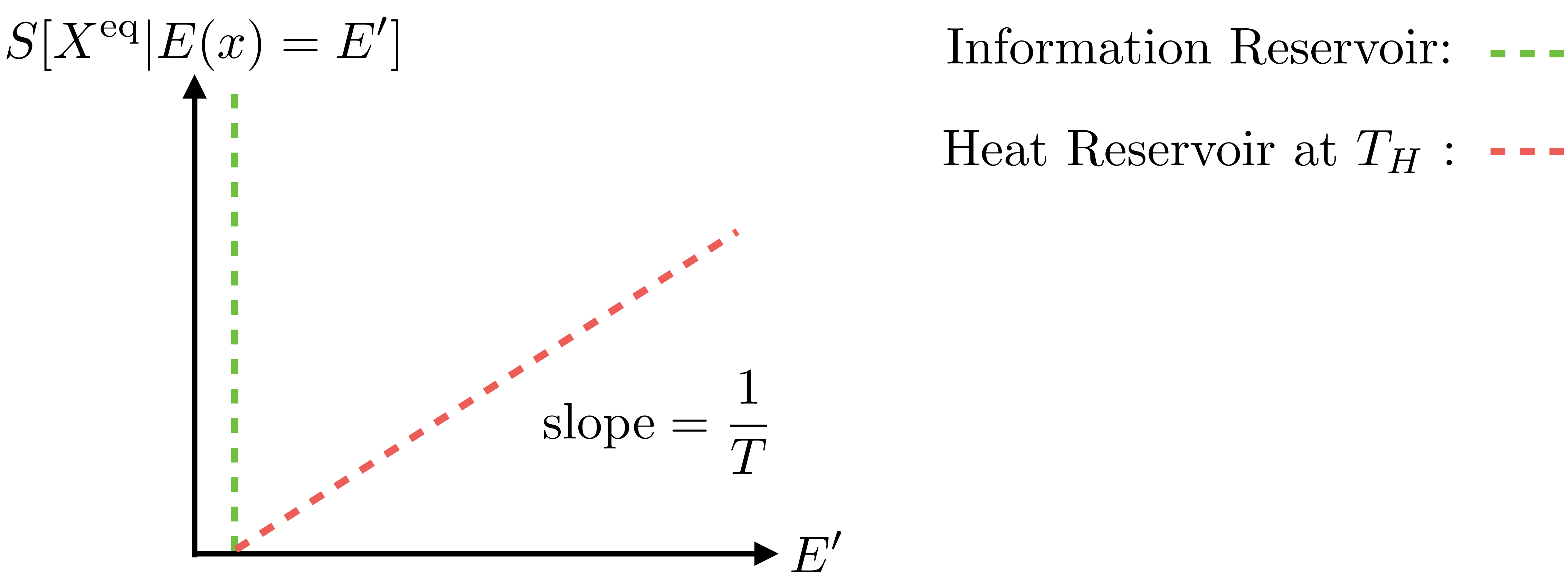

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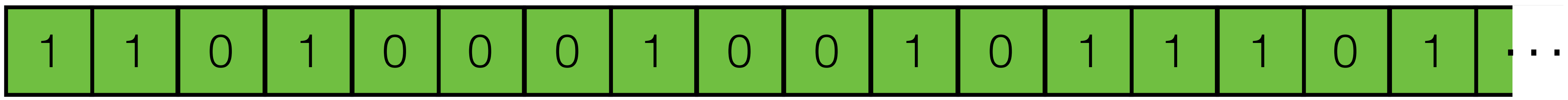

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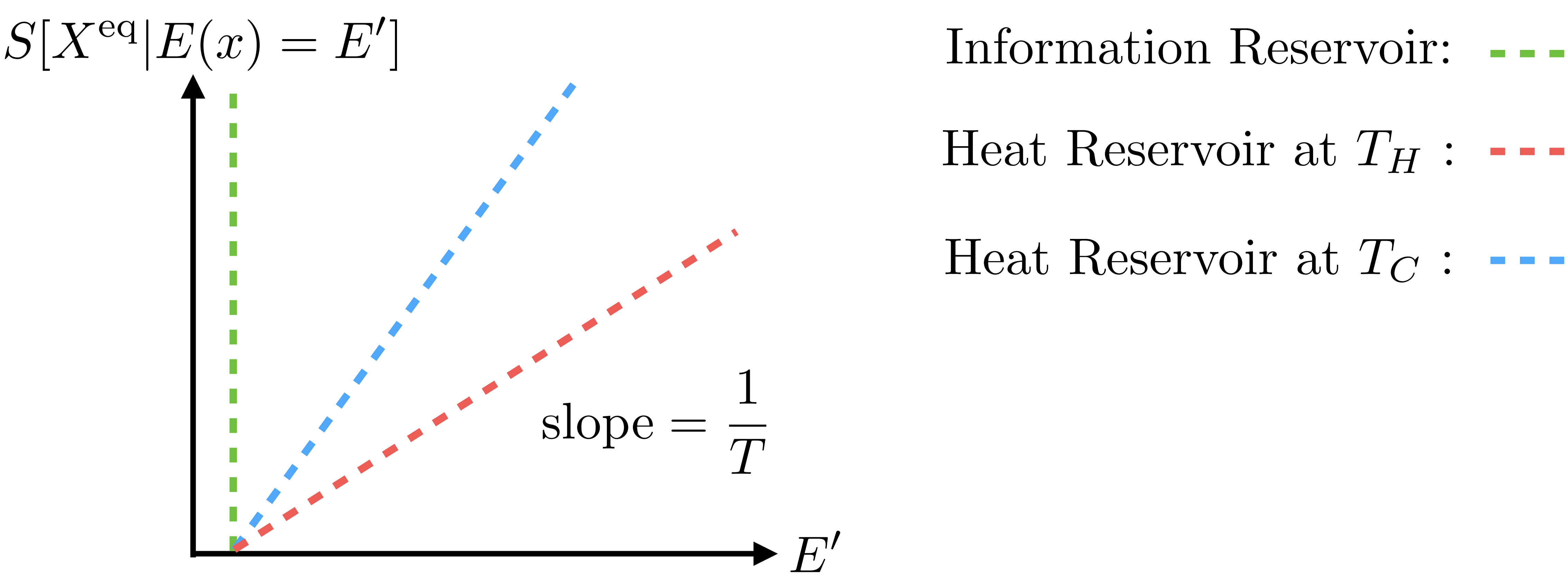

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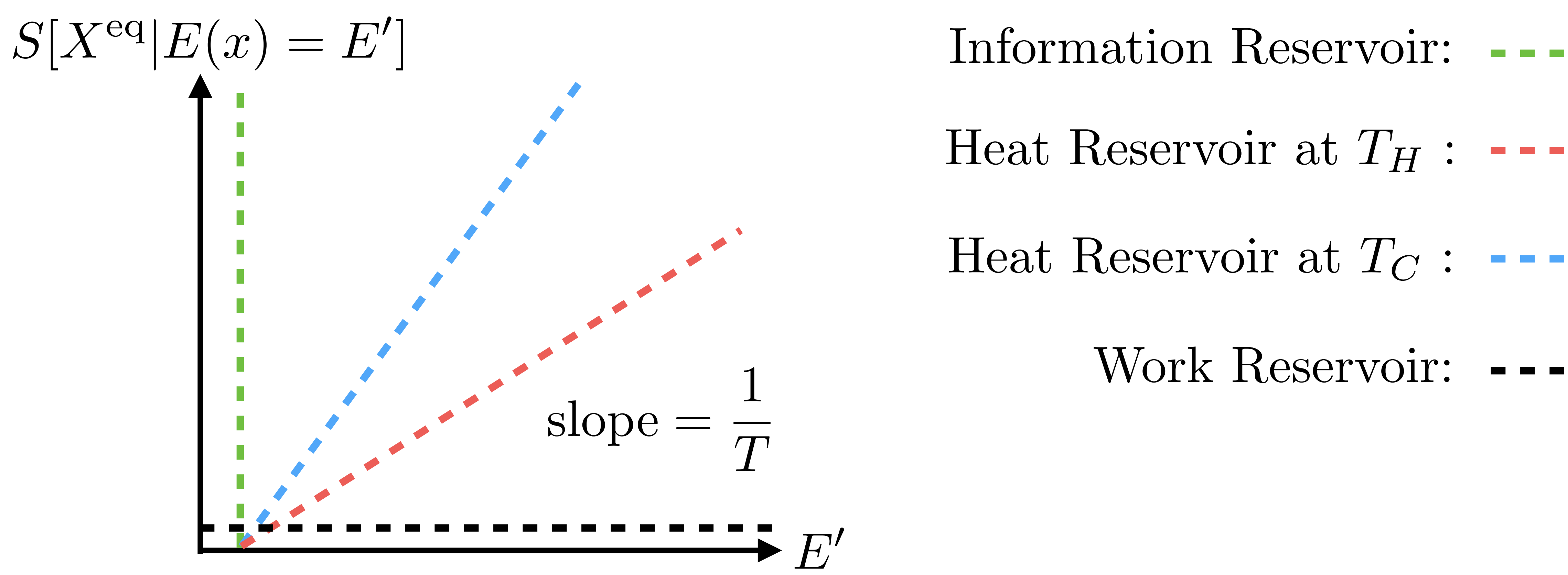

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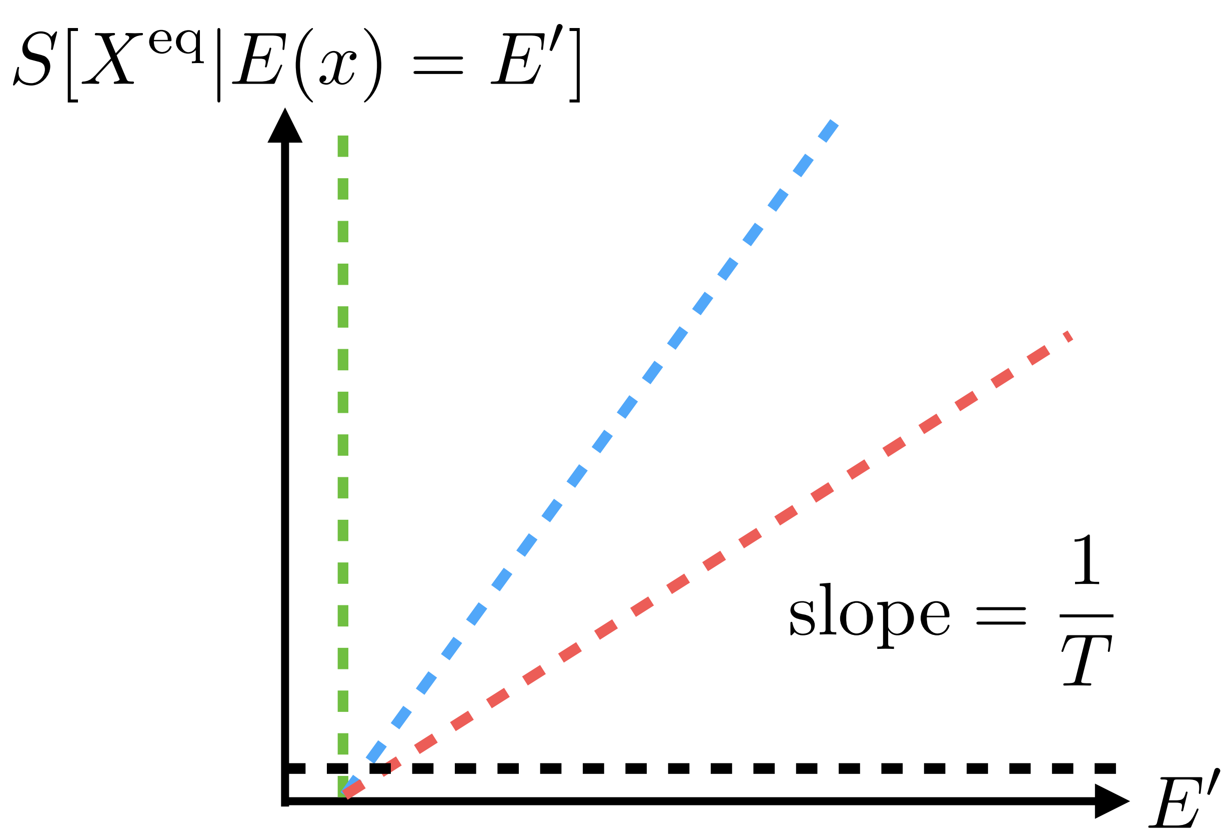

Information Reservoir: - - -

Heat Reservoir at T_H : - - -

Heat Reservoir at T_C : - - -

Work Reservoir: - - -

$$T_{\text{Information Reservoir}} = 0$$

$$T_{\text{Work Reservoir}} = \infty$$

Physical Information

What is the bit in an information string?

1	1	0	1	0	0	0	1	0	0	1	0	1	1	1	0	1	....

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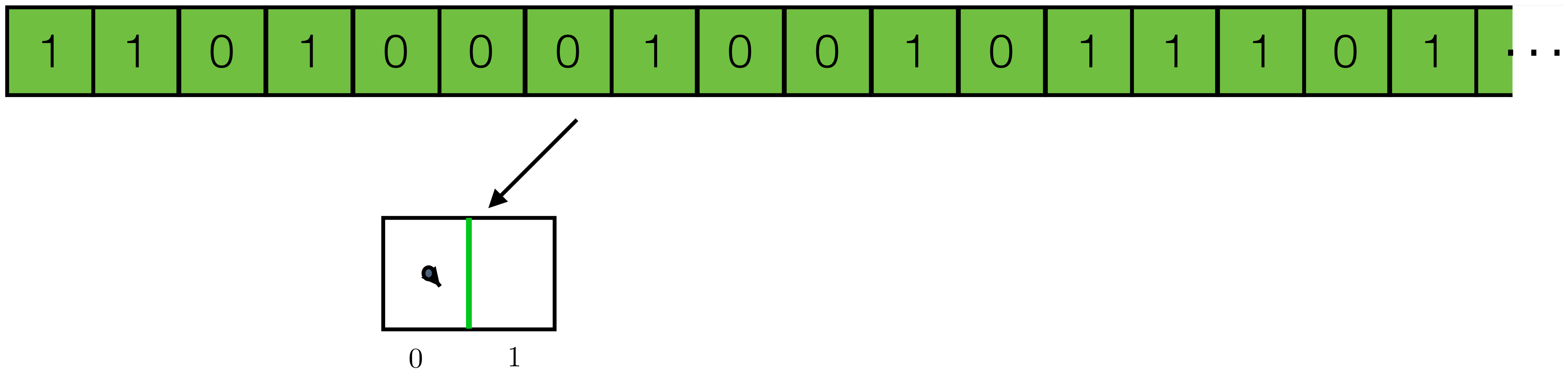

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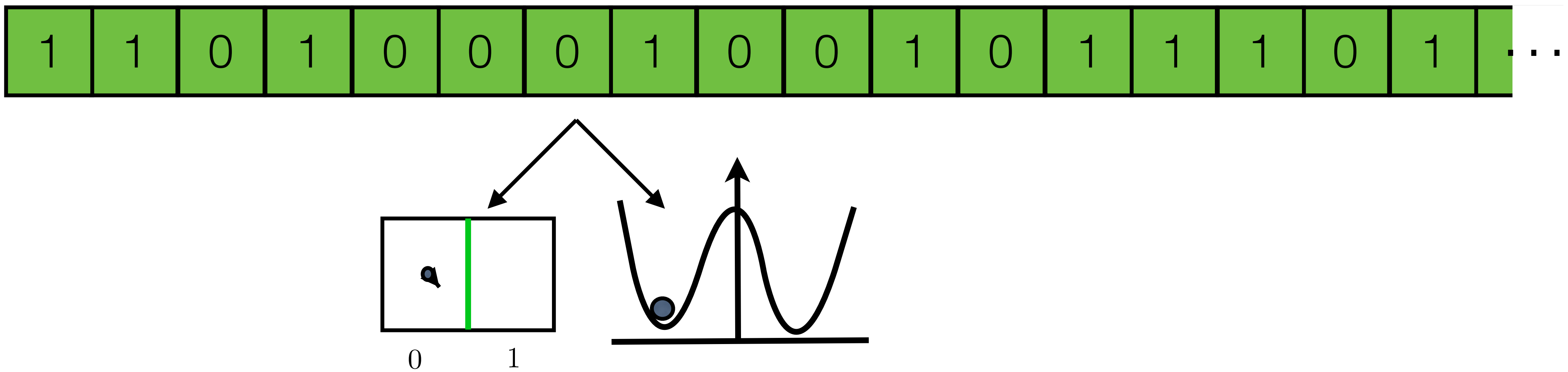

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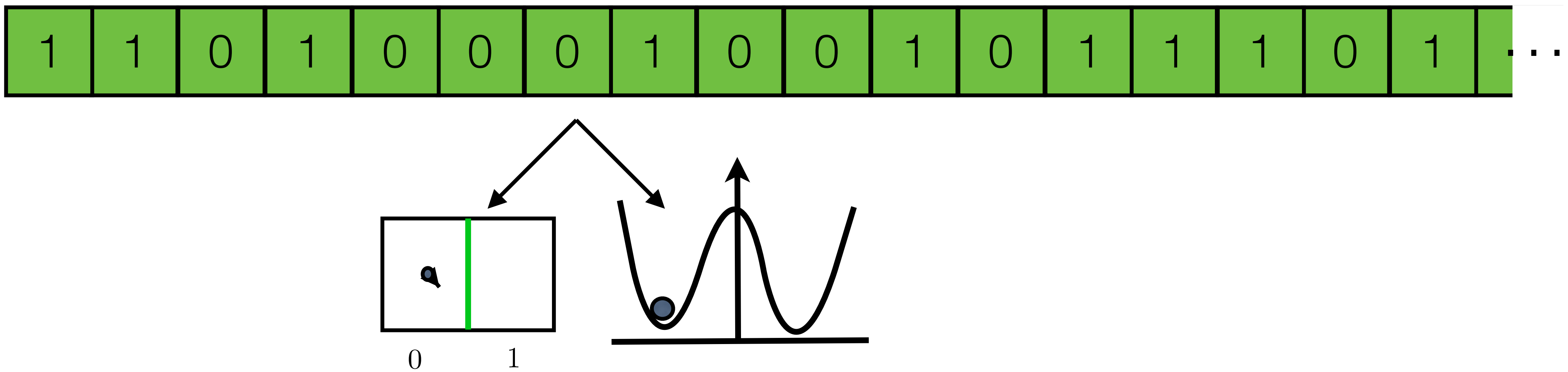
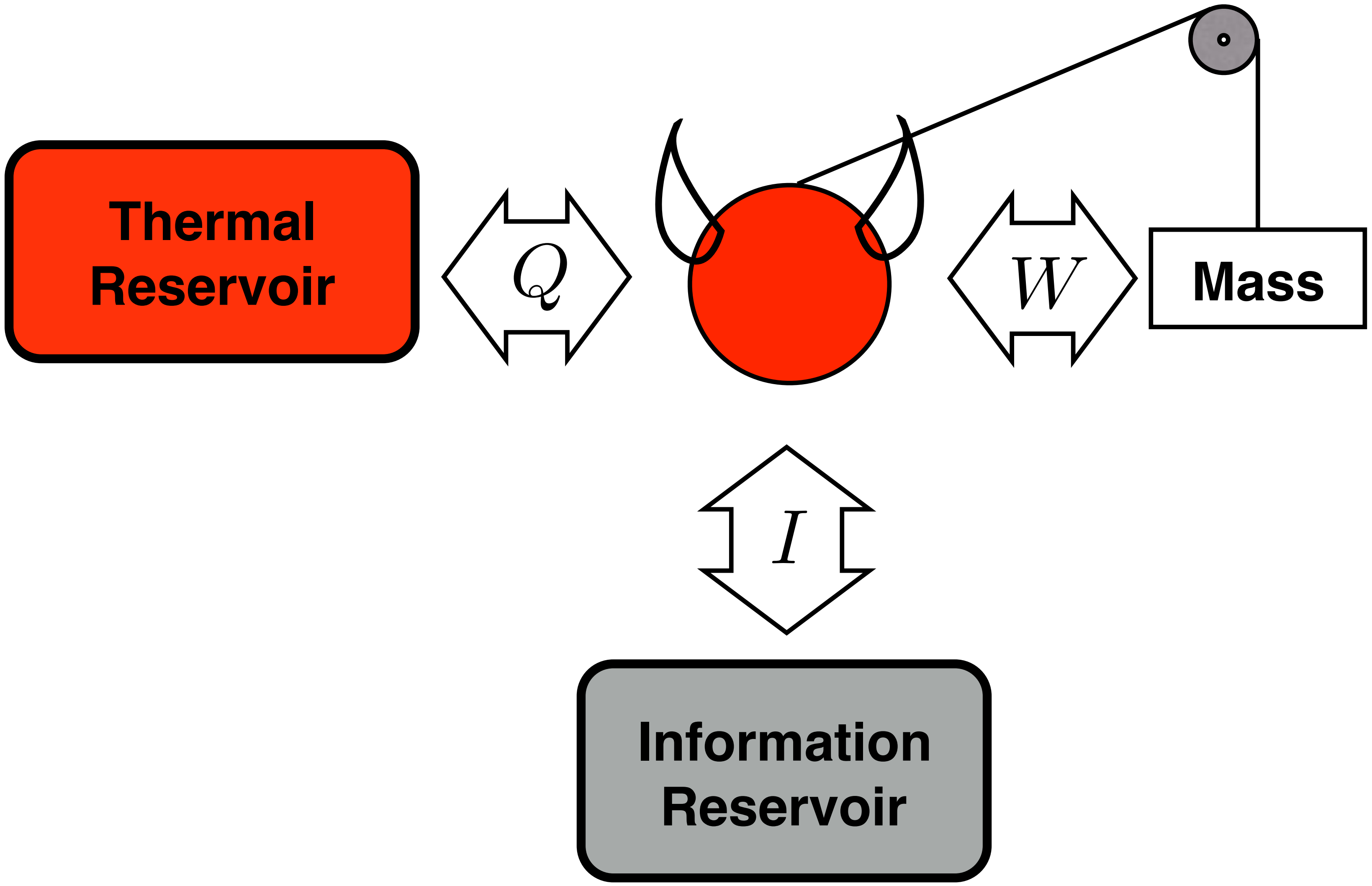


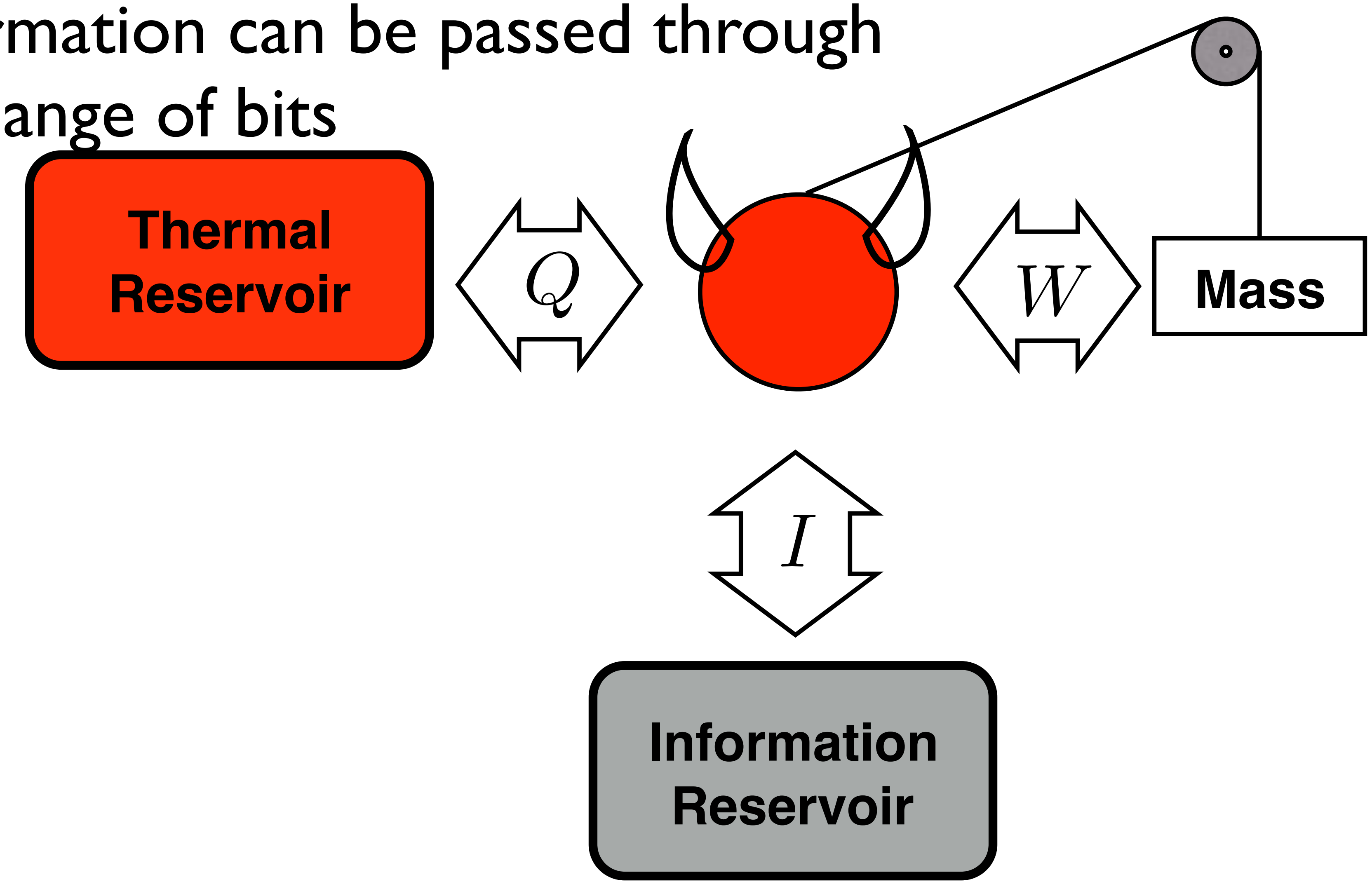
Metastable systems with equal energies.

Information Engine Energy Production



Information Engine Energy Production

Information can be passed through exchange of bits



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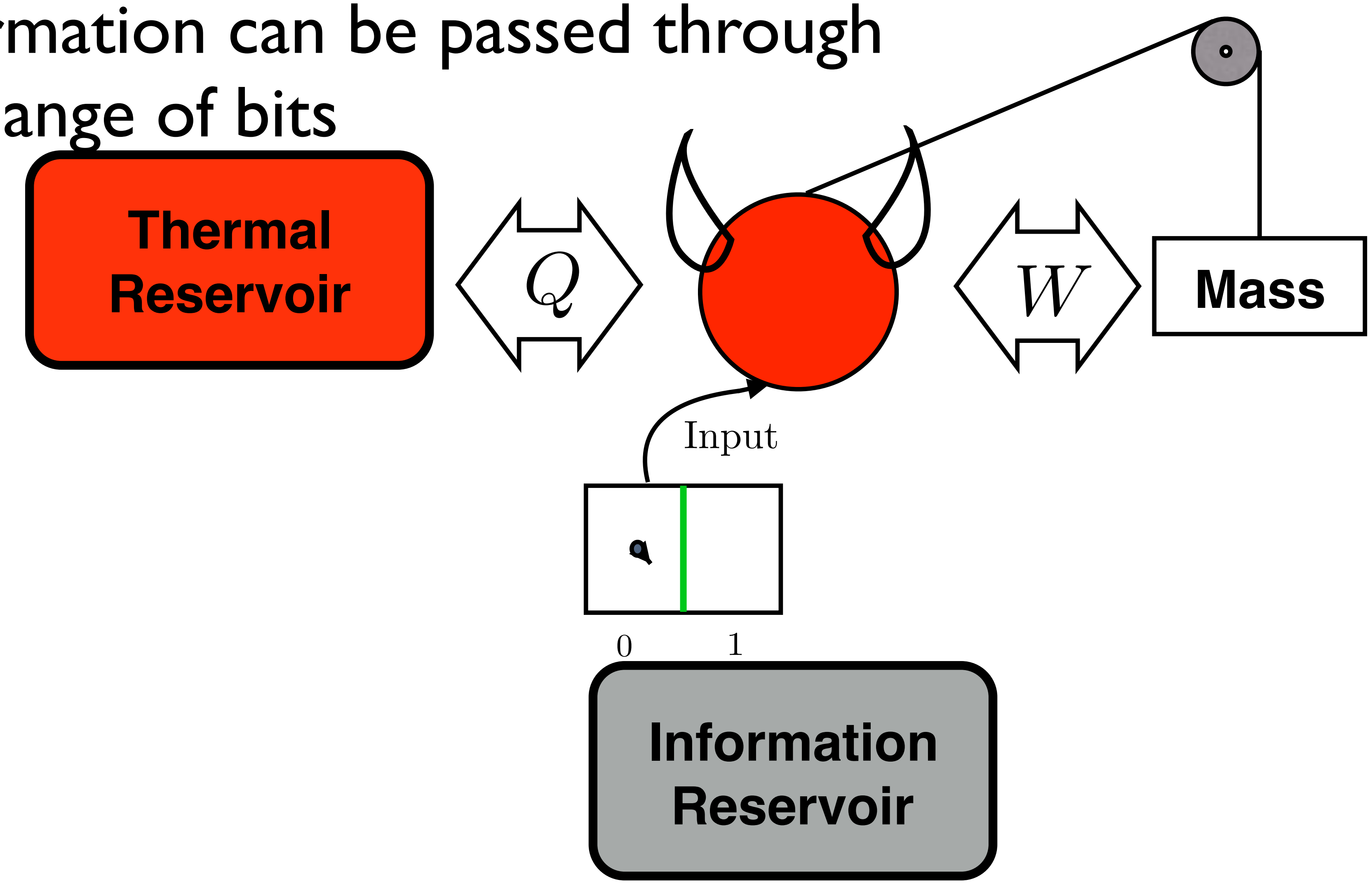

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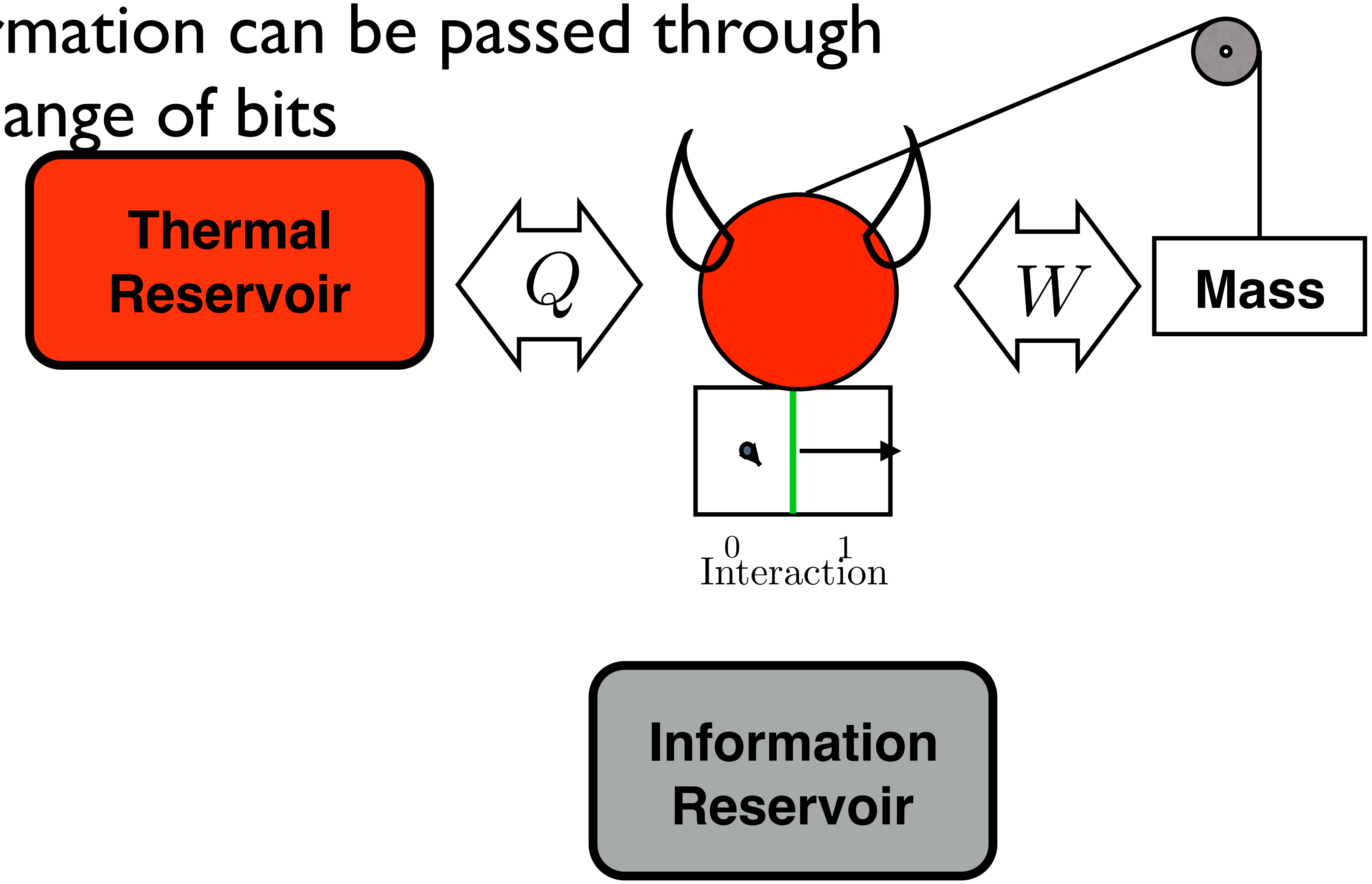

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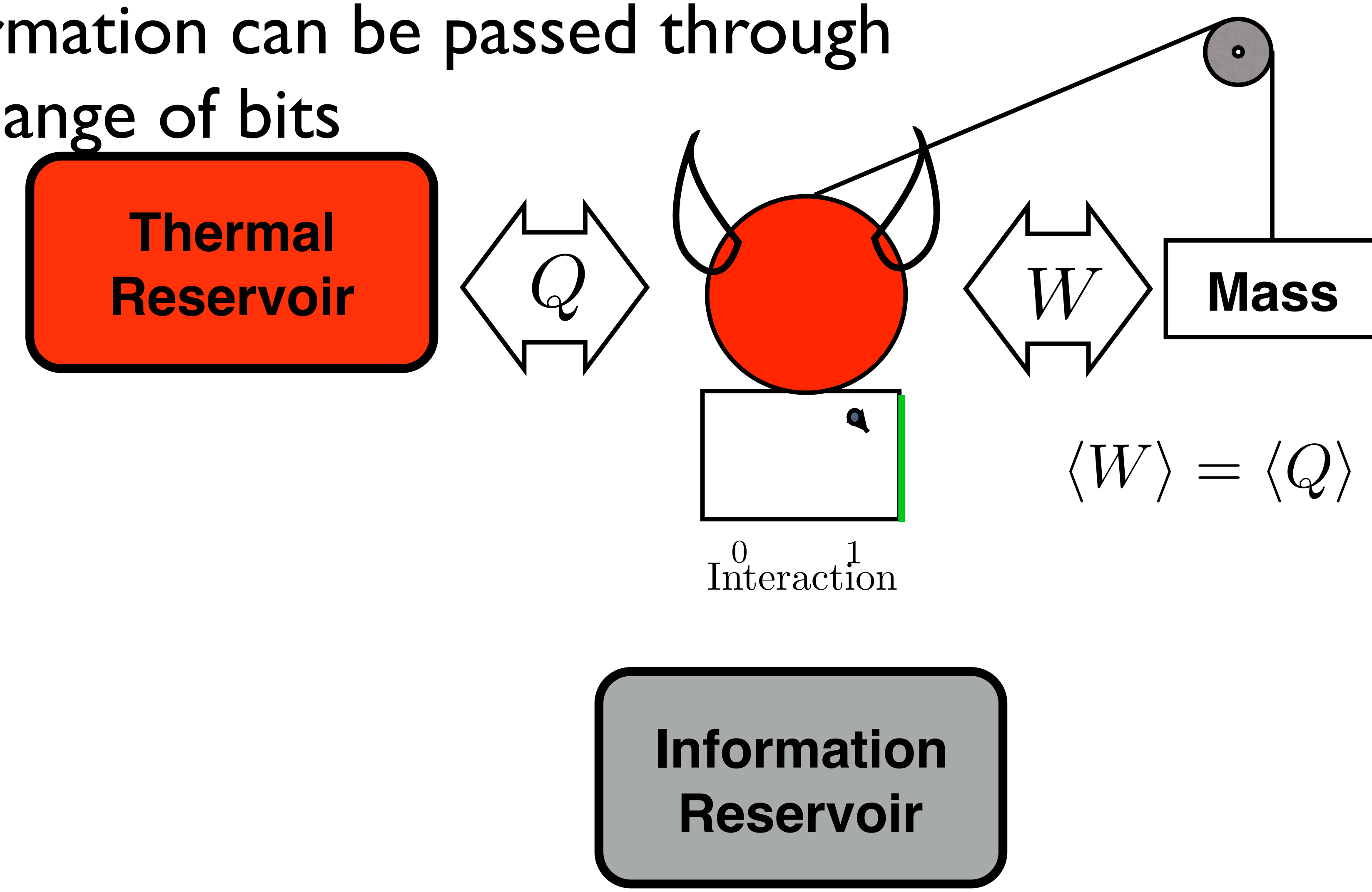

$$\langle W \rangle = \langle Q \rangle = -k_B T \ln 2$$

Information Engine Energy Production

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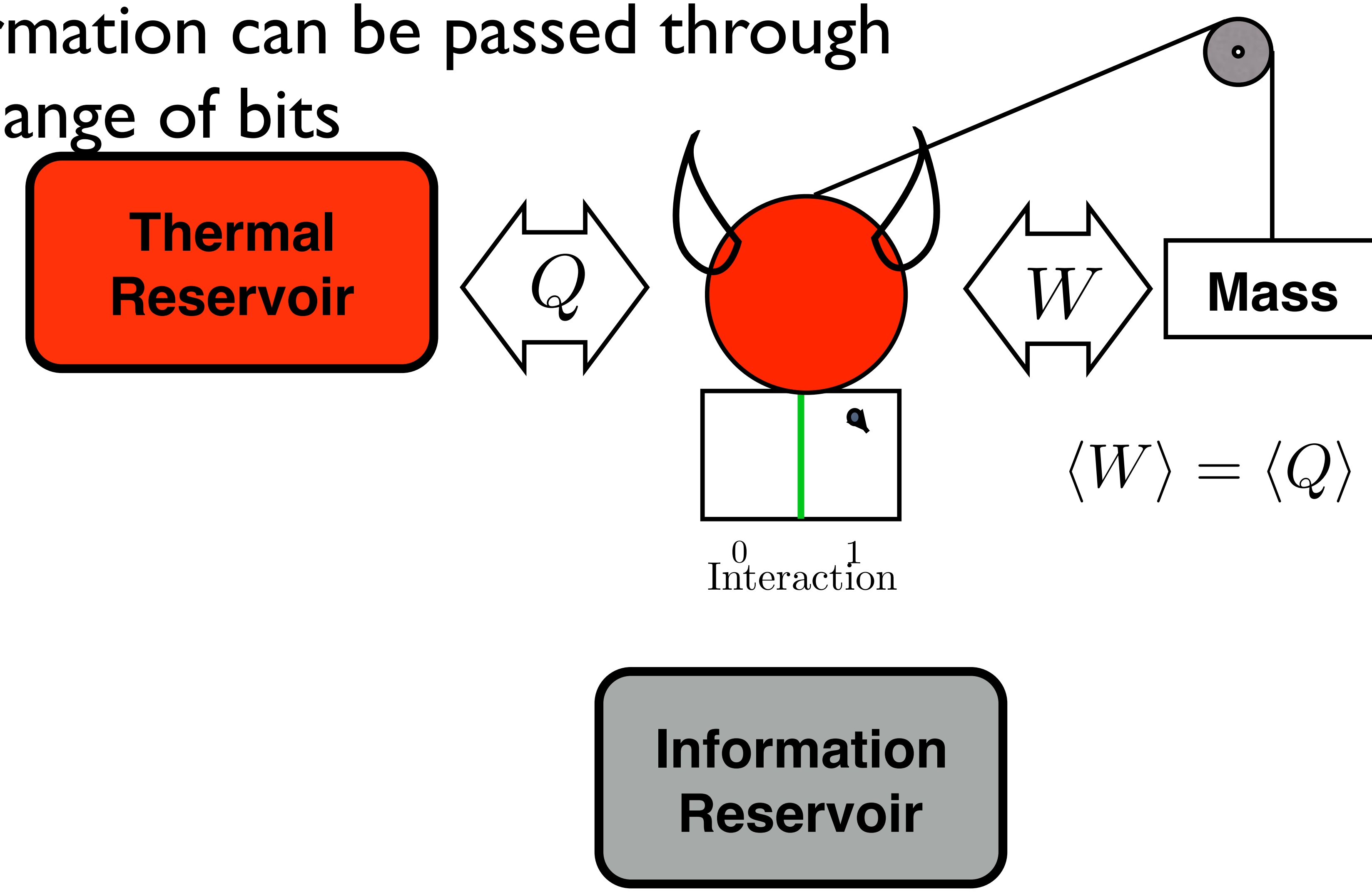

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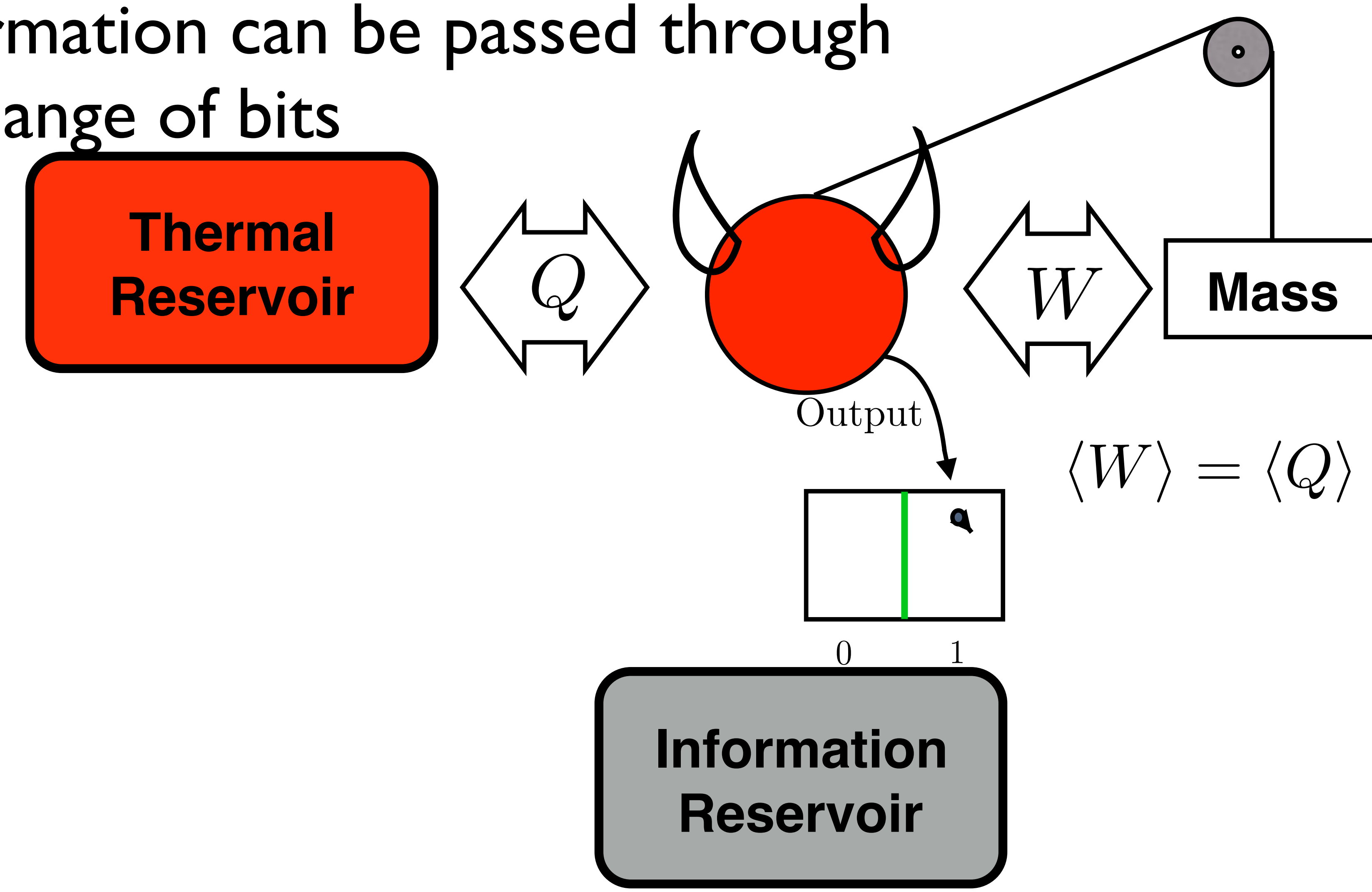

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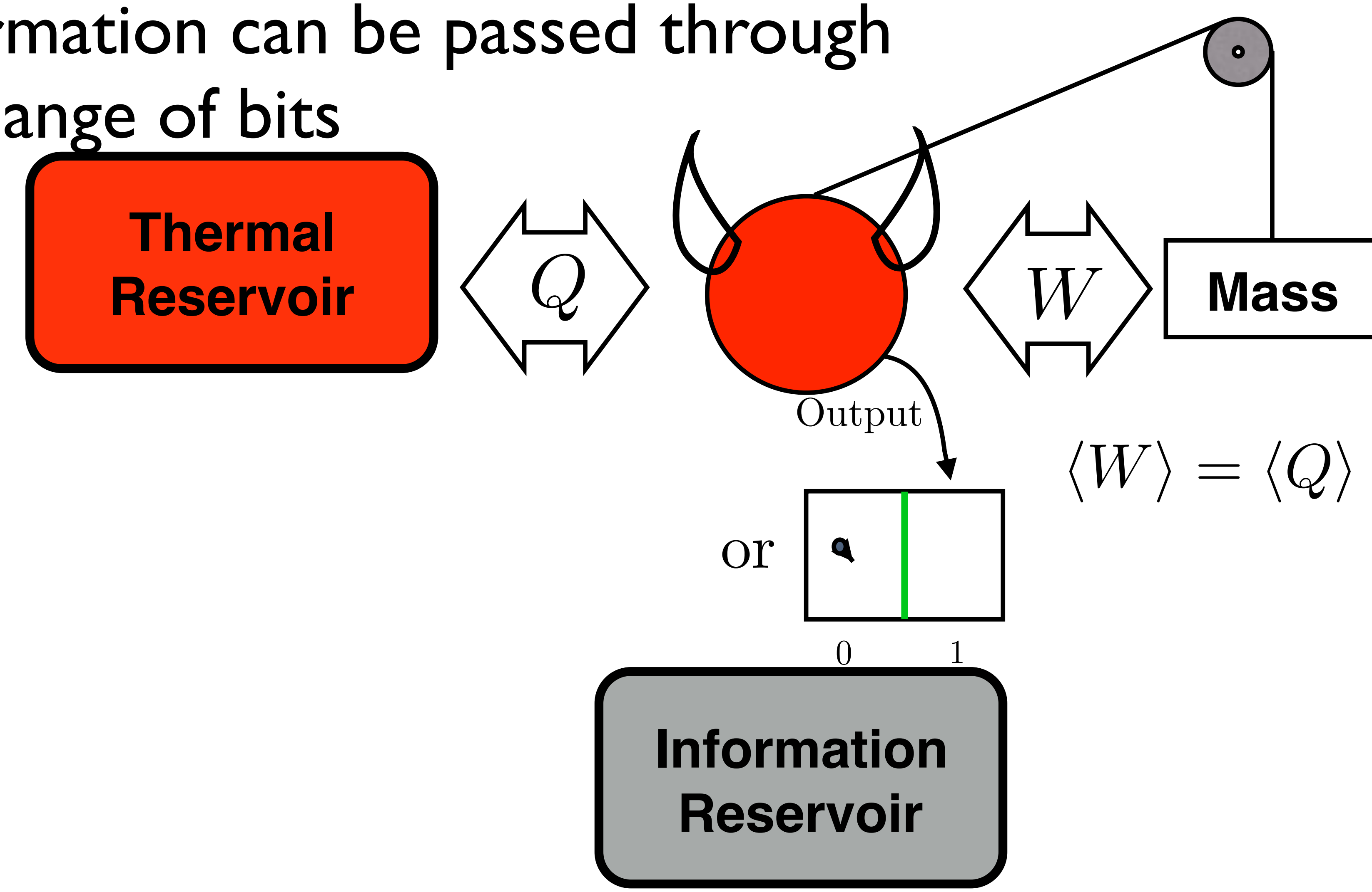

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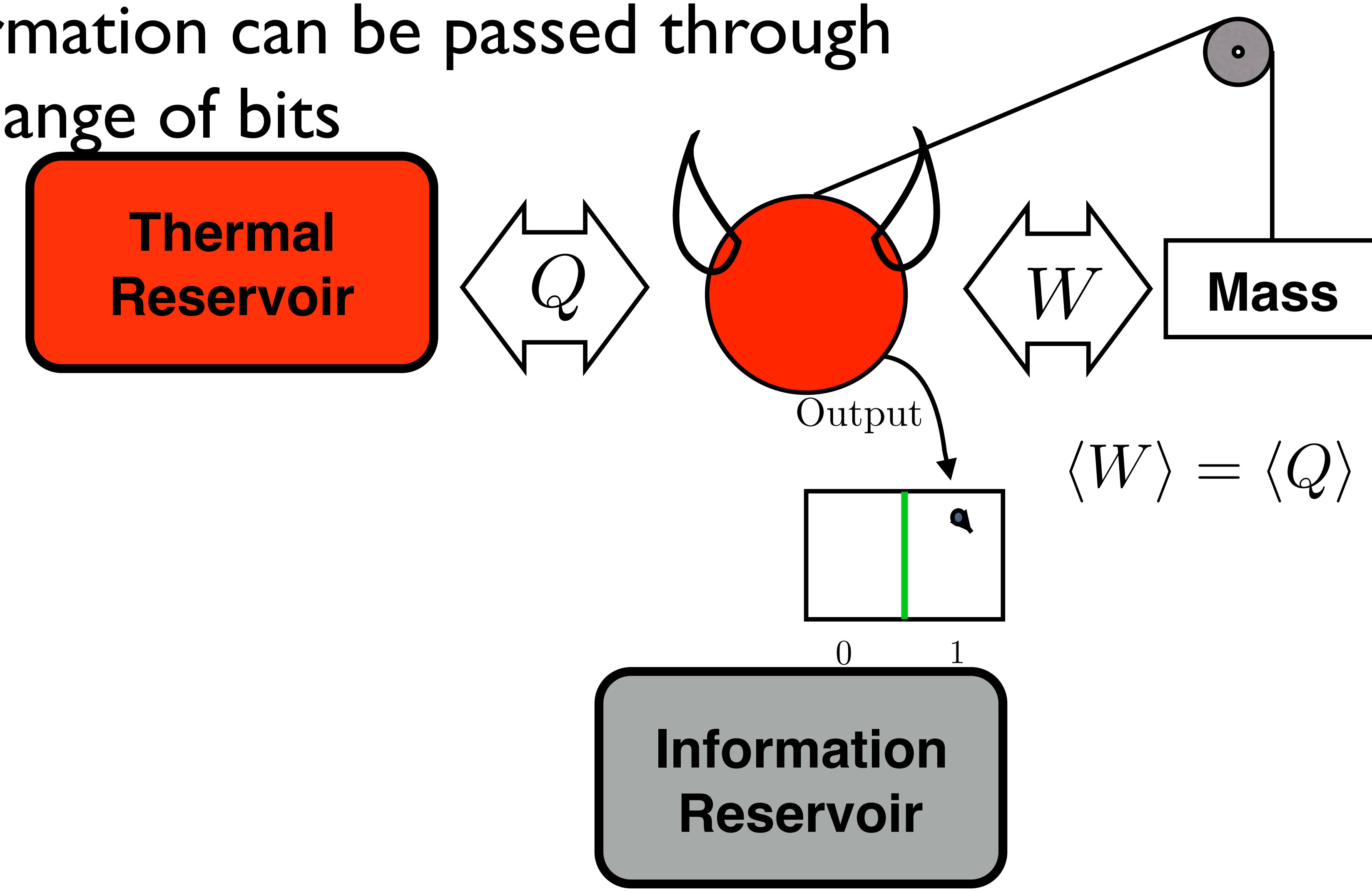

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Randomize the inputs to perpetually produce work.

Information Engine Energy Production

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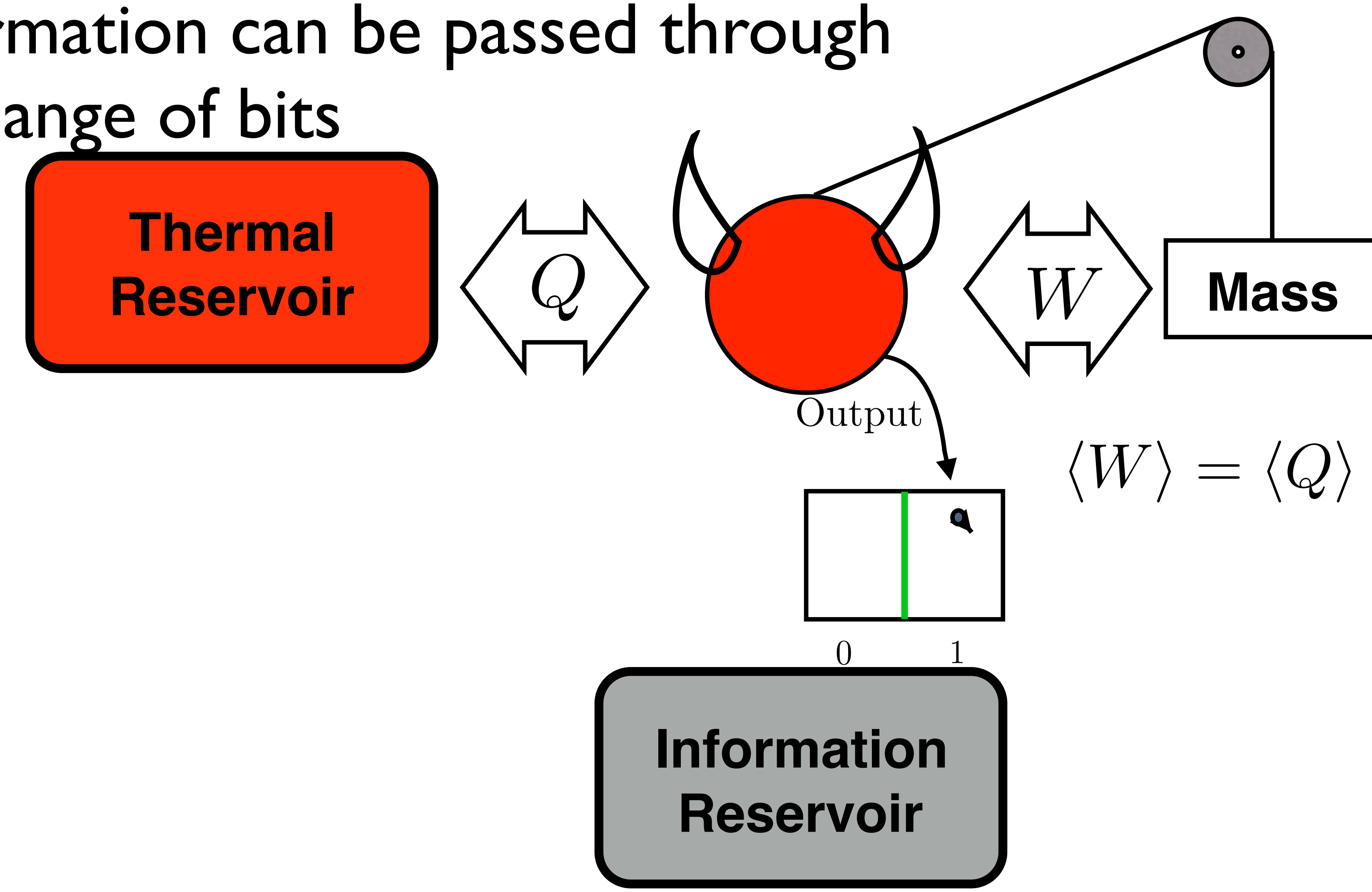

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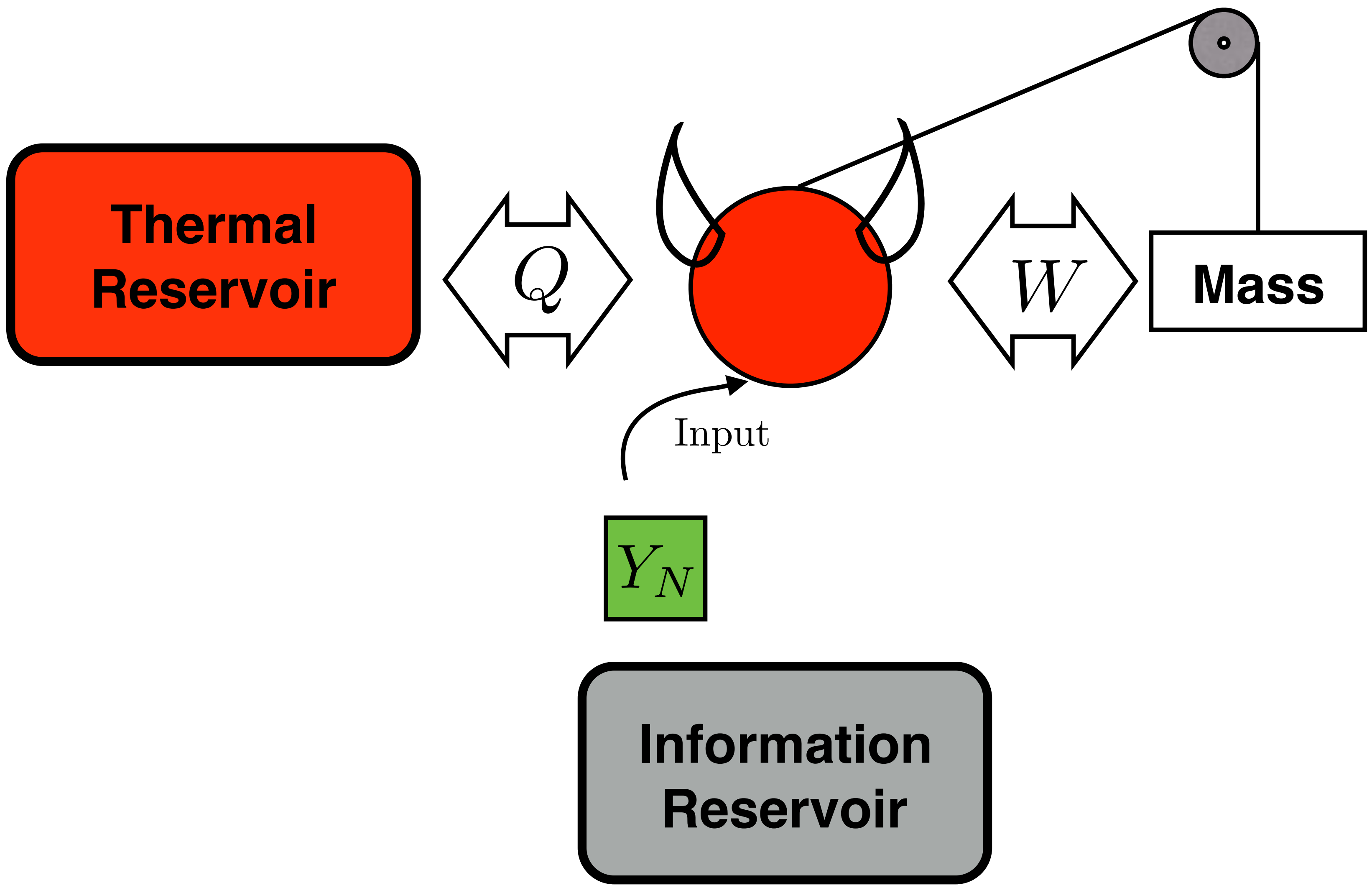
Randomize the inputs to perpetually produce work.

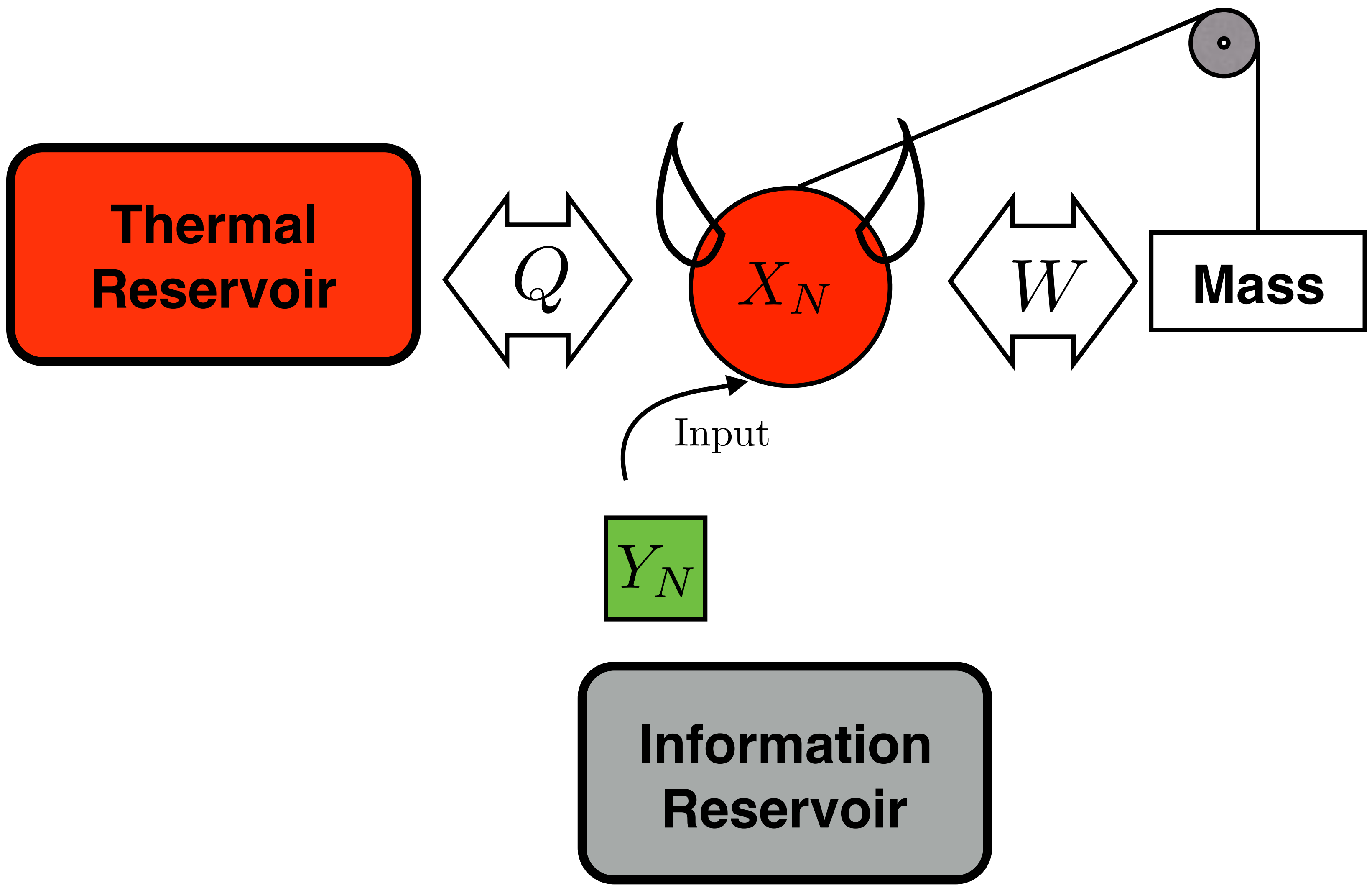
If the states have same energies, we need only describe the symbols.

Information Engine Energy Production



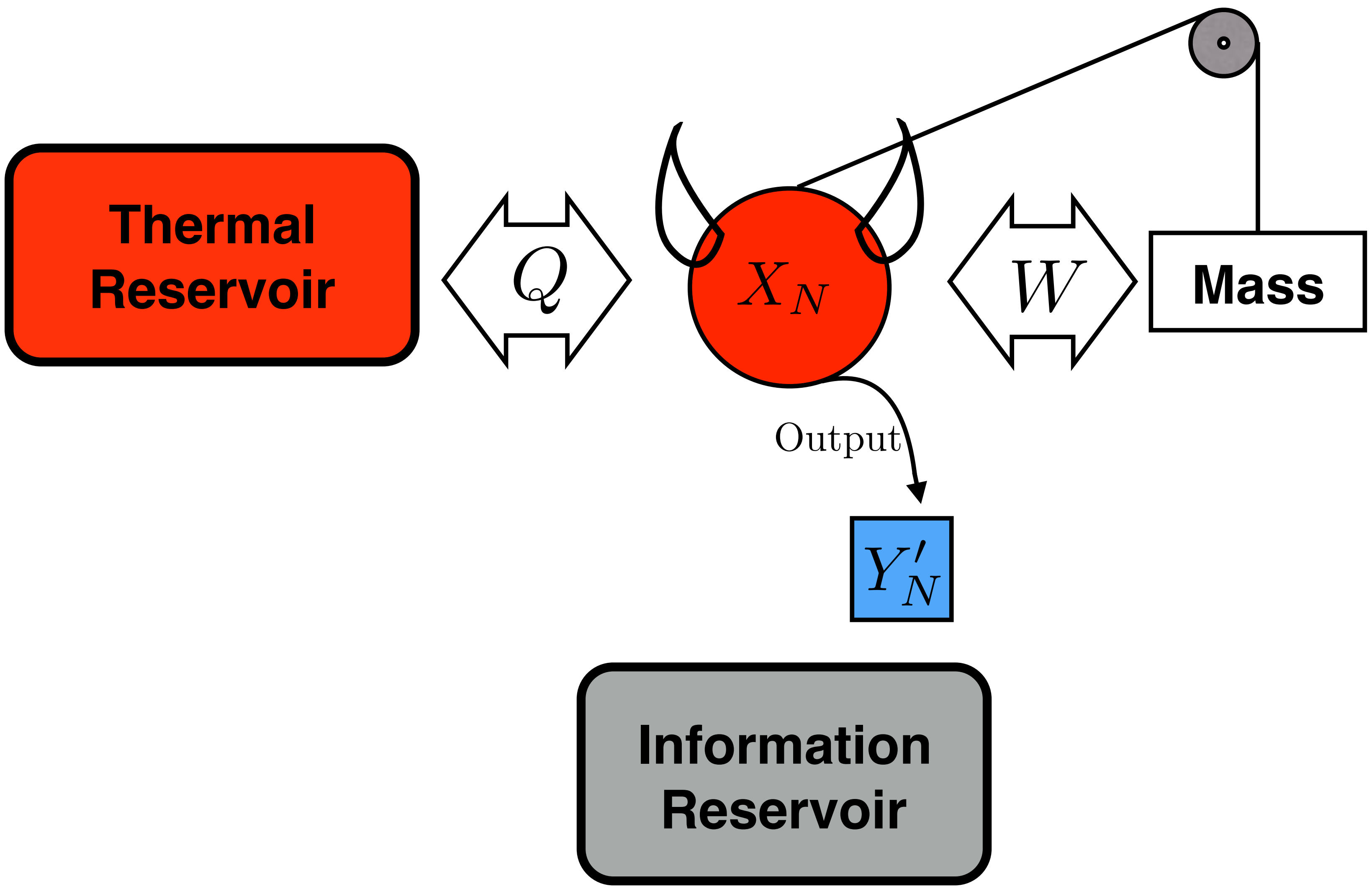
Take an input from the alphabet $\mathcal{Y}$,

Information Engine Energy Production



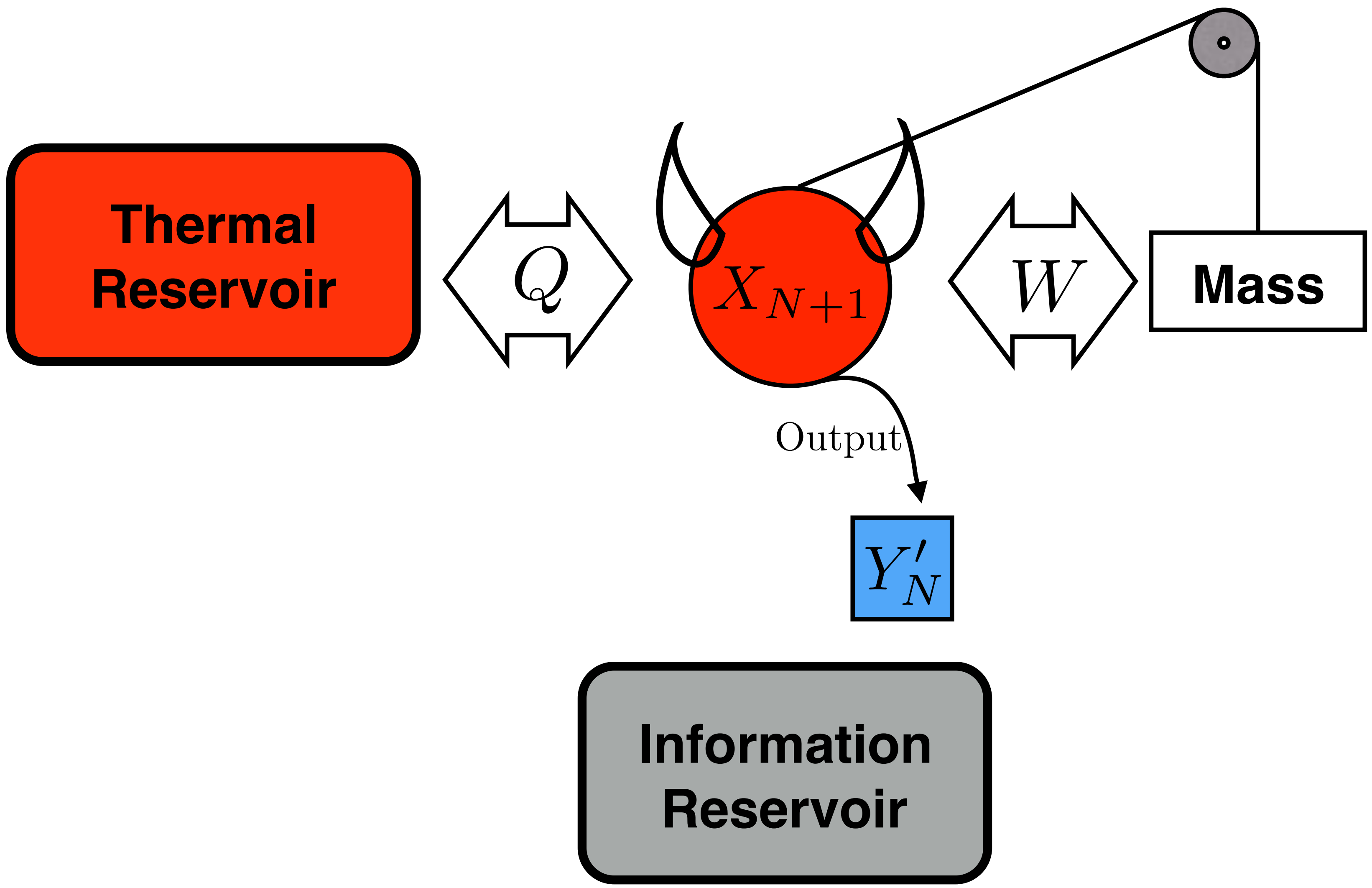
Take an input from the alphabet $\mathcal{Y}$, and, using the demon's internal state at X_N time $t = N\tau$

Information Engine Energy Production



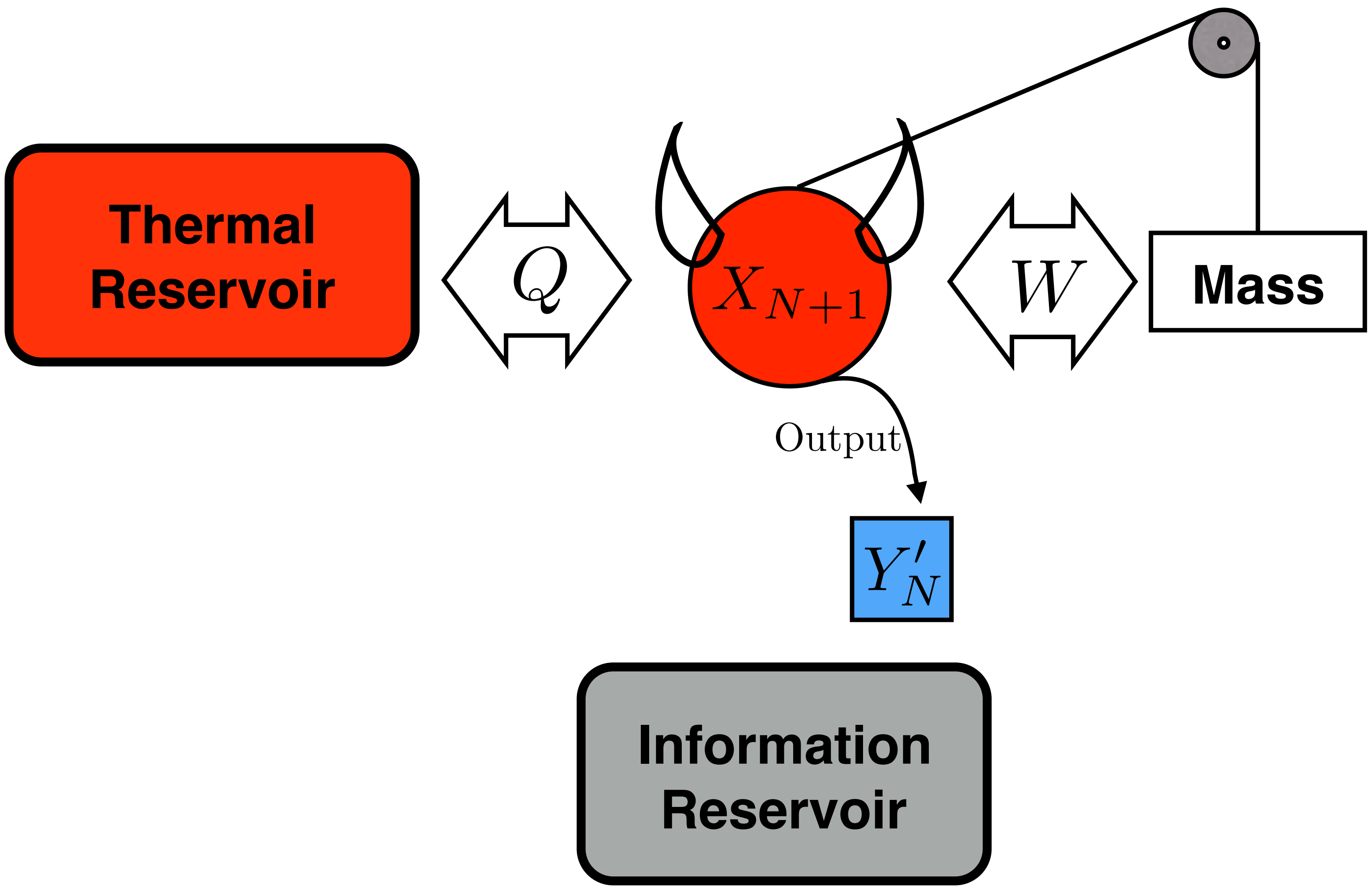
Take an input from the alphabet $\mathcal{Y}$, and, using the demon's internal state at X_N time $t = N\tau$, transform the input to an output in the same alphabet

Information Engine Energy Production



Take an input from the alphabet $\mathcal{Y}$, and, using the demon's internal state at X_N time $t = N\tau$, transform the input to an output in the same alphabet, while also updating your internal state to X_{N+1} at time $t = (N+1)\tau$

Information Engine Energy Production



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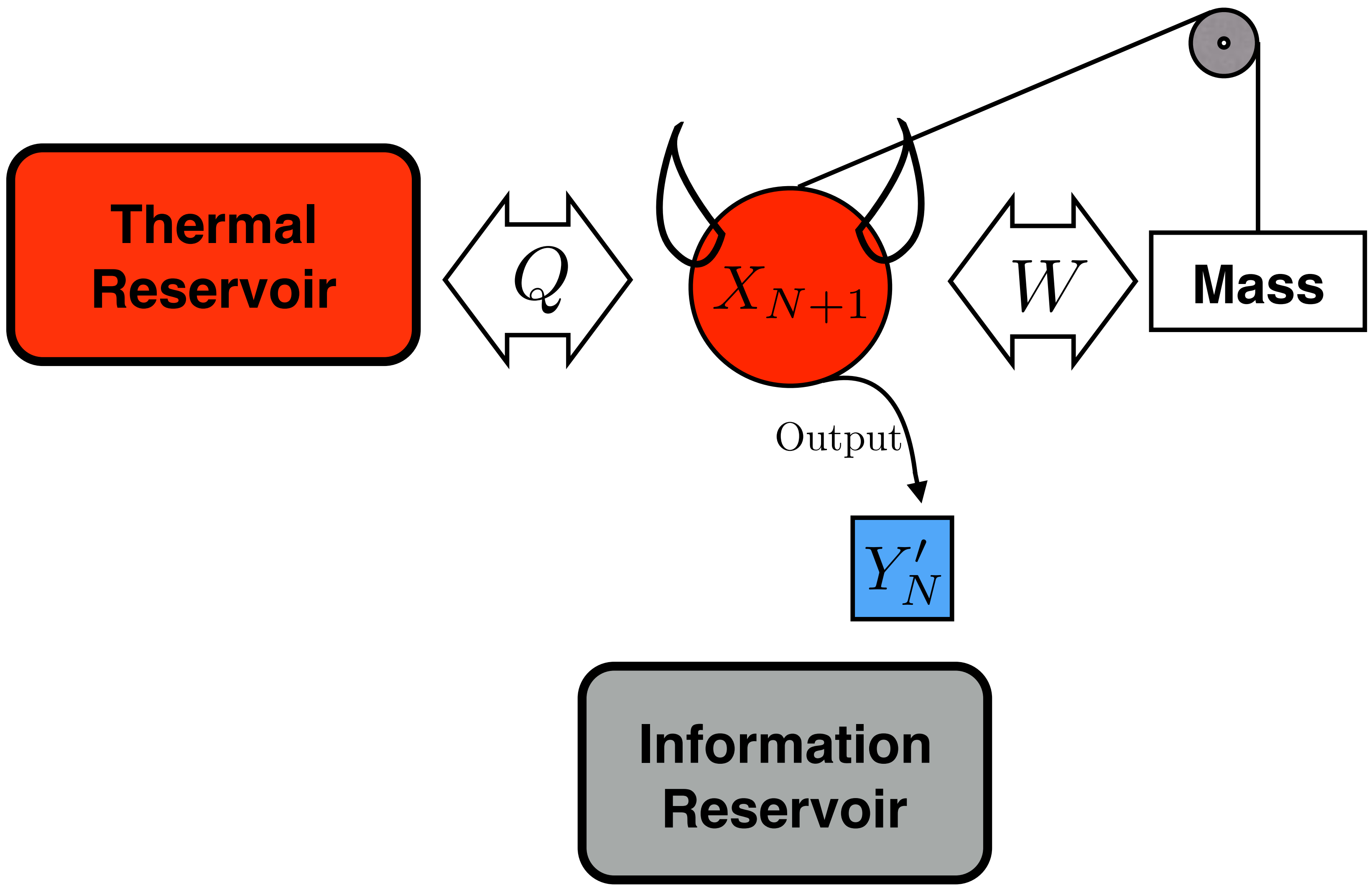
Information Engine Energy Production



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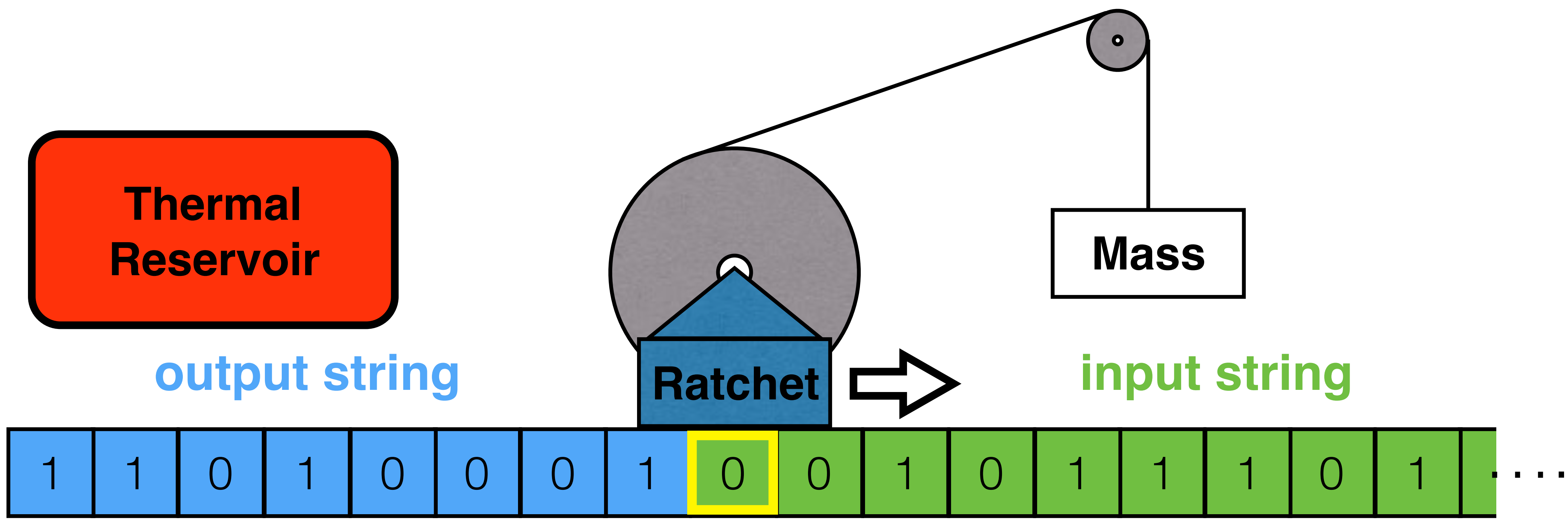
$$M_{x_N, y_N \to x_{N+1}, y'_N} = \Pr(X_{N+1} = x_{N+1}, Y'_N = y'_N | X_N = x_N, Y_N = y_N)$$

Information Ratchet

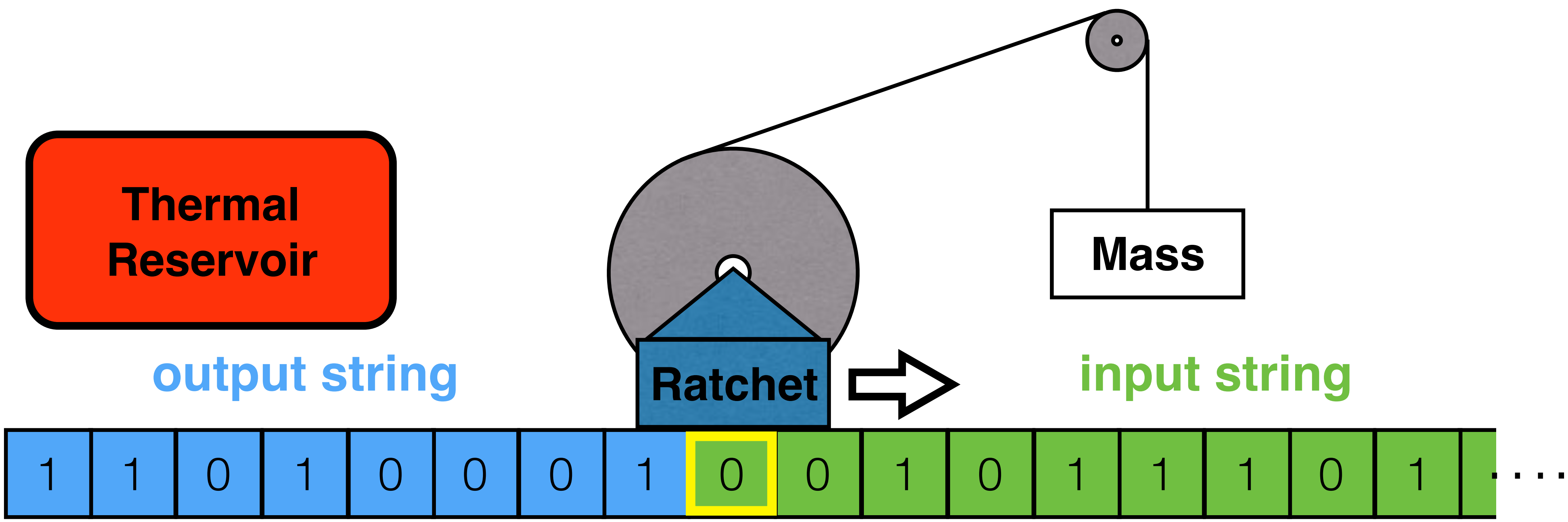
D. Mandal and C. Jarzynski. Work and information processing in a solvable model of Maxwell's demon. Proc. Natl. Acad. Sci. USA, 109(29):11641–11645, 2012.



Information Ratchet

Exactly solvable autonomous Maxwellian demon/information engine

D. Mandal and C. Jarzynski. Work and information processing in a solvable model of Maxwell's demon. Proc. Natl. Acad. Sci. USA, 109(29):11641–11645, 2012.



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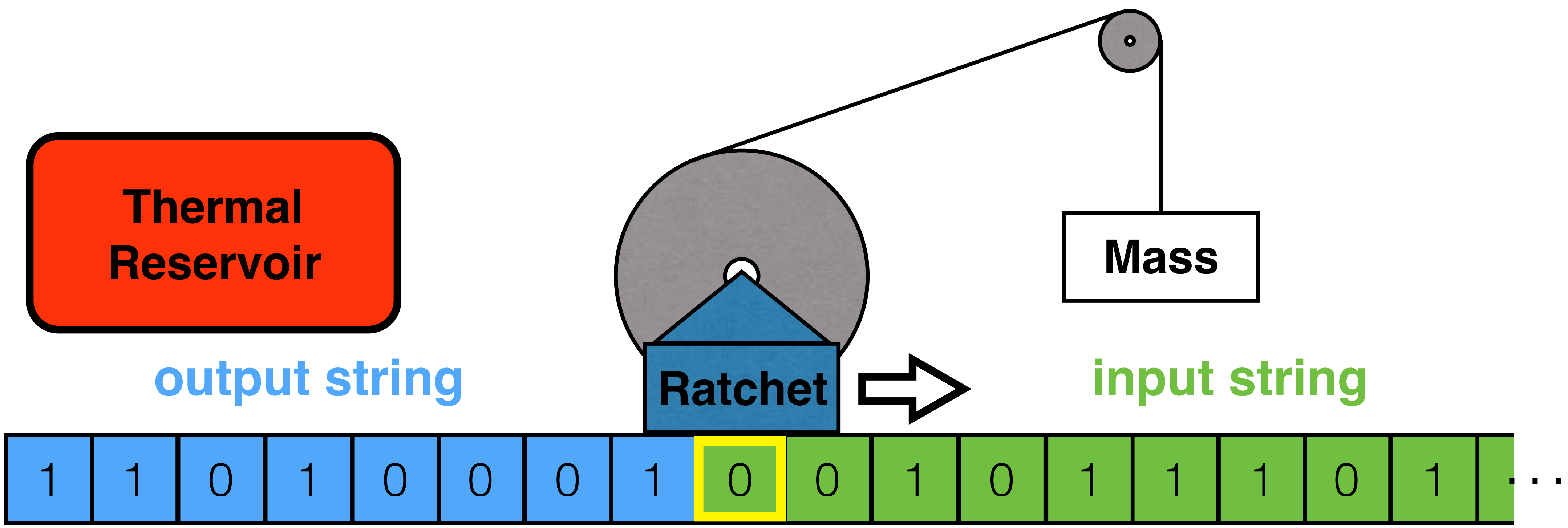
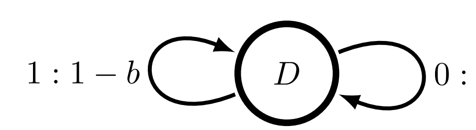


HMM Input Generator: 

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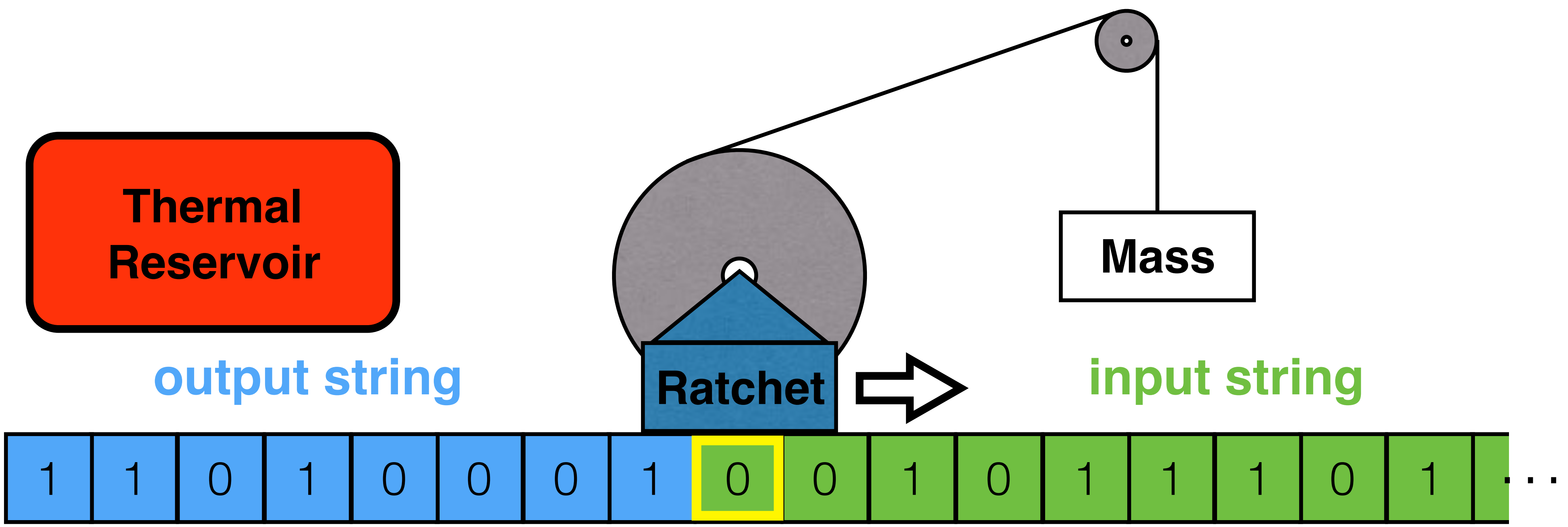
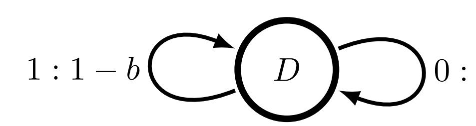
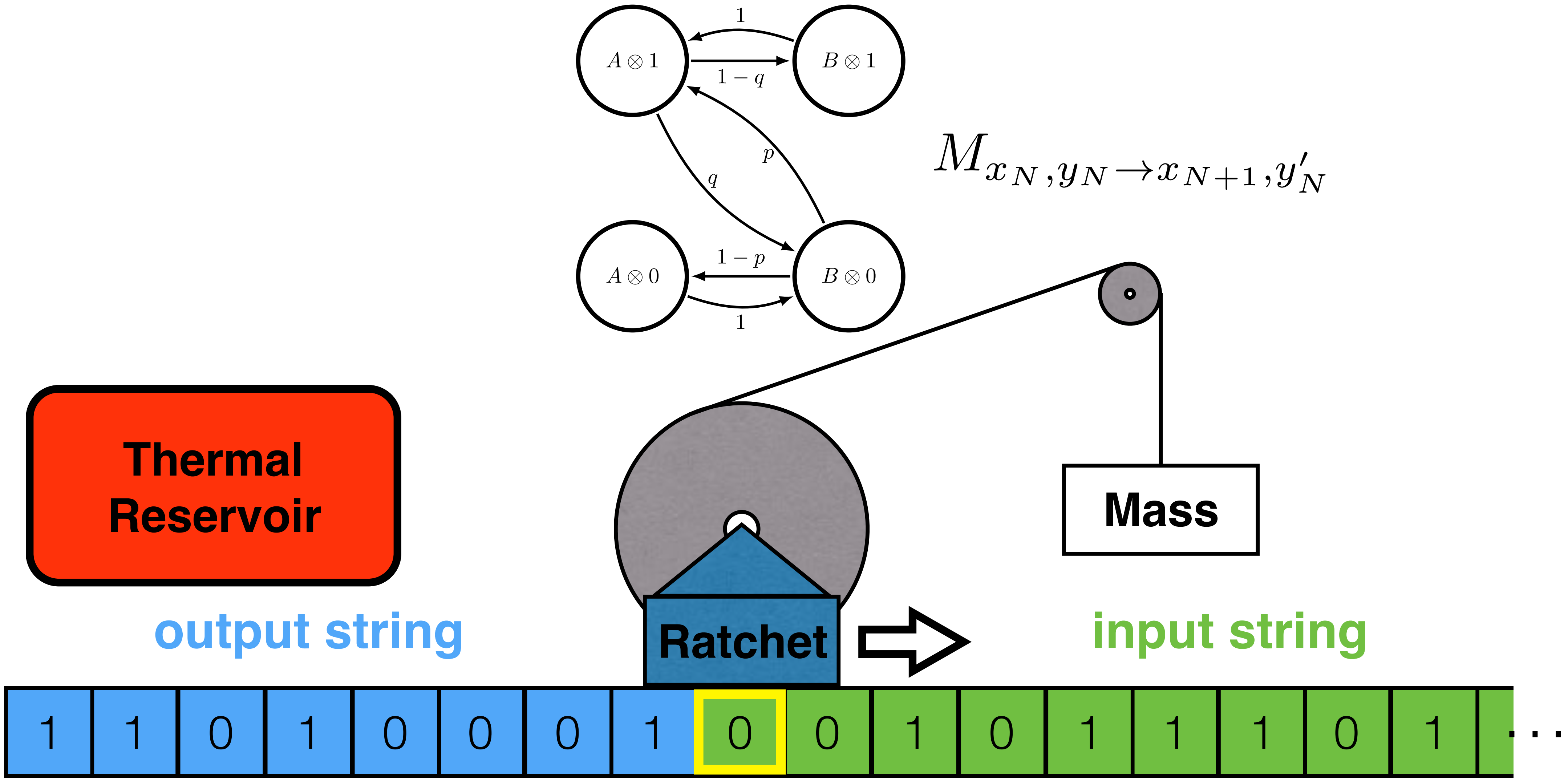
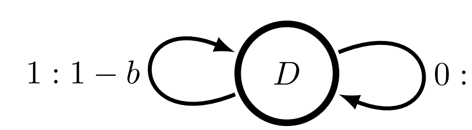


HMM Input Generator: 

$$T^{(y_N)}_{s_N \to s_{N+1}} = \Pr(S_{N+1} = s_{N+1}, Y_N = y_N | S_N = s_N)$$

Information Ratchet



HMM Input Generator: 

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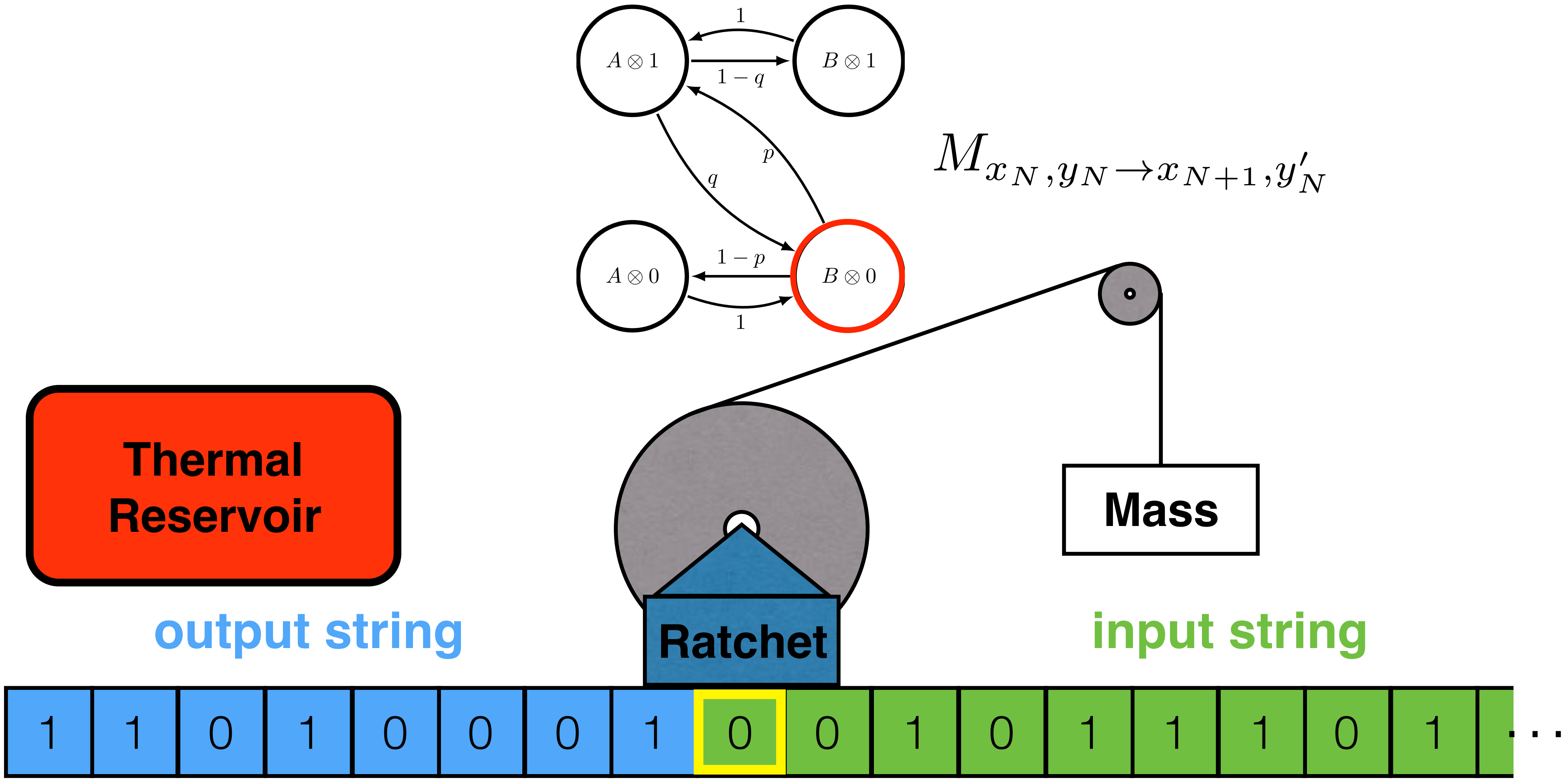
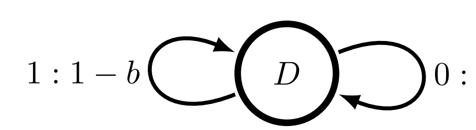
Memoryful Ratchet Interaction



HMM Input Generator: 

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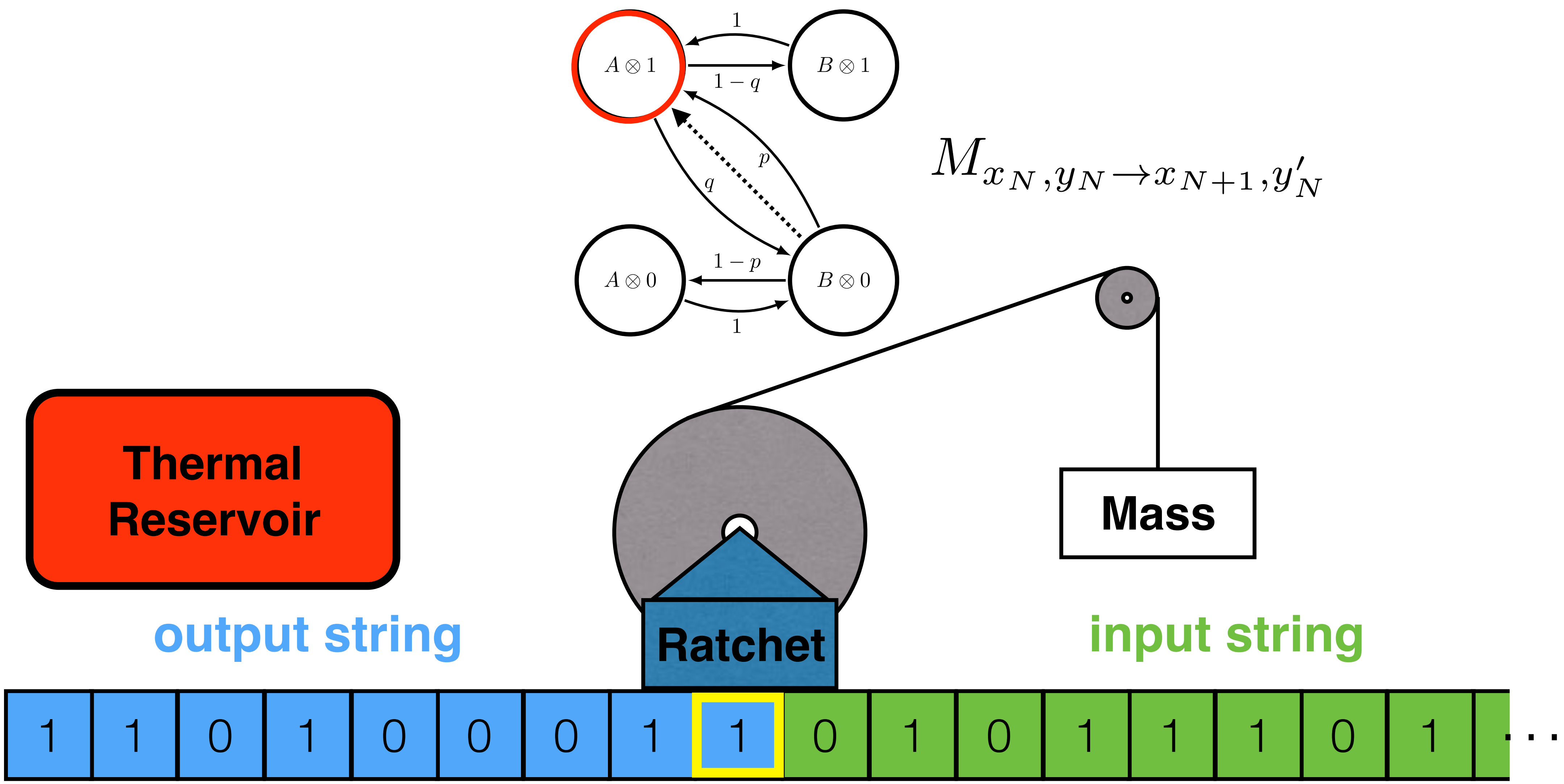
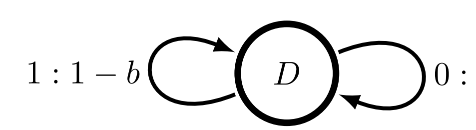


HMM Input Generator: 

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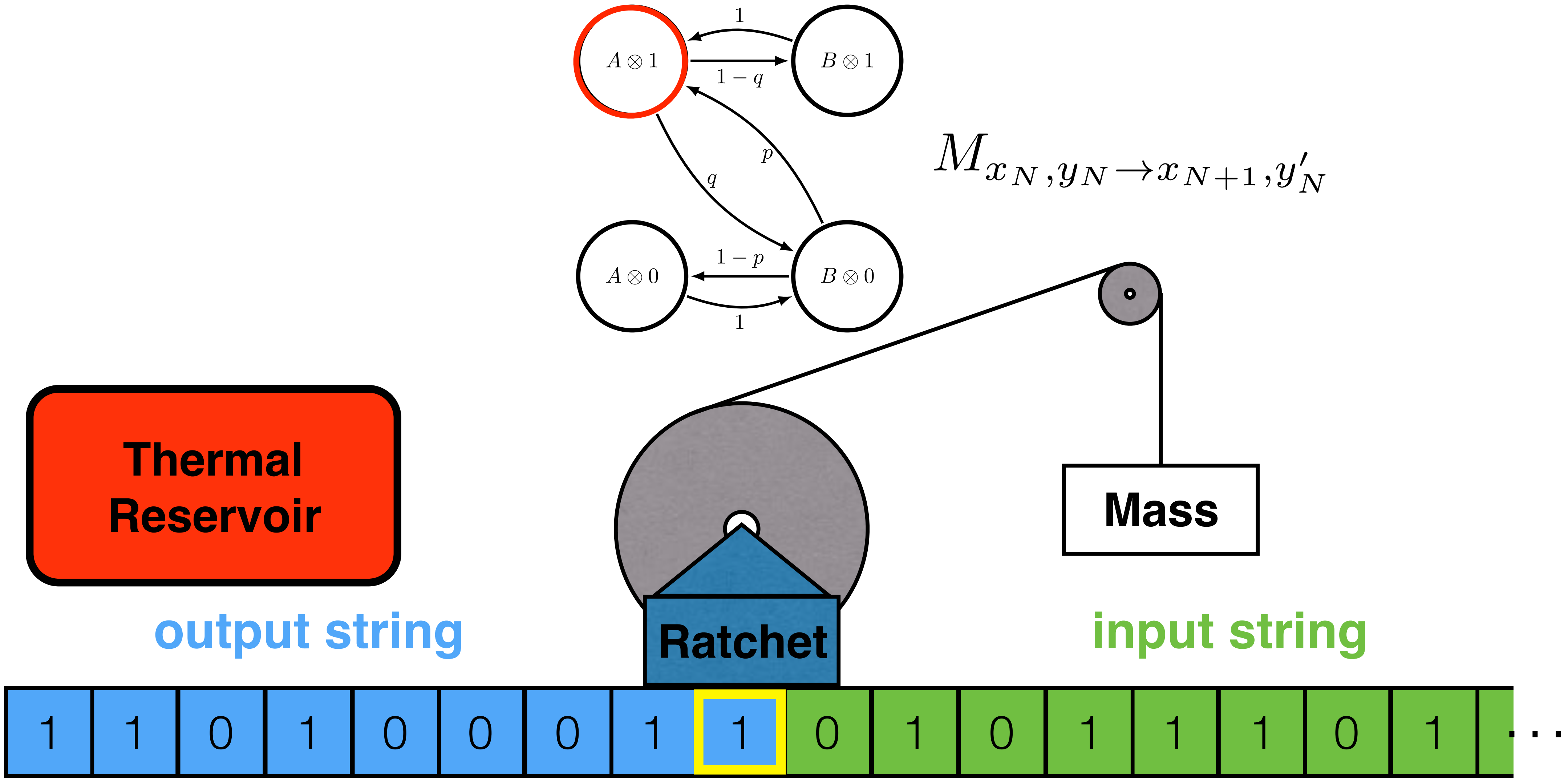
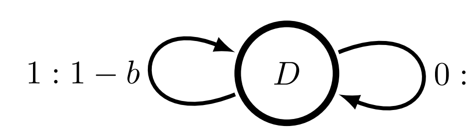
Memoryful Ratchet Switching



HMM Input Generator:



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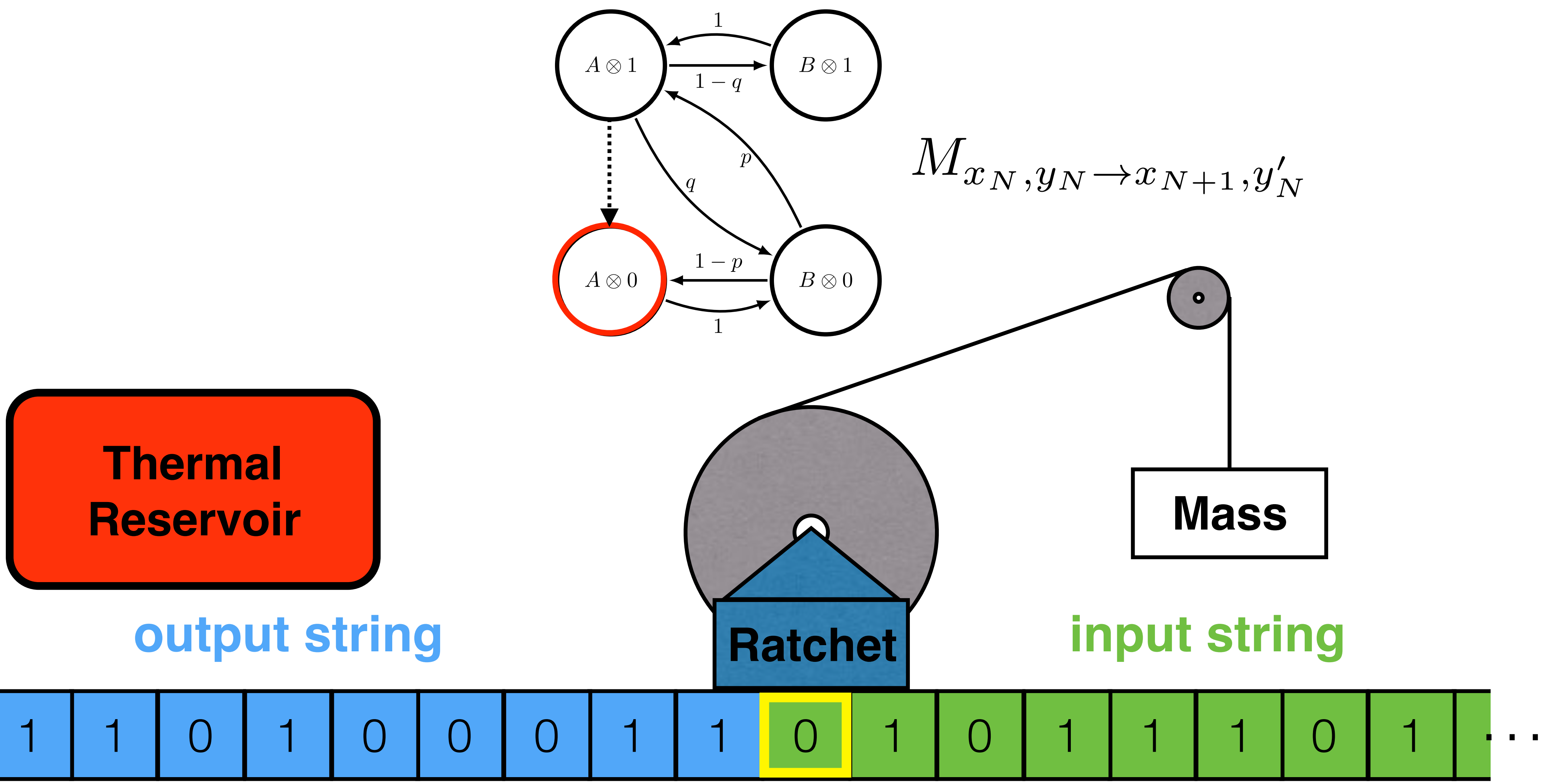
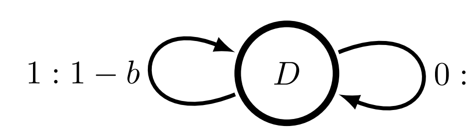


$$M_{x_N, y_N \to x_{N+1}, y'_N}$$

HMM Input Generator: 

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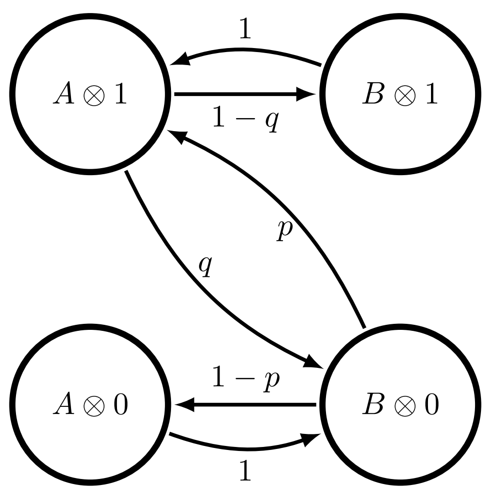
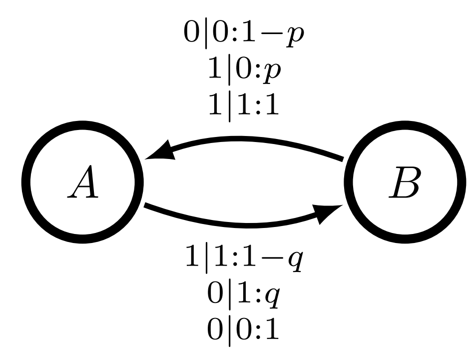
Physical Information Transduction

Channel representing ratchet is also memoryful (multiple hidden states):

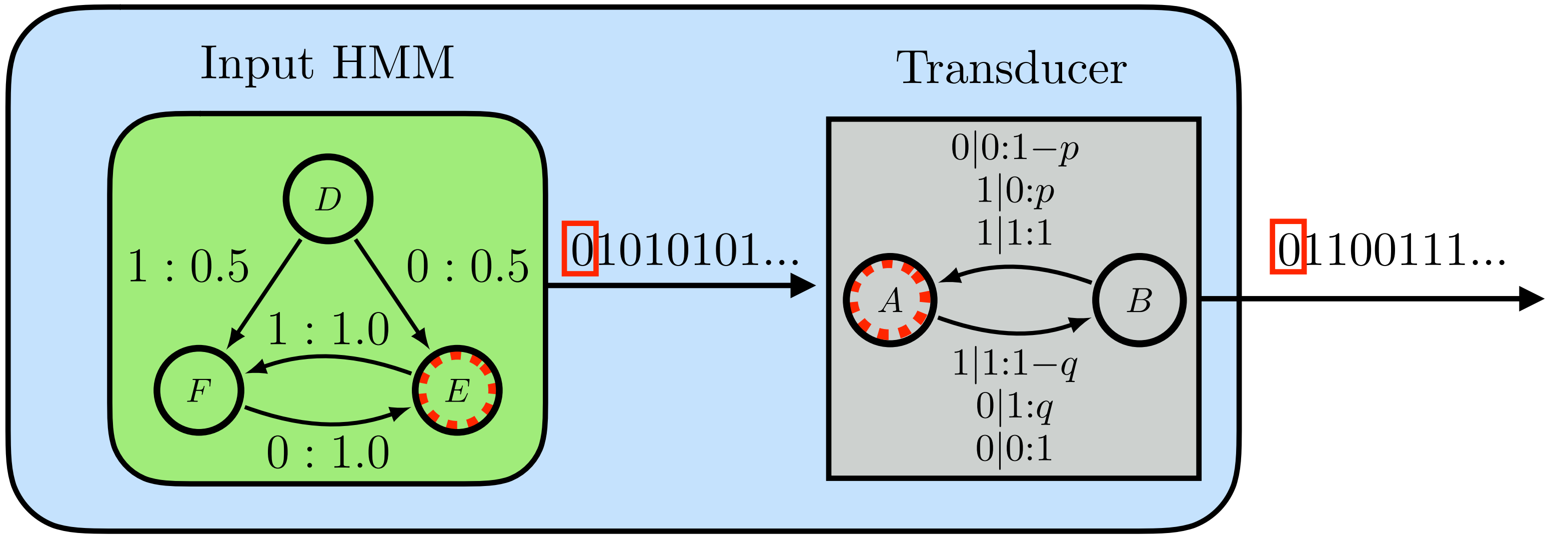


Channel:



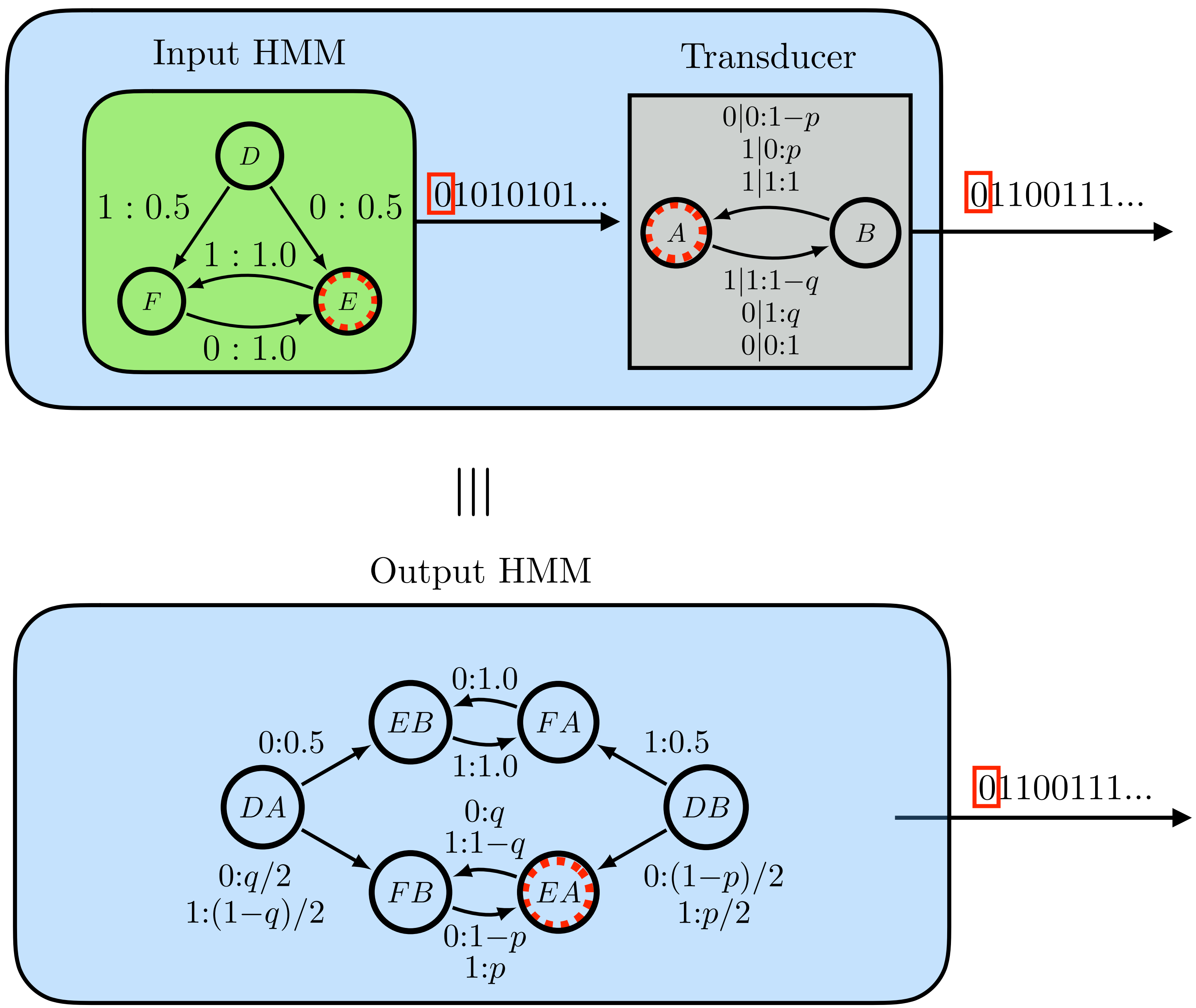
Transducers

Memoryful channel determines output process:

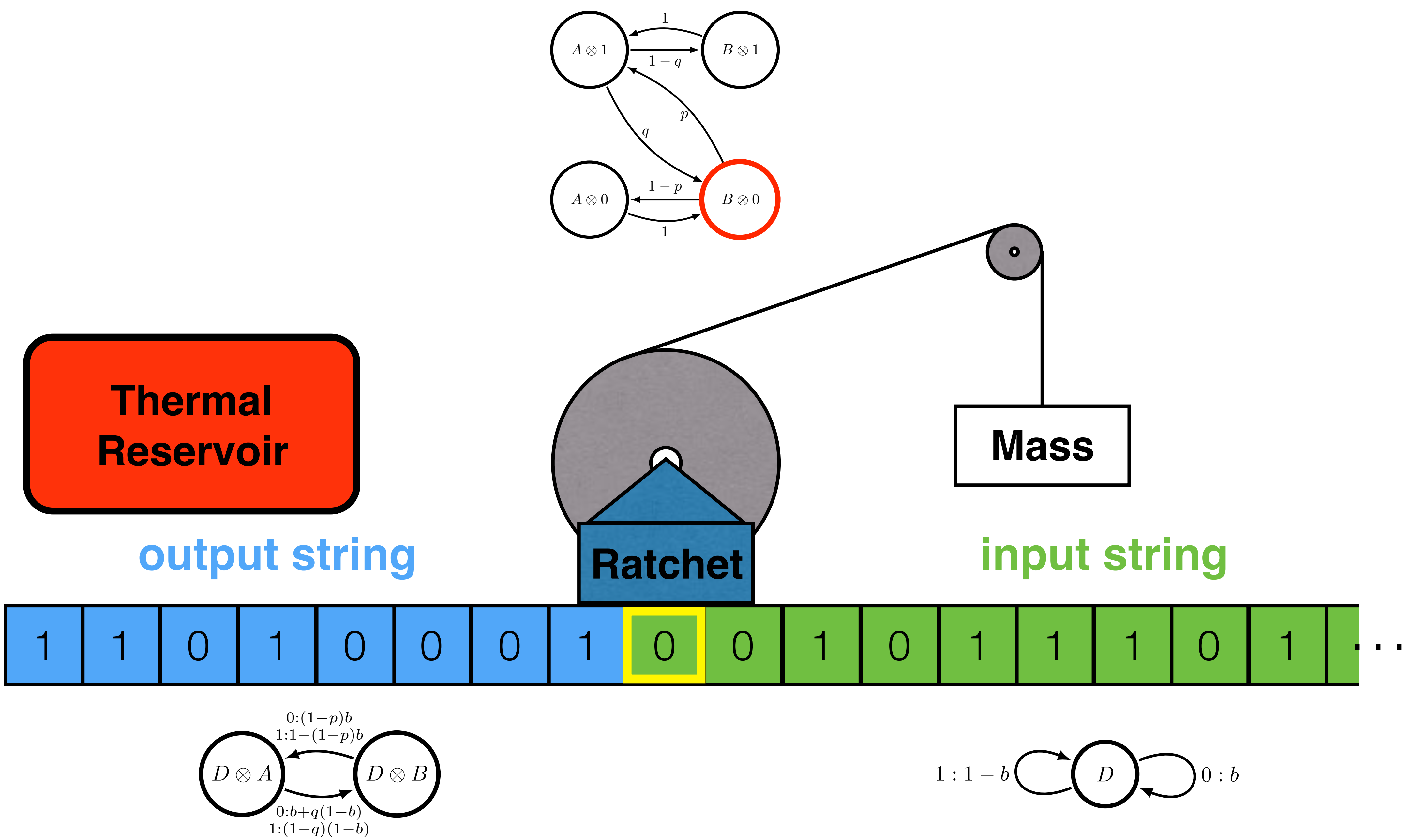


Transducers

Paradigm for information processing

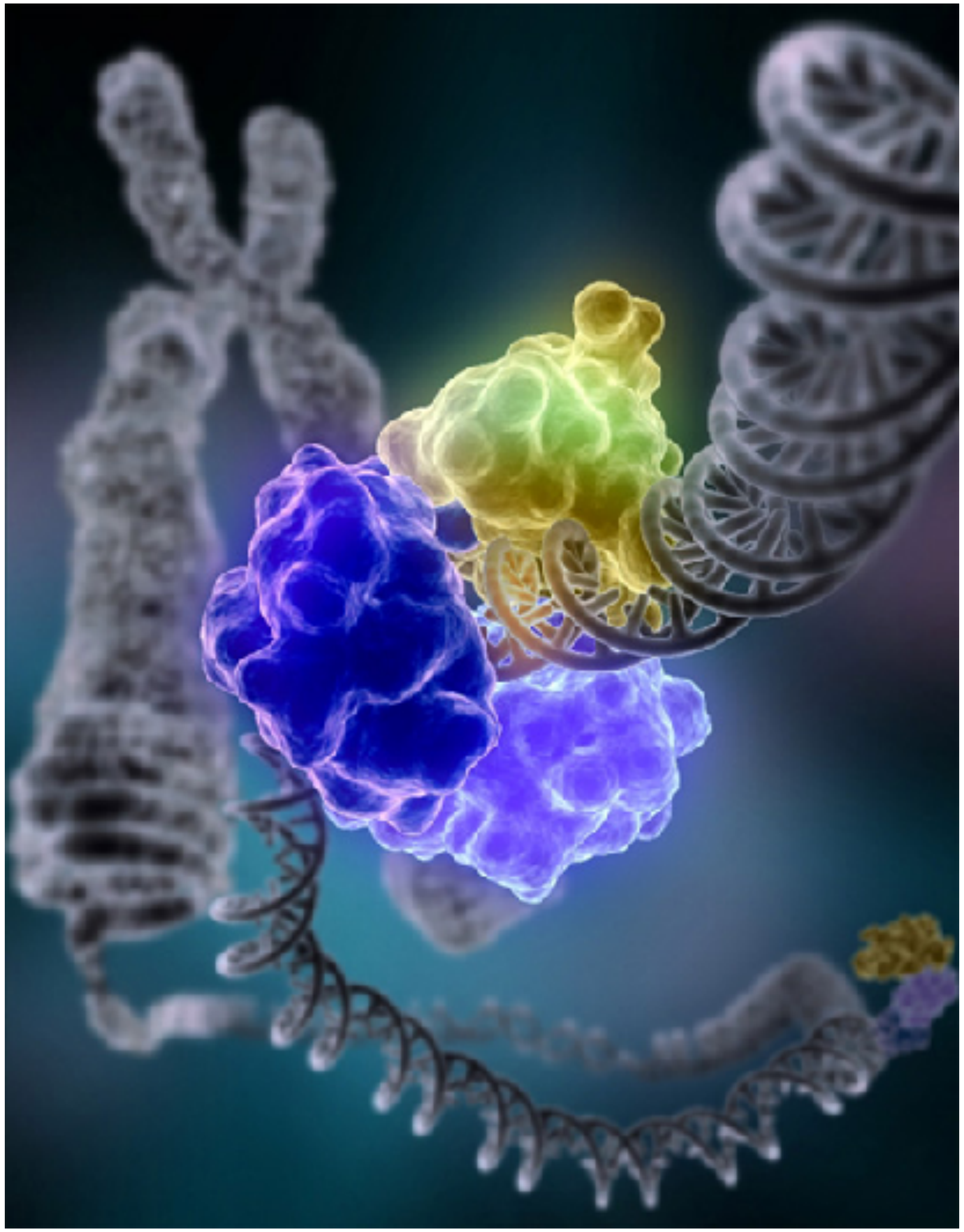


Memoryful Ratchet Interaction



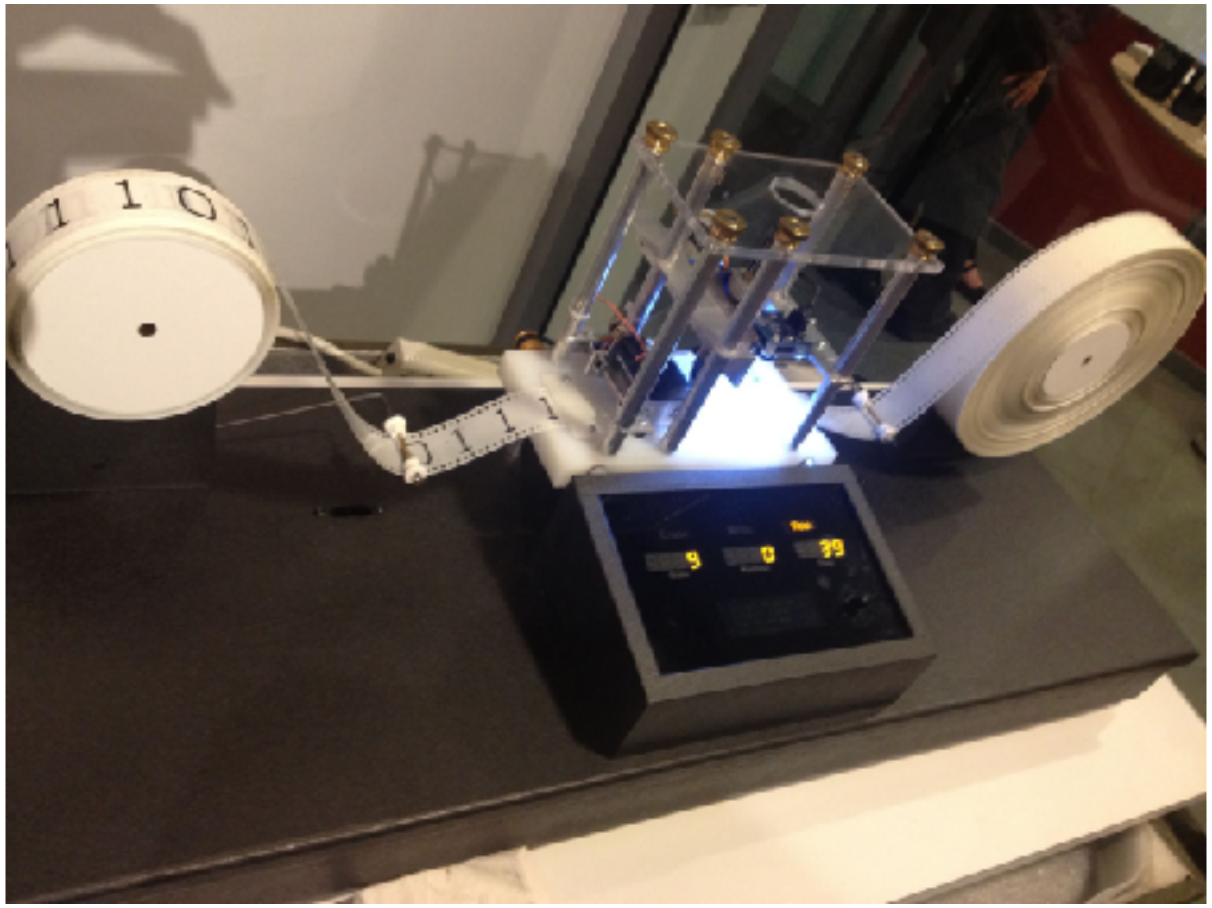
Physical/Computational Comparisons

DNA Ligase



https://en.wikipedia.org/wiki/DNA_ligase

Turing Machine



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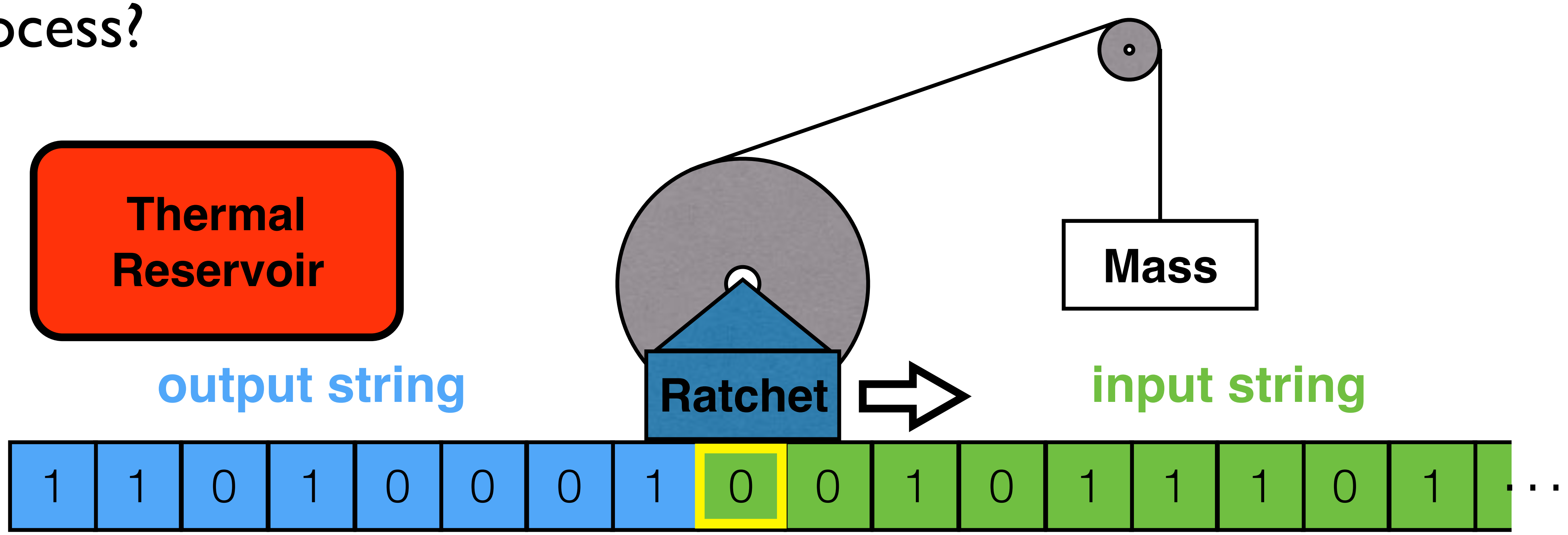
Thermodynamics of Transducers

How much work must we invest to execute an input-ouput process?



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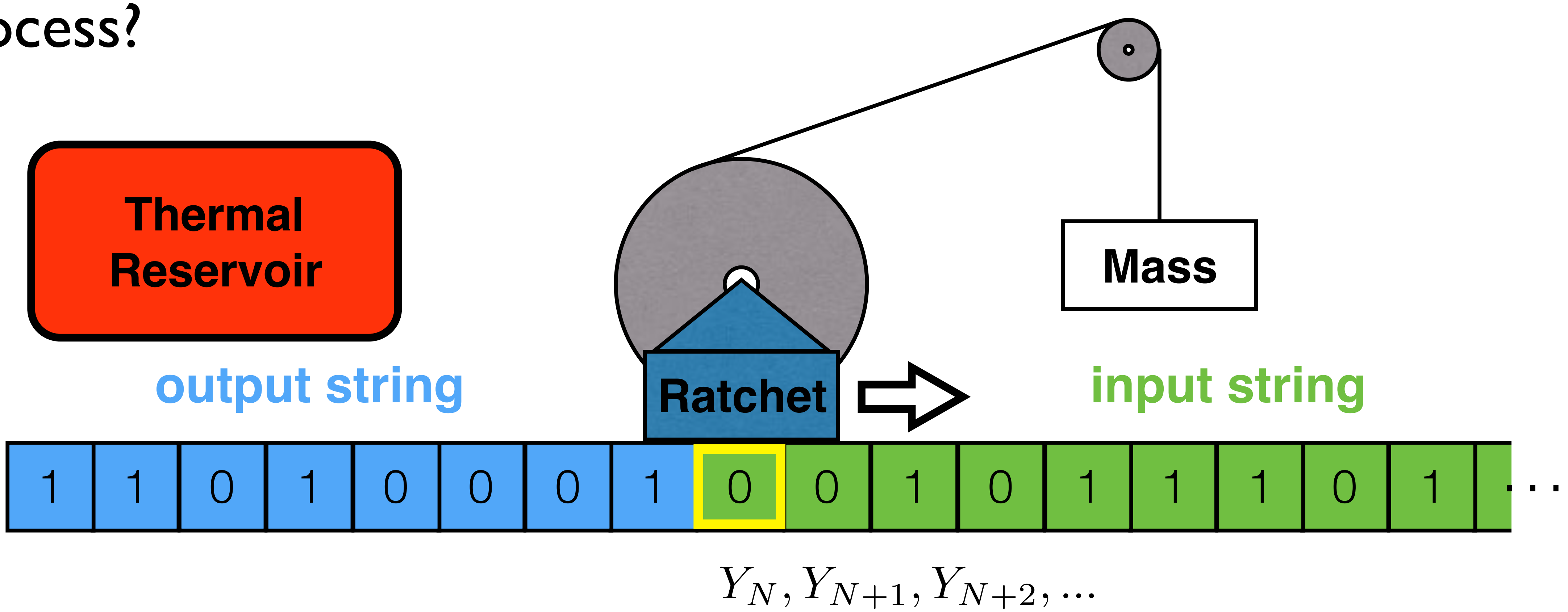

Information reservoir includes inputs $Y_N, Y_{N+1}, Y_{N+2}, \ldots$

Thermodynamics of Transducers

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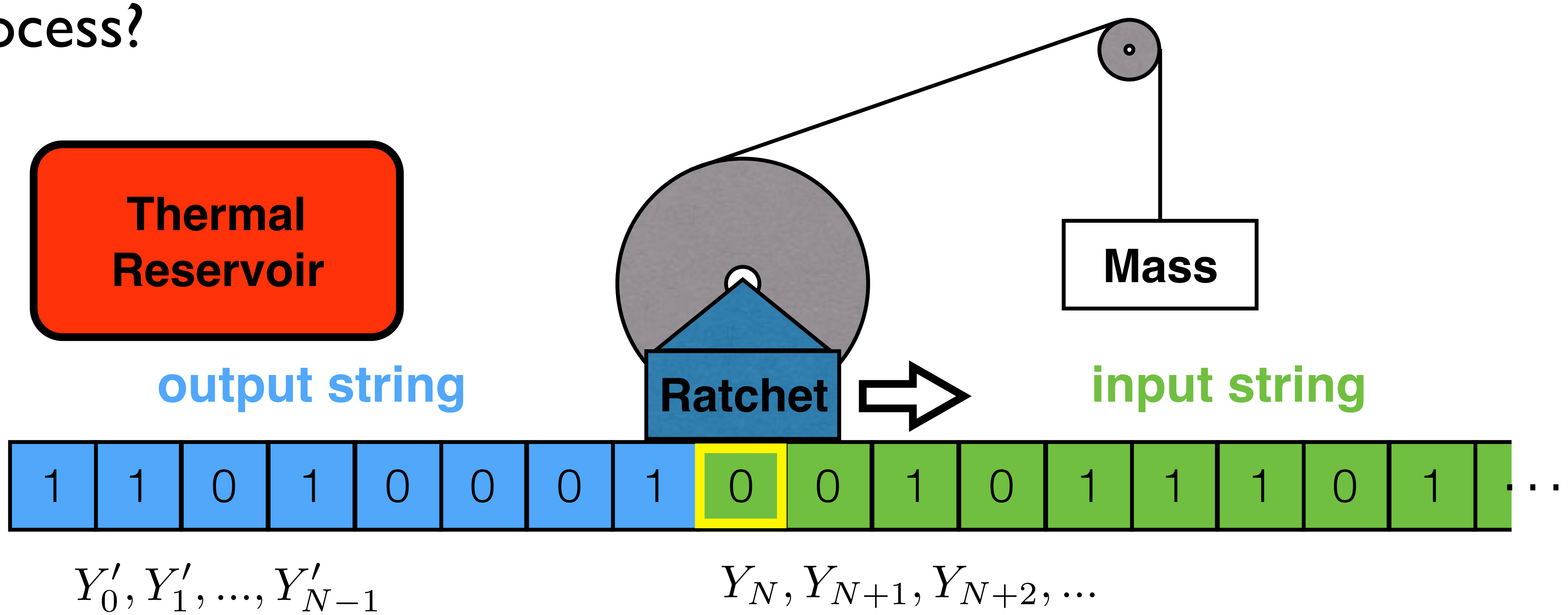

Information reservoir includes inputs $Y_N, Y_{N+1}, Y_{N+2}, ...$, outputs $Y'_0, Y'_1, ..., Y'_{N-1}$

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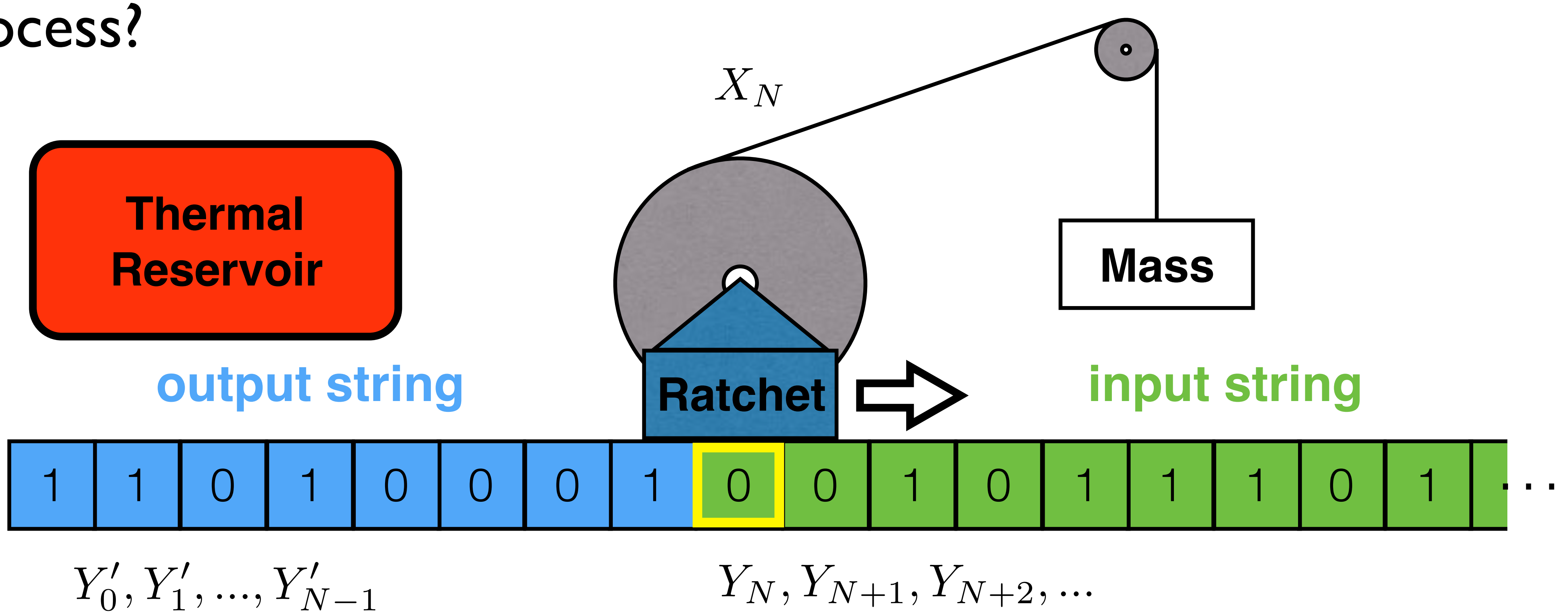

Information reservoir includes inputs $Y_N, Y_{N+1}, Y_{N+2}, ...$, outputs $Y'_0, Y'_1, ..., Y'_{N-1}$, and ratchet/demon state X_N.

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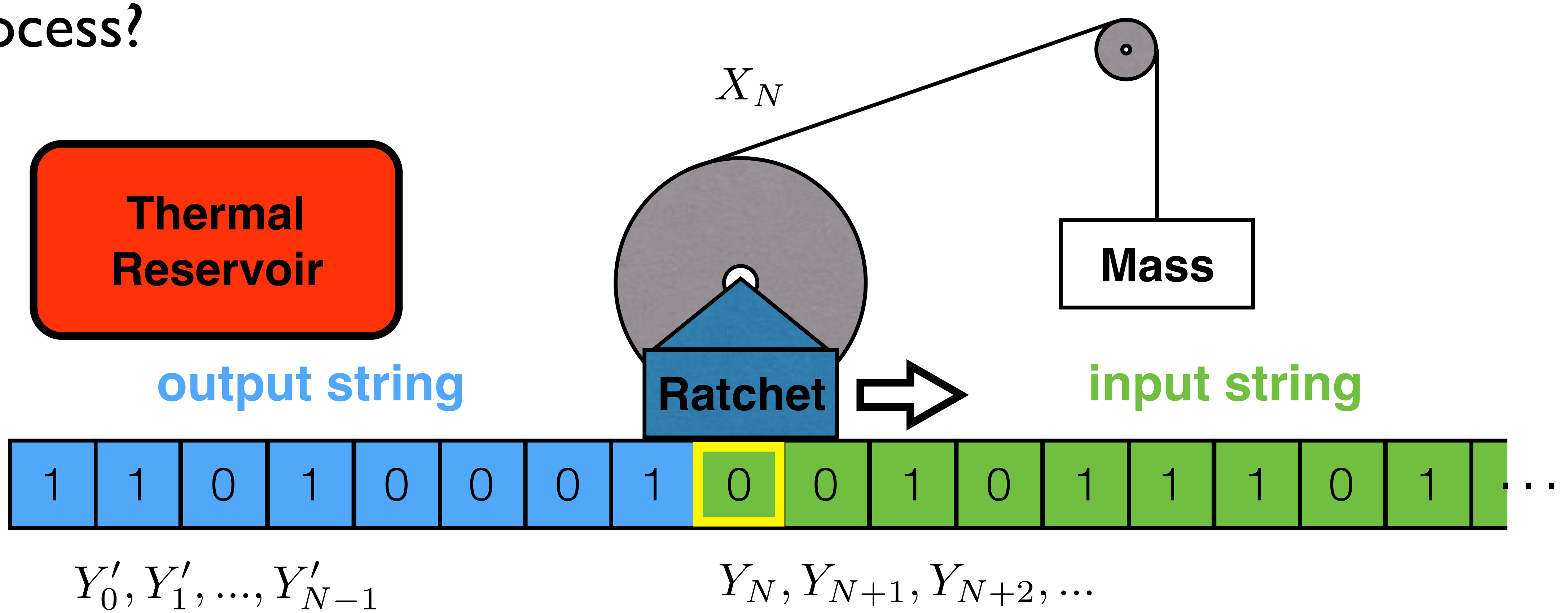


Information reservoir includes inputs $Y_N, Y_{N+1}, Y_{N+2}, ...$, outputs $Y'_0, Y'_1, ..., Y'_{N-1}$, and ratchet/demon state X_N.

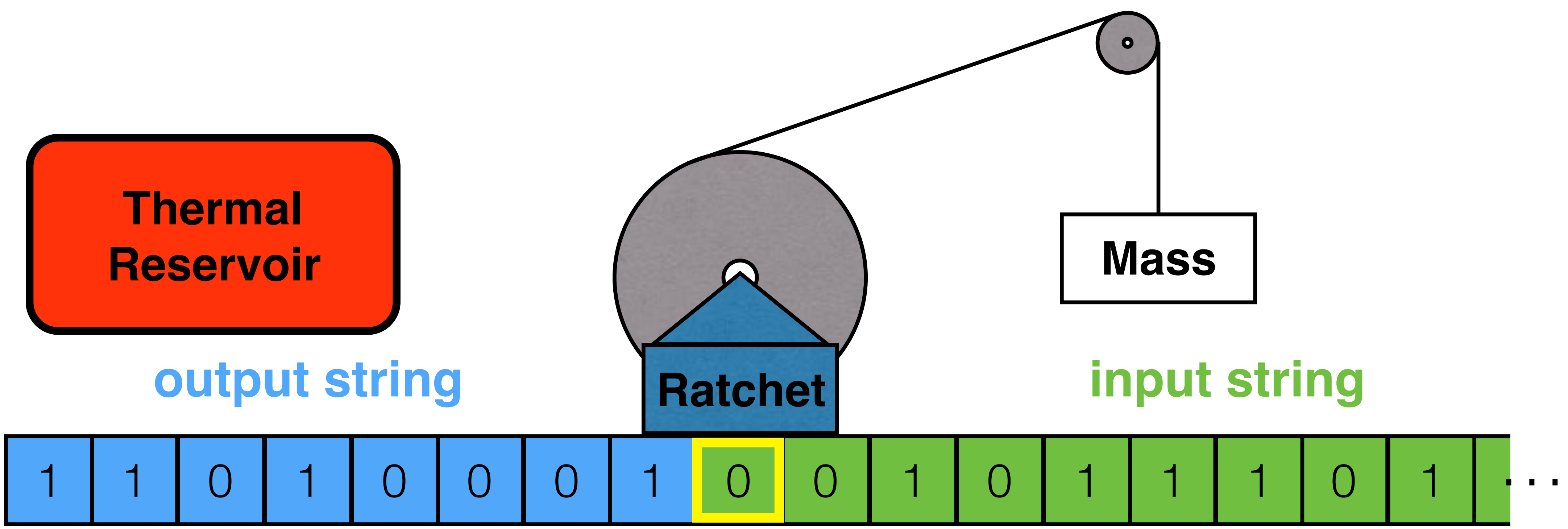
The change in free energy is:

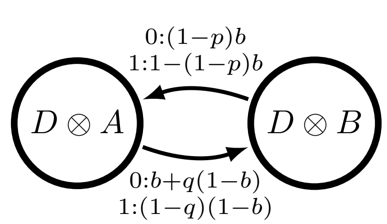
$$\Delta F_{t:N\tau \to N'\tau} = k_B T \ln 2 (H[X_N, Y_{N:\infty}, Y'_{0:N}] - H[X_{N'}, Y_{N':\infty}, Y'_{0:N'}])$$

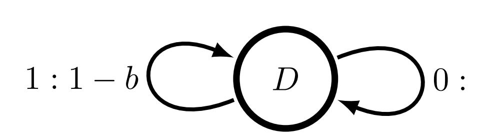
Asymptotic Work Invested

The work investment is bounded by non-equilibrium free energy

$\langle W_{t:N\tau \to N'\tau} \rangle \geq \Delta F_{t:N\tau \to N'\tau}$



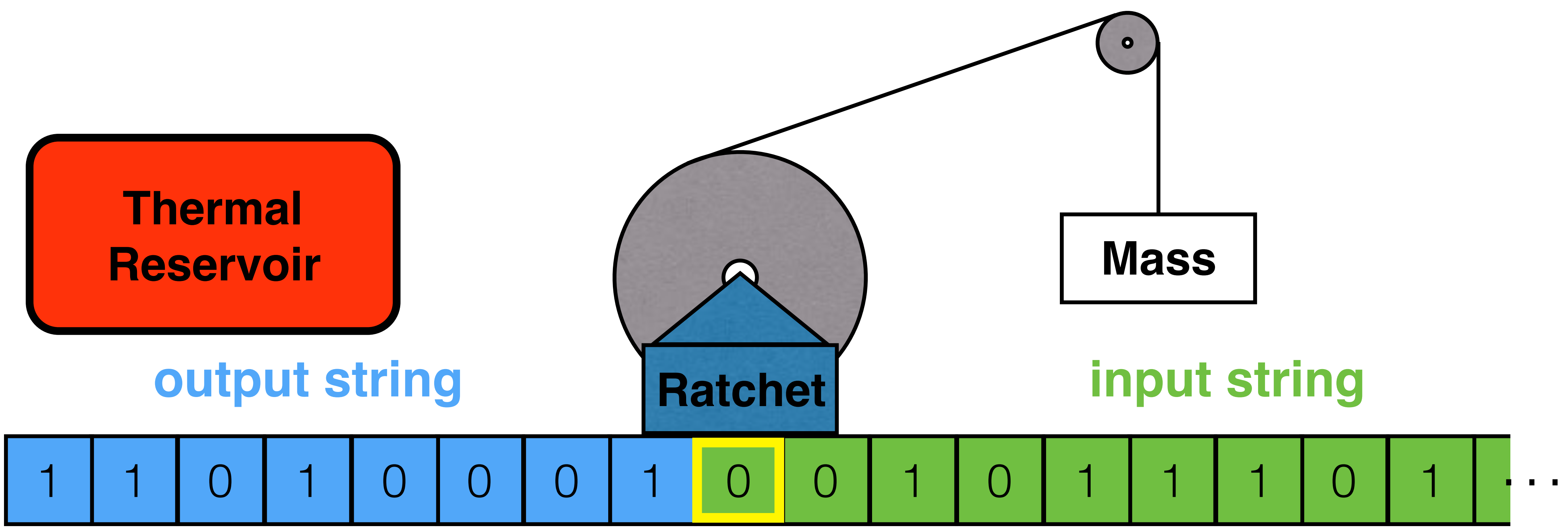


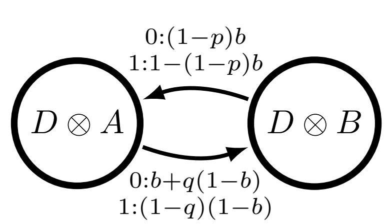


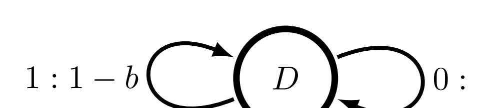
Asymptotic Work Invested

The work investment is bounded by non-equilibrium free energy

$$
\begin{aligned}
\langle W_{t:N\tau \to N'\tau} \rangle &\geq \Delta F_{t:N\tau \to N'\tau} \\
&= k_B T \ln 2 (H[X_N, Y_{N:\infty}, Y'_{0:N}] - H[X_{N'}, Y_{N':\infty}, Y'_{0:N'}])
\end{aligned}
$$







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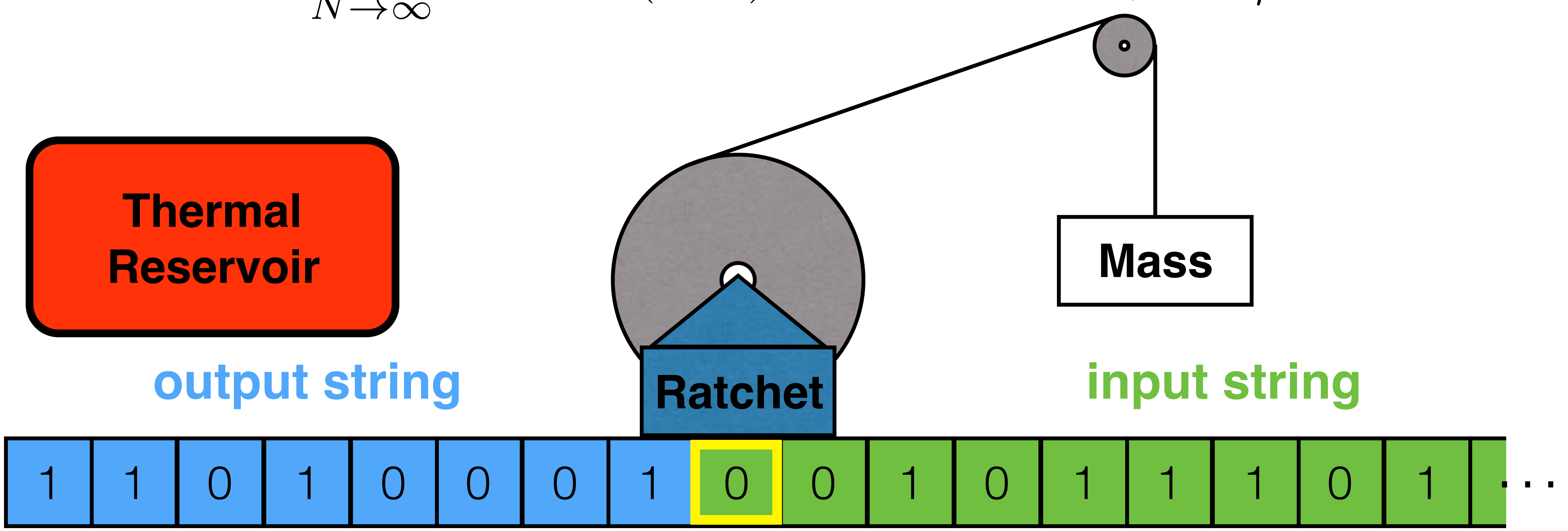
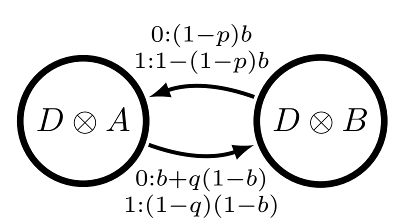
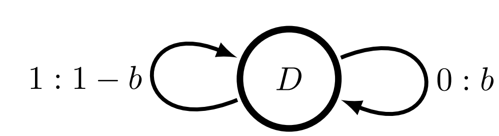
Asymptotically, the work investment is bounded by the difference in entropy rates of the input and output process

$$\text{IPSL:} \quad \lim_{N \to \infty} \langle W_{t:N\tau \to (N+1)\tau} \rangle \geq k_B T \ln 2 (h_\mu - h'_\mu)$$







Asymptotic Work Invested

The work investment is bounded by non-equilibrium free energy

$$\langle W_{t:N\tau \to N'\tau} \rangle \geq \Delta F_{t:N\tau \to N'\tau}$$
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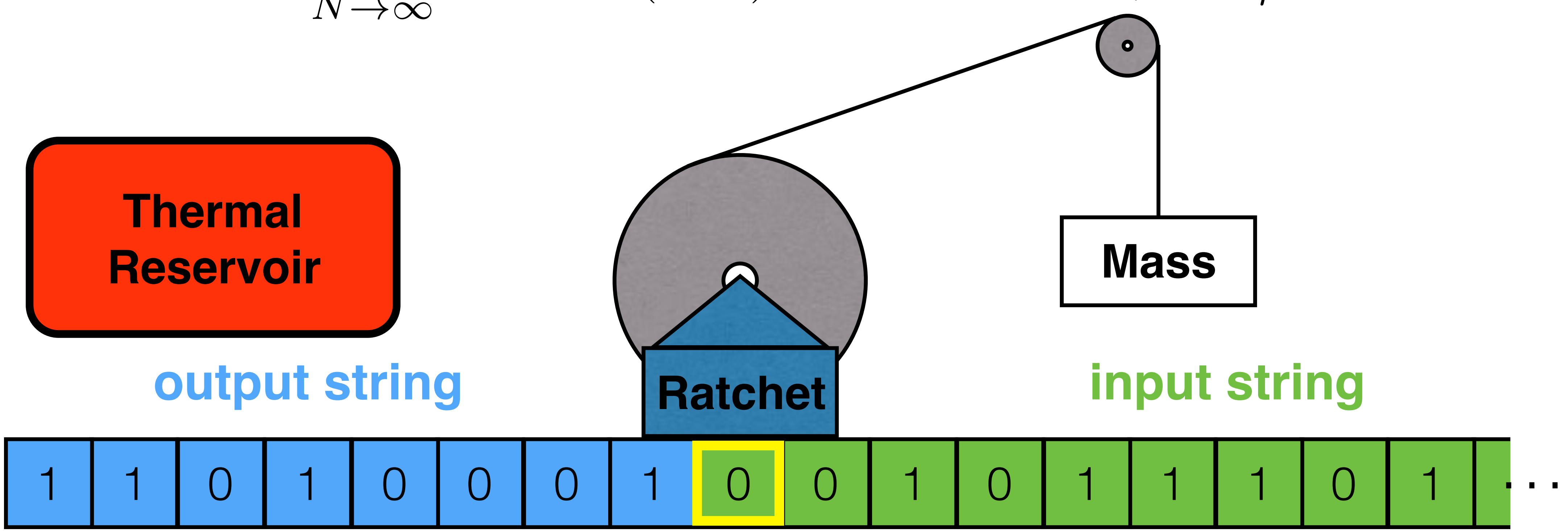
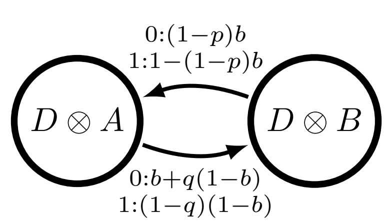
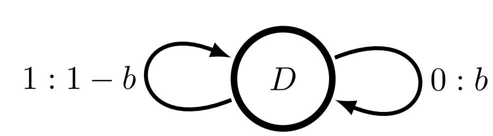




$: h'_\mu$

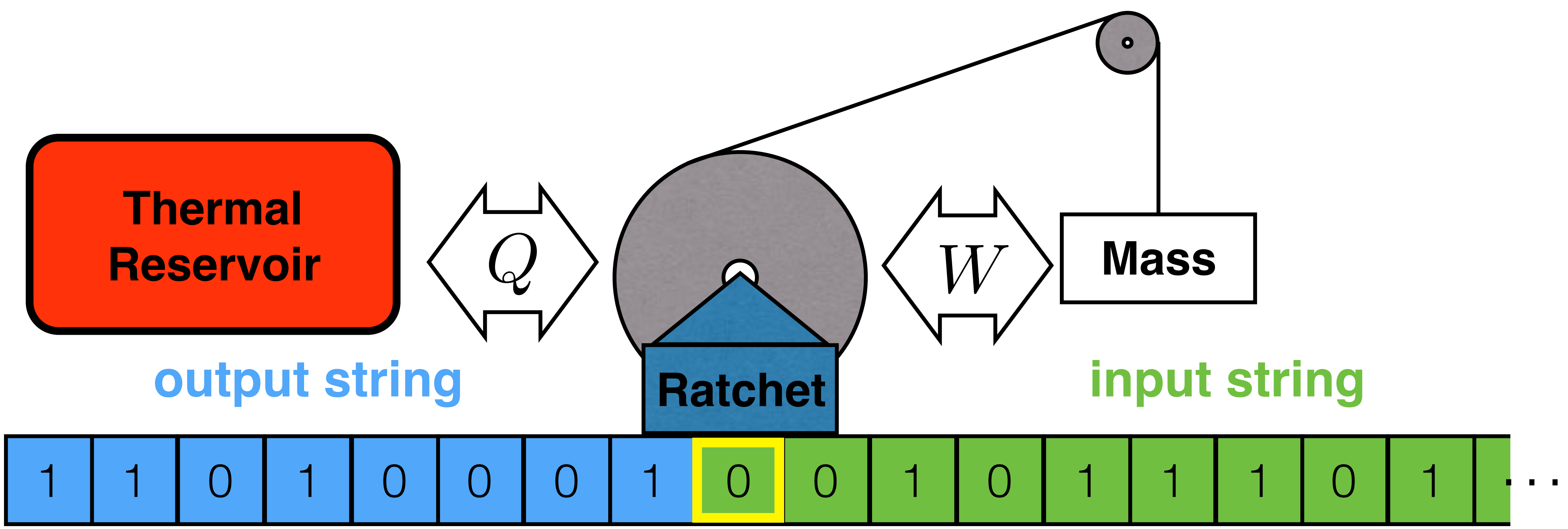


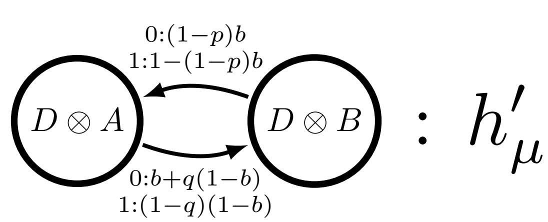
$: h_\mu$

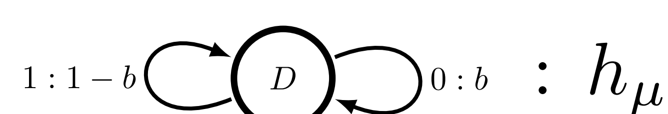
Engines and Erasers

There are three possible types of functionality:

Engine: $W < 0$ and $h_\mu < h'_\mu$







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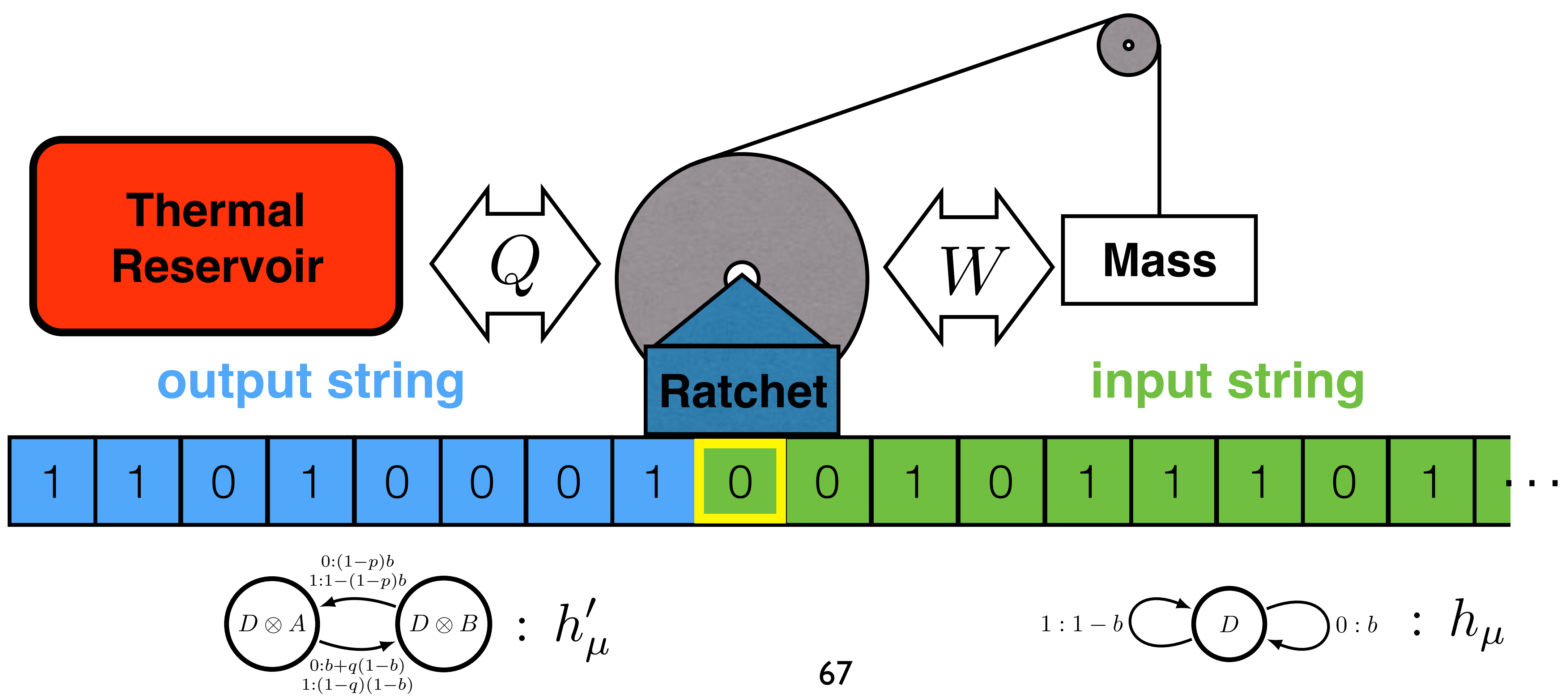

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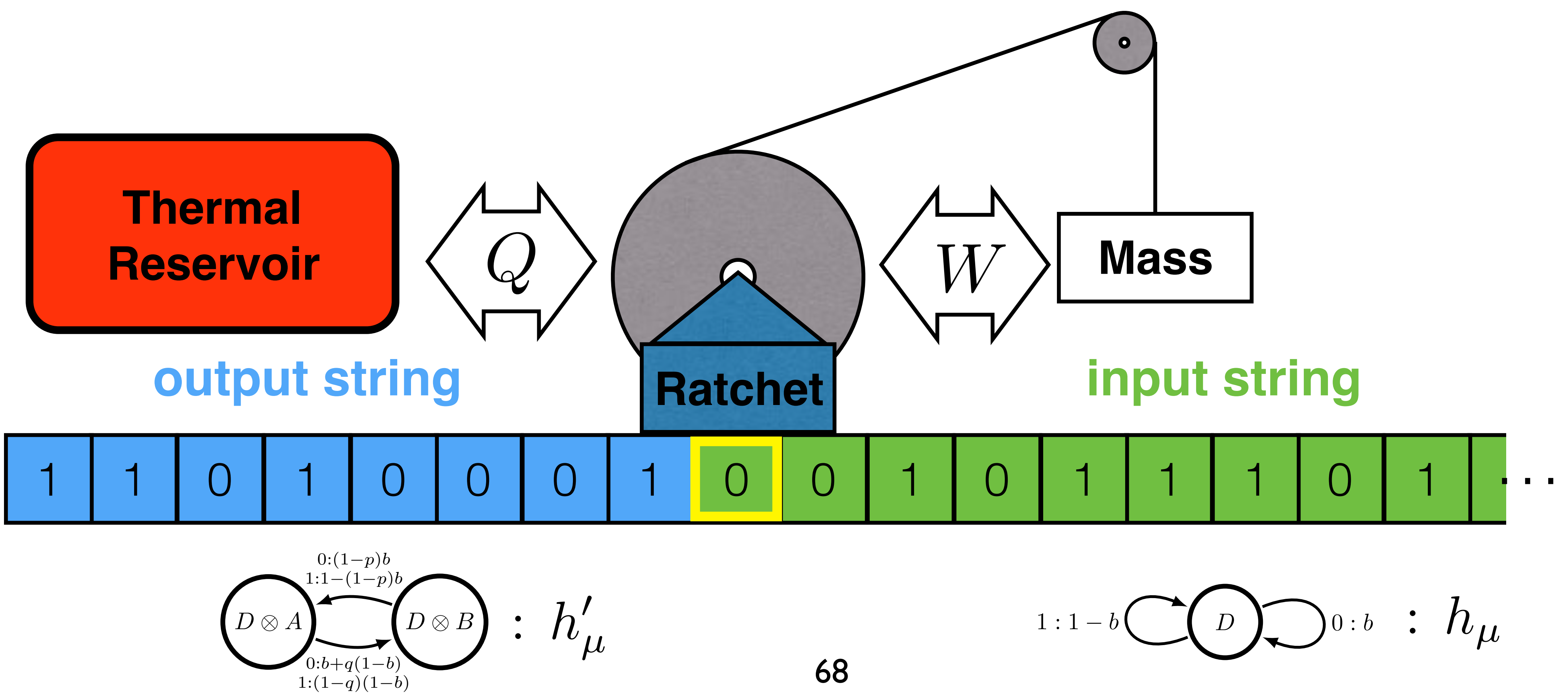

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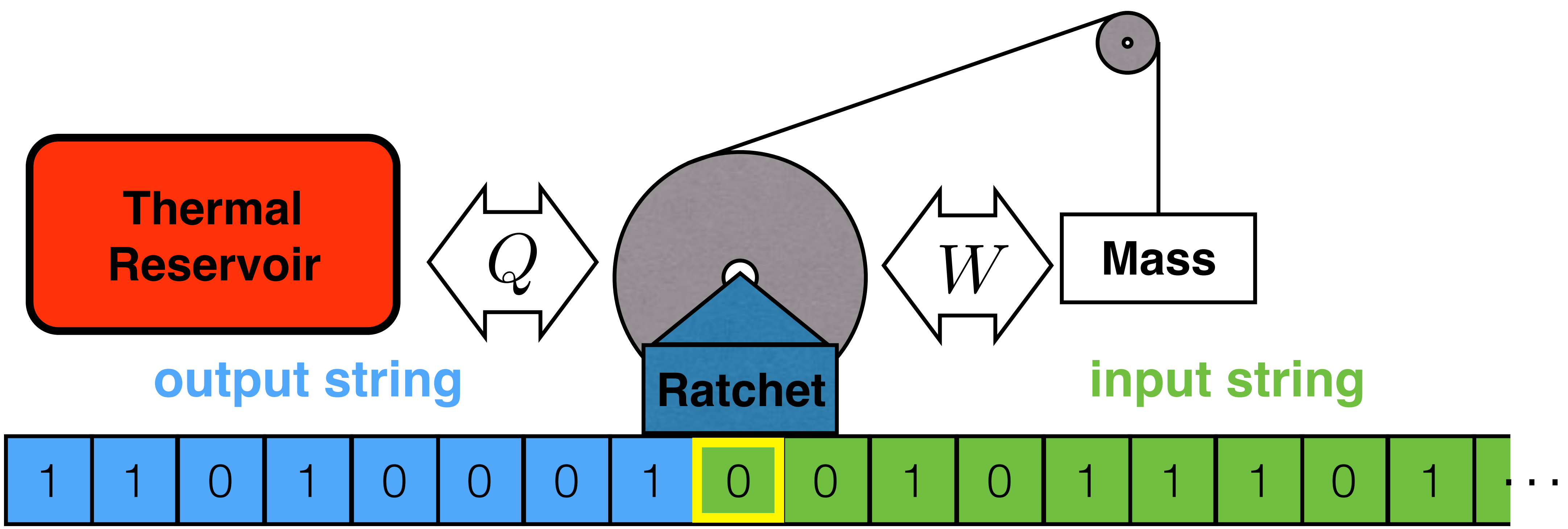
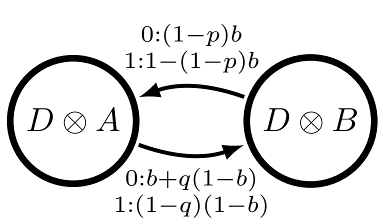
$$\text{Eraser: } W > 0 \text{ and } h_\mu > h'_\mu$$

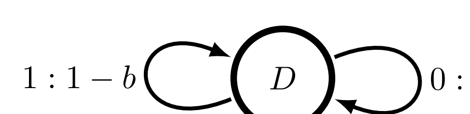
$$\text{Dud: } W > 0 \text{ and } h_\mu < h'_\mu$$

How do we determine the work?



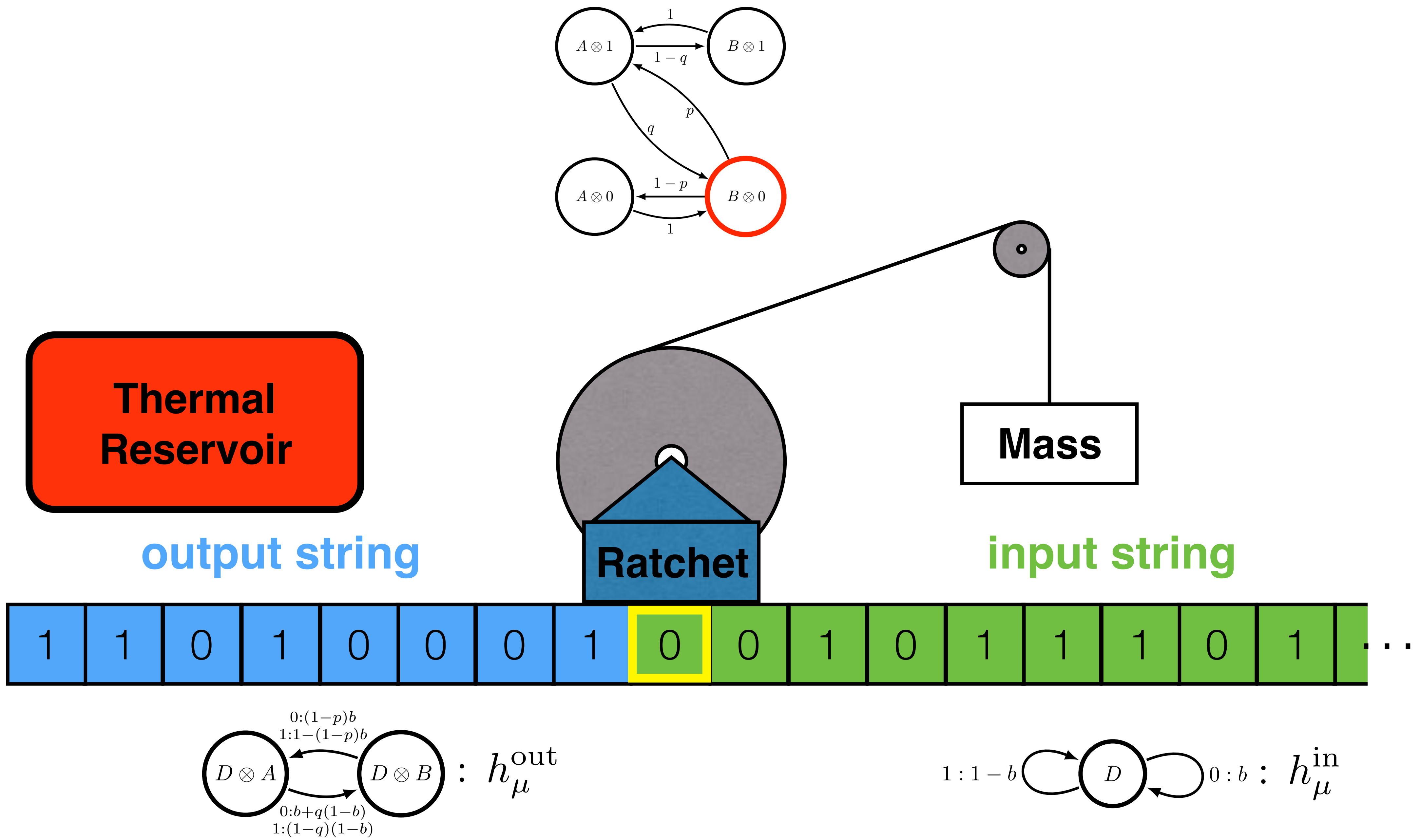


$: h'_\mu$



$: h_\mu$

Non-Equilibrium Ratchet Example



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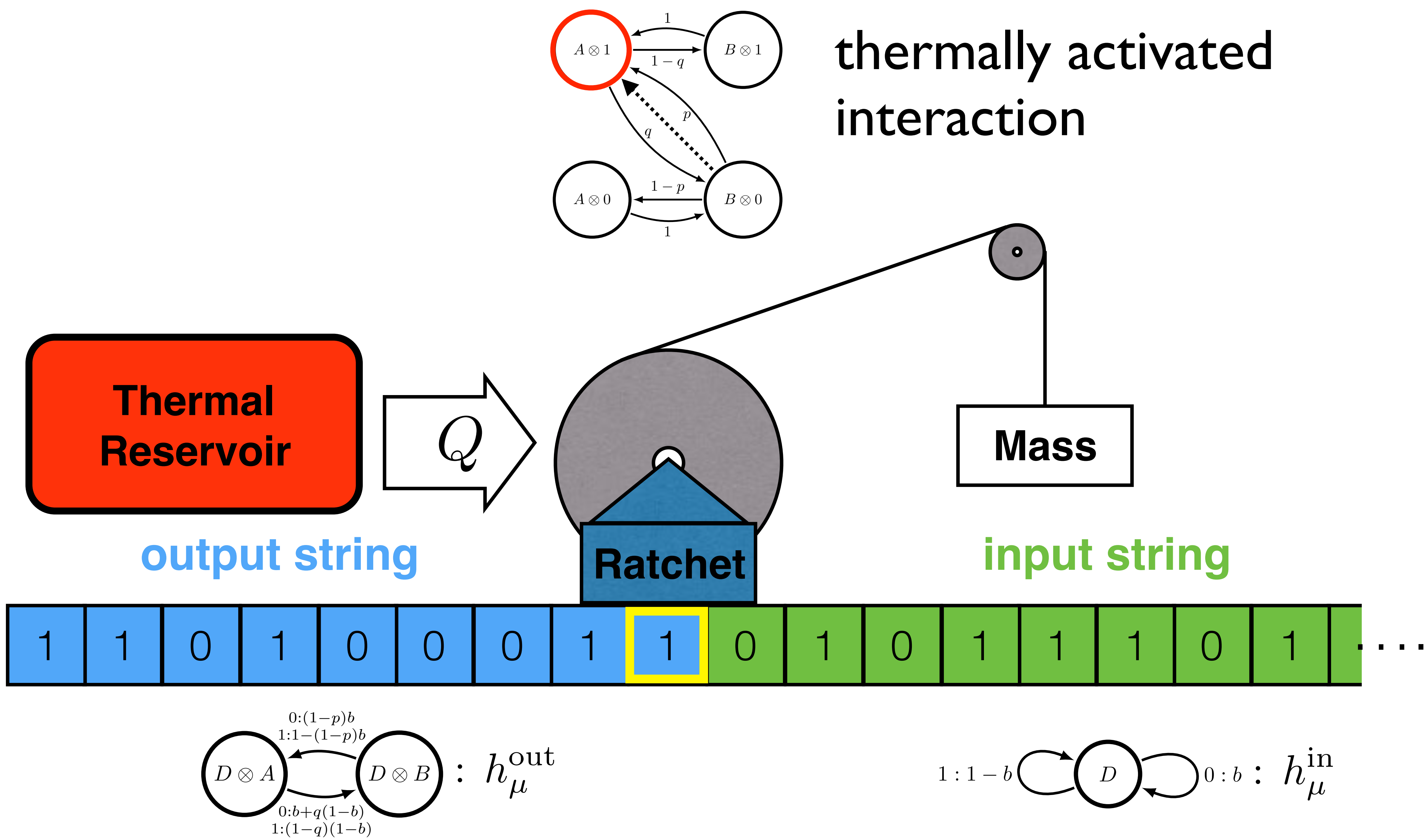
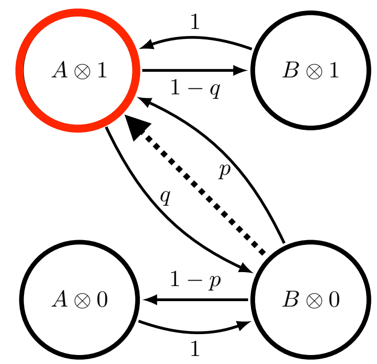


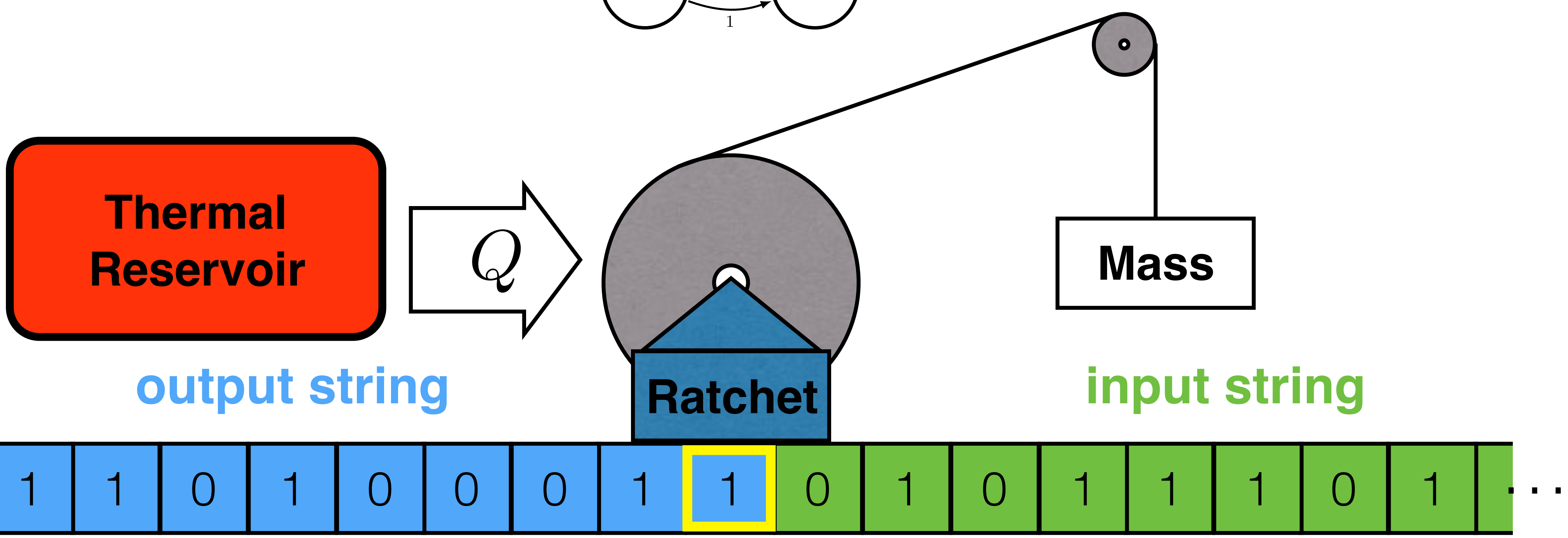
thermally activated interaction

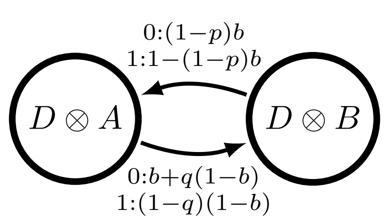
Non-Equilibrium Ratchet Example

$$E_{A\otimes 1} - E_{B\otimes 0} = k_B T \ln \frac{M_{A\otimes 1 \to B\otimes 0}}{M_{B\otimes 0 \to A\otimes 1}}$$

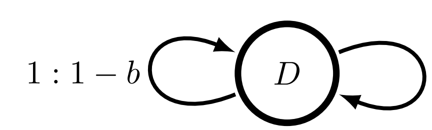


thermally activated interaction





$: h_\mu^{\text{out}}$



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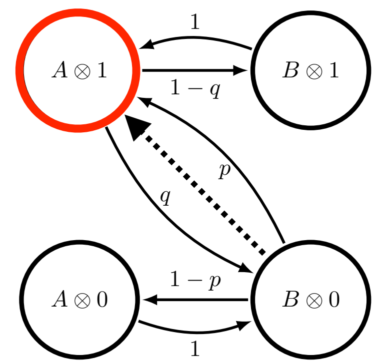
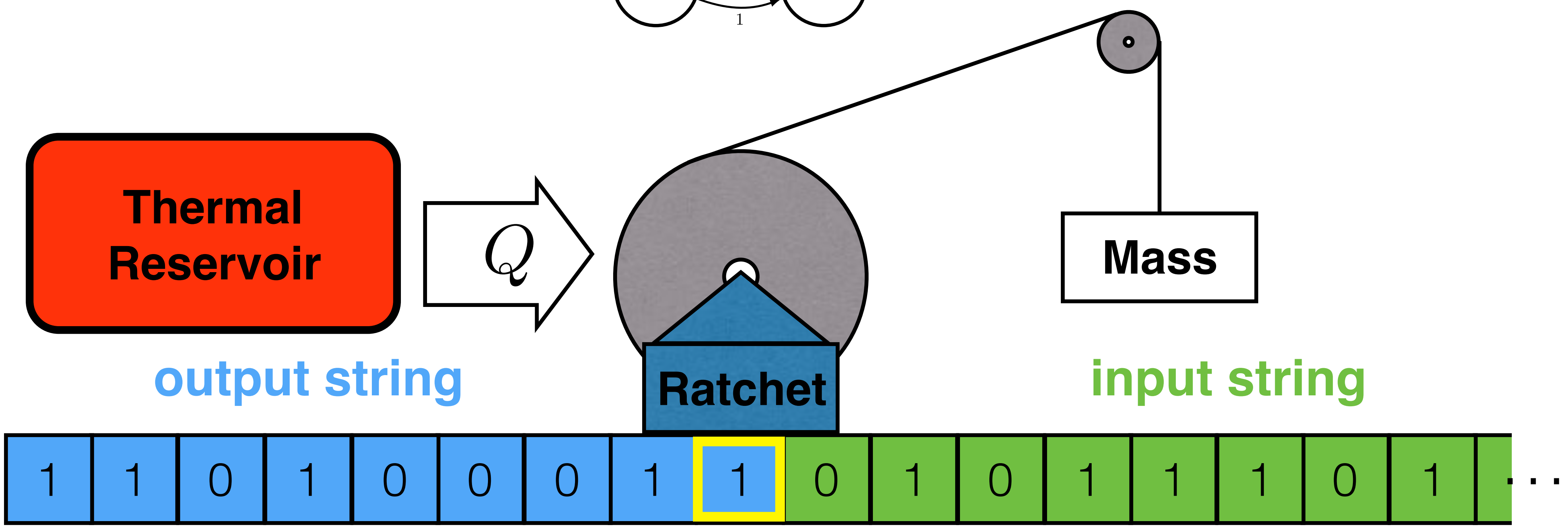
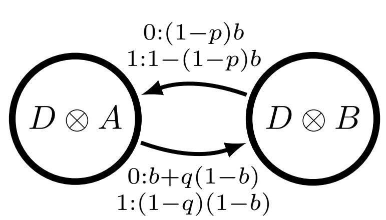


thermally activated

$$Q = E_{B\otimes 0} - E_{A\otimes 1}$$





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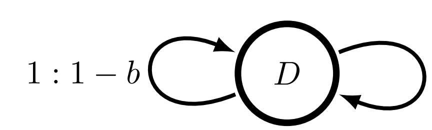

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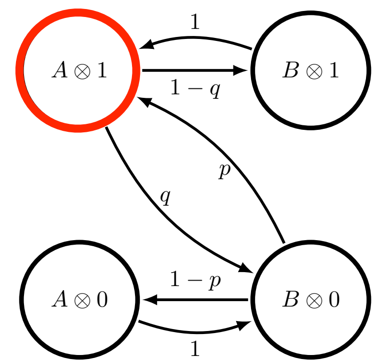

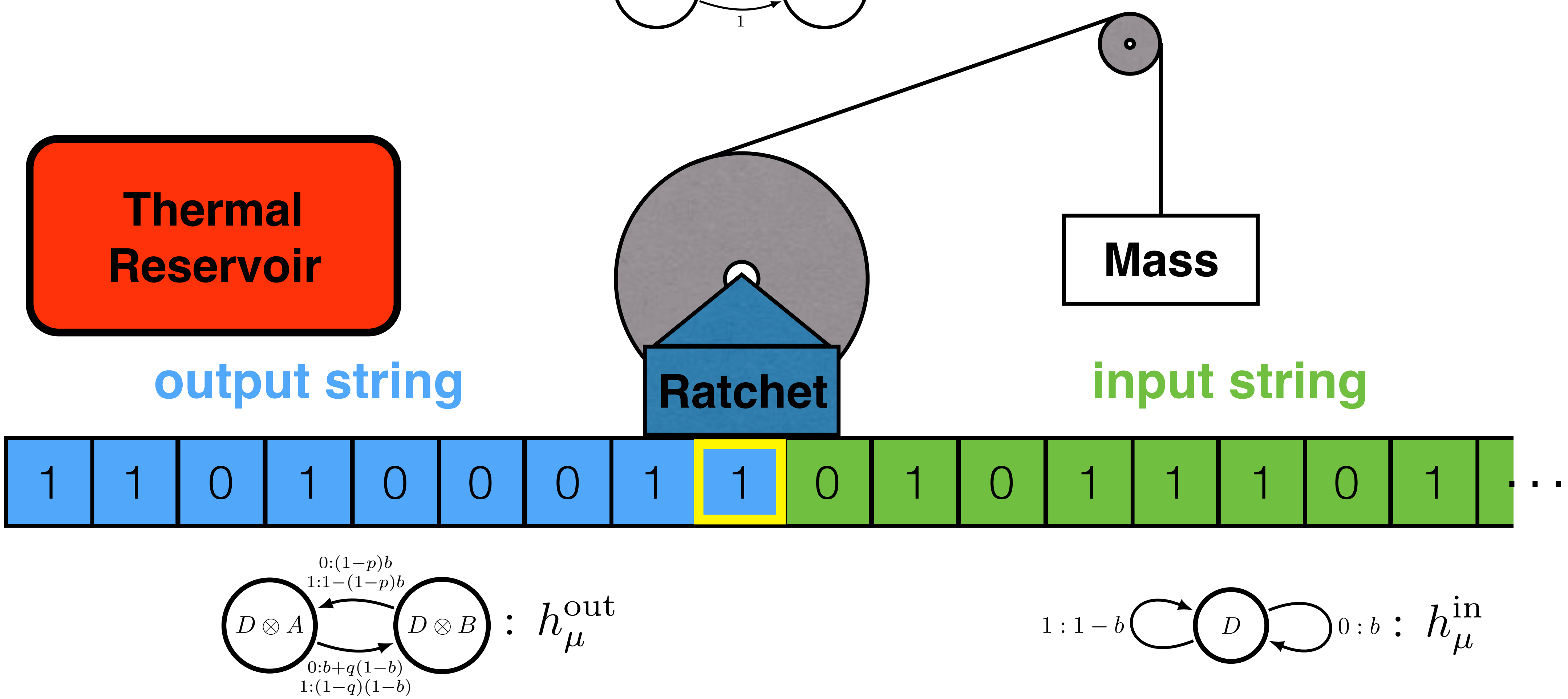

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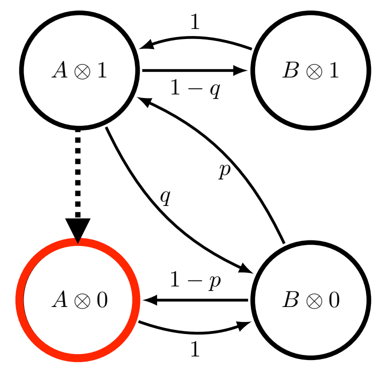
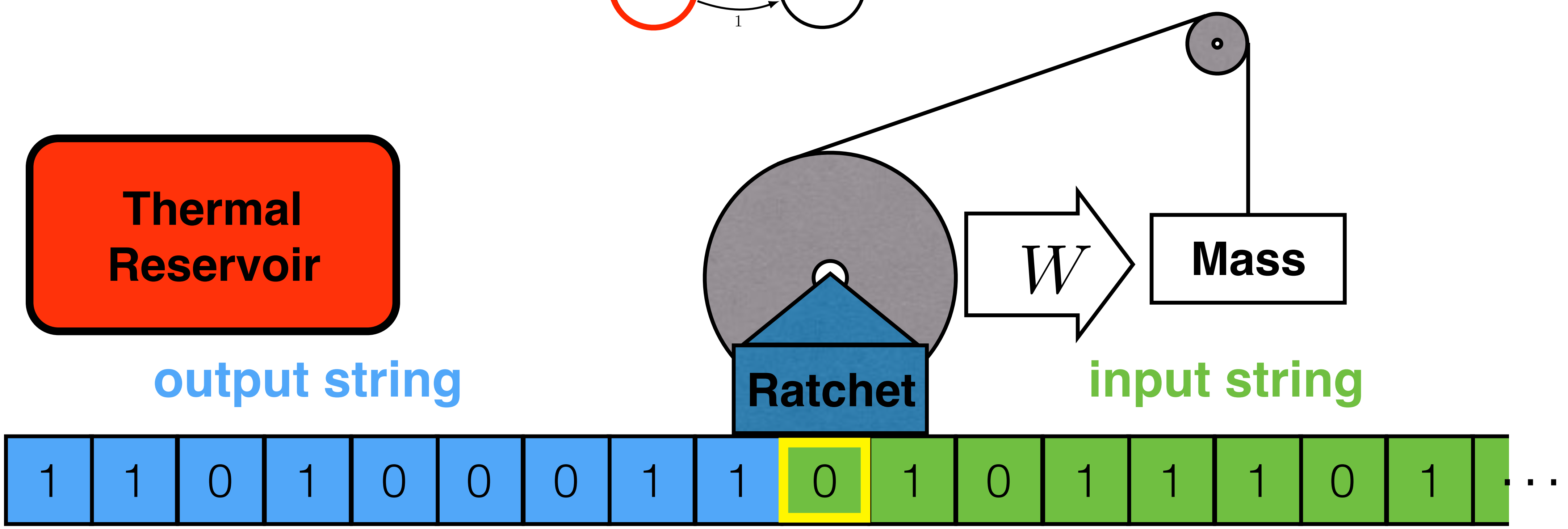
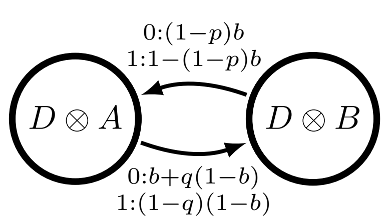


driven switching





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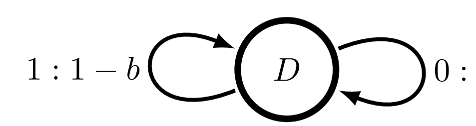

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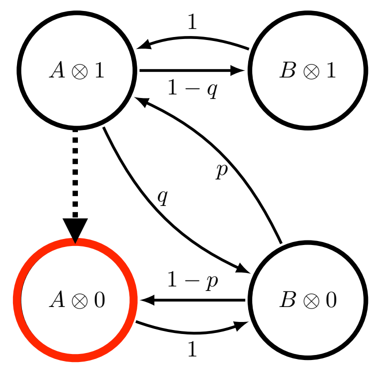
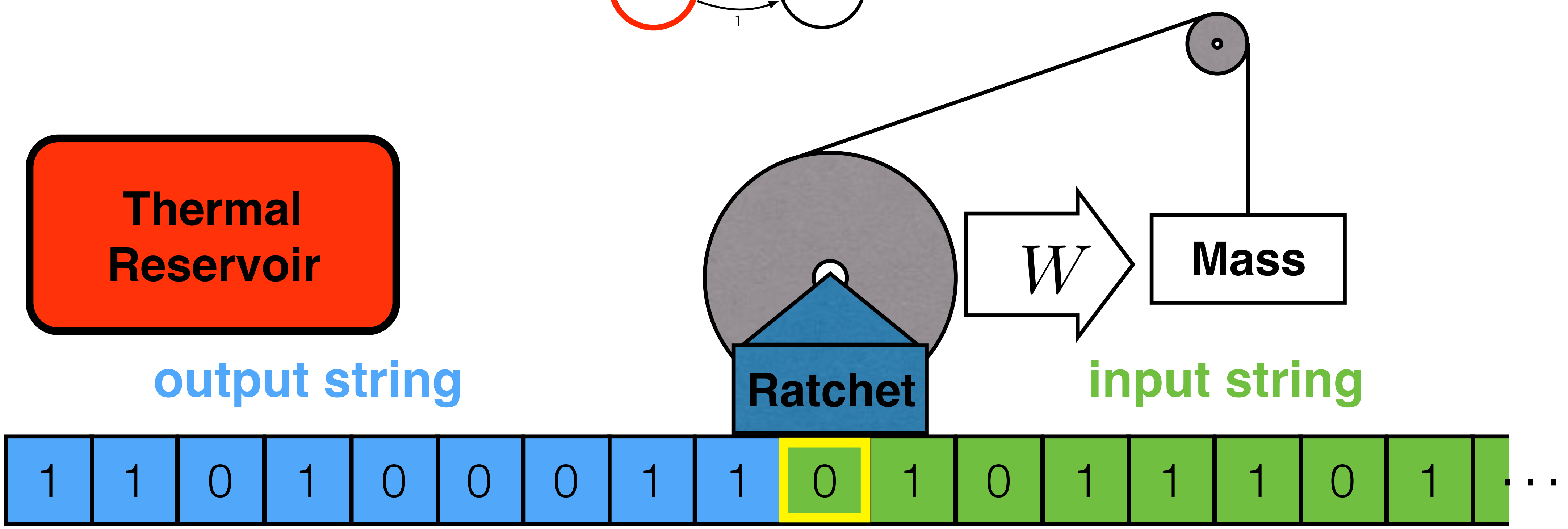
Non-Equilibrium Ratchet Example

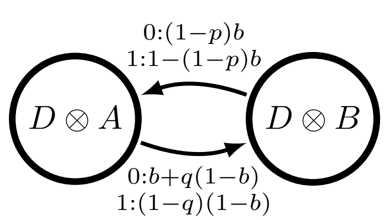
$$E_{A\otimes 1} - E_{B\otimes 0} = k_B T \ln \frac{M_{A\otimes 1 \to B\otimes 0}}{M_{B\otimes 0 \to A\otimes 1}}$$



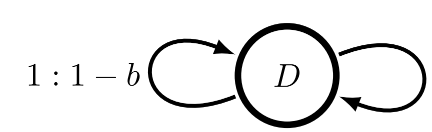
driven switching

$$W = E_{A\otimes 0} - E_{A\otimes 1}$$





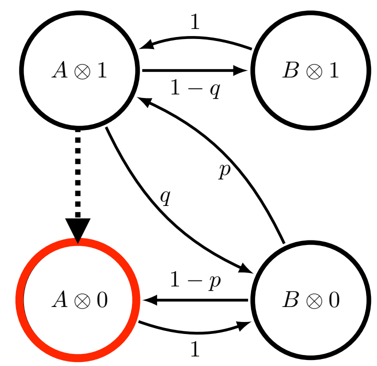
$: h_\mu^{\text{out}}$



$: h_\mu^{\text{in}}$

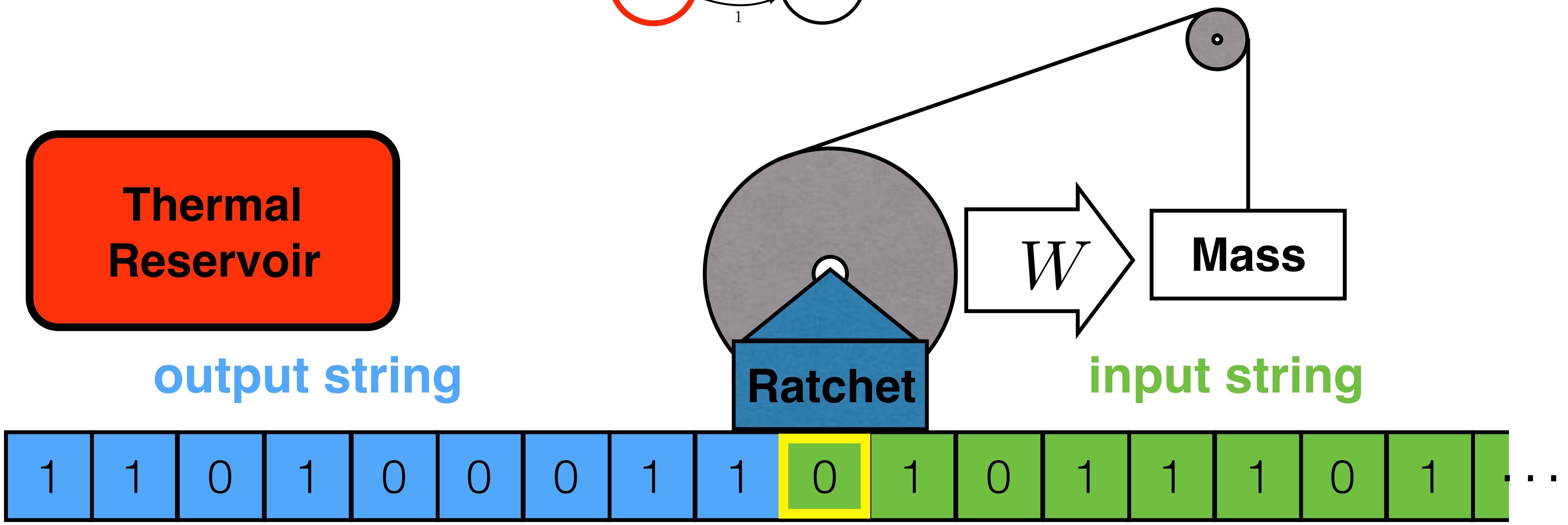
Non-Equilibrium Ratchet Example

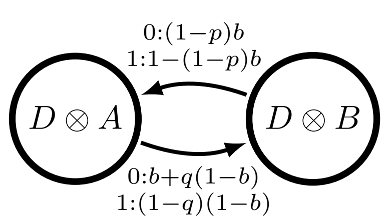
$$E_{A\otimes 1} - E_{B\otimes 0} = k_B T \ln \frac{M_{A\otimes 1 \to B\otimes 0}}{M_{B\otimes 0 \to A\otimes 1}}$$



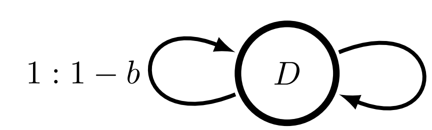
driven switching

$$W = E_{A\otimes 0} - E_{A\otimes 1}$$





$: \; h_\mu^{\text{out}}$



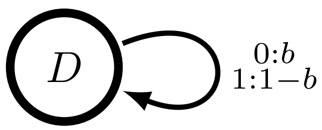
$: \; h_\mu^{\text{in}}$

$$\langle Q \rangle = \langle W \rangle = k_B T \sum_{x,x',y,y'} \Pr(X_N = x, Y_N = y) M_{x\otimes y \to x'\otimes y'} \ln \frac{M_{x'\otimes y' \to x\otimes y}}{M_{x\otimes y \to x'\otimes y'}}$$

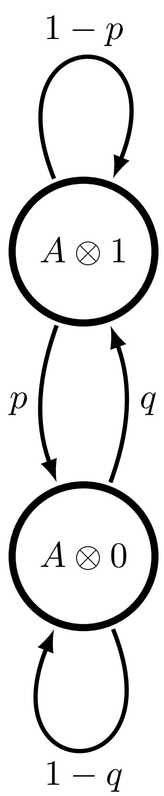
A. B. Boyd, D. Mandal, and J. P. Crutchfield, “Correlation-powered information engines and the thermodynamics of self-correction”, PRE (2017)

Different Types of Order

simple inputs

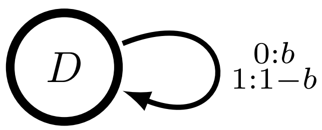


simple engine

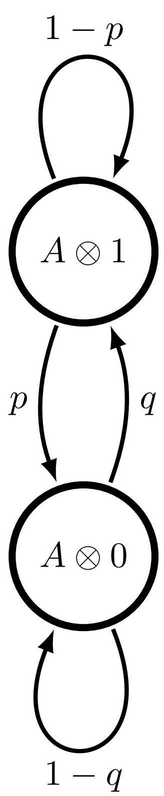


Different Types of Order

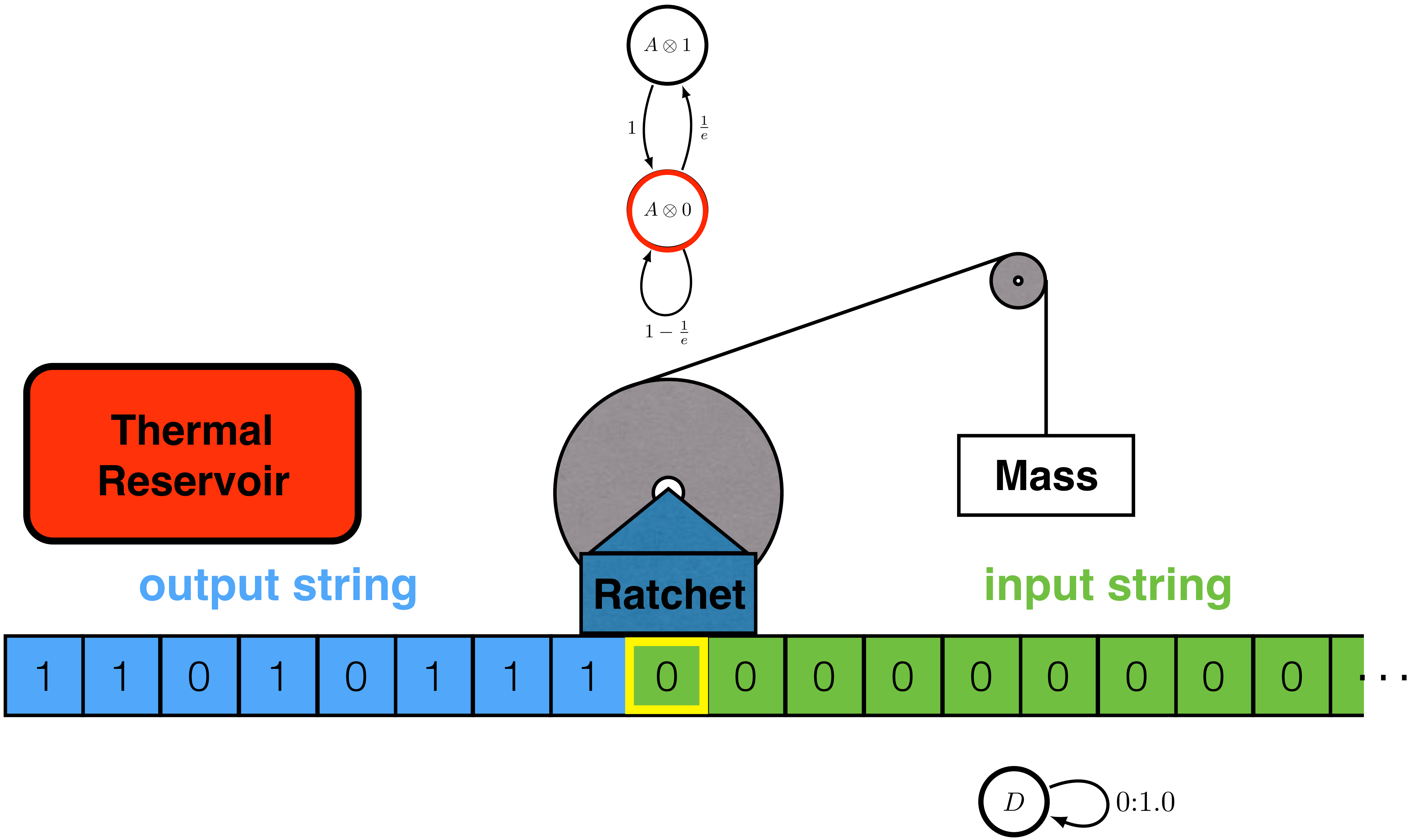
simple inputs: creates no temporal correlations



simple engine: insensitive to temporal correlations

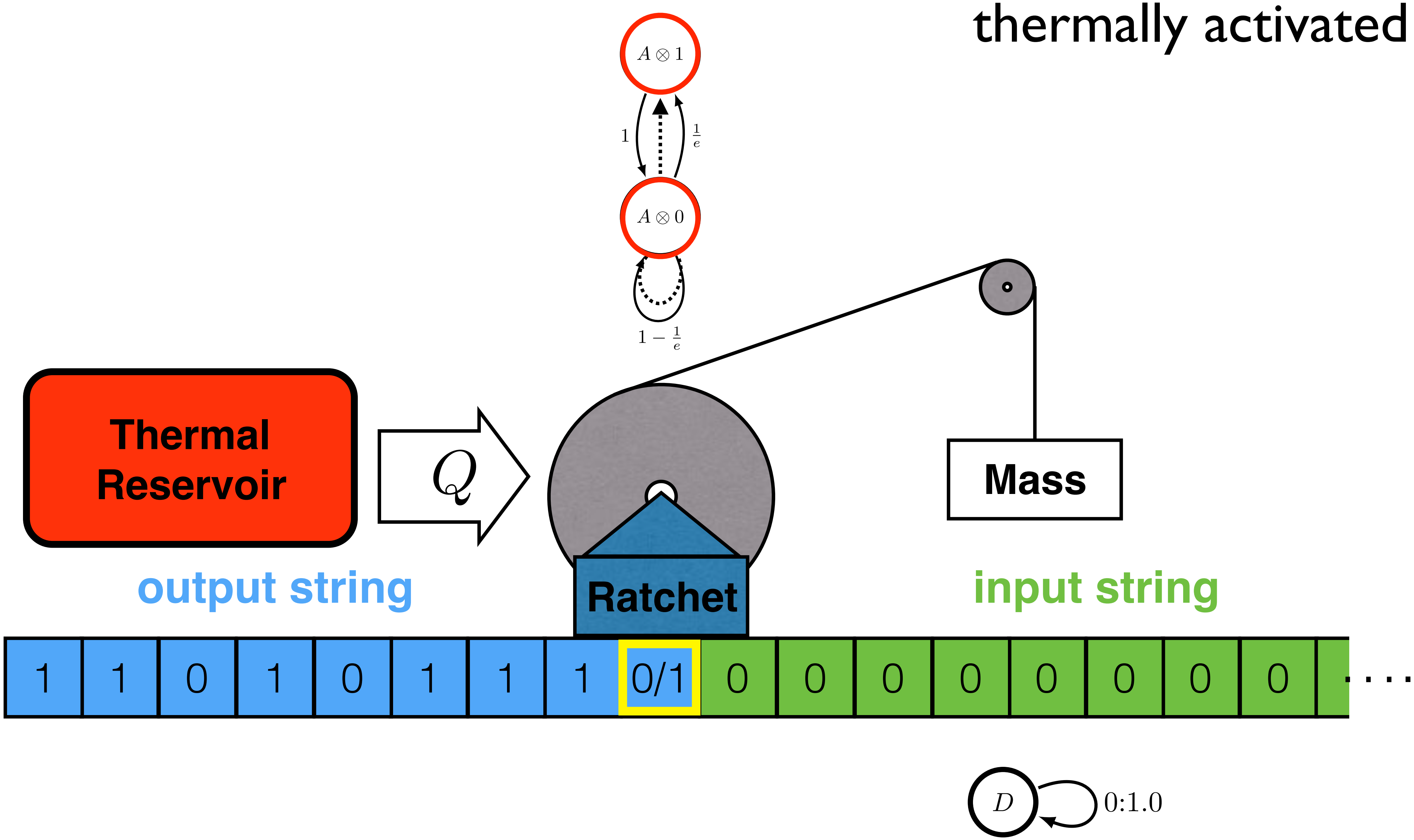


Memoryless Ratchet Operation



Memoryless Ratchet Operation

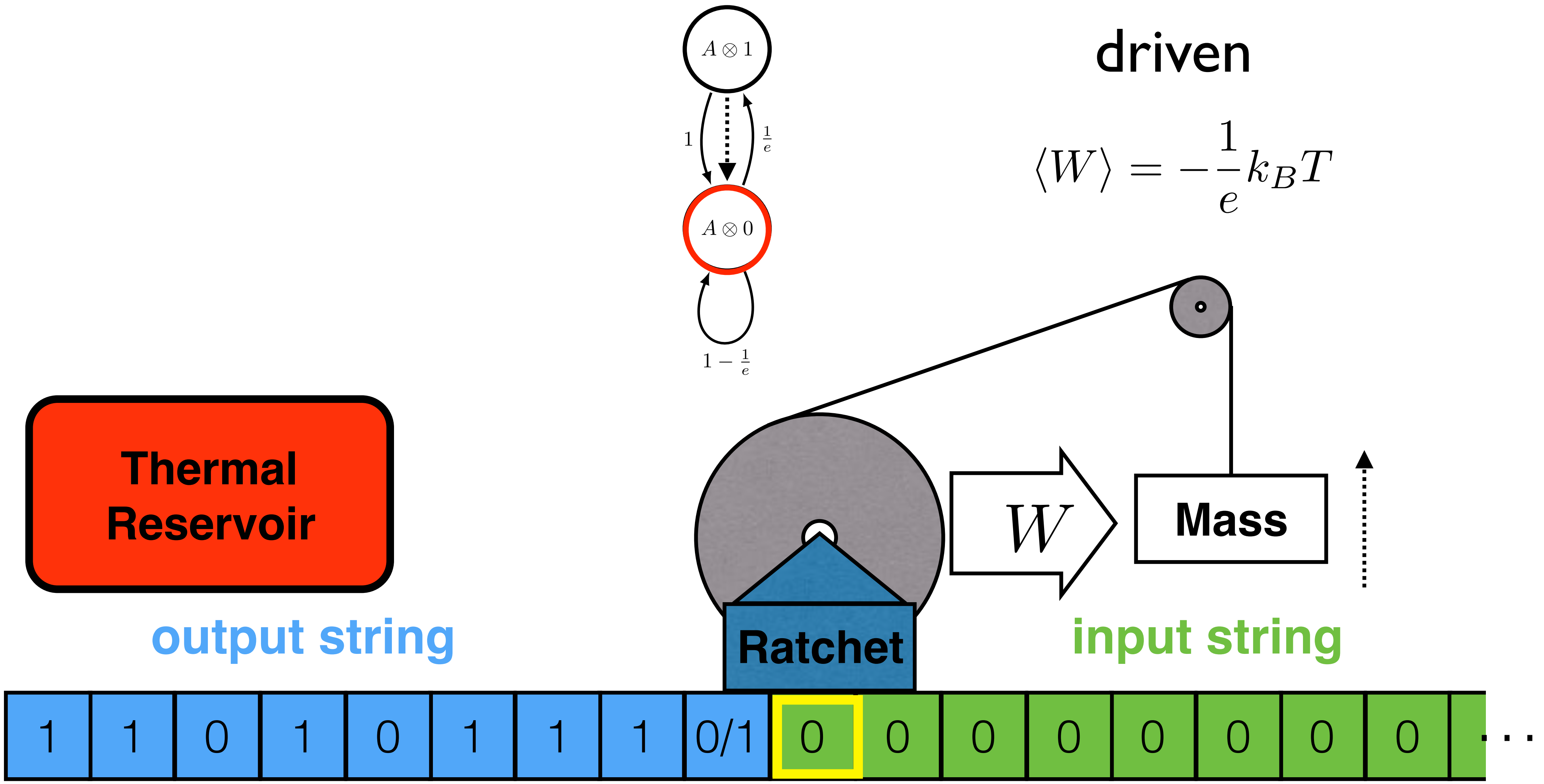
thermally activated



Memoryless Ratchet Operation

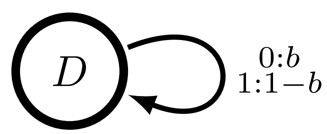
driven

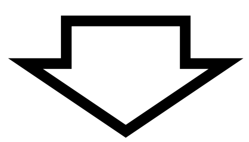
$$\langle W \rangle = -\frac{1}{e} k_B T$$



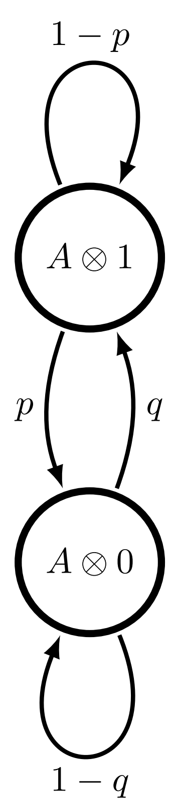
Different Types of Order

simple inputs

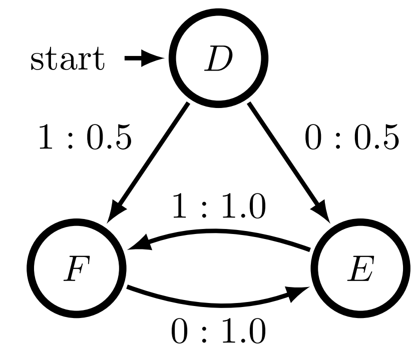




simple engine

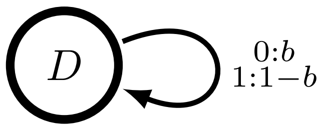


complex inputs

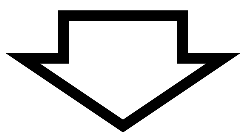


Different Types of Order

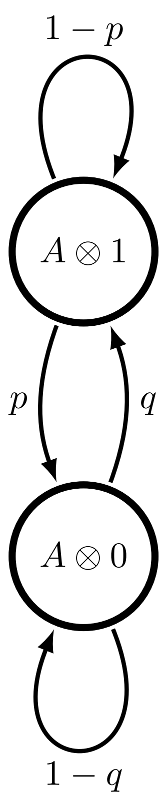
simple inputs



$C_\mu = H[S_N^+] = 0$

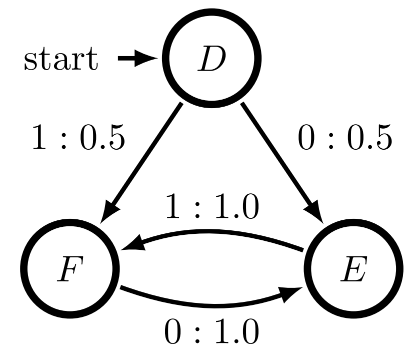


simple engine



$H[X_N] = 0$

complex inputs

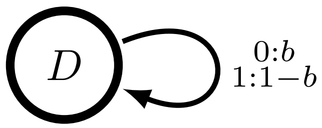


$C_\mu = H[S_N^+] = 1 > 0$

A. B. Boyd, D. Mandal, and J. P. Crutchfield, "Above and Beyond the Landauer Bound: Thermodynamics of Modularity", arXiv:1708.03030 (2017)

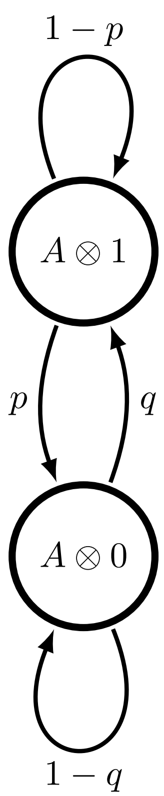
Different Types of Order

simple inputs



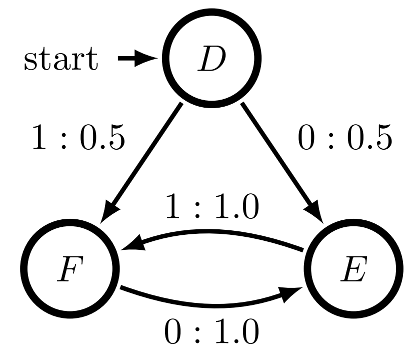
$$C_\mu = H[S_N^+] = 0$$

simple engine



$$H[X_N] = 0$$

complex inputs



$$C_\mu = H[S_N^+] = 1 > 0$$

order is temporal correlations:

$$H[Y_N] - h_\mu = 1$$

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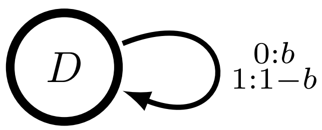
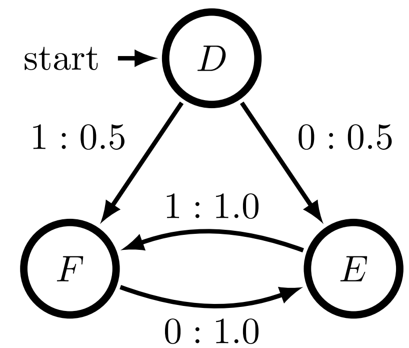
Different Types of Order

simple inputs



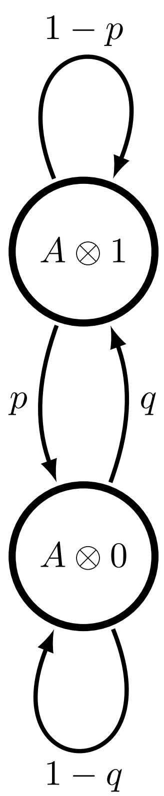
$$C_\mu = H[S_N^+] = 0$$

complex inputs



$$C_\mu = H[S_N^+] = 1 > 0$$

simple engine



$$H[X_N] = 0$$

order is temporal correlations:

$$H[Y_N] - h_\mu = 1$$

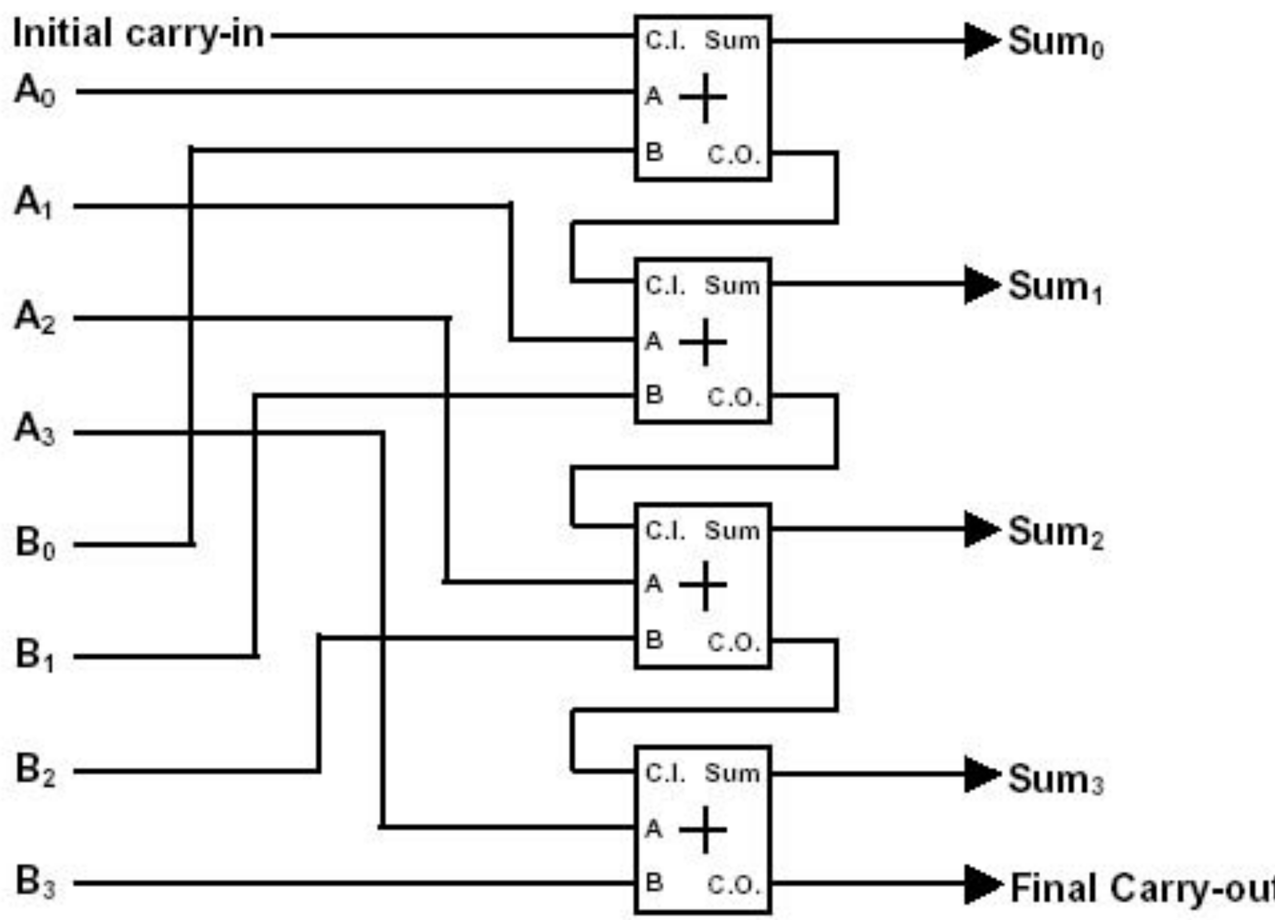
Memoryless ratchet dissipates all temporal correlations, because of **modularity**.

A. B. Boyd, D. Mandal, and J. P. Crutchfield, "Above and Beyond the Landauer Bound: Thermodynamics of Modularity", arXiv:1708.03030 (2017)

"Complex" Computation

$$A + B =?$$

information states: $\mathcal{Z}$



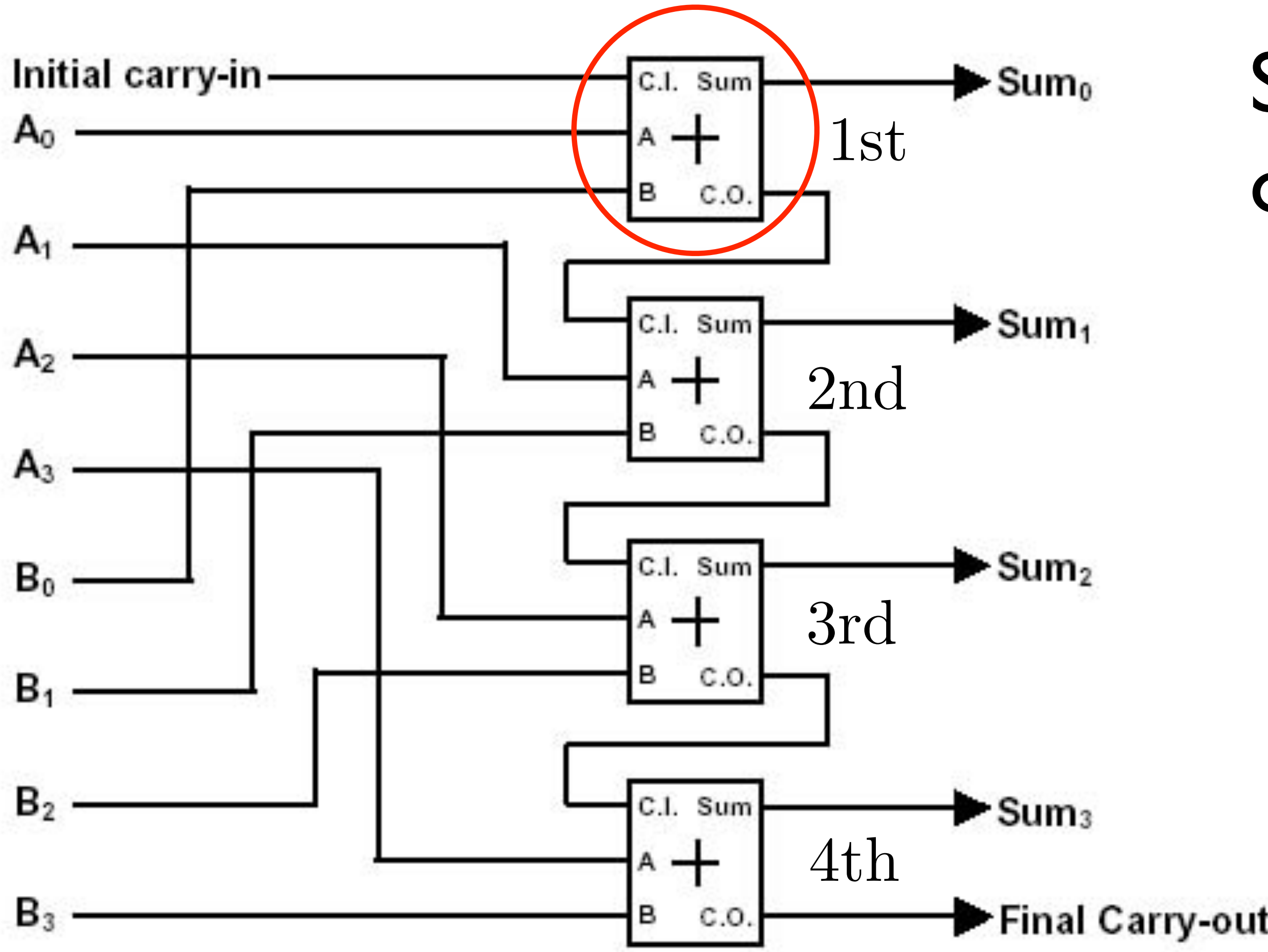
http://www.science.smith.edu/dftwiki/images/9/93/4BitAdderBlockDiagram.jpg

$$\langle W \rangle_{\min} = k_B T \ln 2 (H[Z_t] - H[Z_{t+\tau}])?$$

Modular Design

$$A + B = ?$$

information states: $\mathcal{Z}$

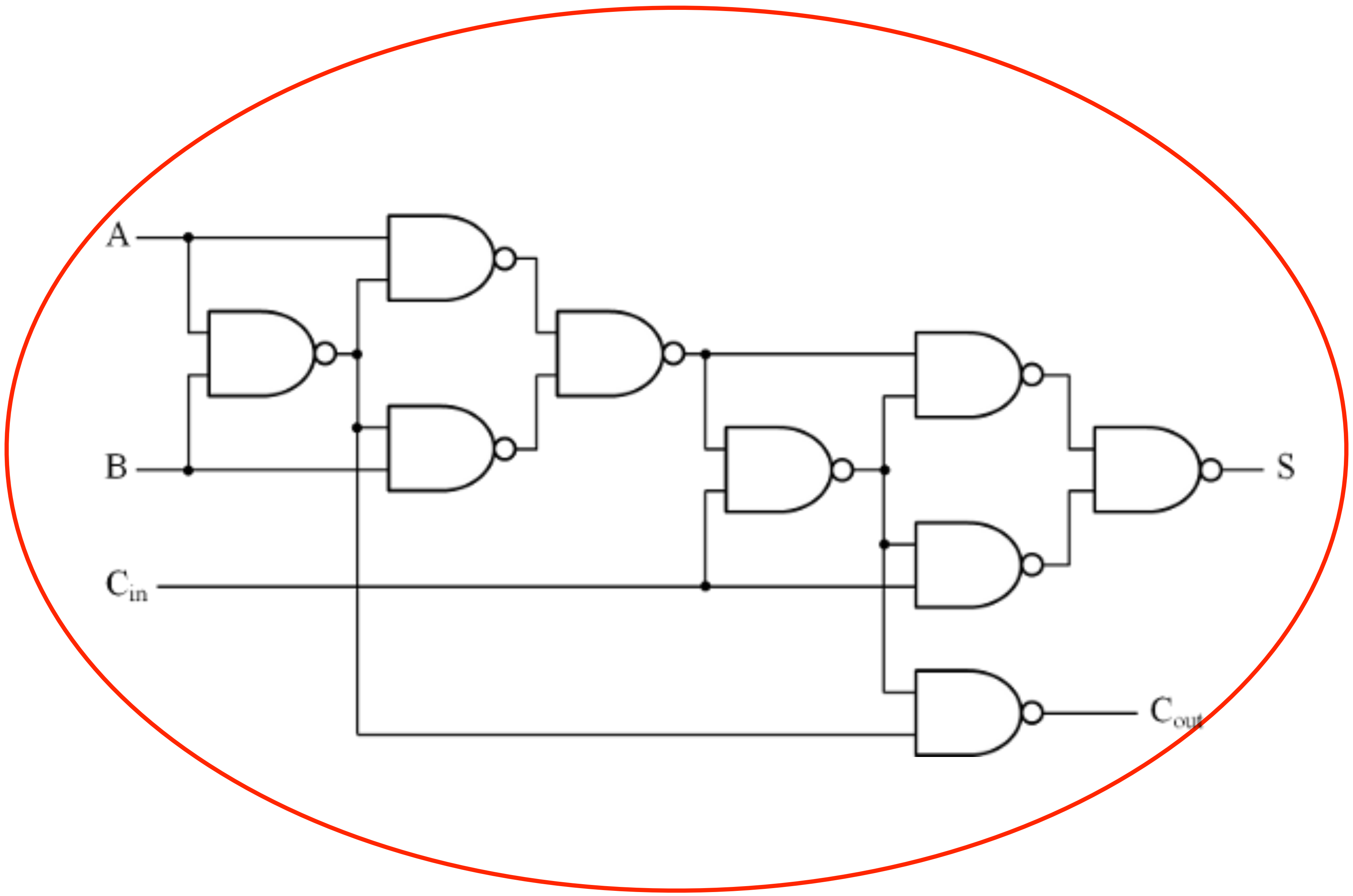


http://www.science.smith.edu/dftwiki/images/9/93/4BitAdderBlockDiagram.jpg

Sequence of simpler operations

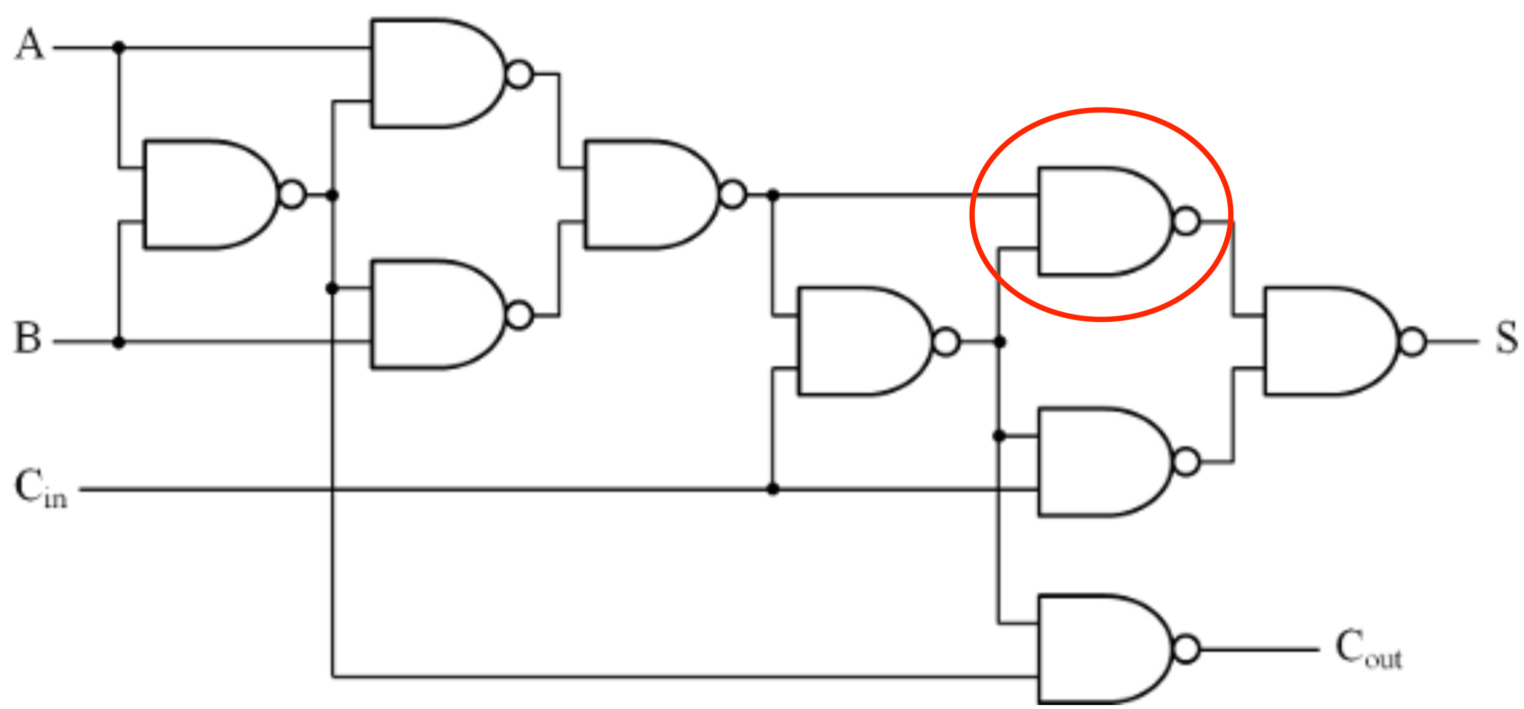
$$\langle W \rangle_{\min} = k_B T \ln 2 (H[Z_t] - H[Z_{t+\tau}])?$$

Modular Design



http://gateoverflow.in/84564/digitial

Modular Design



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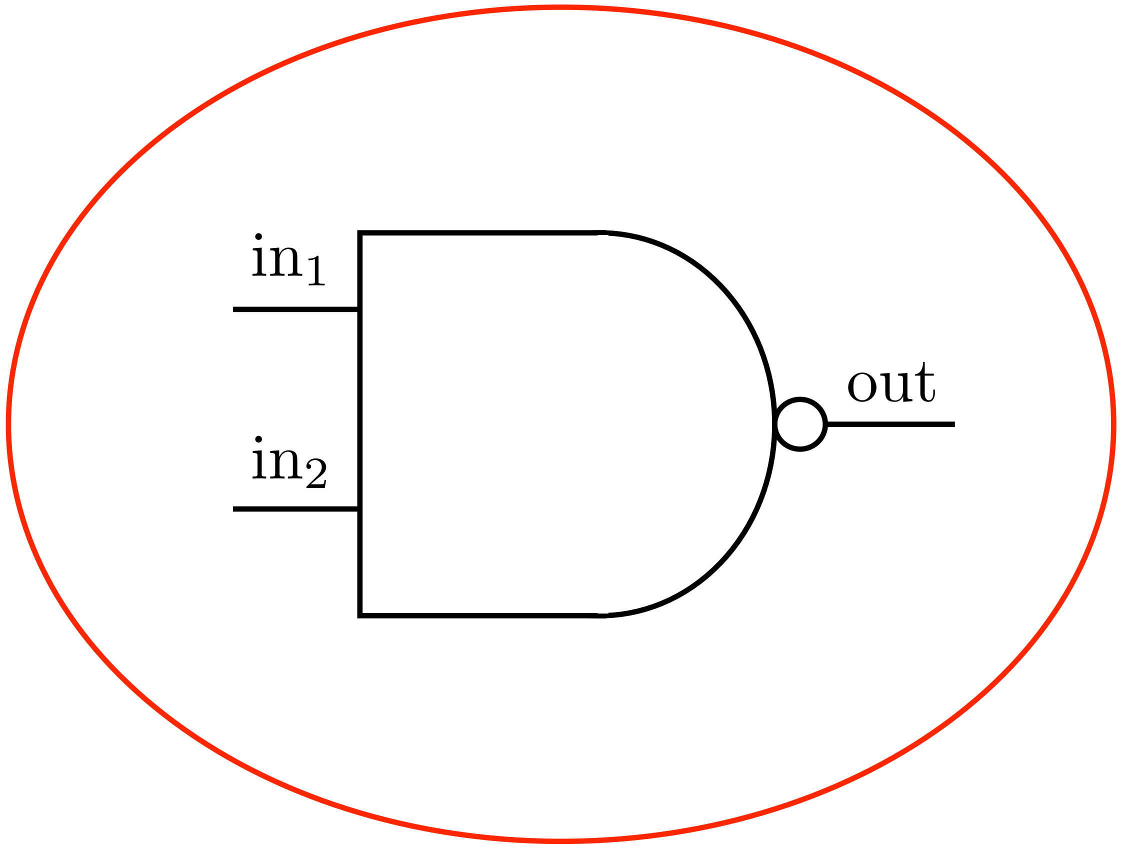
Local Elementary Operations



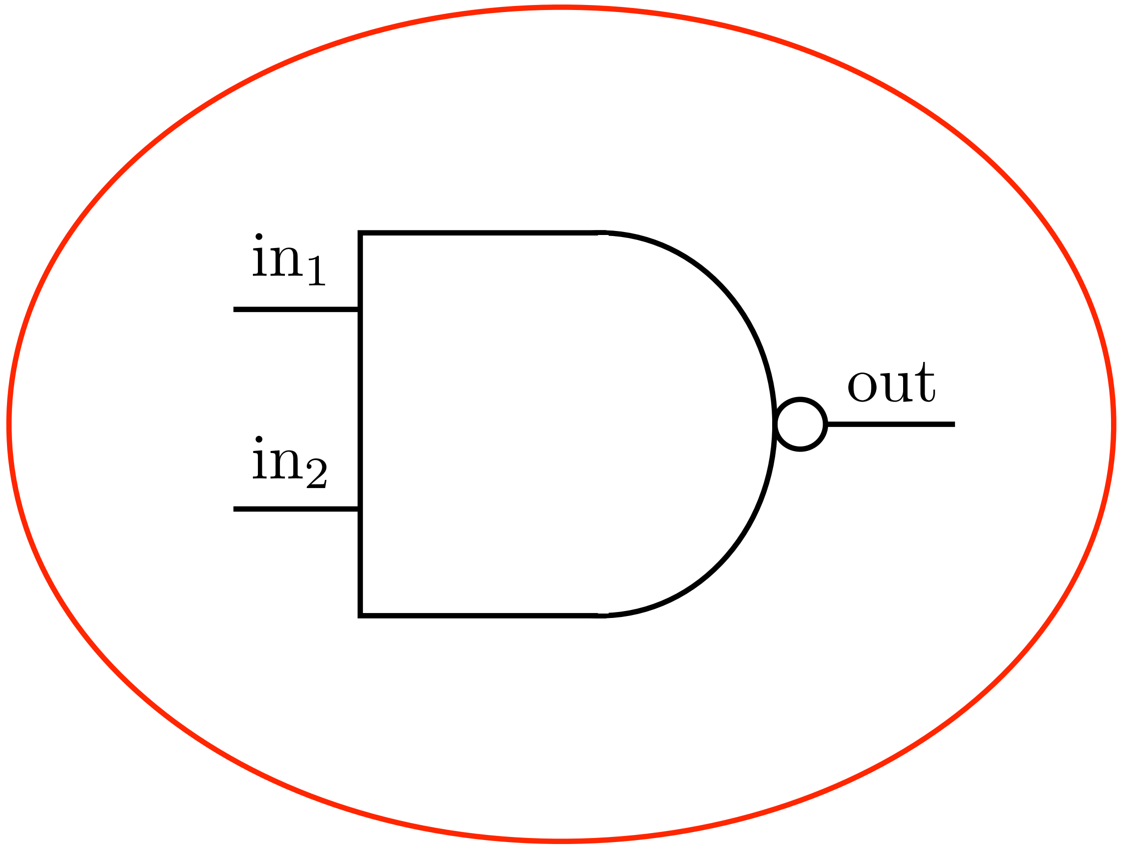
local information states: $\mathcal{Z}^i = \{0,1\} \otimes \{0,1\} \otimes \{0,1\}$

in$_1$	in$_2$	out
0	0	1
0	1	1
1	0	1
1	1	0

random variable: Z^i

Local Elementary Operations

local information states: $\mathcal{Z}^i = \{0,1\} \otimes \{0,1\} \otimes \{0,1\}$



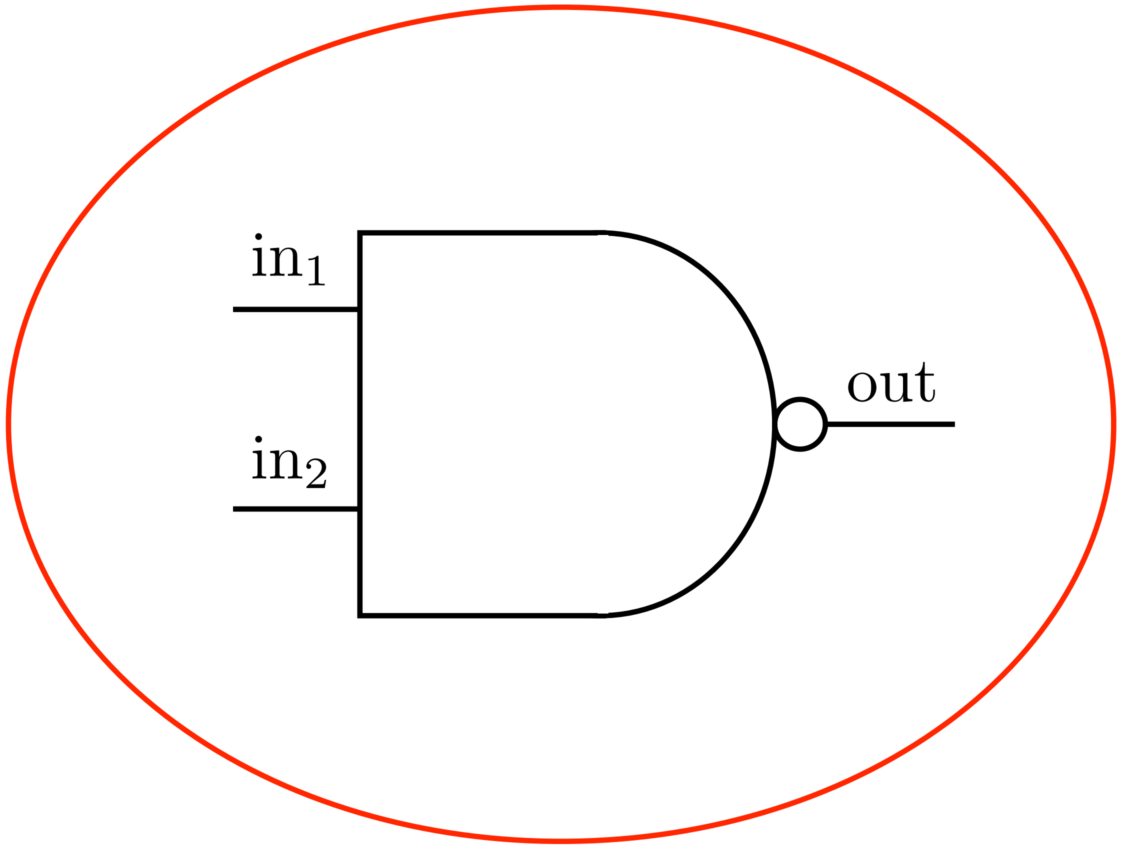
in_1	in_2	out
0	0	1
0	1	1
1	0	1
1	1	0

random variable: Z^i

local Markov dynamics: $M^{\text{local}}_{z^i_t \to z^i_{t+\tau}} = \Pr(Z^i_{t+\tau} = z^i_{t+\tau} | Z^i_t = z^i_t)$

Local Elementary Operations

local information states: $\mathcal{Z}^i = \{0,1\} \otimes \{0,1\} \otimes \{0,1\}$



in_1	in_2	out
0	0	1
0	1	1
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1	1	0

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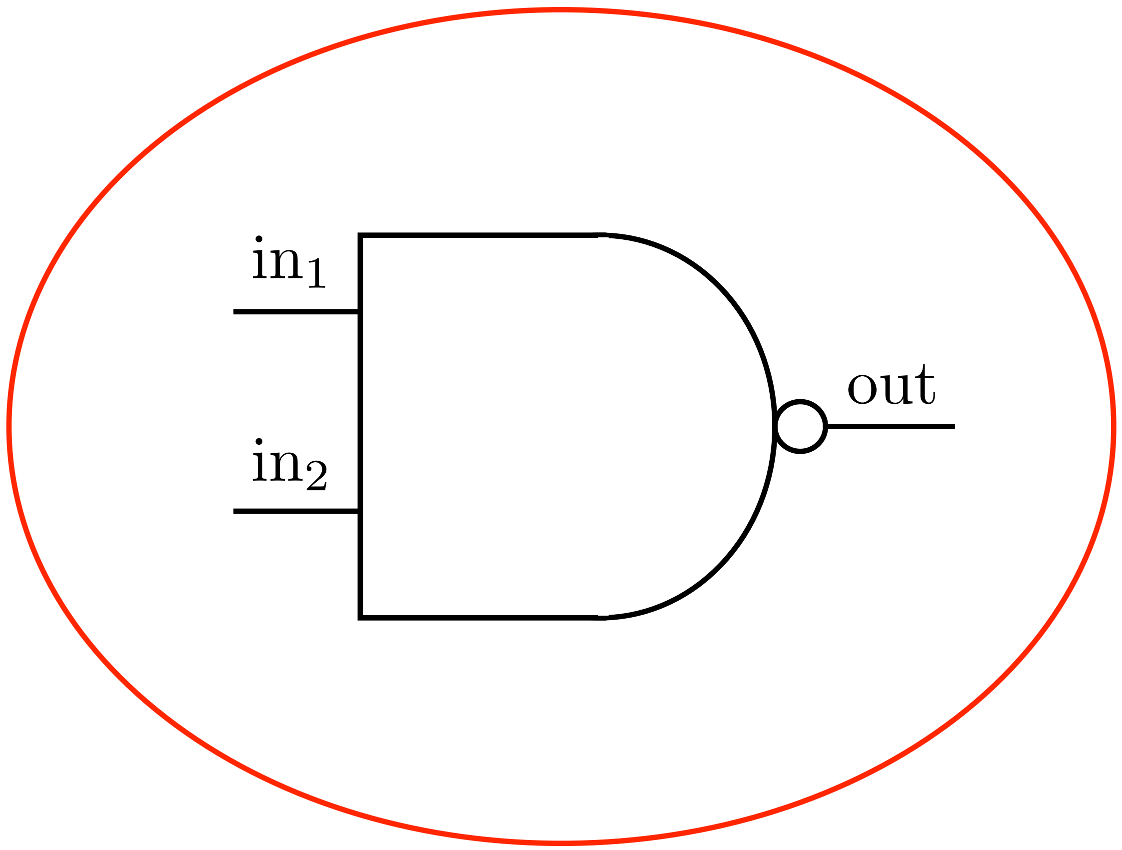
localized control: $\langle W^{\text{local}} \rangle_{\min} = k_B T \ln 2 (H[Z^i_t] - H[Z^i_{t+\tau}])$

Local Elementary Operations

local information states: $\mathcal{Z}^i = \{0,1\} \otimes \{0,1\} \otimes \{0,1\}$



in_1	in_2	out
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1	1	0

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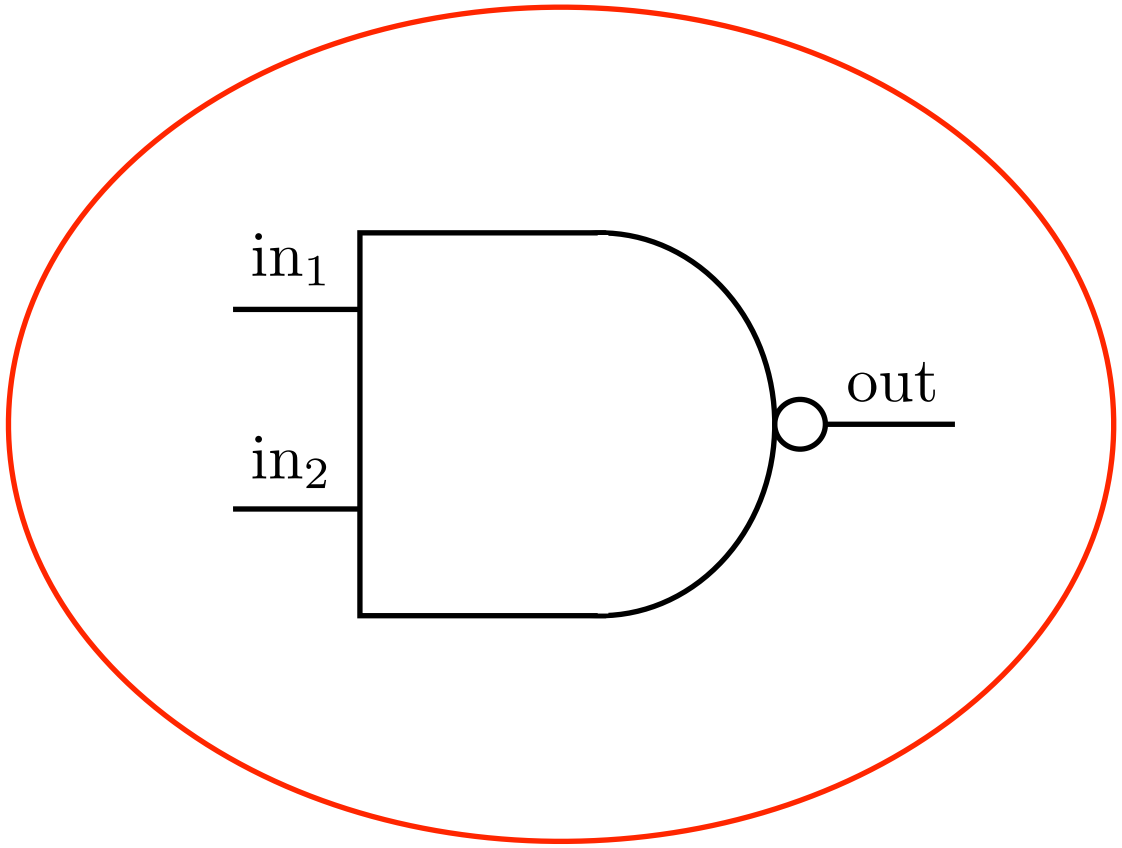
$$\text{localized control: } \langle W^{\text{local}} \rangle_{\min} = k_B T \ln 2 (H[Z^i_t] - H[Z^i_{t+\tau}])$$
$$\geq \Delta F^{\text{NEQ}}$$

Local Elementary Operations

local information states: $\mathcal{Z}^i = \{0,1\} \otimes \{0,1\} \otimes \{0,1\}$



in_1	in_2	out
0	0	1
0	1	1
1	0	1
1	1	0

random variable: Z^i

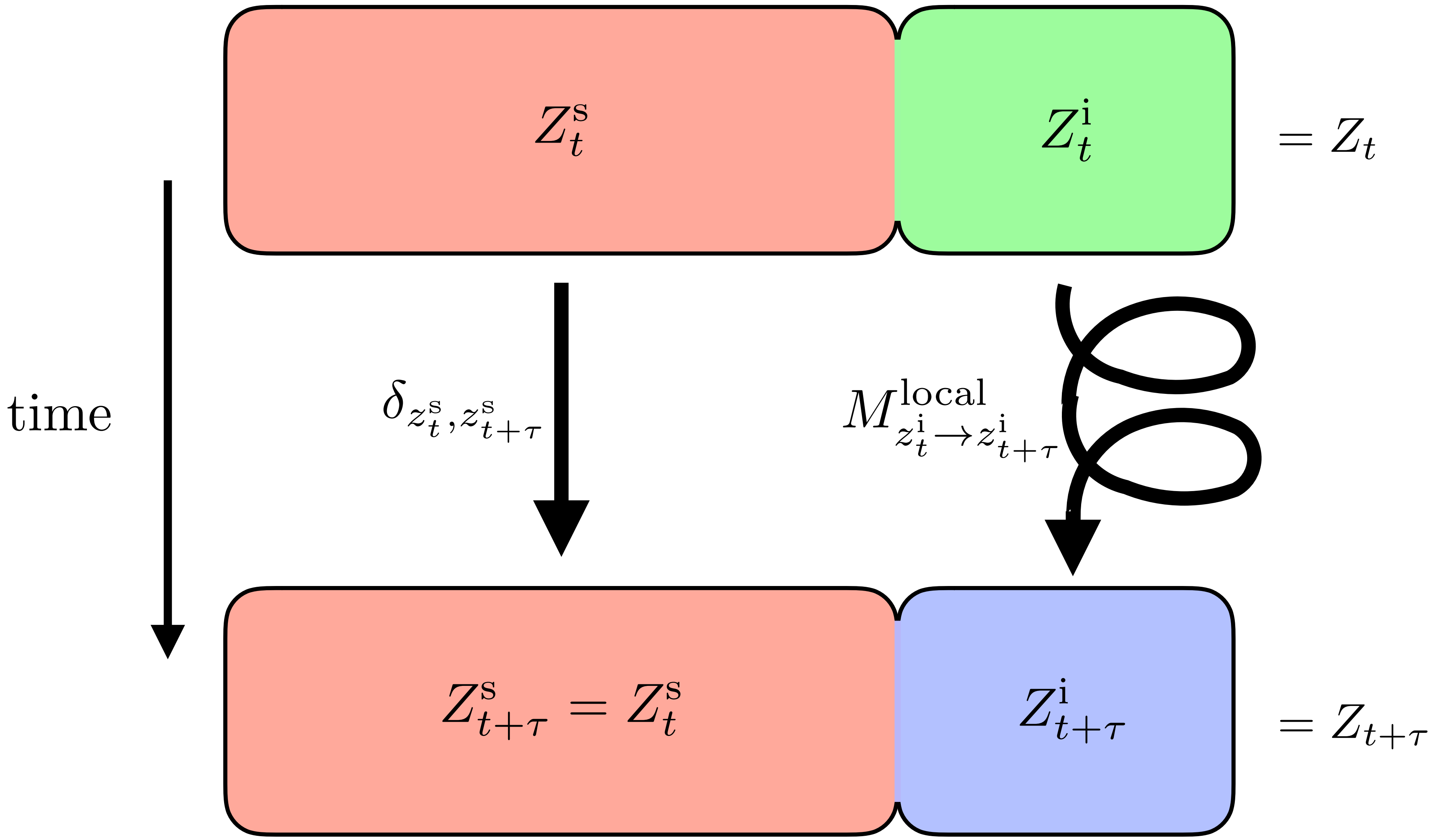
local Markov dynamics: $M^{\mathrm{local}}_{z^i_t \to z^i_{t+\tau}} = \Pr(Z^i_{t+\tau} = z^i_{t+\tau} | Z^i_t = z^i_t)$

$$\text{localized control: } \langle W^{\mathrm{local}} \rangle_{\min} = k_B T \ln 2 (H[Z^i_t] - H[Z^i_{t+\tau}]) \\ \geq \Delta F^{\mathrm{NEQ}}$$

global change in free energy: $\Delta F^{\mathrm{NEQ}} = k_B T \ln 2 (H[Z_t] - H[Z_{t+\tau}])$

Local vs Global Dynamics

global information states: $\mathcal{Z} = \mathcal{Z}^i \otimes \mathcal{Z}^s$



global Markov dynamics: $M^{\mathrm{global}}_{(z^i_t, z^s_t) \to (z^i_{t+\tau}, z^s_{t+\tau})} = \delta_{z^s_t, z^s_{t+\tau}} M^{\mathrm{local}}_{z^i_t \to z^i_{t+\tau}}$

Modularity Dissipation

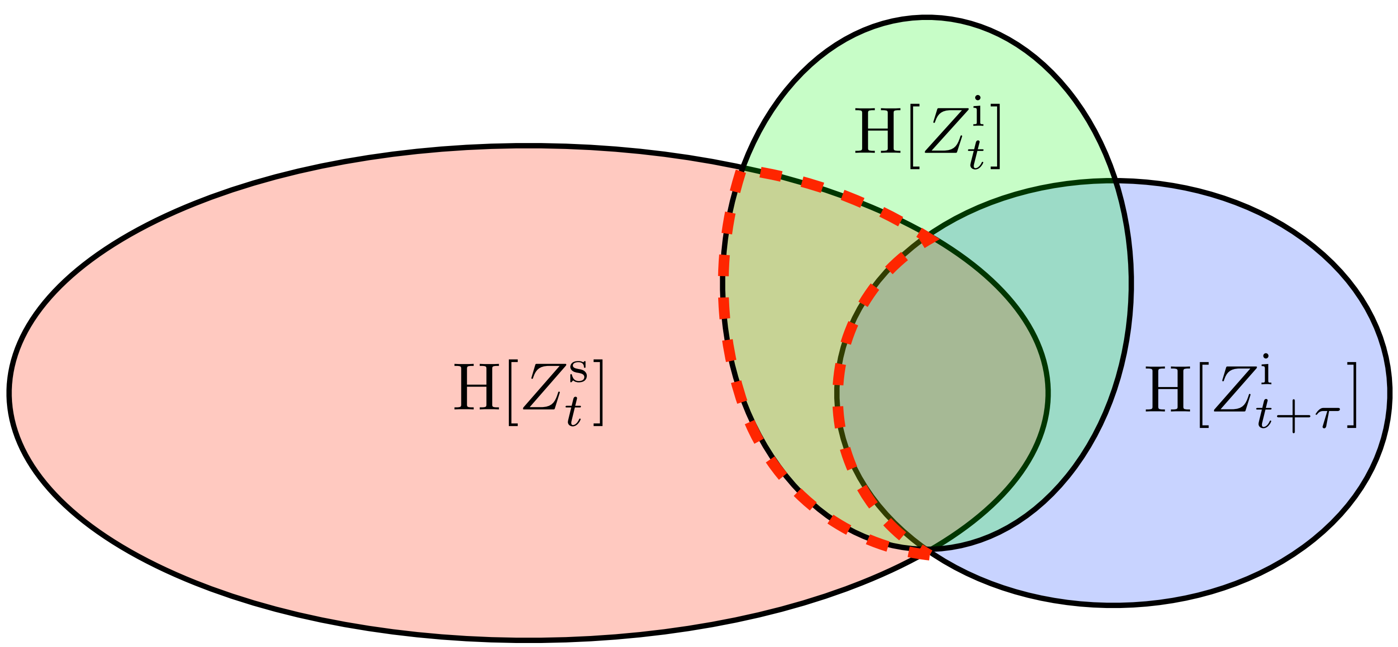
$$T\langle \Sigma^{\mathrm{mod}} \rangle_{\min} = \langle W^{\mathrm{local}} \rangle_{\min} - \Delta F^{\mathrm{NEQ}}$$

Modularity Dissipation

$$
\begin{aligned}
T\langle \Sigma^{\mathrm{mod}} \rangle_{\min} &= \langle W^{\mathrm{local}} \rangle_{\min} - \Delta F^{\mathrm{NEQ}} \\
&= \langle W^{\mathrm{local}} \rangle_{\min} - \langle W^{\mathrm{global}} \rangle_{\min}
\end{aligned}
$$

Modularity Dissipation

$$
\begin{aligned}
T\langle \Sigma^{\mathrm{mod}} \rangle_{\min} &= \langle W^{\mathrm{local}} \rangle_{\min} - \Delta F^{\mathrm{NEQ}} \\
&= \langle W^{\mathrm{local}} \rangle_{\min} - \langle W^{\mathrm{global}} \rangle_{\min} \\
&= k_B T \ln 2 (H[Z^i_t] - H[Z^i_{t+\tau}] - H[Z_t] + H[Z_{t+\tau}])
\end{aligned}
$$



Modularity Dissipation

$$
\begin{aligned}
T\langle \Sigma^{\mathrm{mod}} \rangle_{\min} &= \langle W^{\mathrm{local}} \rangle_{\min} - \Delta F^{\mathrm{NEQ}} \\
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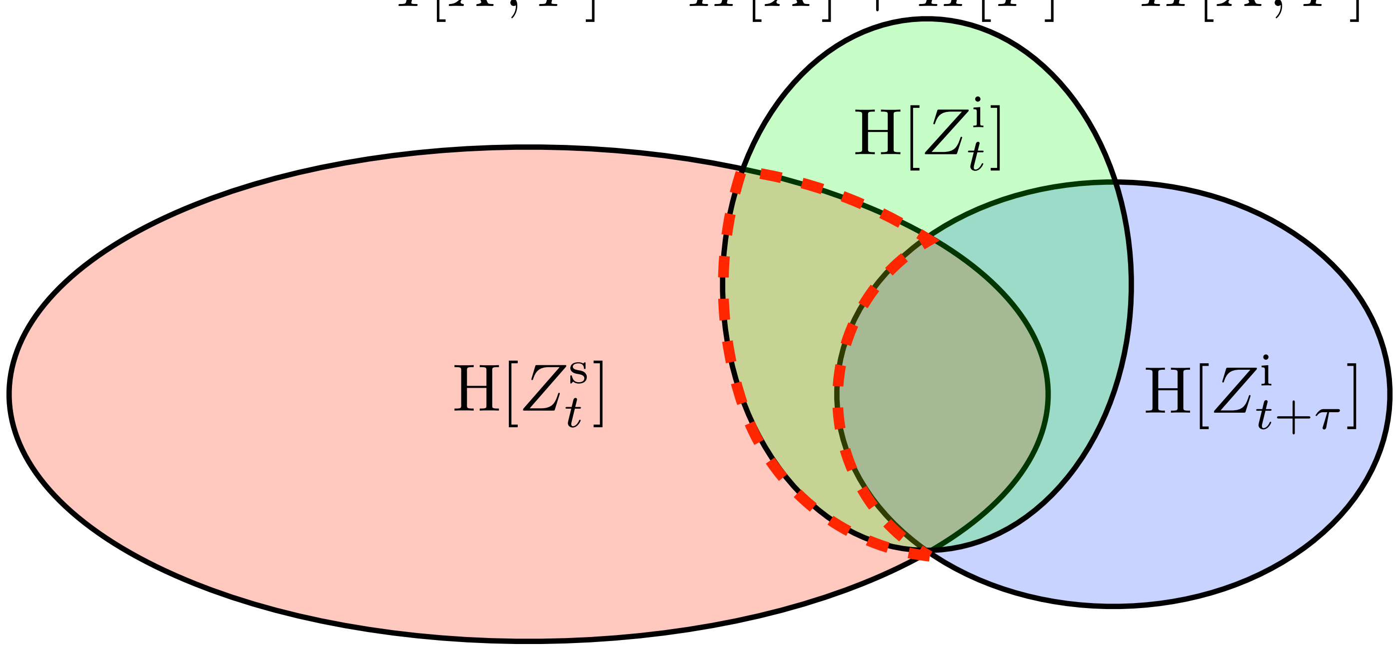
&= k_B T \ln 2 (H[Z_t^i] - H[Z_{t+\tau}^i] - H[Z_t] + H[Z_{t+\tau}]) \\
&= k_B T \ln 2 (I[Z_t^i; Z_t^s] - I[Z_{t+\tau}^i; Z_t^s])
\end{aligned}
$$

$$I[X;Y] = H[X] + H[Y] - H[X,Y]$$

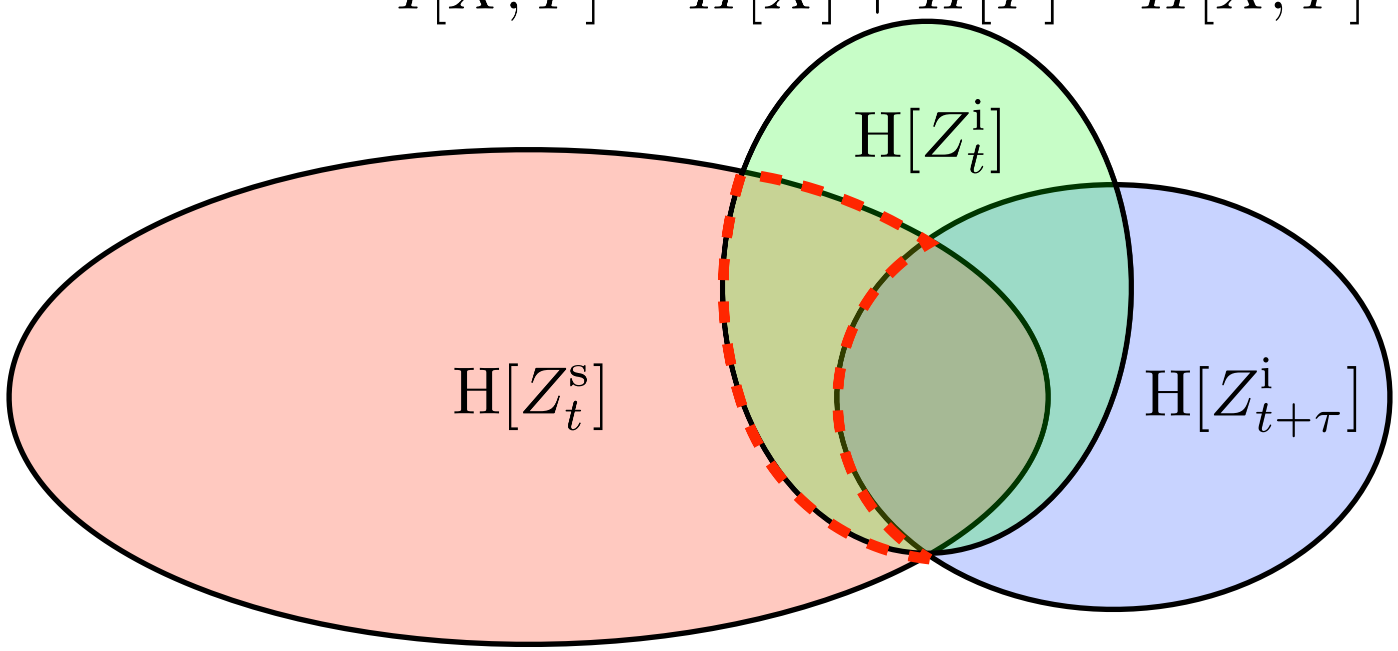


Dissipation of forgotten global correlations

Modularity Dissipation

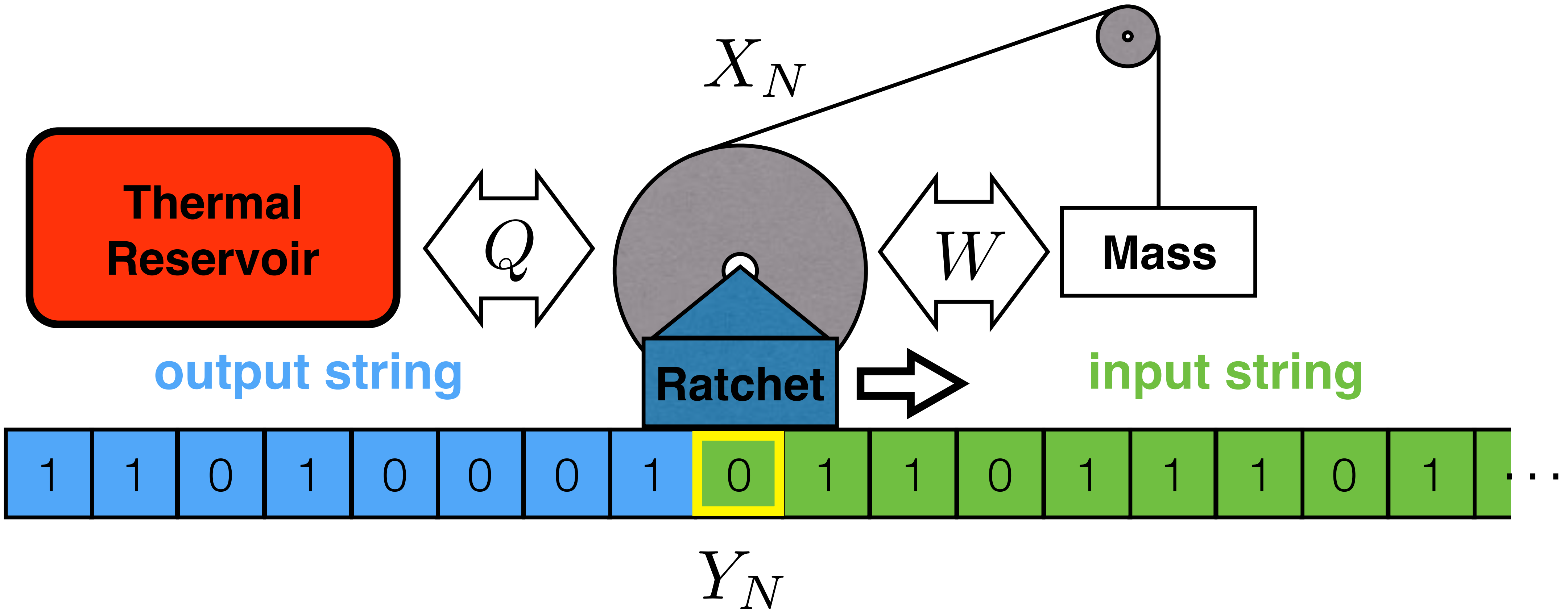
$$\begin{aligned}
T\langle\Sigma^{\text{mod}}\rangle_{\min} &= \langle W^{\text{local}}\rangle_{\min} - \Delta F^{\text{NEQ}} \\
&= \langle W^{\text{local}}\rangle_{\min} - \langle W^{\text{global}}\rangle_{\min} \\
&= k_B T \ln 2 (H[Z^i_t] - H[Z^i_{t+\tau}] - H[Z_t] + H[Z_{t+\tau}]) \\
&= k_B T \ln 2 (I[Z^i_t; Z^s_t] - I[Z^i_{t+\tau}; Z^s_t]) \\
&\boxed{= k_B T \ln 2 I[Z^i_t; Z^s_t | Z^i_{t+\tau}]}
\end{aligned}$$

$$I[X;Y] = H[X] + H[Y] - H[X,Y]$$



Dissipation of forgotten global correlations

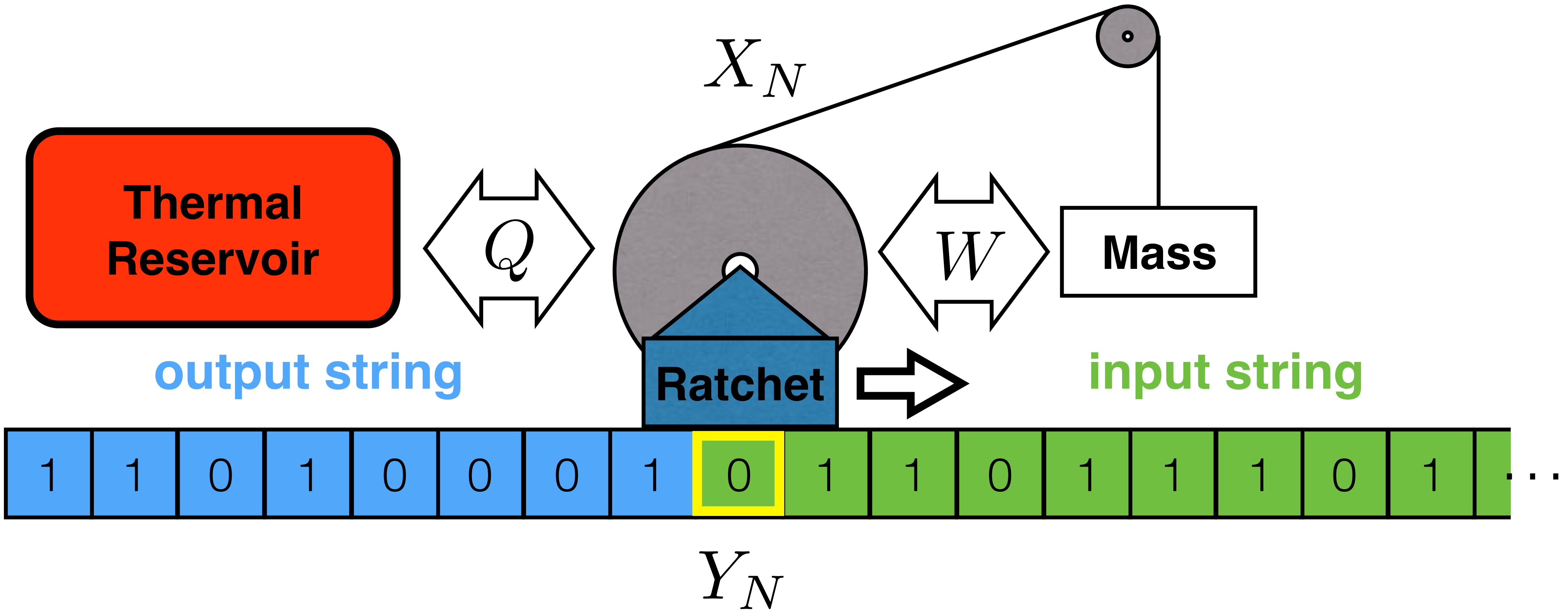
Modular Thermodynamic Transducers



Ratchet and interaction bit are locally changing elements:

$$\langle W_N^{\text{local}} \rangle_{\min} = k_B T \ln 2 (H[X_N, Y_N] - H[X_{N+1}, Y'_N])$$

Modular Thermodynamic Transducers



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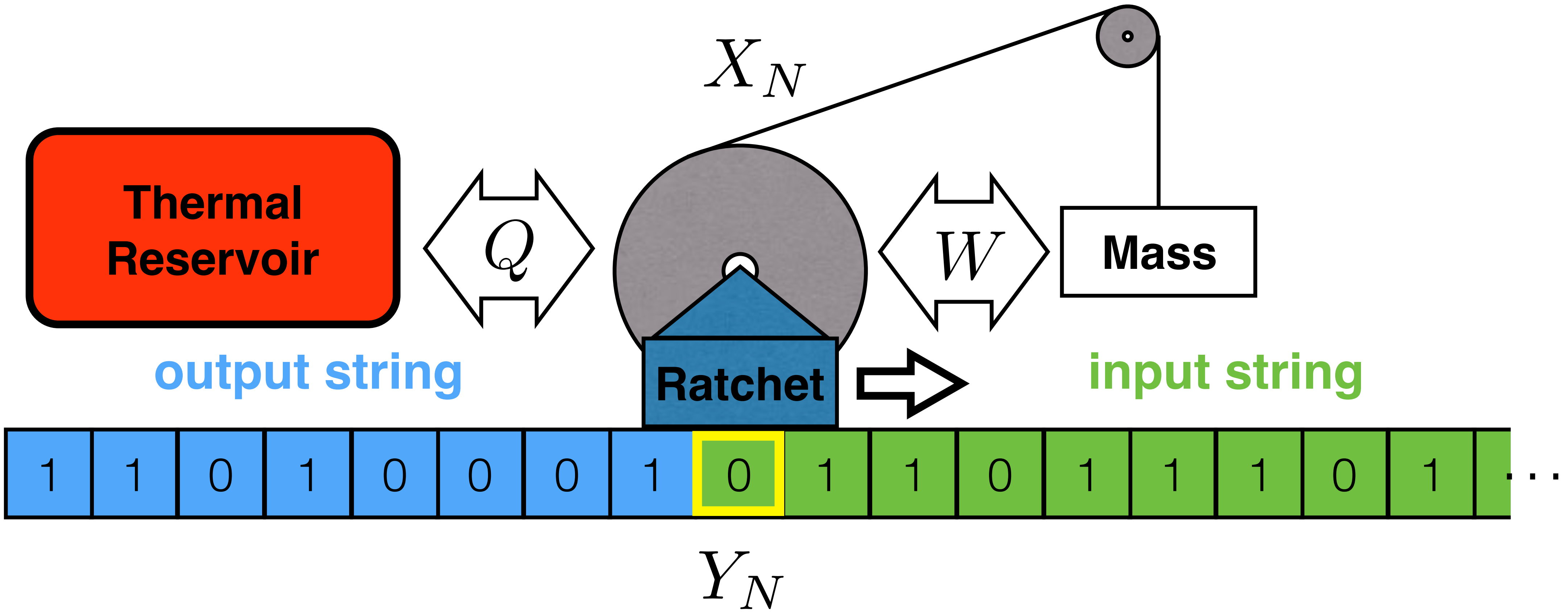
Locally operating ratchets necessarily dissipate:

$$\begin{aligned} T \langle \Sigma_N^{\text{mod}} \rangle_{\min} &= \langle W_N^{\text{local}} \rangle_{\min} - \Delta F^{\text{NEQ}} \\ &= k_B T \ln 2 I[X_N Y_N ; Y'_{0:N} Y_{N+1:\infty} | X_{N+1} Y'_N] \end{aligned}$$

Modular Thermodynamic Transducers



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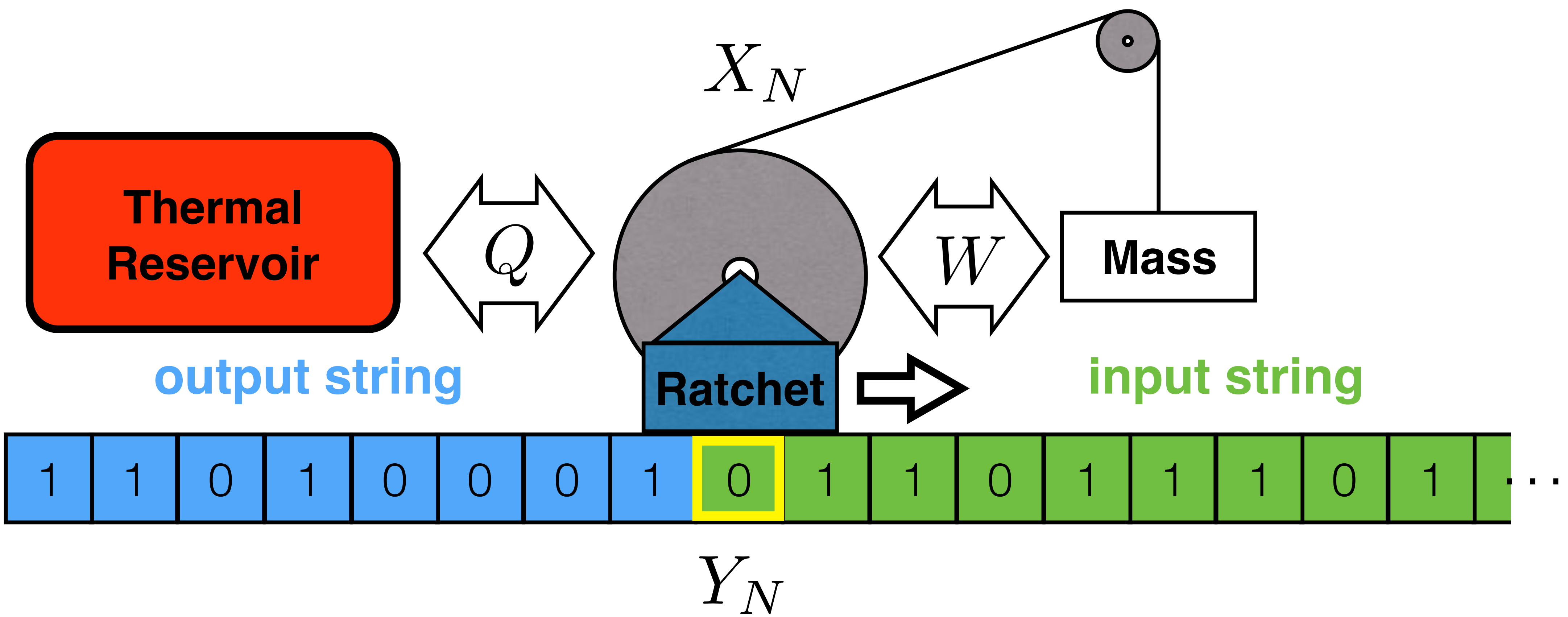
Can be seen by plugging into formula for modularity dissipation:

Modular Thermodynamic Transducers



Ratchet and interaction bit are locally changing elements:

$$\langle W_N^{\text{local}} \rangle_{\min} = k_B T \ln 2 (H[X_N, Y_N] - H[X_{N+1}, Y'_N])$$

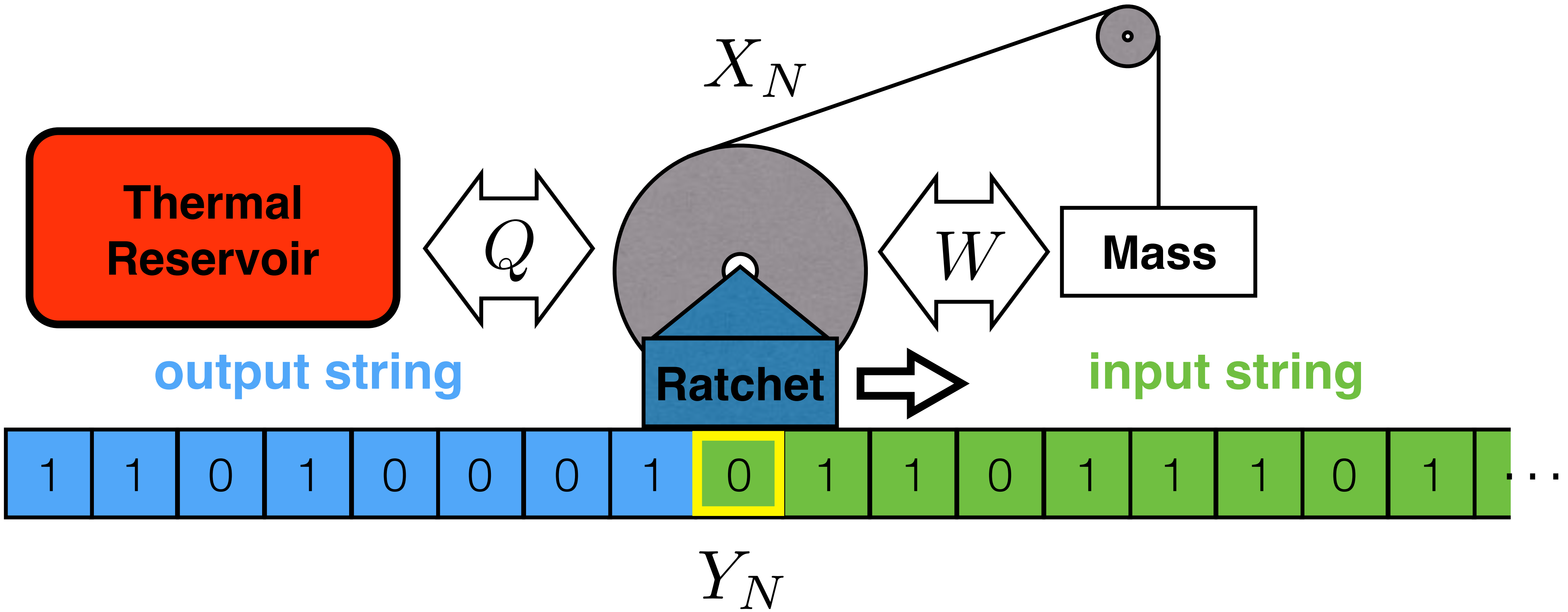
Locally operating ratchets necessarily dissipate:

$$\begin{aligned} T\langle \Sigma_N^{\text{mod}} \rangle_{\min} &= \langle W_N^{\text{local}} \rangle_{\min} - \Delta F^{\text{NEQ}} \\ &= k_B T \ln 2 I[\underline{X_N Y_N}; Y'_{0:N} Y_{N+1:\infty} | X_{N+1} Y'_N] \end{aligned}$$

Can be seen by plugging into formula for modularity dissipation:

$$\underline{Z^i_{N\tau} = X_N Y_N}$$

Modular Thermodynamic Transducers



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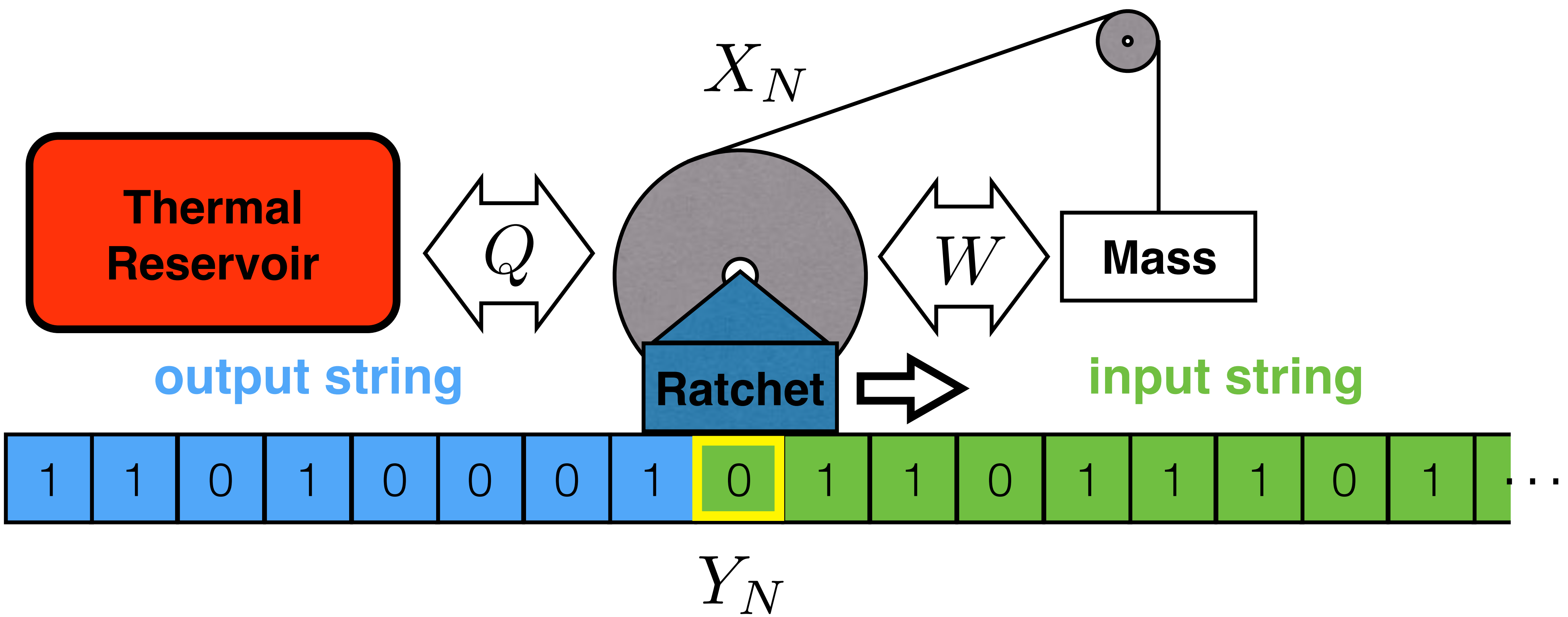
Modular Thermodynamic Transducers



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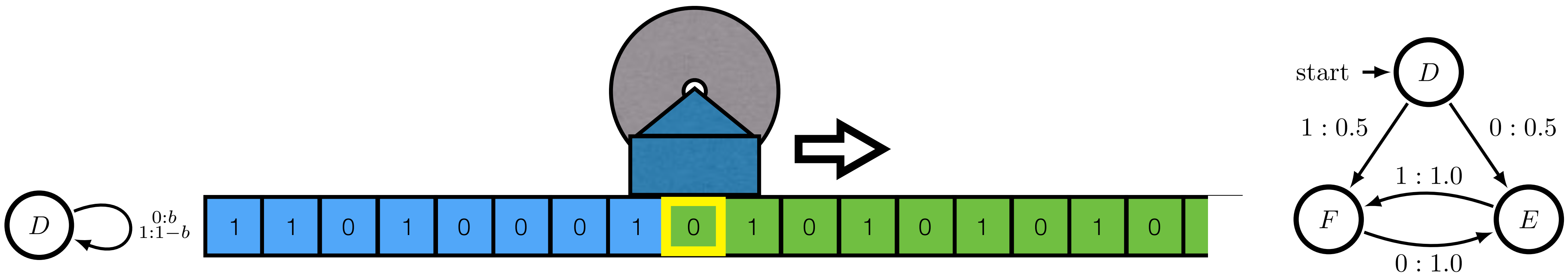
Can be seen by plugging into formula for modularity dissipation:

$\underline{Z^i_{N\tau} = X_N Y_N}$ $\underline{Z^i_{(N+1)\tau} = X_{N+1} Y'_N}$ $\underline{Z^s_{N\tau} = Y'_{0:N} Y_{N+1:\infty}}$

Modular Extractors

Pattern Extractors: correlated inputs, uncorrelated outputs



$$\Pr(Y'_{a:b} = y'_{a:b}) = \prod_{i=a}^{b-1} \Pr(Y'_i = y'_i)$$

$$\Pr(Y_{a:b} = y_{a:b}) \neq \prod_{i=a}^{b-1} \Pr(Y_i = y_i)$$

$$C'_\mu = H[S'^{+}_{N}] = 0$$

$$C_\mu = H[S^{+}_{N}] \geq 0$$

Efficient Extractors

Perfect efficiency: $\langle \Sigma_N^{\mathrm{mod}} \rangle_{\min} = 0 \text{ for all } N$

$$\Updownarrow$$

Predictive ratchet: $I[X_N; Y_{N:\infty} | Y_{0:N}] = 0 \text{ and } I[Y_{0:N}; Y_{N:\infty} | X_N] = 0$

Efficient Extractors

Perfect efficiency: $\langle \Sigma_N^{\mathrm{mod}} \rangle_{\min} = 0$ for all N

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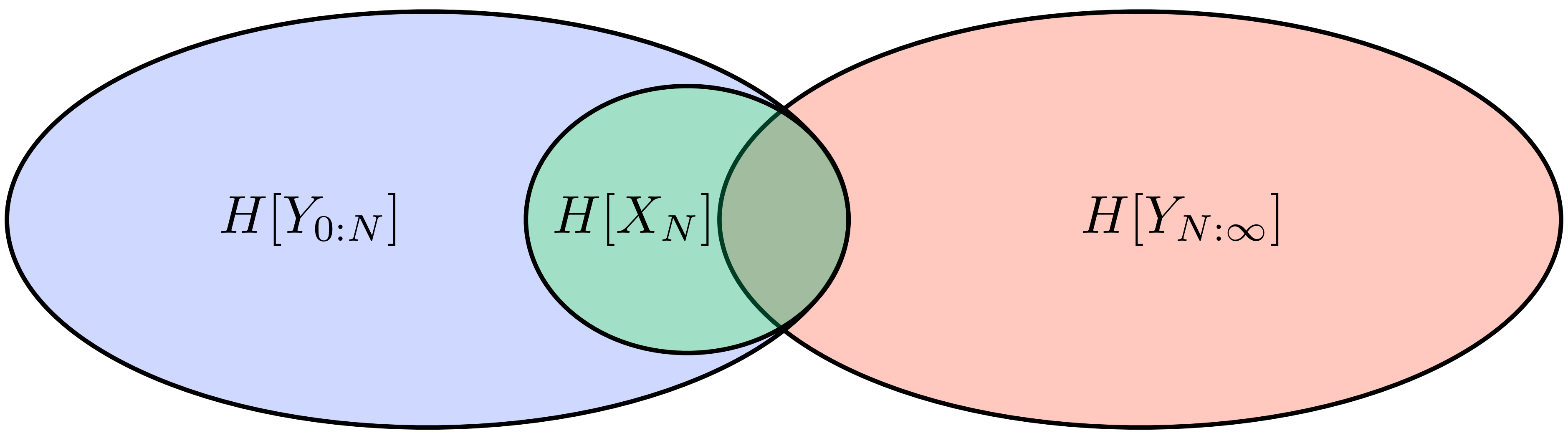

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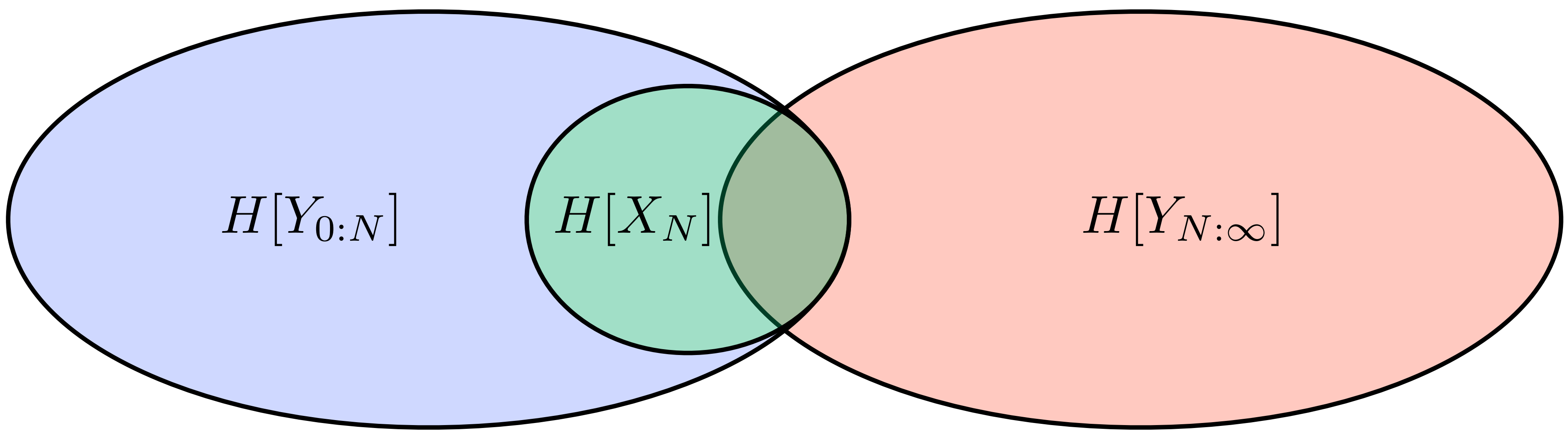


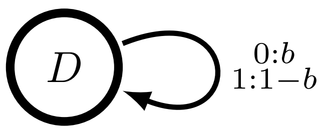
Implies principle of requisite complexity: an efficient extractor must at least match the statistical complexity of its input.

$$\lim_{N \to \infty} H[X_N] \geq C_\mu$$

A. B. Boyd, D. Mandal, and J. P. Crutchfield, “Leveraging Environmental Correlations: The Thermodynamics of Requisite Variety”, J. Stat. Phys. (2017)

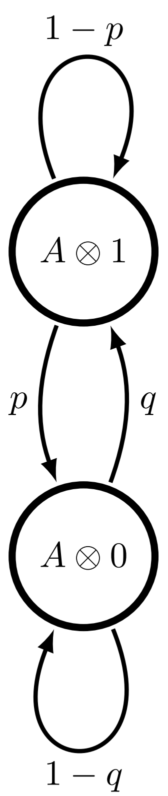
Different Types of Order

simple inputs



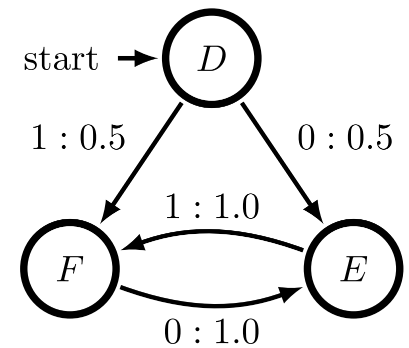
$$C_\mu = H[S_N^+] = 0$$

simple engine



$$H[X_N] = 0$$

complex inputs



$$C_\mu = H[S_N^+] = 1 > 0$$

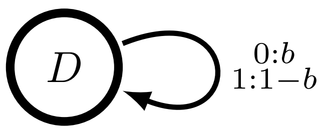
order is temporal correlations:

$$H[Y_N] - h_\mu = 1$$

Memoryless ratchet dissipates all temporal correlations, because of modularity:

Different Types of Order

simple inputs



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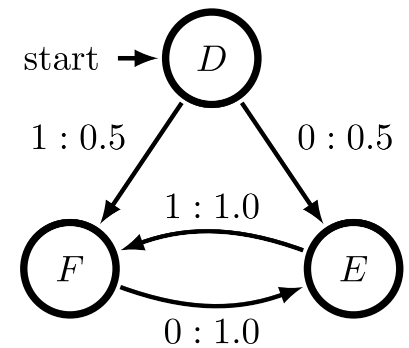

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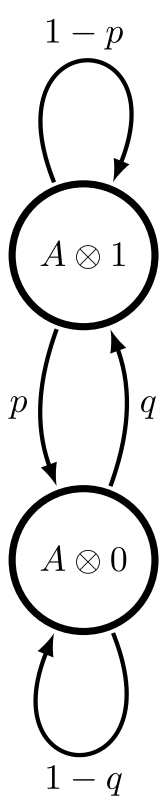

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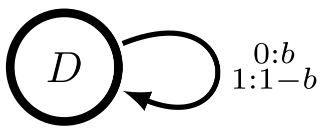
Different Types of Order

simple inputs



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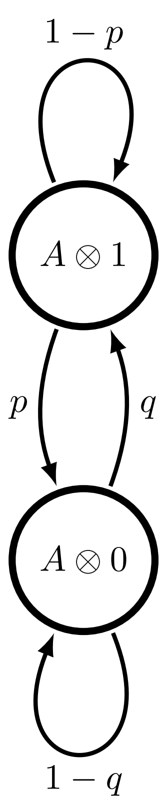

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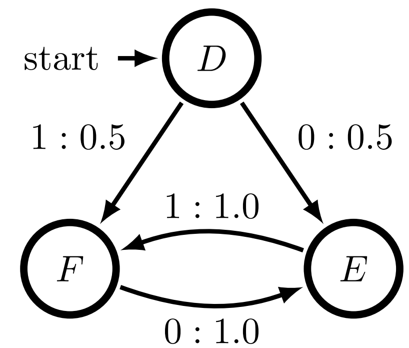

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Memoryless ratchet dissipates all temporal correlations, because of modularity: $\langle \Sigma_N^{\mathrm{mod}} \rangle_{\min} = k_B T \ln 2 \, (H[Y_N] - h_\mu)$

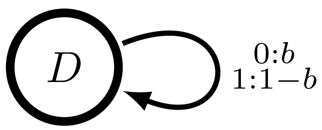
Modularity dissipation is only minimized when the ratchet is ***predictive***, matching ***statistical complexity*** of the input.

$$H[X_N] \geq C_\mu$$

A. B. Boyd, D. Mandal, and J. P. Crutchfield, “Above and Beyond the Landauer Bound: Thermodynamics of Modularity”, arXiv:1708.03030 (2017)

Different Types of Order

simple inputs



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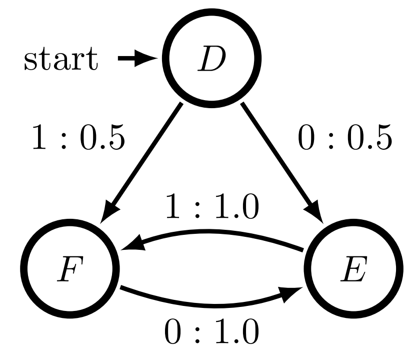

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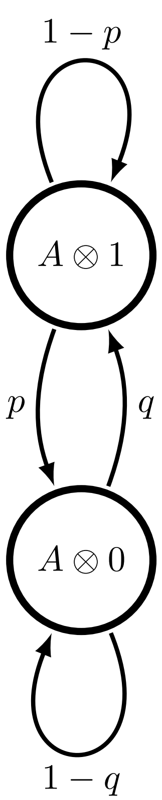
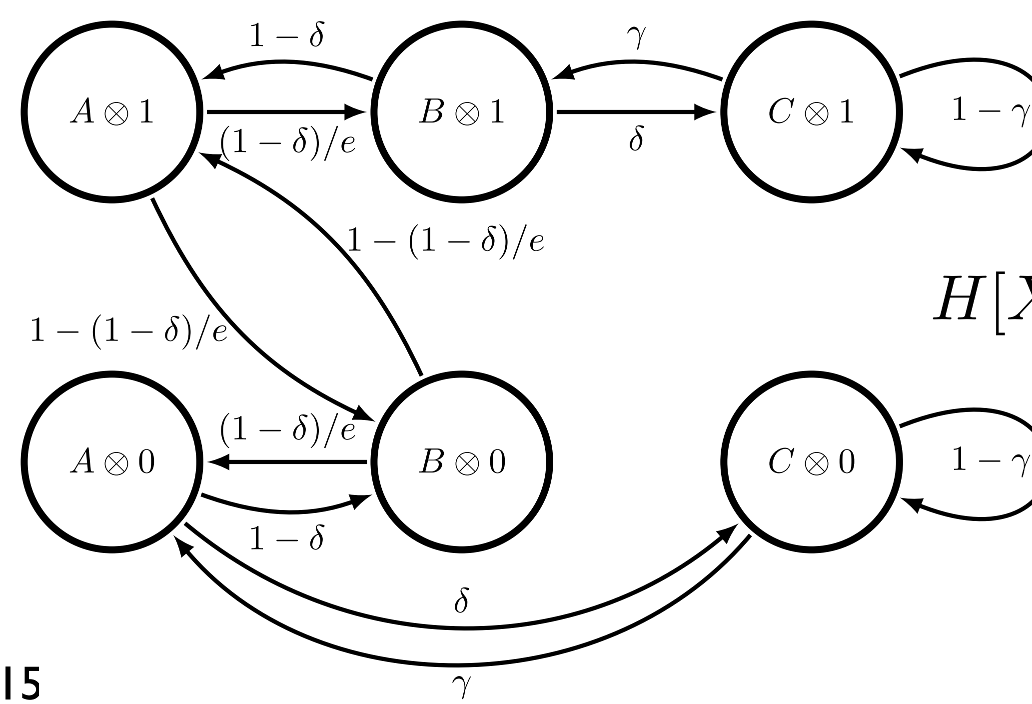


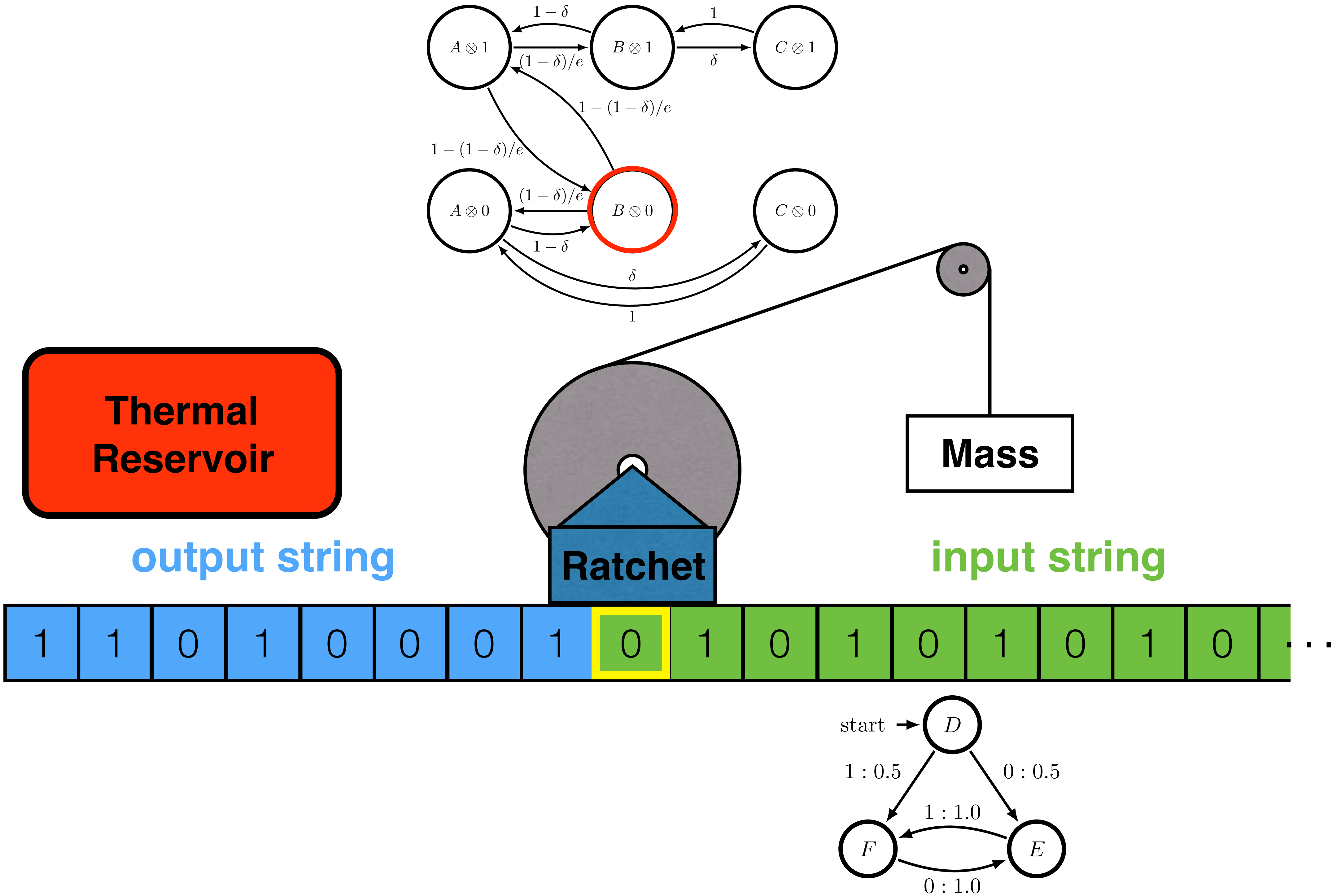
$$H[X_N] = 0$$

complex engine



$$H[X_N] = 1$$

Memoryful Ratchet Operation



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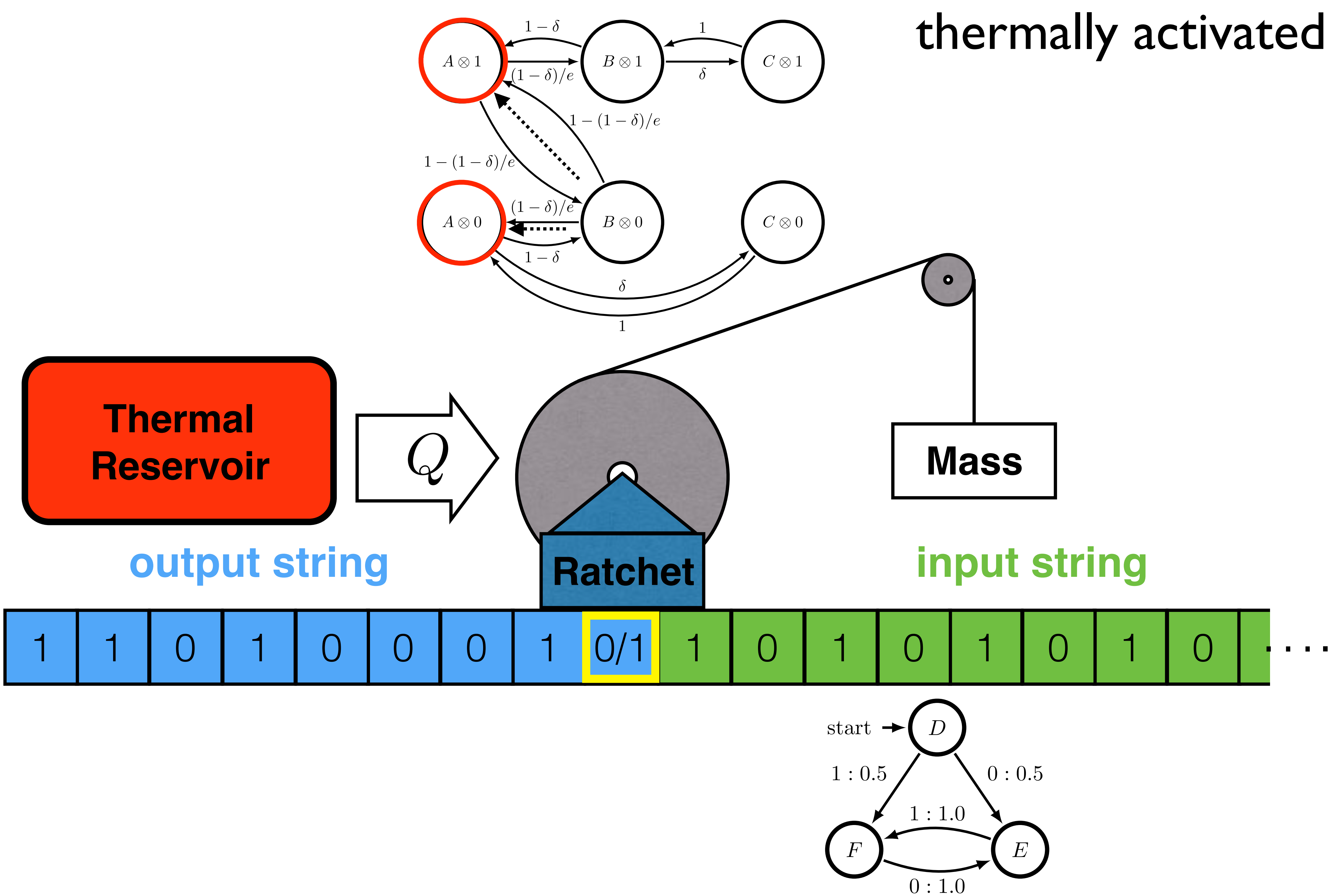

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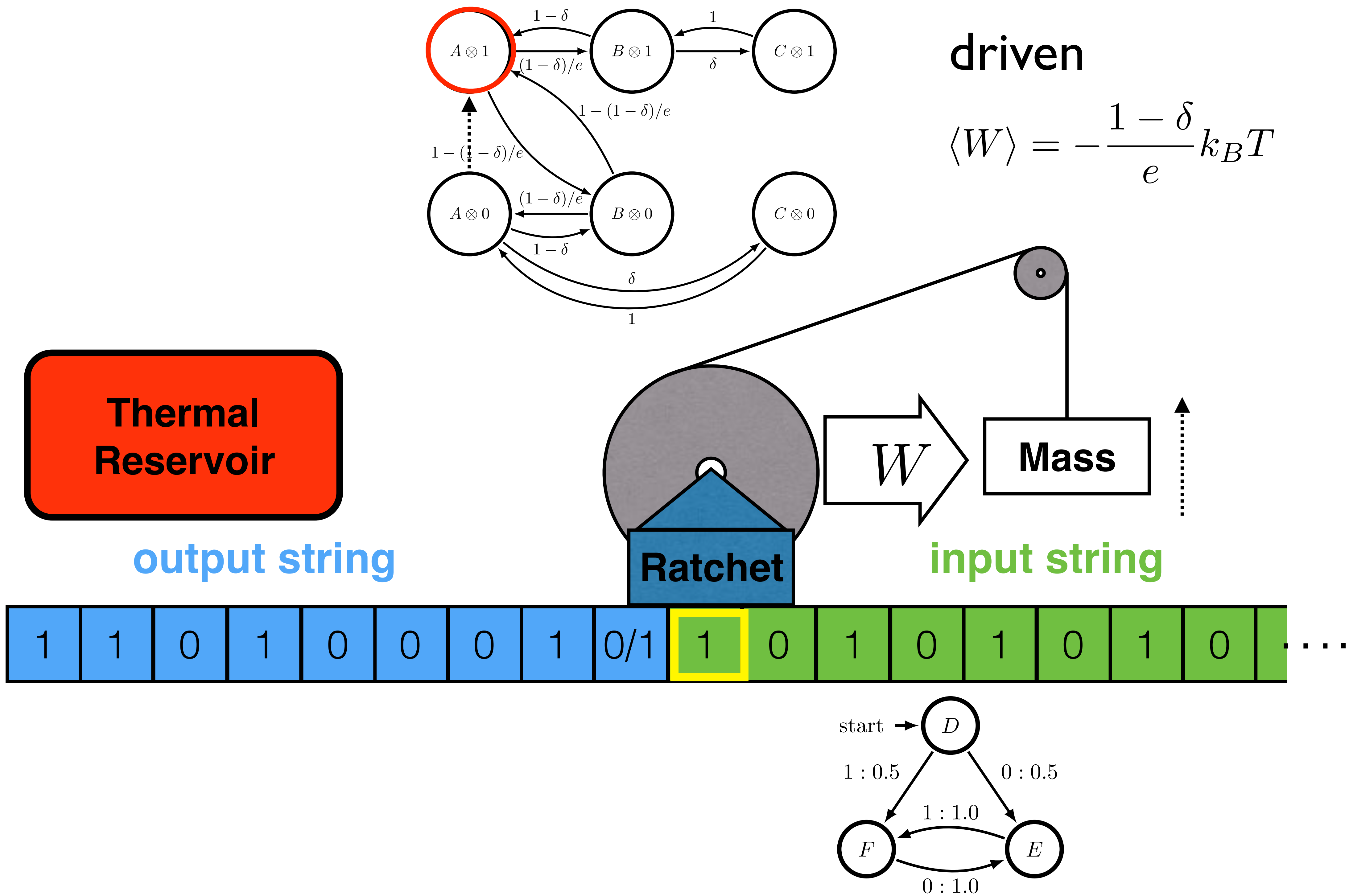
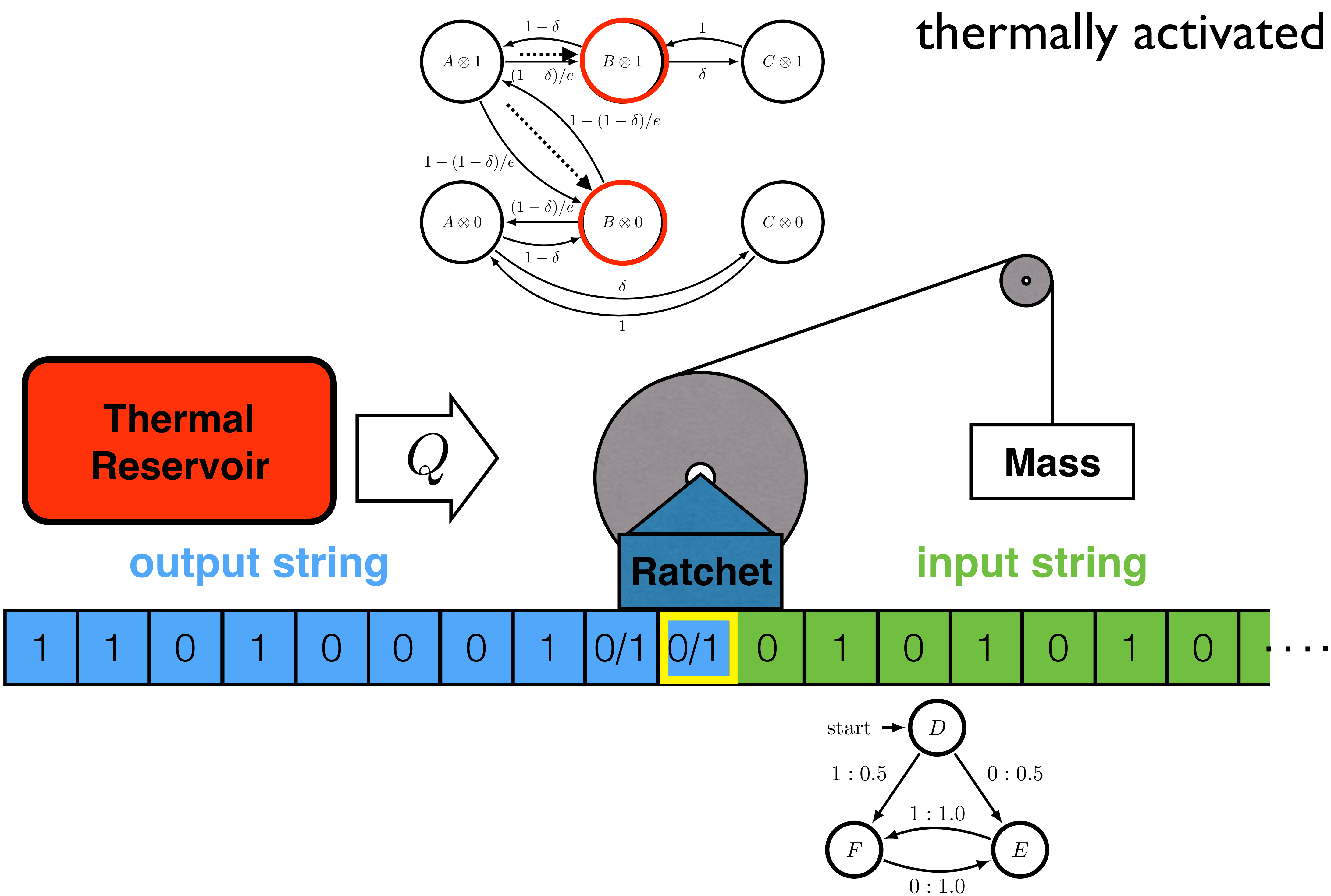


Memoryful Ratchet Operation



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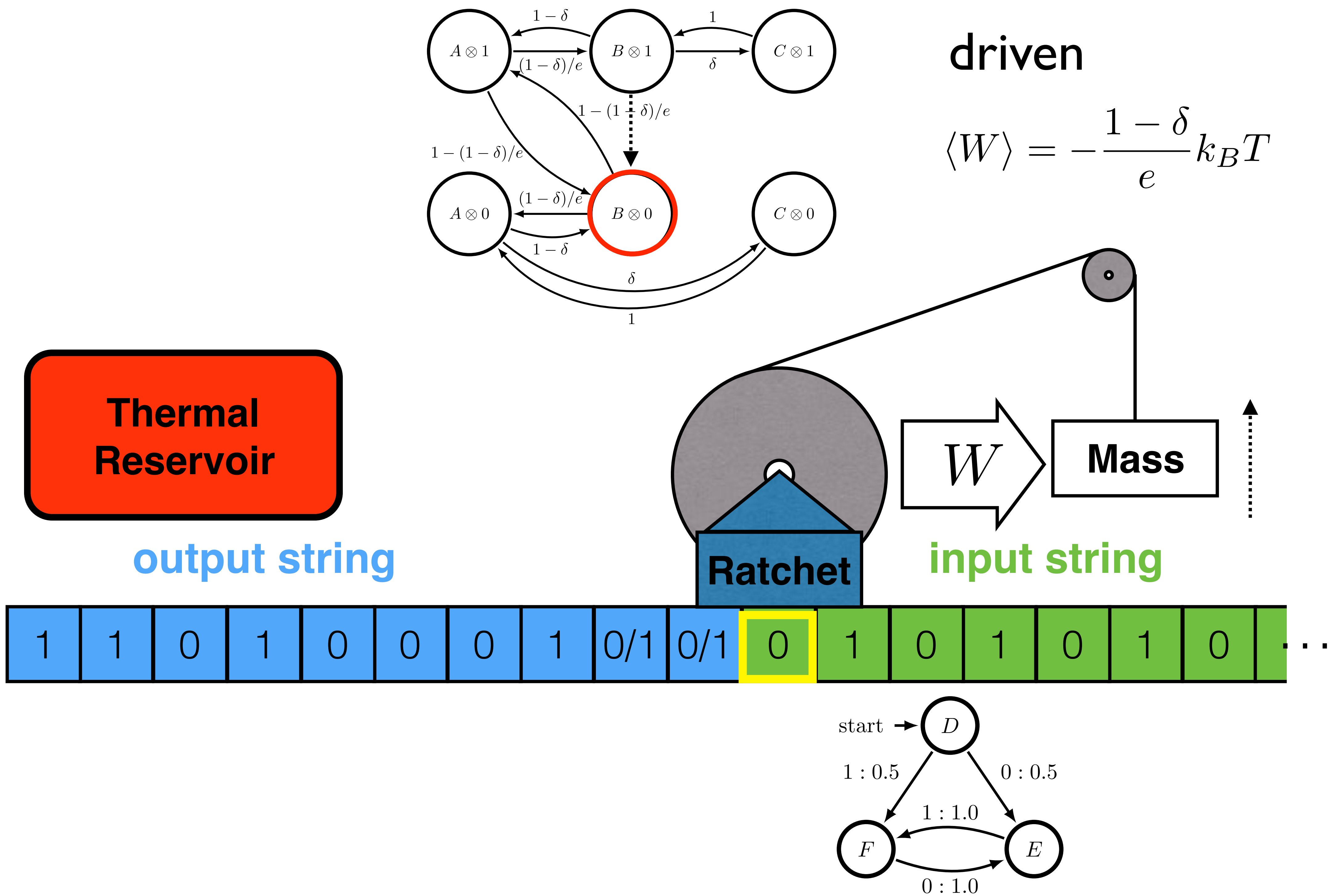

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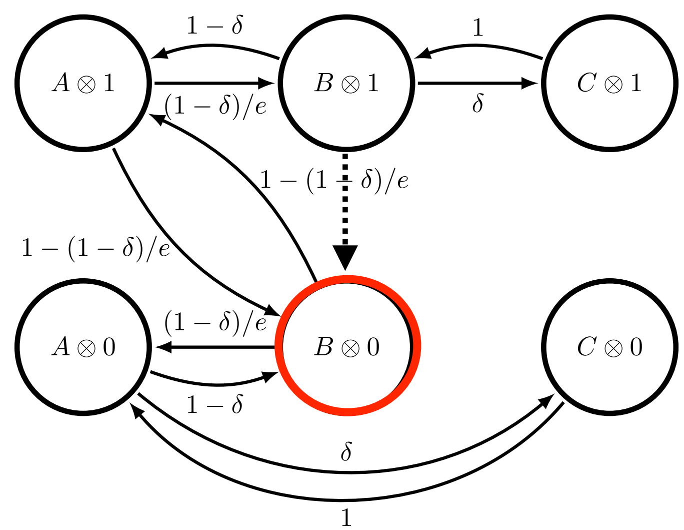
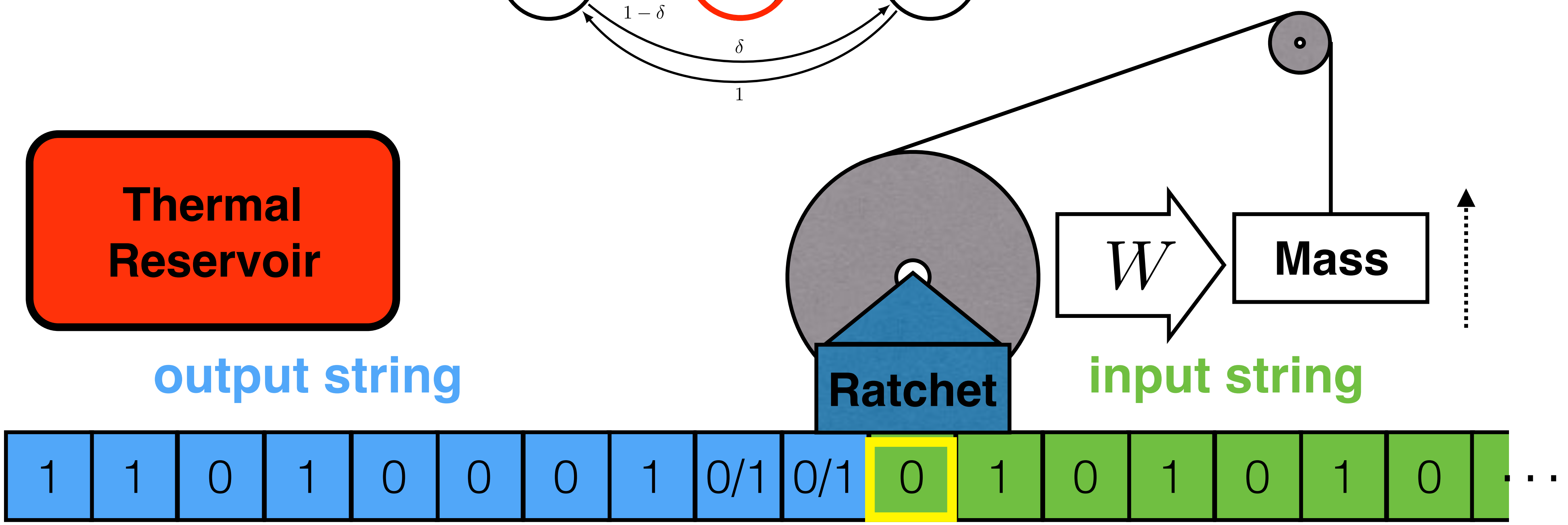
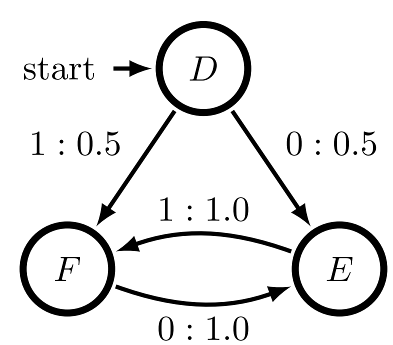


driven

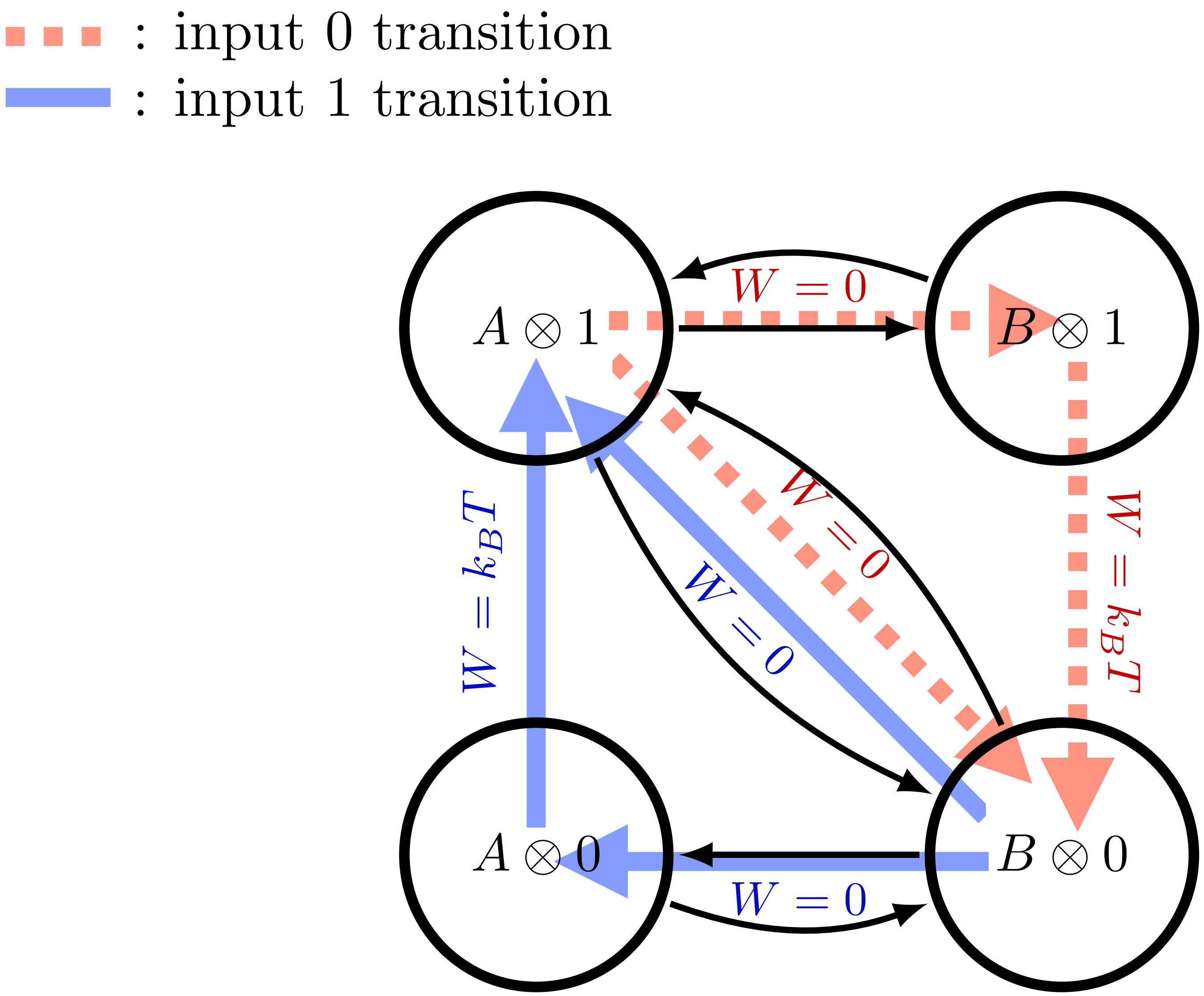
$$\langle W \rangle = -\frac{1-\delta}{e} k_B T$$





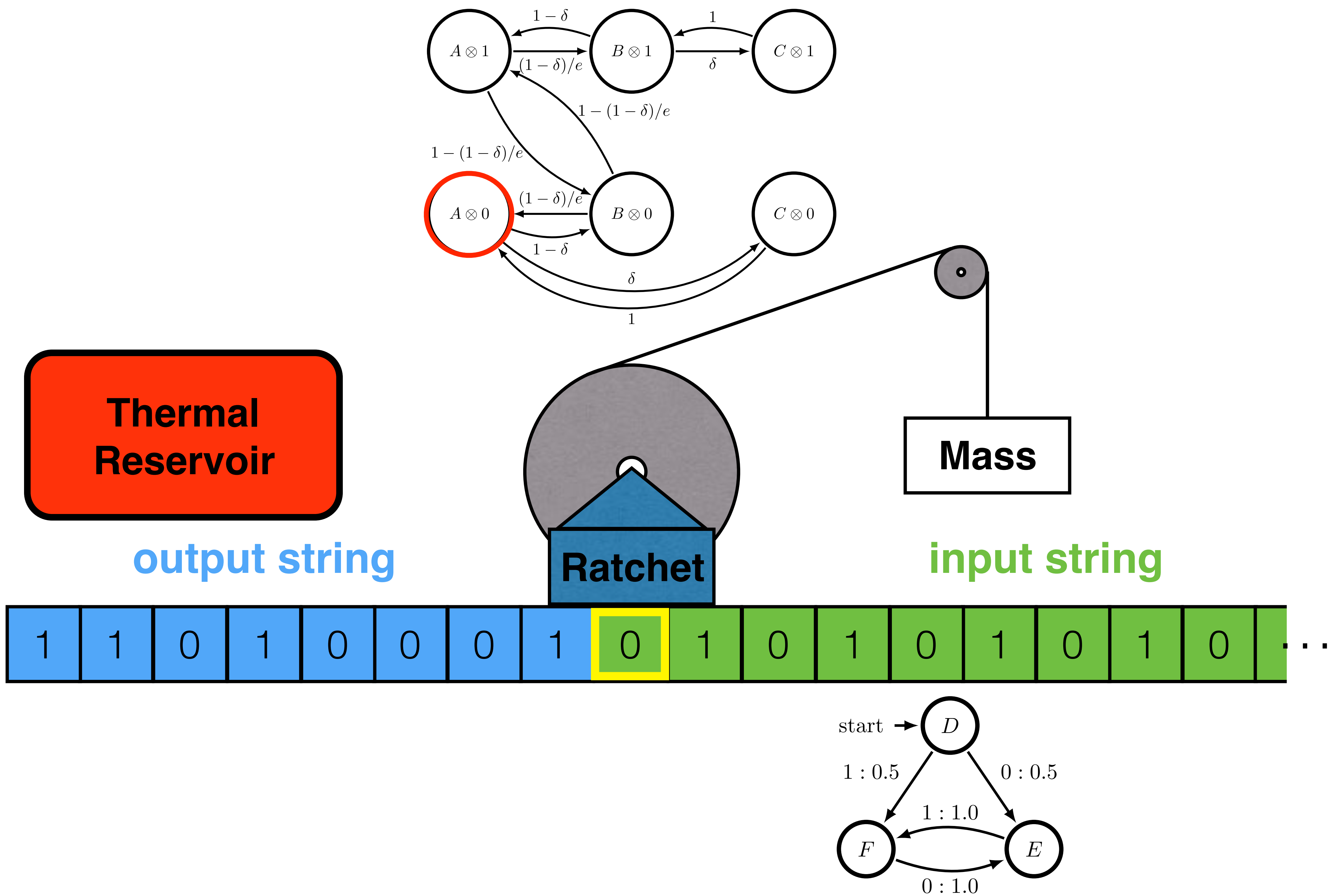
Stationary work producing dynamical phase

Dynamical Phase



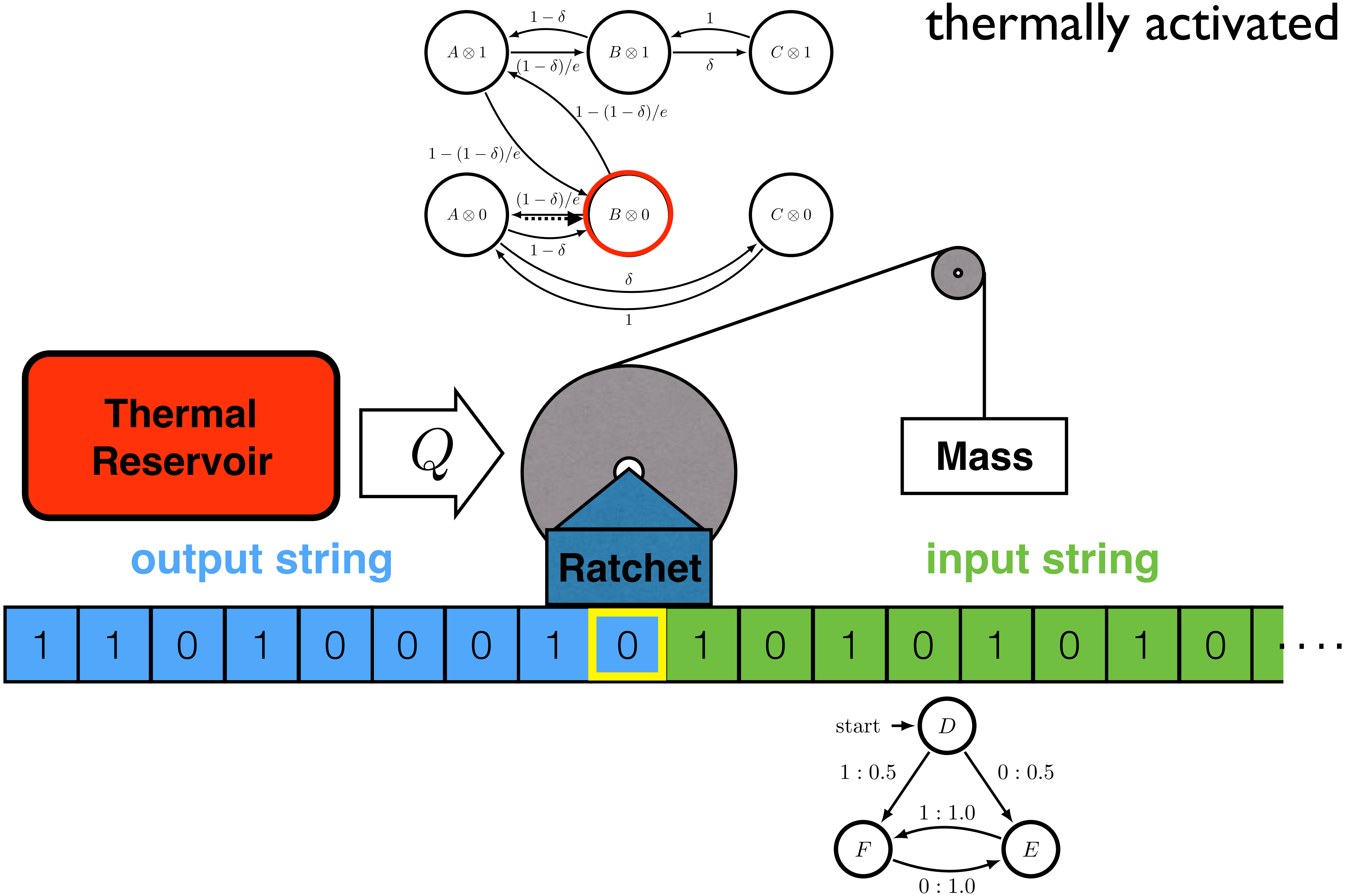
$$\langle W \rangle_{\text{clockwise}} = -k_B T / e$$

Memoryful Ratchet Operation

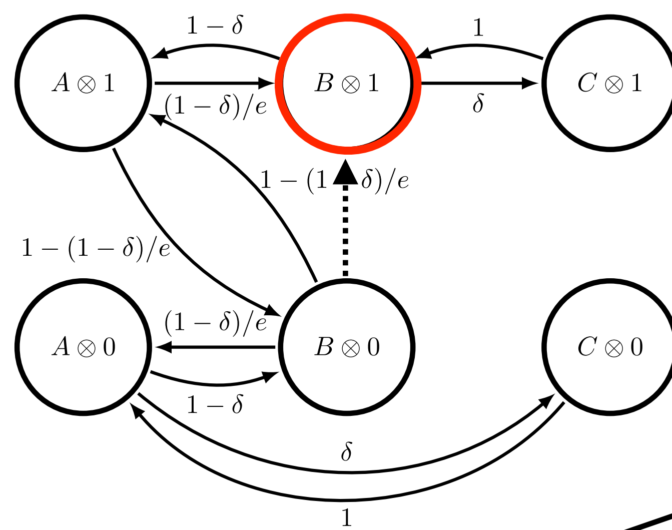


Memoryful Ratchet Operation

thermally activated

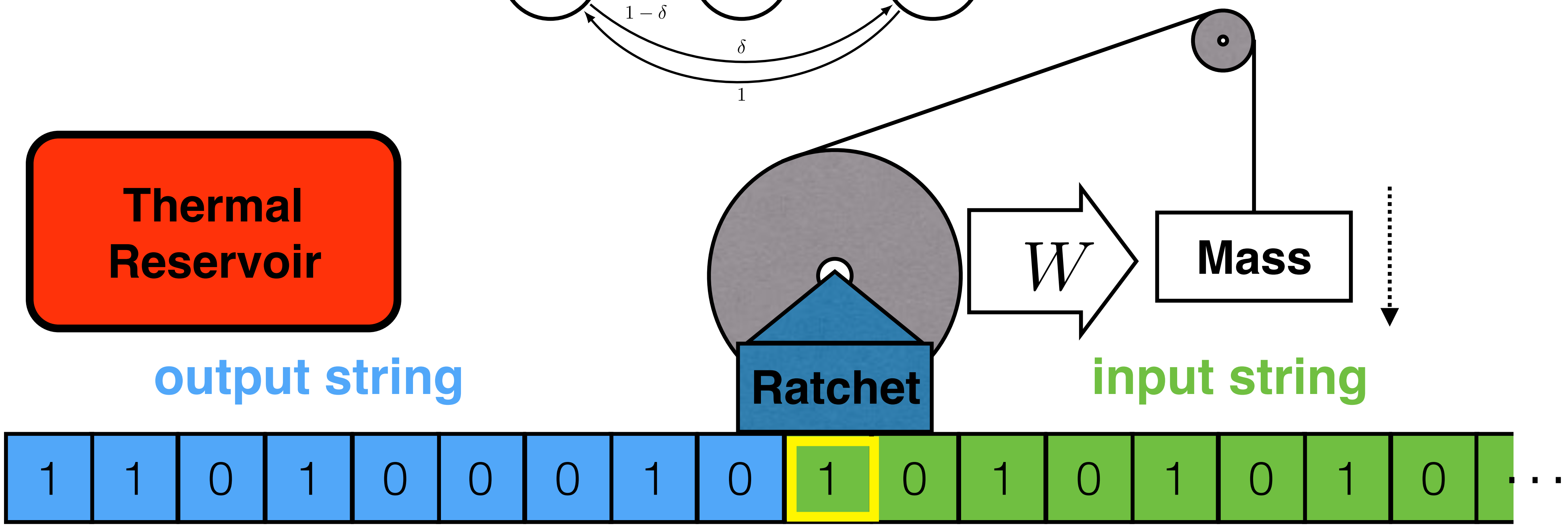


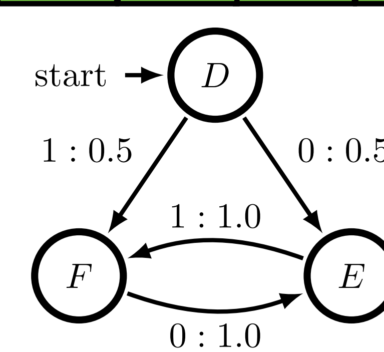
Memoryful Ratchet Operation



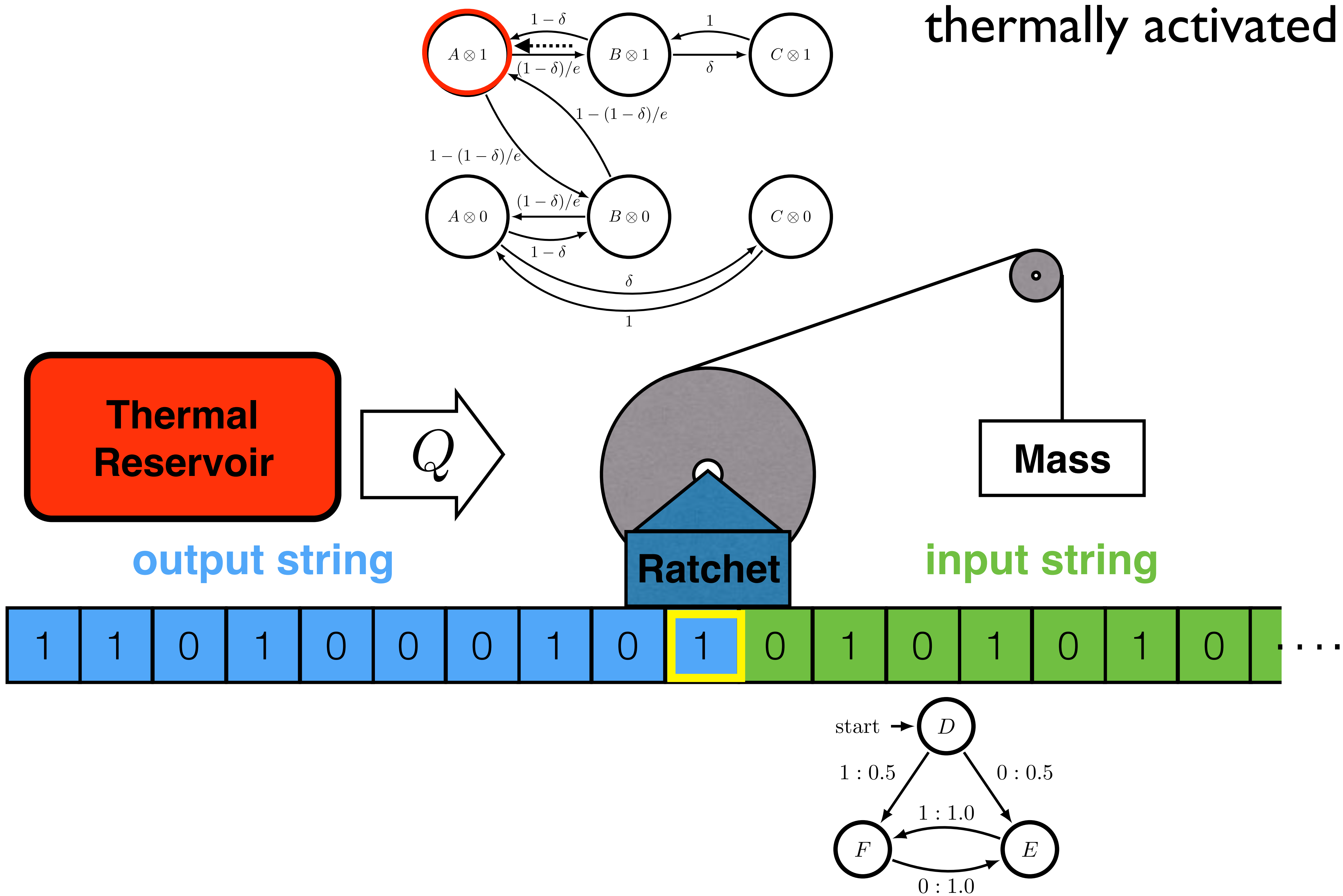
driven

$$\langle W \rangle \approx \Delta E_{B\otimes 0 \to B\otimes 1} = -k_B T$$

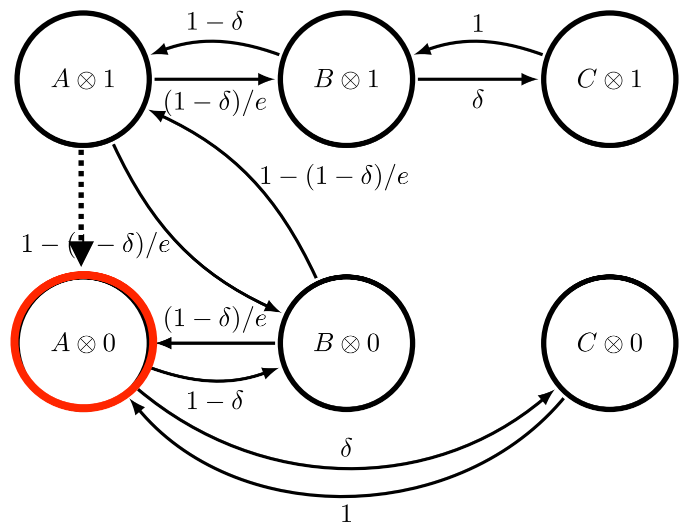




Memoryful Ratchet Operation

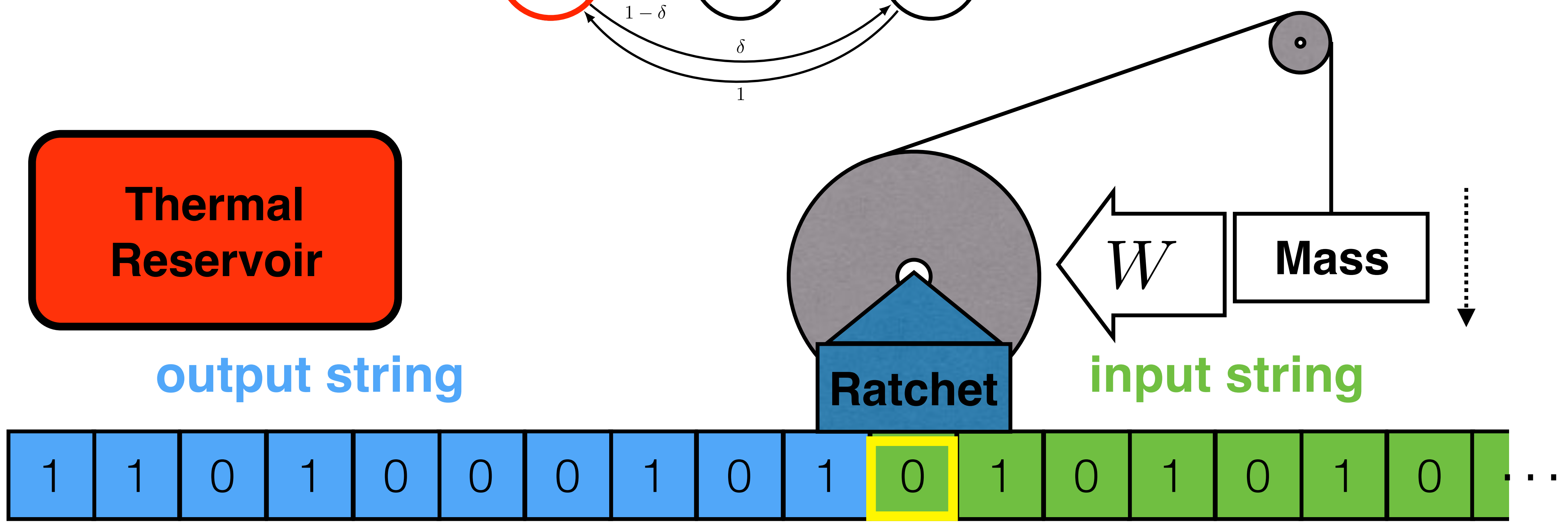


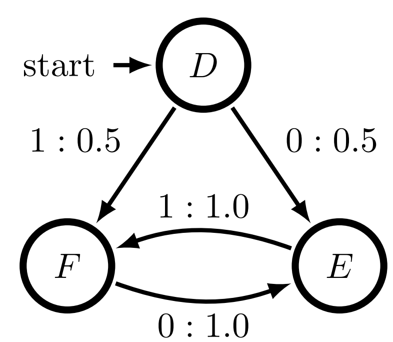
Memoryful Ratchet Operation



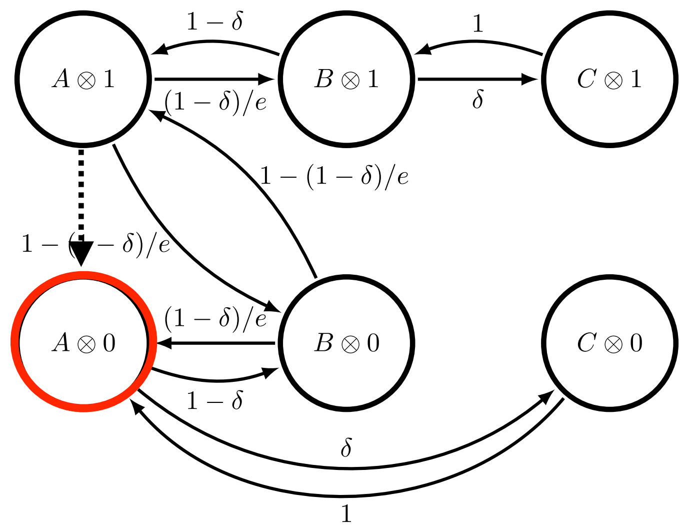
driven

$$\langle W \rangle \approx -k_B T$$





Memoryful Ratchet Operation



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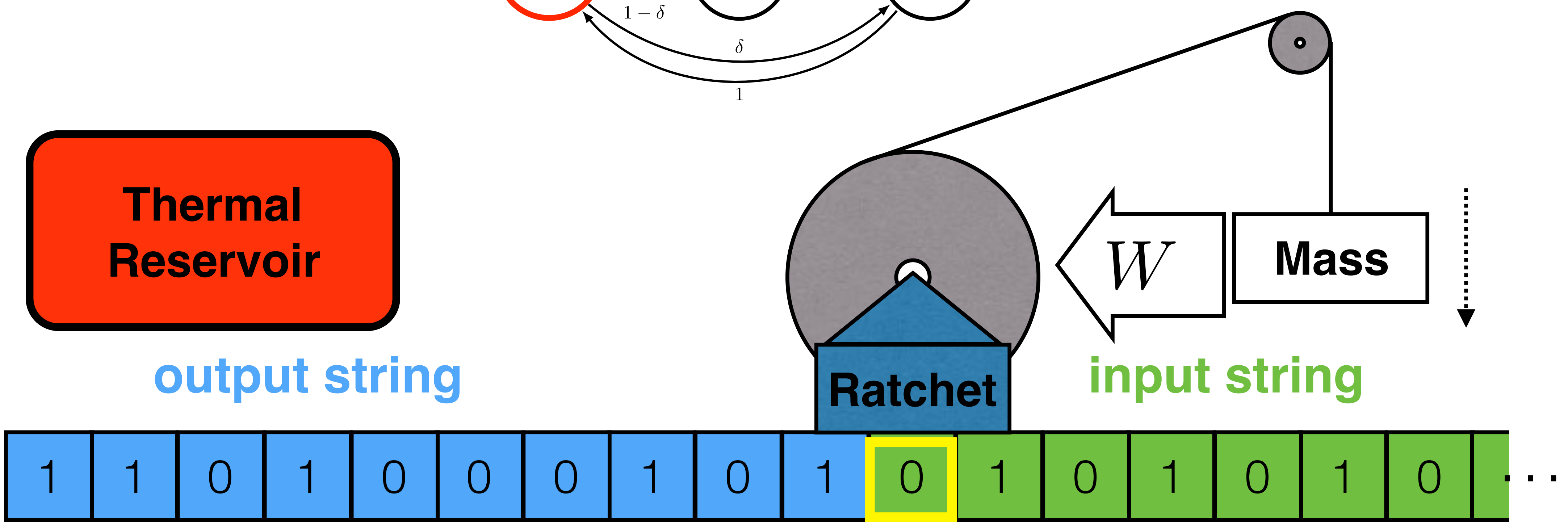
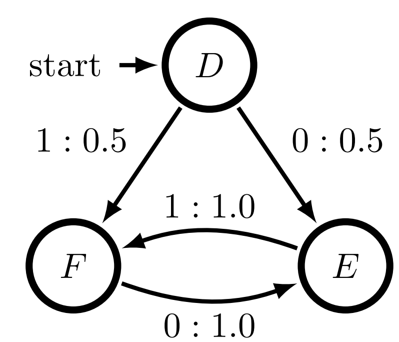
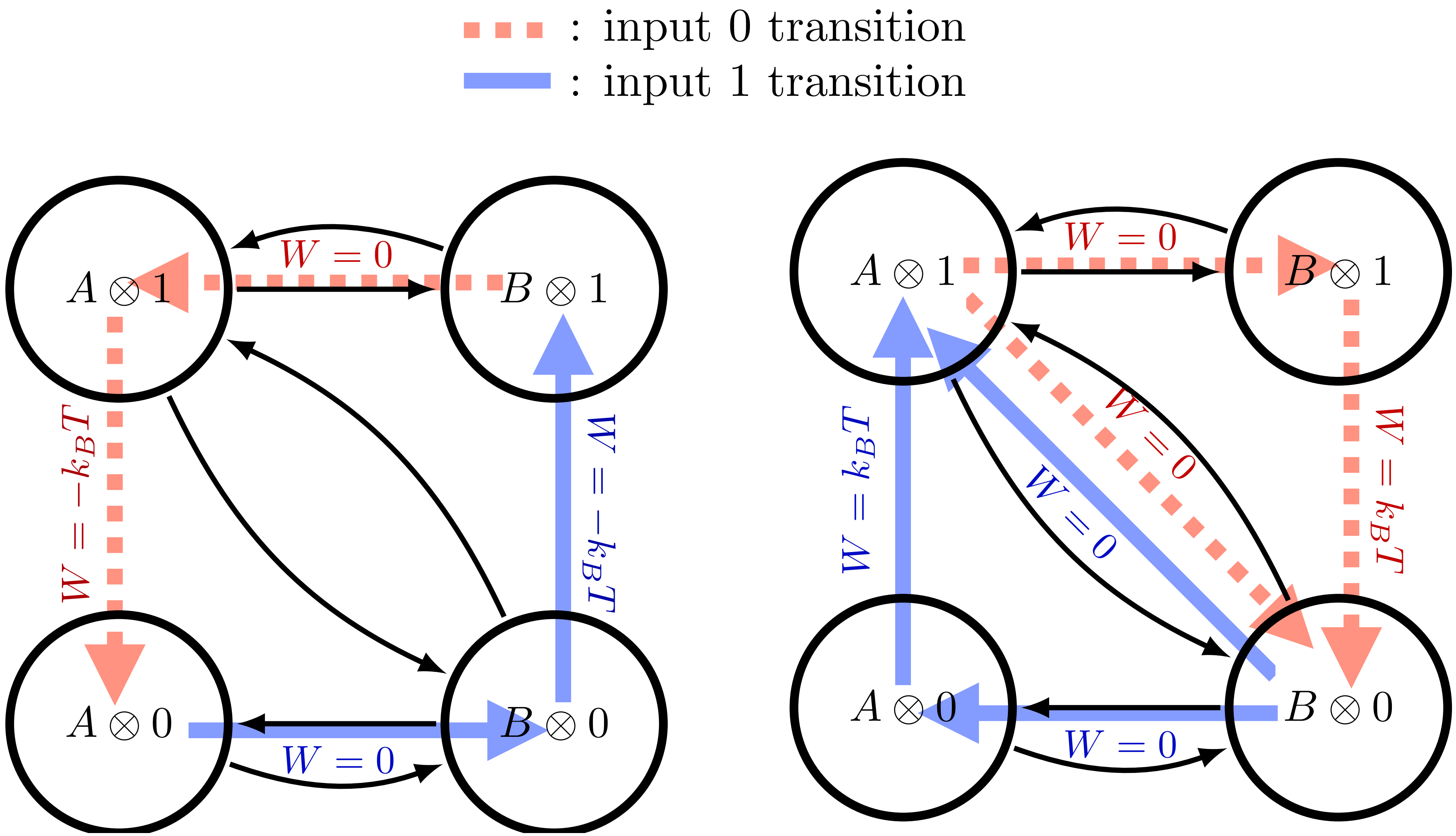




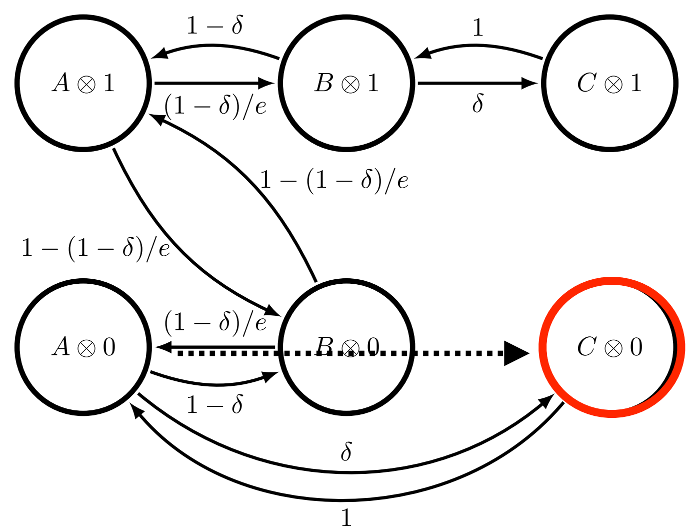
Work dissipating cycle

Two Dynamical Phases

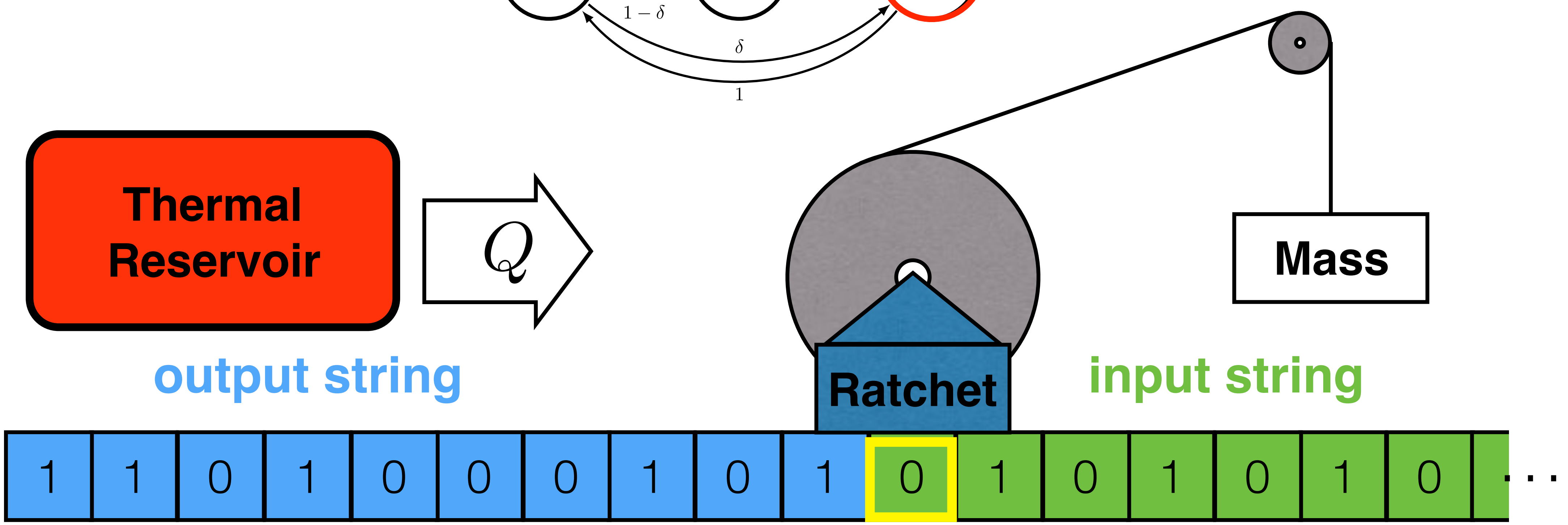


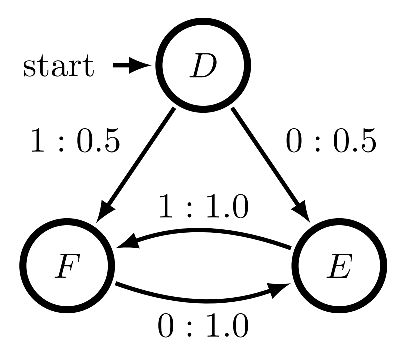
$$\langle W \rangle_{\text{counterclockwise}} = k_B T \qquad \langle W \rangle_{\text{clockwise}} = -k_B T / e$$

Memoryful Ratchet Operation



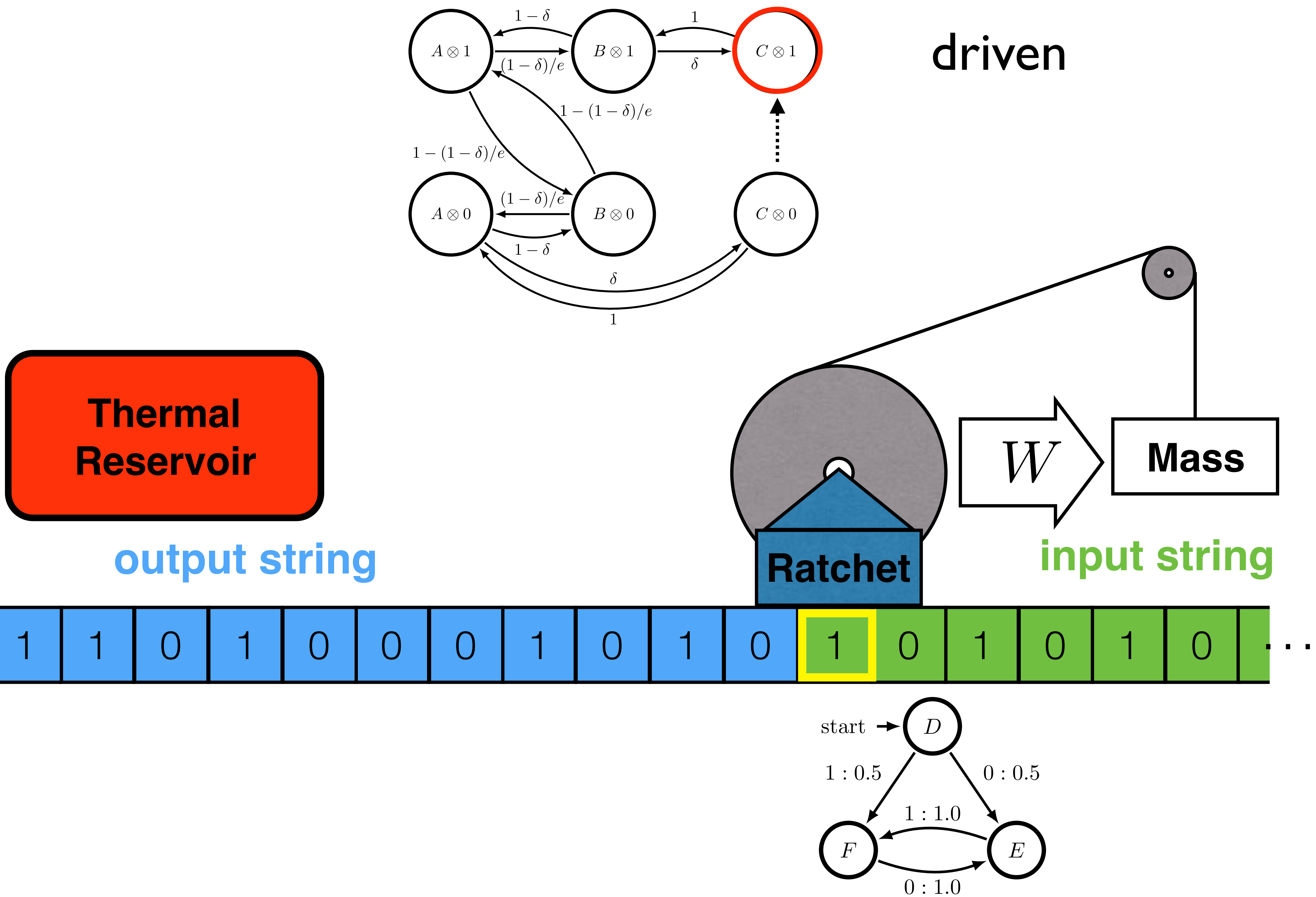
thermally activated





Work dissipating cycle is unstable

Memoryful Ratchet Operation



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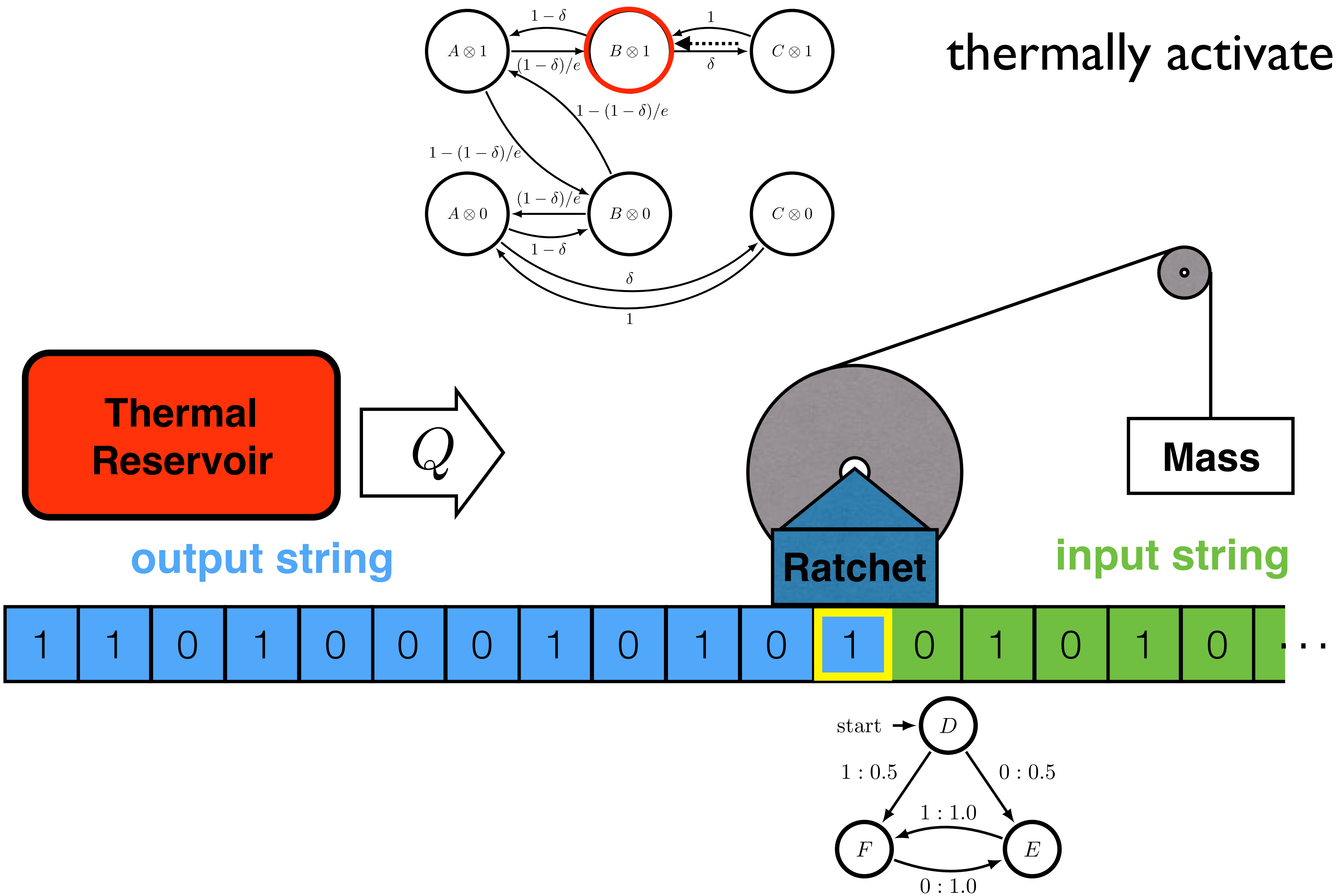

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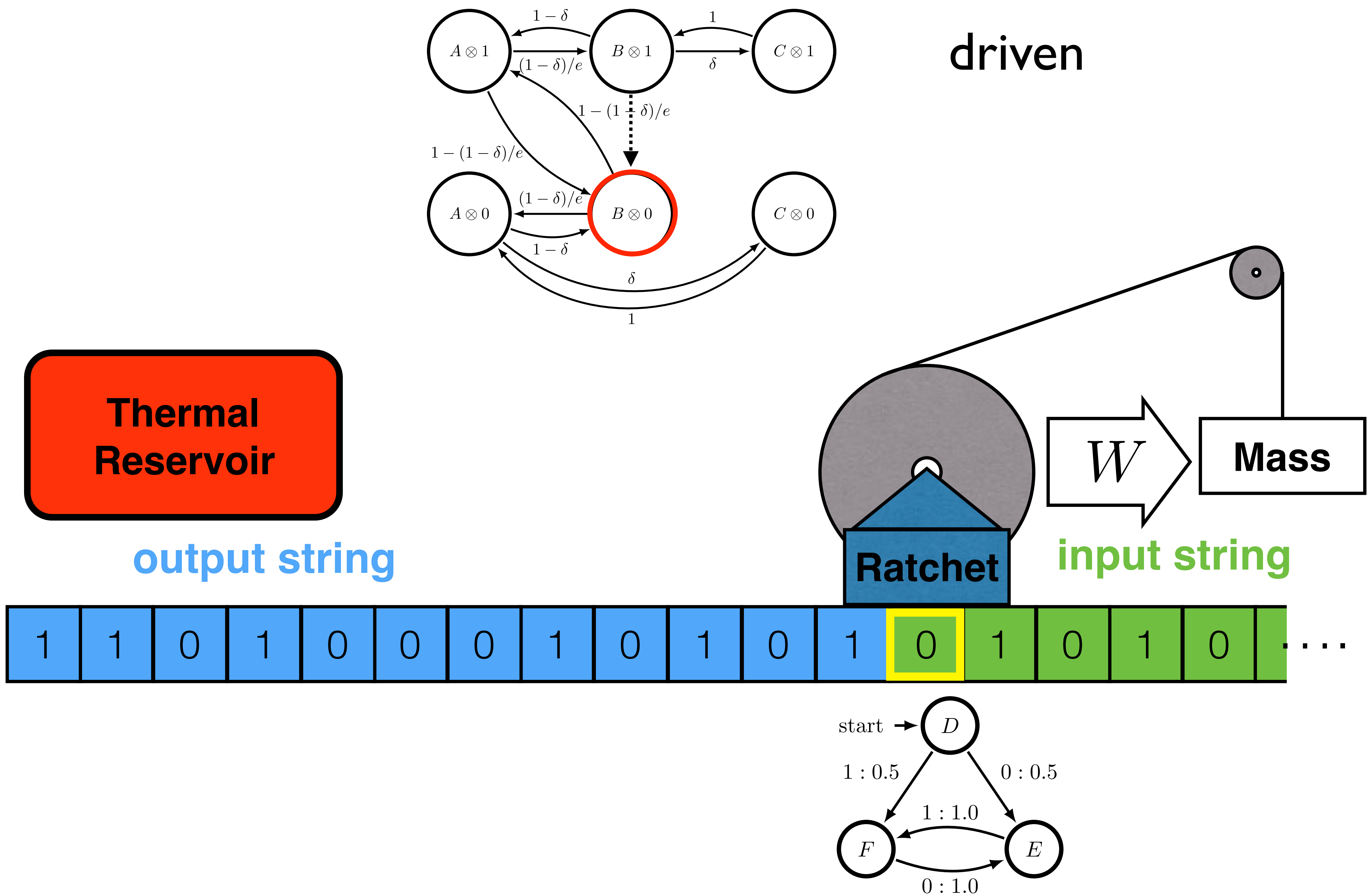
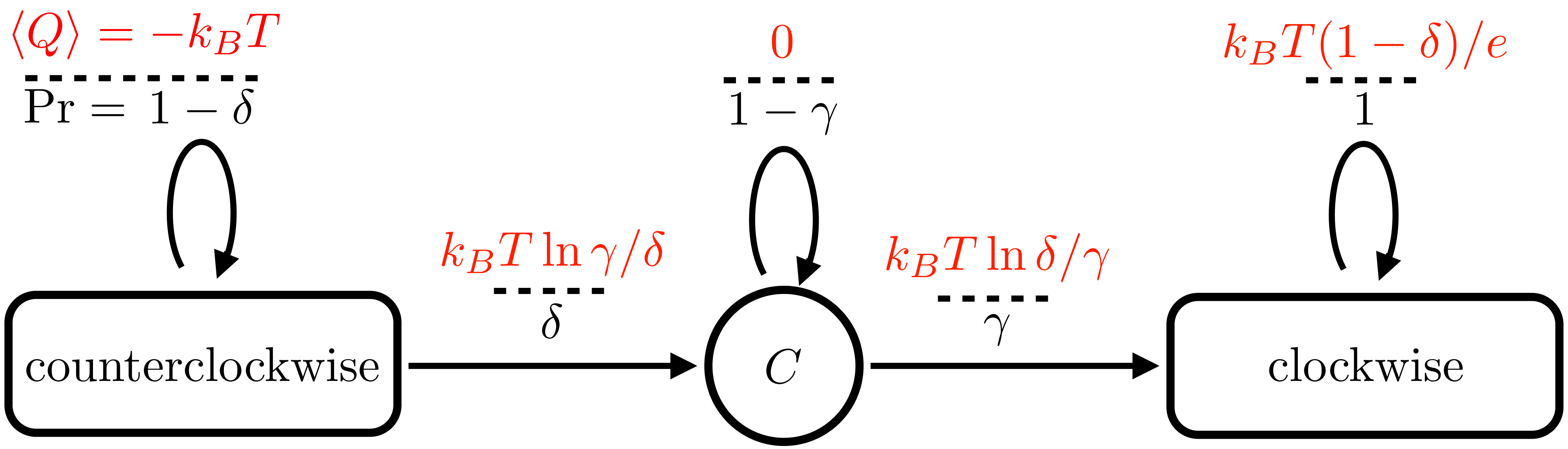


synchronizes to work producing dynamical phase

Non-Ergodic Dynamical Phases

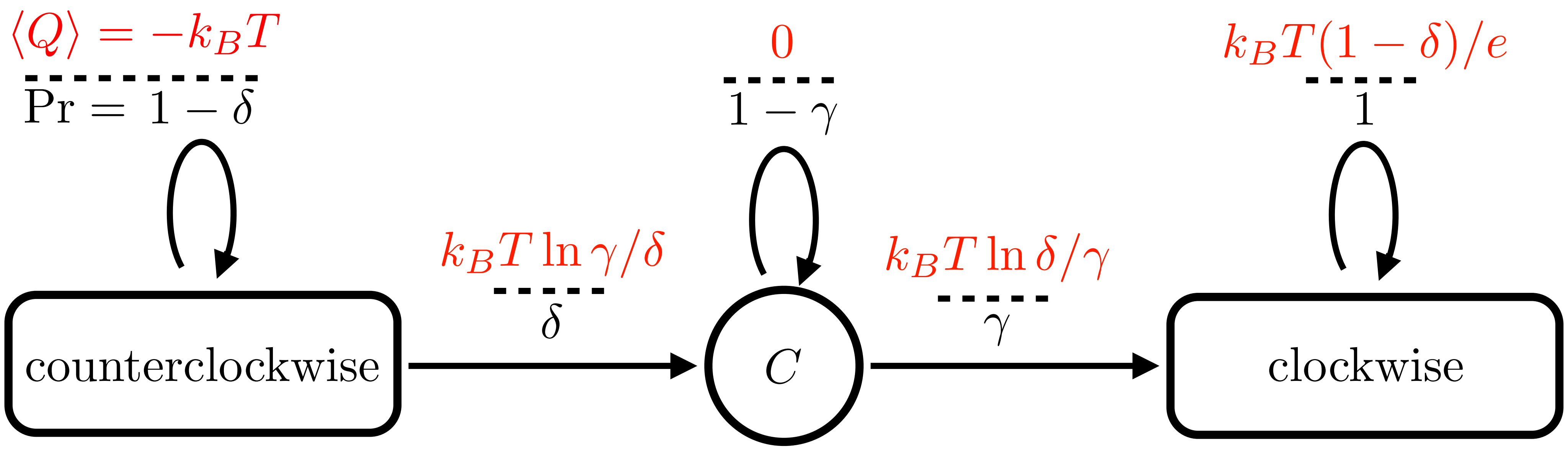


Synchronizes to predictive states of input to generate work from temporal correlations:

$$\lim_{t\to\infty} \langle W \rangle_t = \langle W \rangle_{\text{clockwise}} = -\frac{1-\delta}{e} k_B T$$

A. B. Boyd, D. Mandal, and J. P. Crutchfield, "Correlation-powered information engines and the thermodynamics of self-correction", PRE (2017)

Non-Ergodic Dynamical Phases

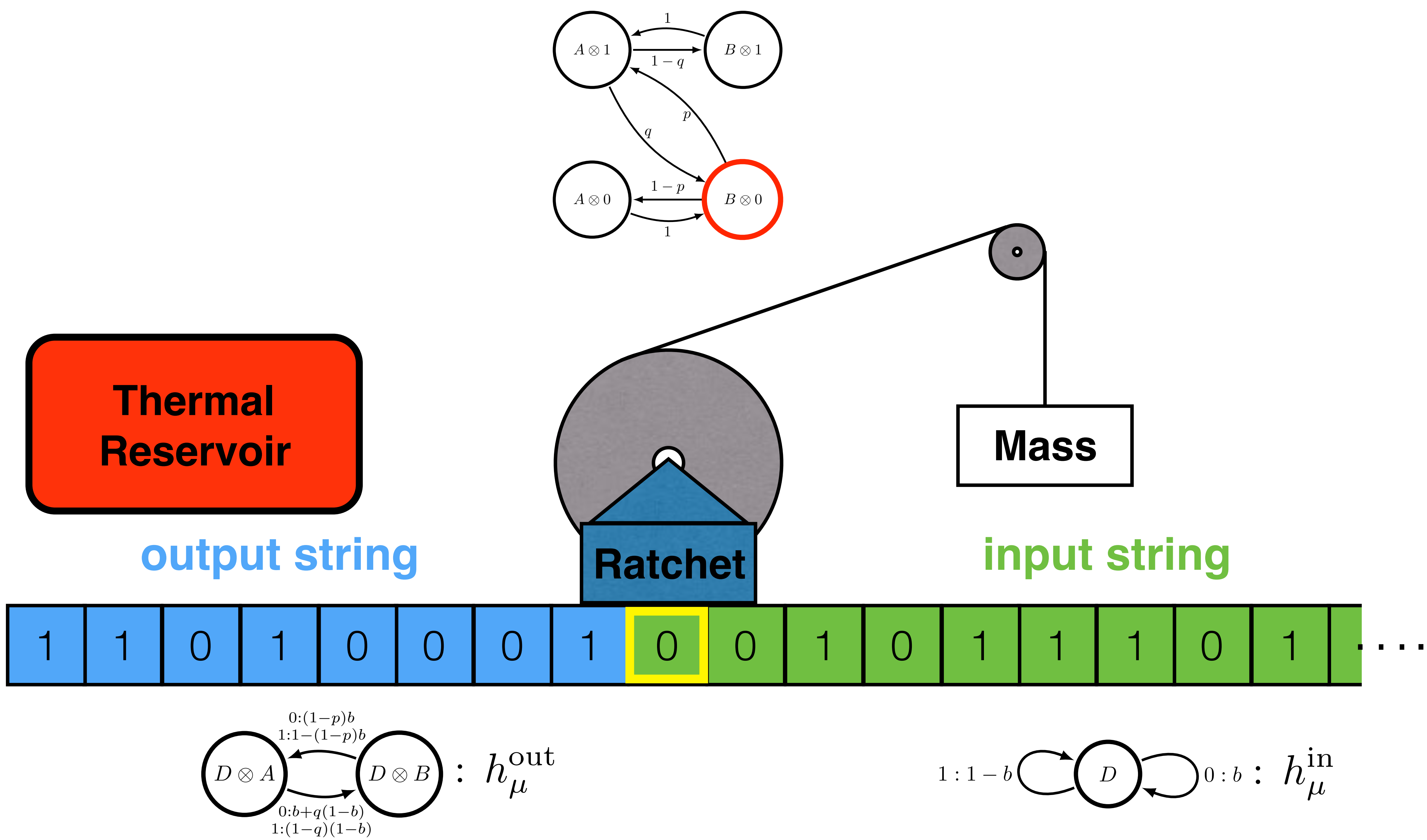


Synchronizes to predictive states of input to generate work from temporal correlations:

$$\lim_{t \to \infty} \langle W \rangle_t = \langle W \rangle_{\text{clockwise}} = -\frac{1-\delta}{e} k_B T$$
$$\geq k_B T \ln 2 \, \Delta h_\mu$$

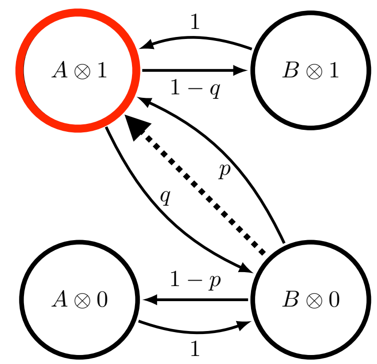
A. B. Boyd, D. Mandal, and J. P. Crutchfield, "Correlation-powered information engines and the thermodynamics of self-correction", PRE (2017)

Efficient Quasistatic Transducer

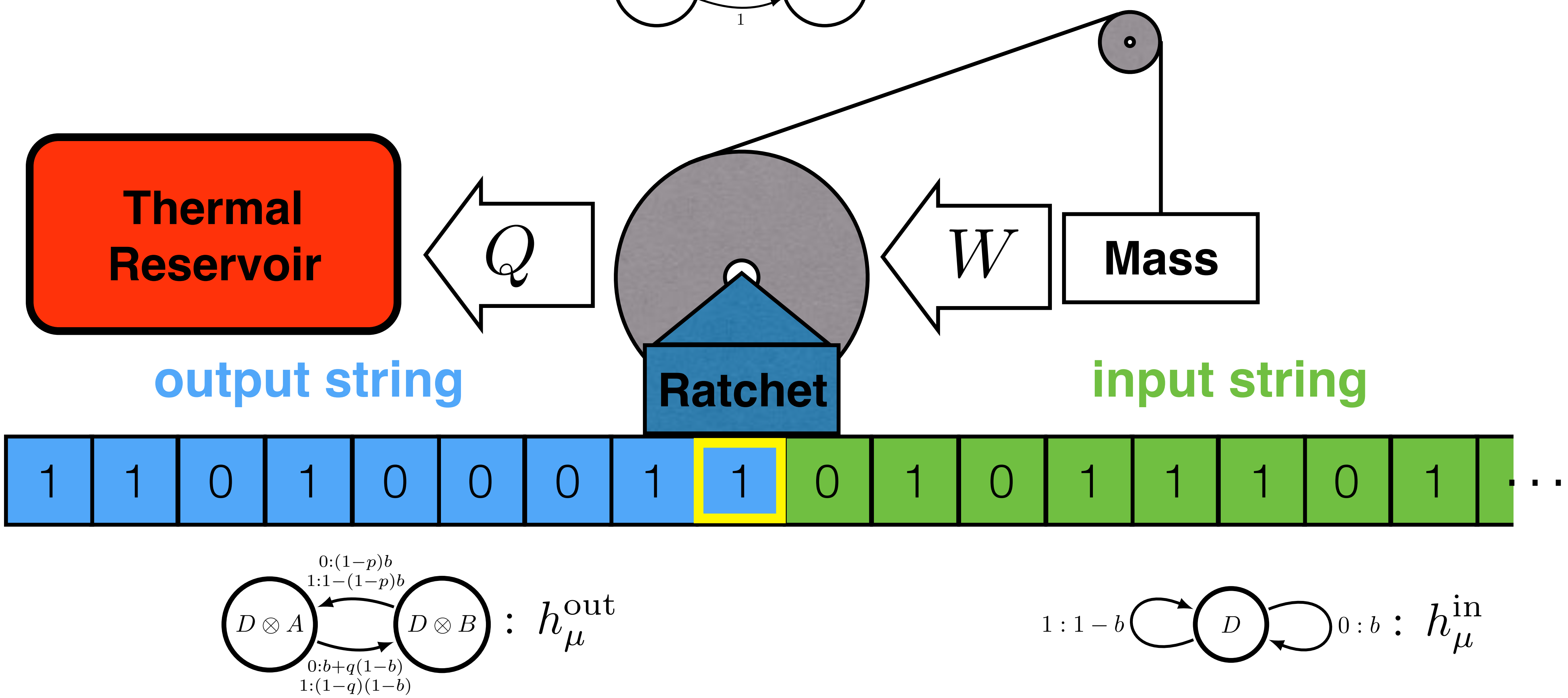


Efficient Quasistatic Transducer

$$W_{B\otimes 0\to A\otimes 1} = k_B T \ln \frac{\Pr(B\otimes 0)}{\Pr(A\otimes 1)}$$
$$= Q_{B\otimes 0\to A\otimes 1}$$



Slow simultaneous exchange of work and heat



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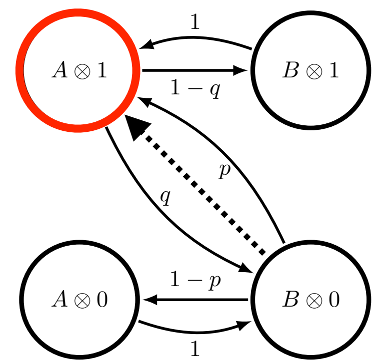

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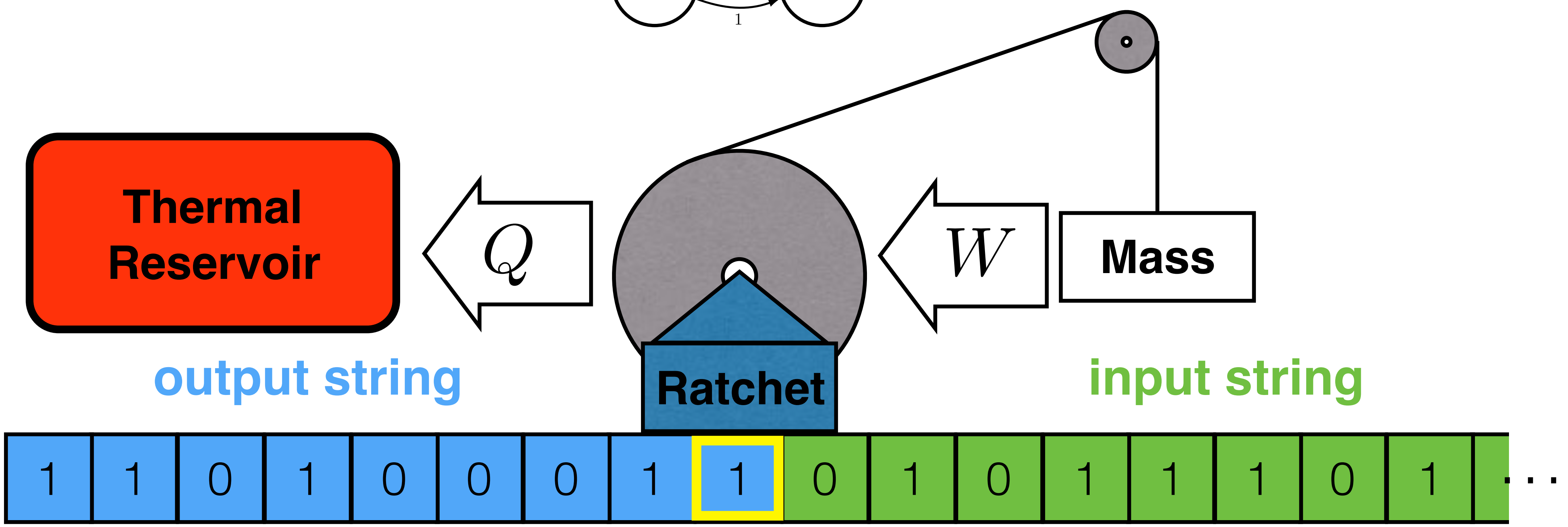
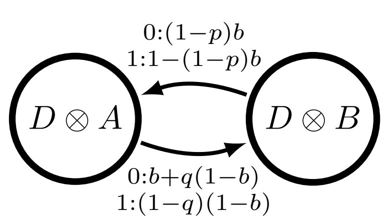
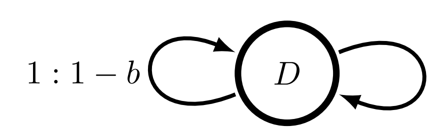




$: \; h_\mu^{\text{out}}$

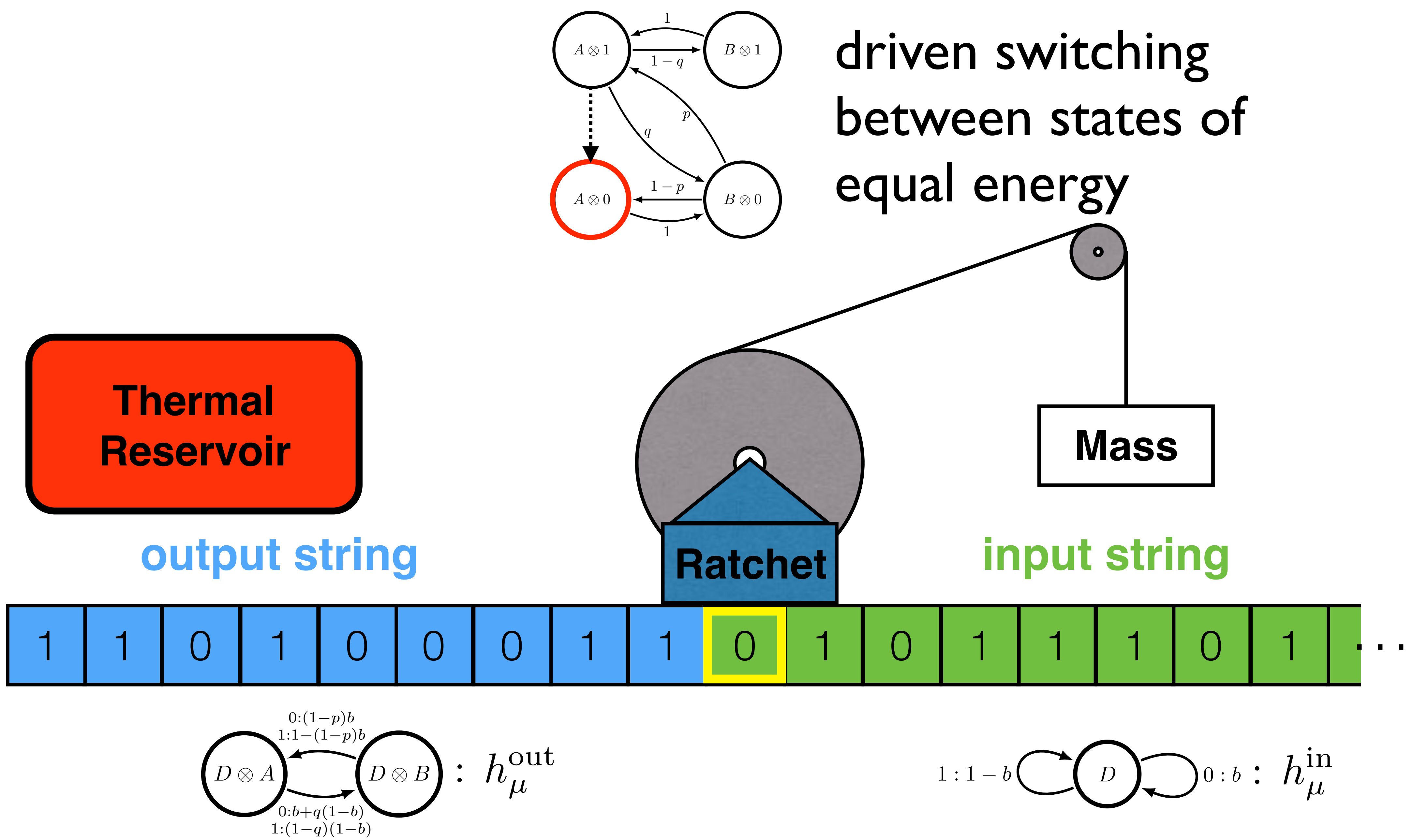


$: \; h_\mu^{\text{in}}$

$$\langle W\rangle = \langle Q\rangle = k_B T \ln 2\,(H[X_N, Y_N] - H[X_{N+1}, Y'_N])$$

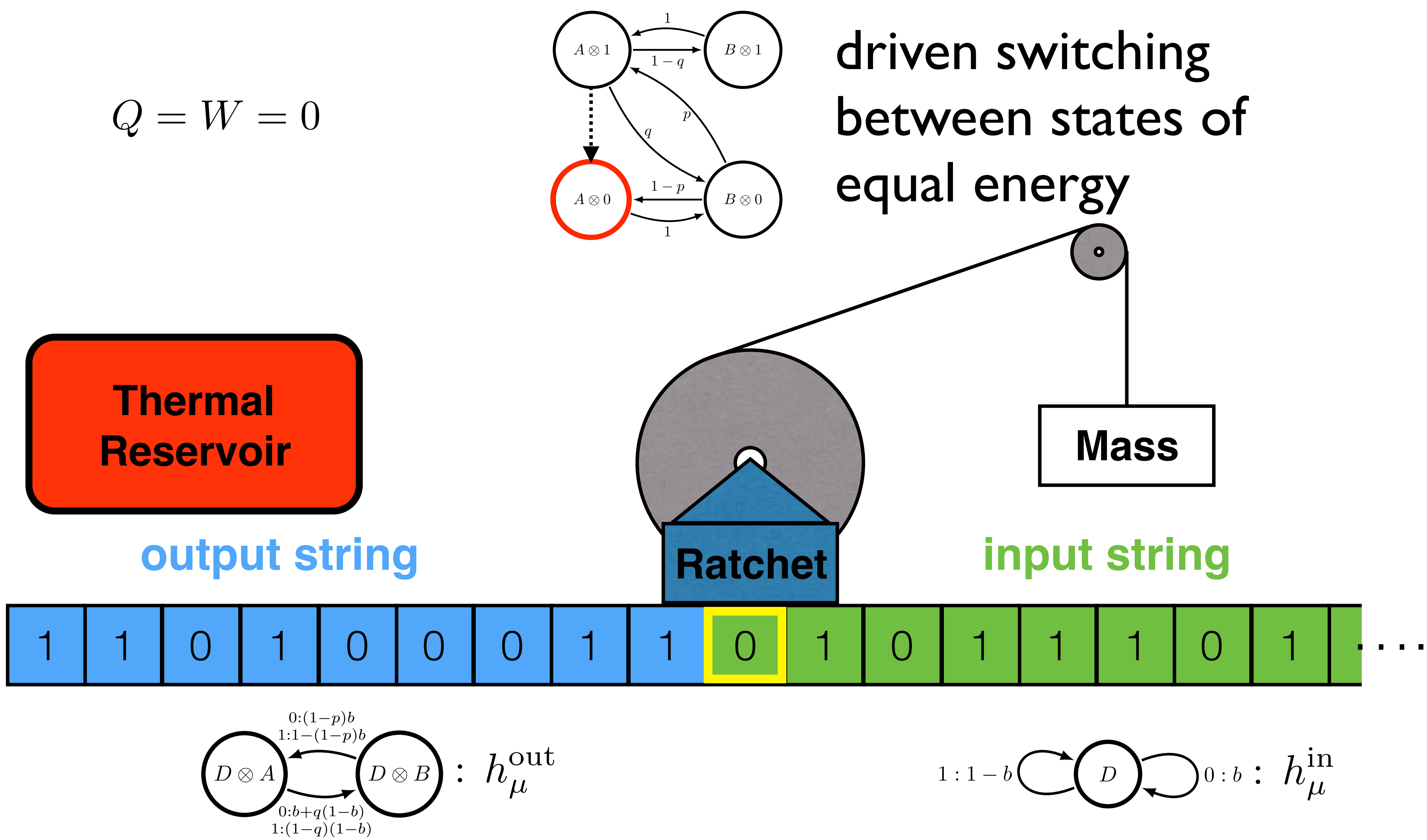
Efficient Quasistatic Transducer

driven switching between states of equal energy



Efficient Quasistatic Transducer

$$Q = W = 0$$



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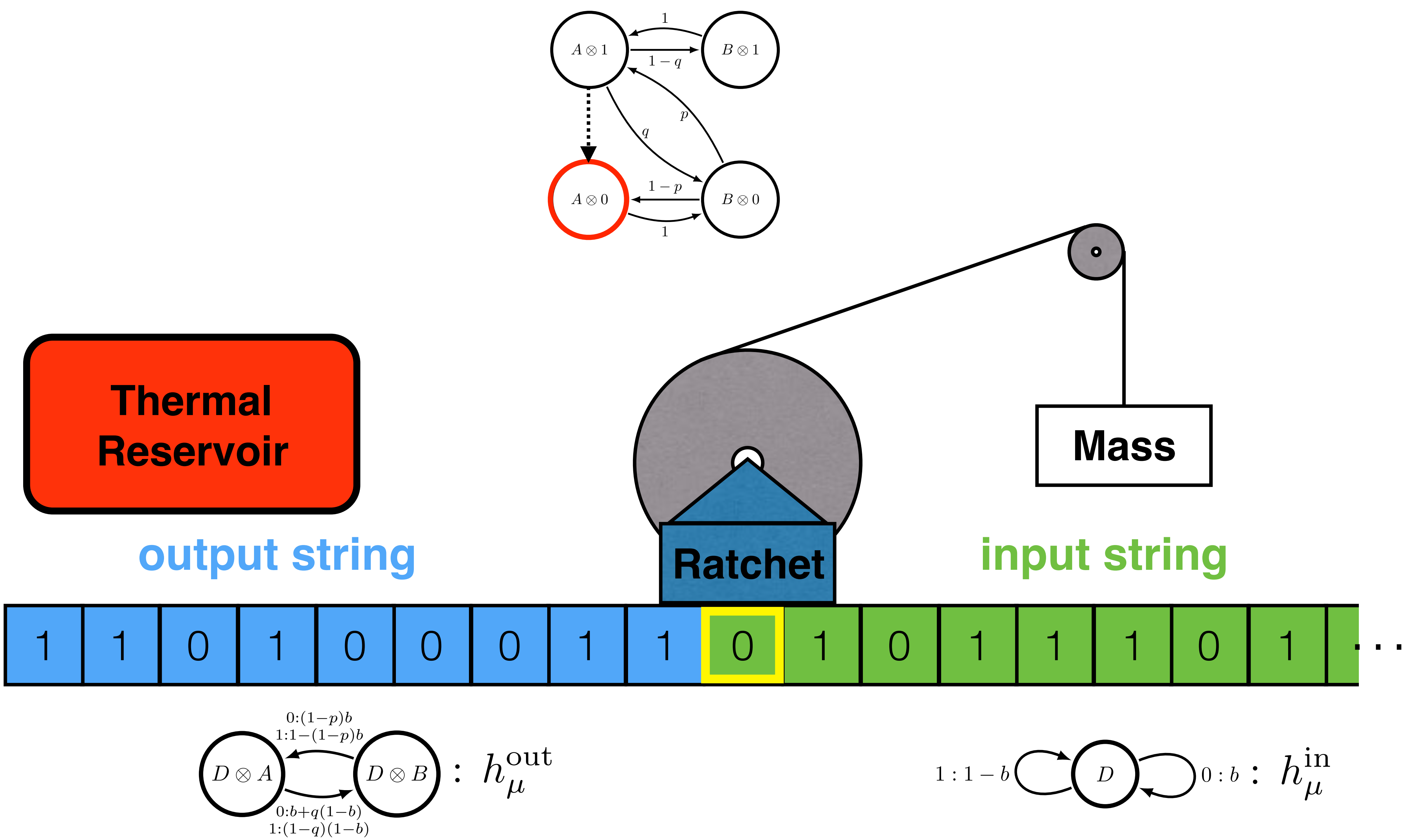

$$\langle W \rangle = \langle Q \rangle = k_B T \ln 2 \left(H[X_N, Y_N] - H[X_{N+1}, Y'_N] \right)$$

Achieves IPSL if modularity dissipation is minimized.

Golden Mean Extractor

Minimize modularity dissipation by storing globally relevant correlations in ratchet.

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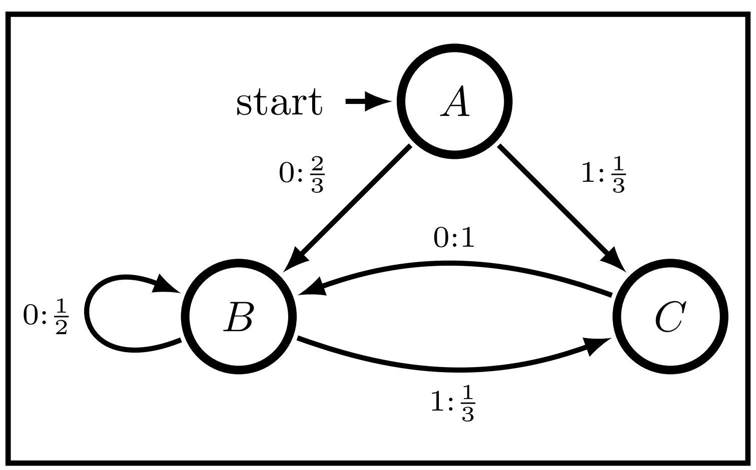
Input epsilon-machine gives prescription for designing optimal quasistatic ratchet:

input ϵ-machine



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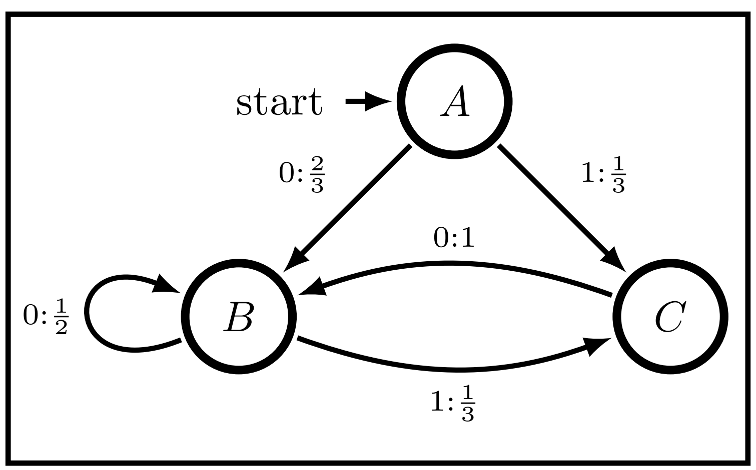

$$T^{(y)}_{s \to s'}$$

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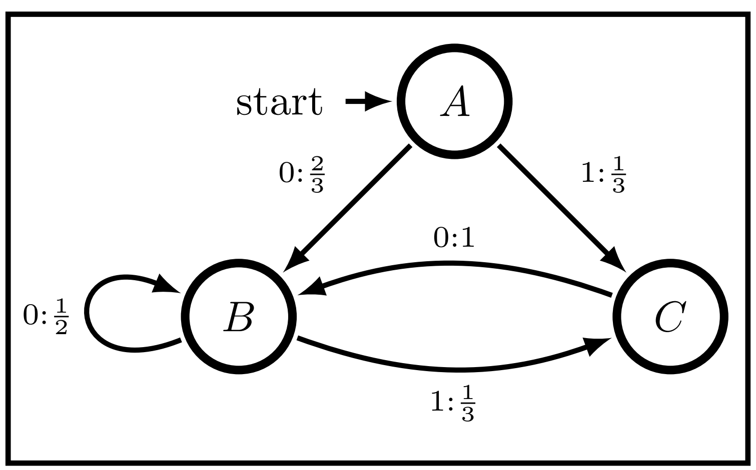

$T^{(y)}_{s \to s'}$

$$M^{\text{local}}_{(x,y)\to(x',y')} = \begin{cases} b, & \text{if } T^{(y)}_{x\to x'} \neq 0 \text{ and } y' = 0 \\ 1-b, & \text{if } T^{(y)}_{x\to x'} \neq 0 \text{ and } y' = 1 \end{cases}$$

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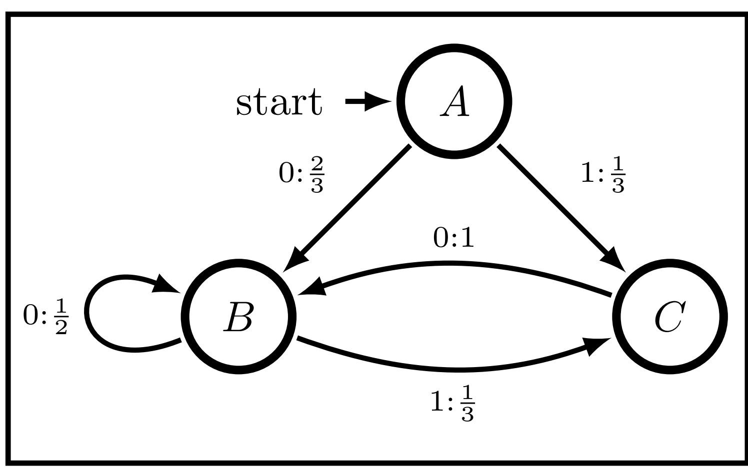
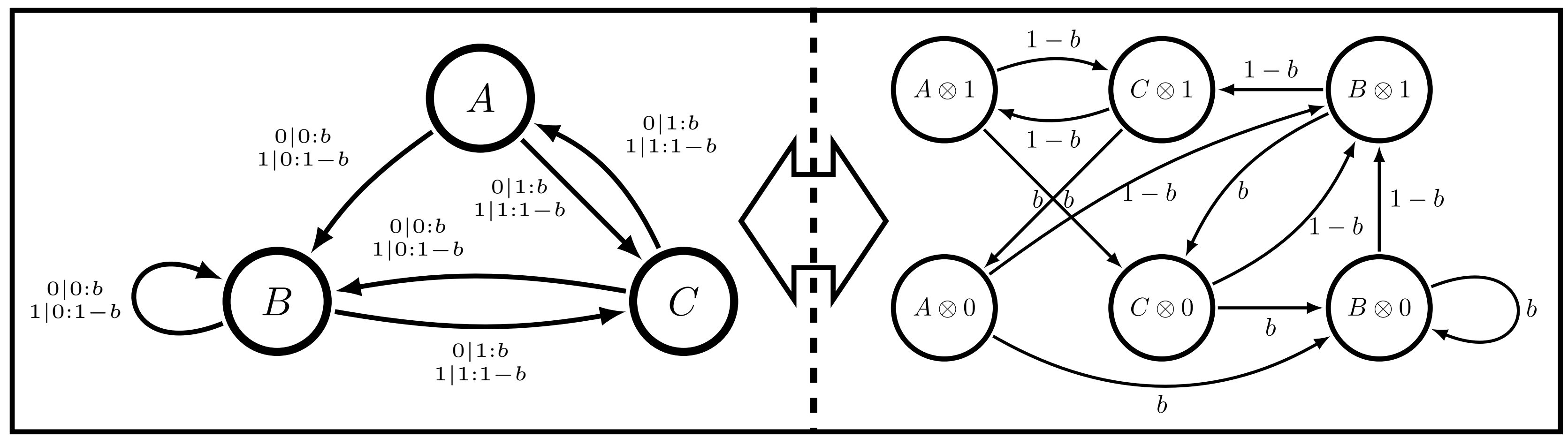


predictive efficient extractor: $\langle \Sigma_\infty^{\text{ext}} \rangle_{\min} = 0$



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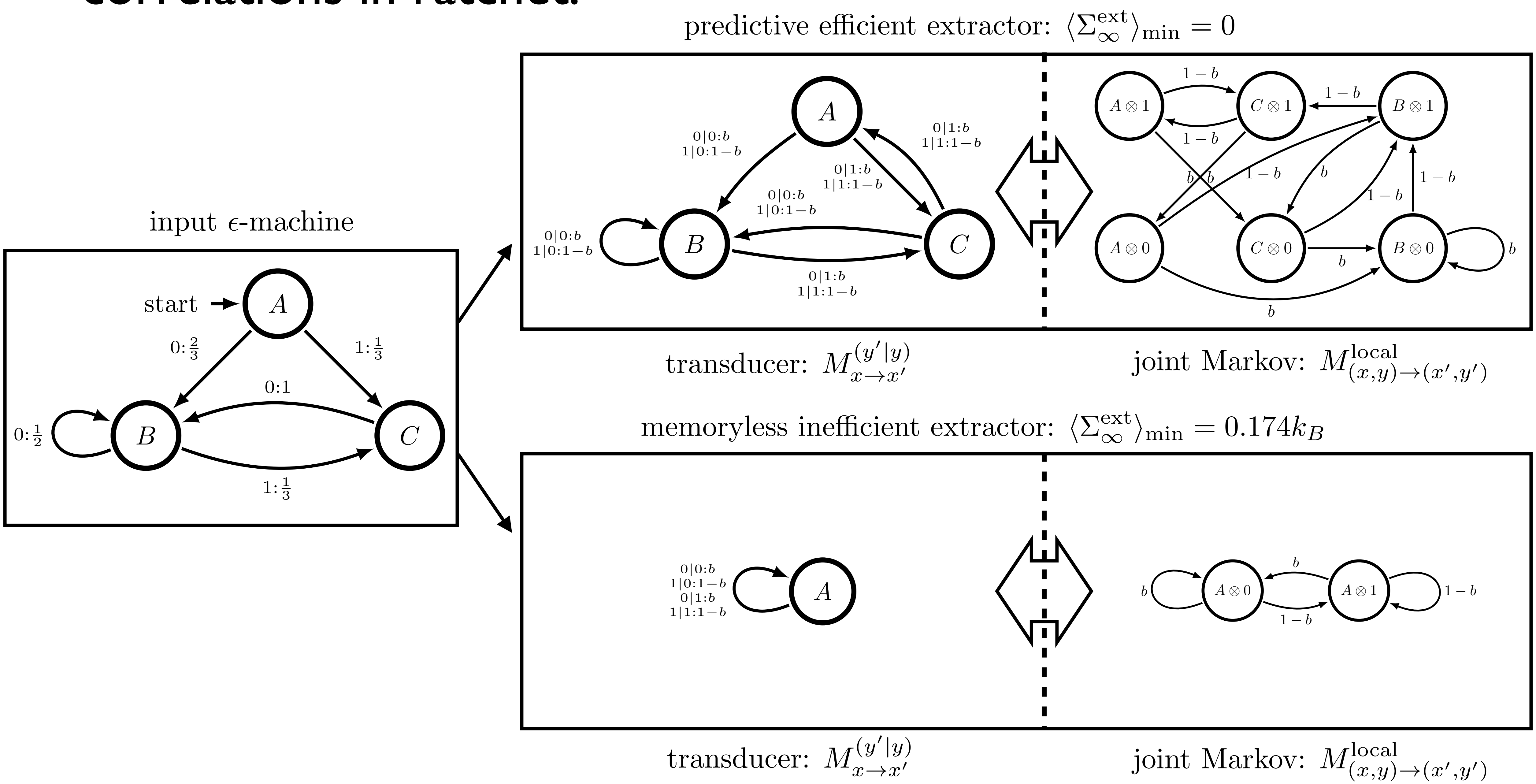

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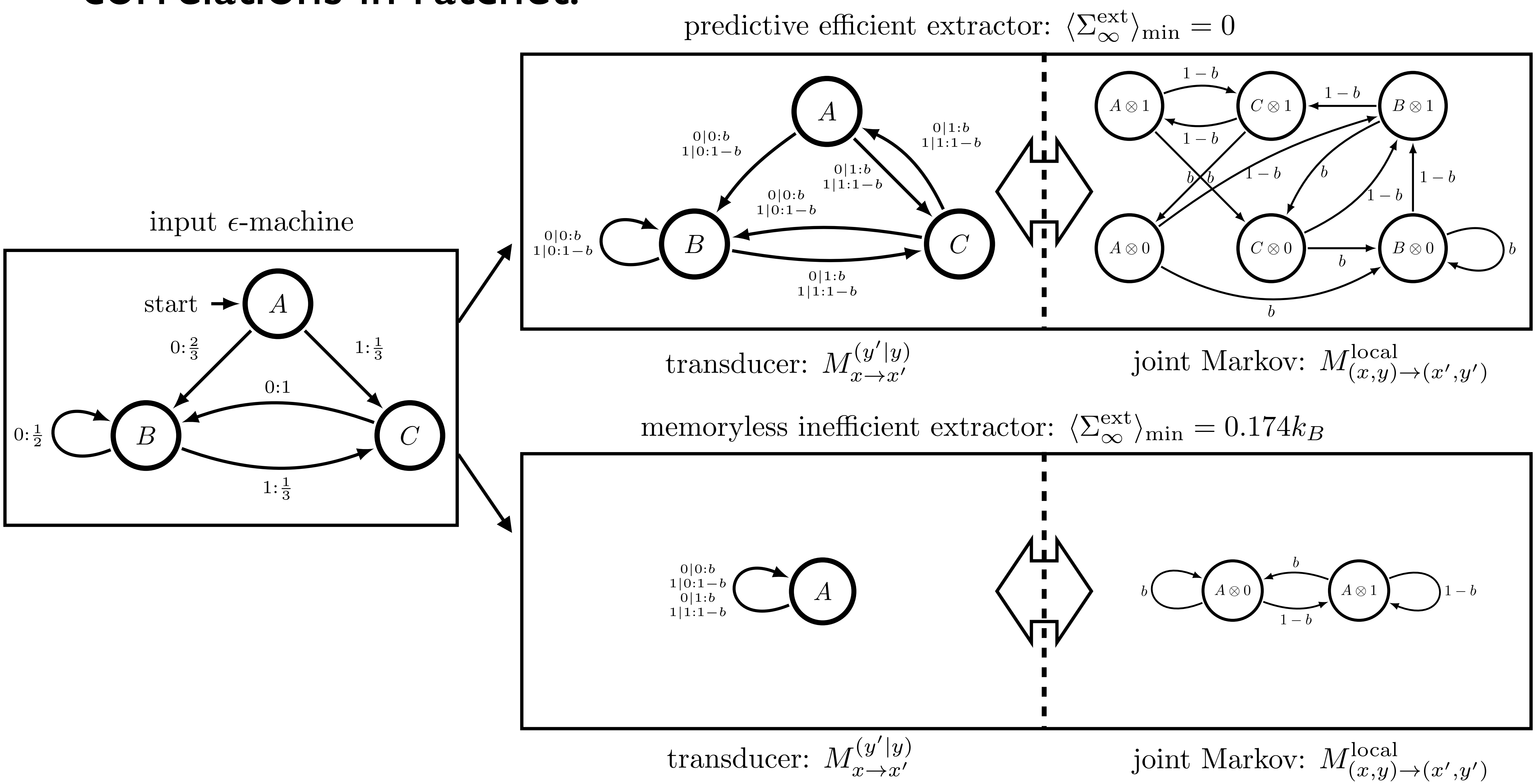

Memoryless ratchet dissipates all temporal correlations:

$$\langle \Sigma^{\text{ext}}_{N} \rangle_{\min} = k_B \ln 2 (H[Y_N] - h_\mu)$$

Golden Mean Extractor

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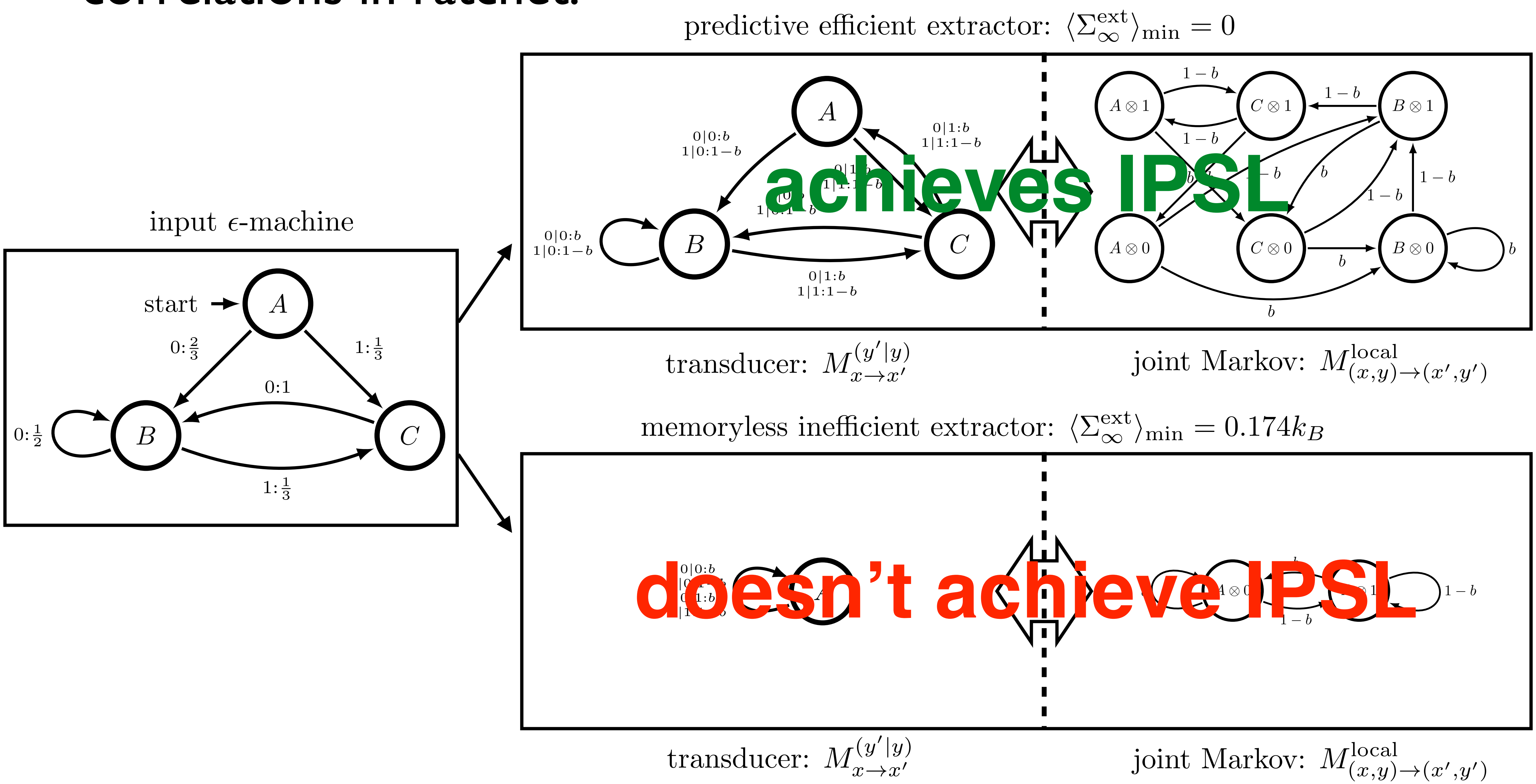

Memoryless ratchet dissipates all temporal correlations:

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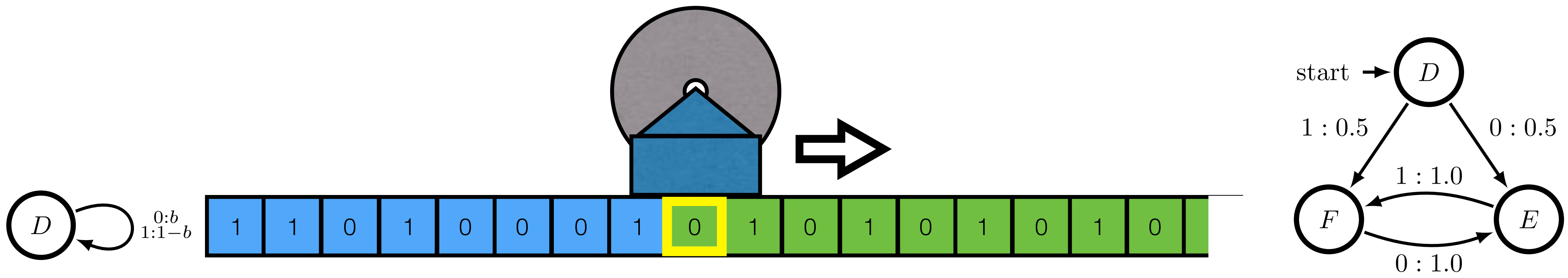
Beyond Pattern Extractors

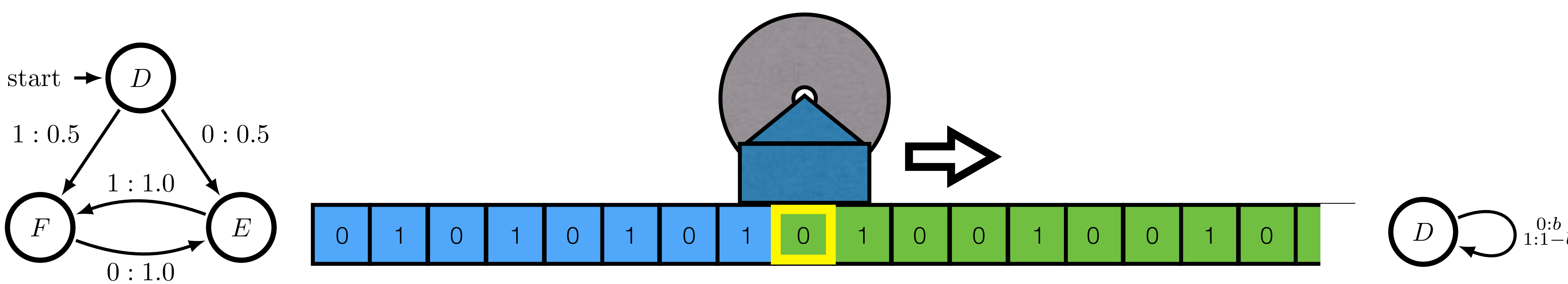
Pattern Extractors: correlated inputs, uncorrelated outputs



$$\Pr(Y'_{a:b} = y'_{a:b}) = \prod_{i=a}^{b-1} \Pr(Y'_i = y'_i)$$

$$\Pr(Y_{a:b} = y_{a:b}) \neq \prod_{i=a}^{b-1} \Pr(Y_i = y_i)$$

Pattern Generators: uncorrelated inputs, correlated outputs



$$\Pr(Y'_{a:b} = y'_{a:b}) \neq \prod_{i=a}^{b-1} \Pr(Y'_i = y'_i)$$

$$\Pr(Y_{a:b} = y_{a:b}) = \prod_{i=a}^{b-1} \Pr(Y_i = y_i)$$

Efficient Generators

Perfect efficiency: $\langle \Sigma_N^{\text{mod}} \rangle_{\min} = 0 \text{ for all } N$

$$\Uparrow$$

Retrodictive ratchet: $I[Y'_{0:N}; Y'_{N:\infty} | X_N] = 0 \text{ and } I[Y'_{0:N}; X_N | Y'_{N:\infty}] = 0$

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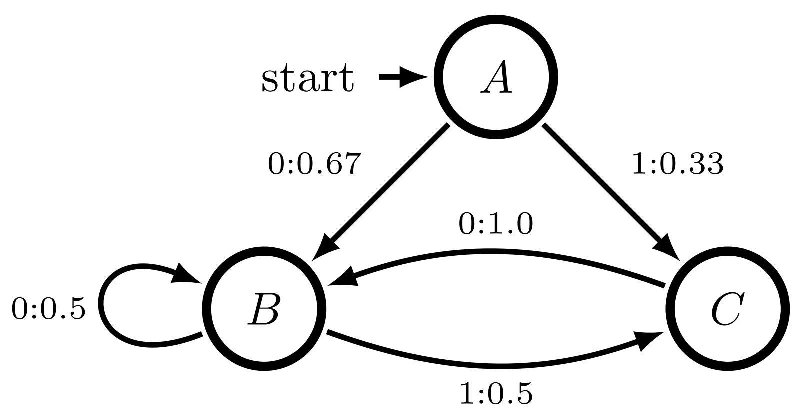
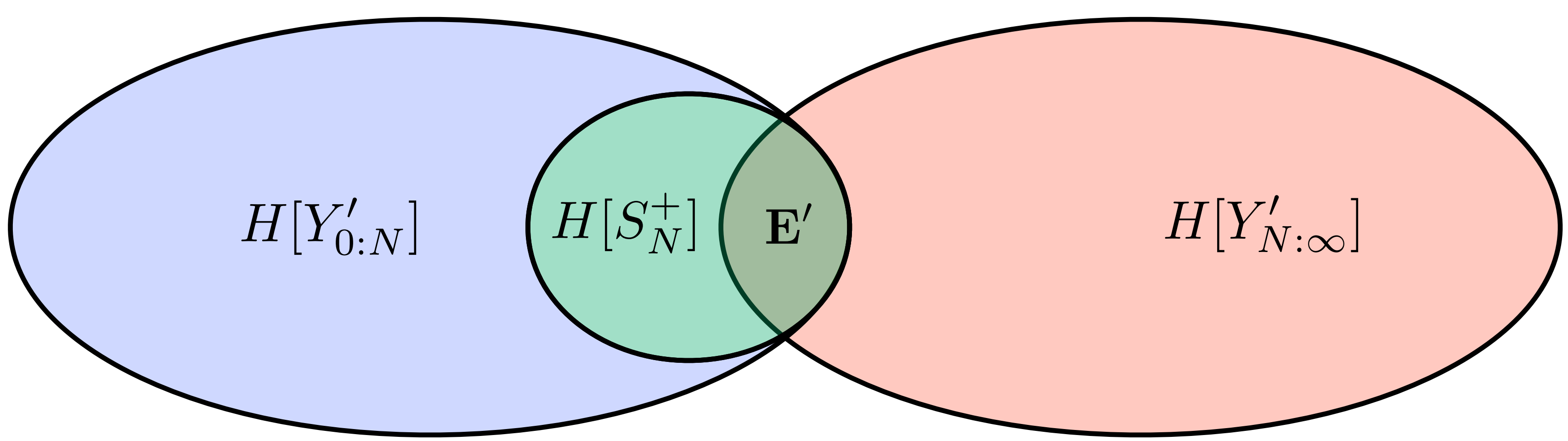
ϵ-machine



minimal predictive states: S_N^+



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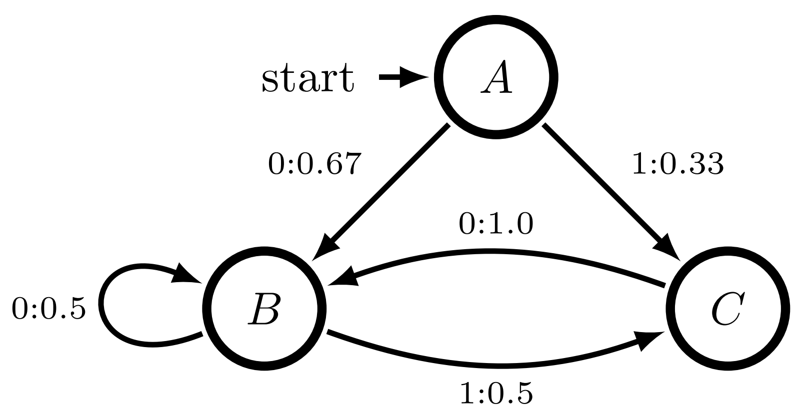
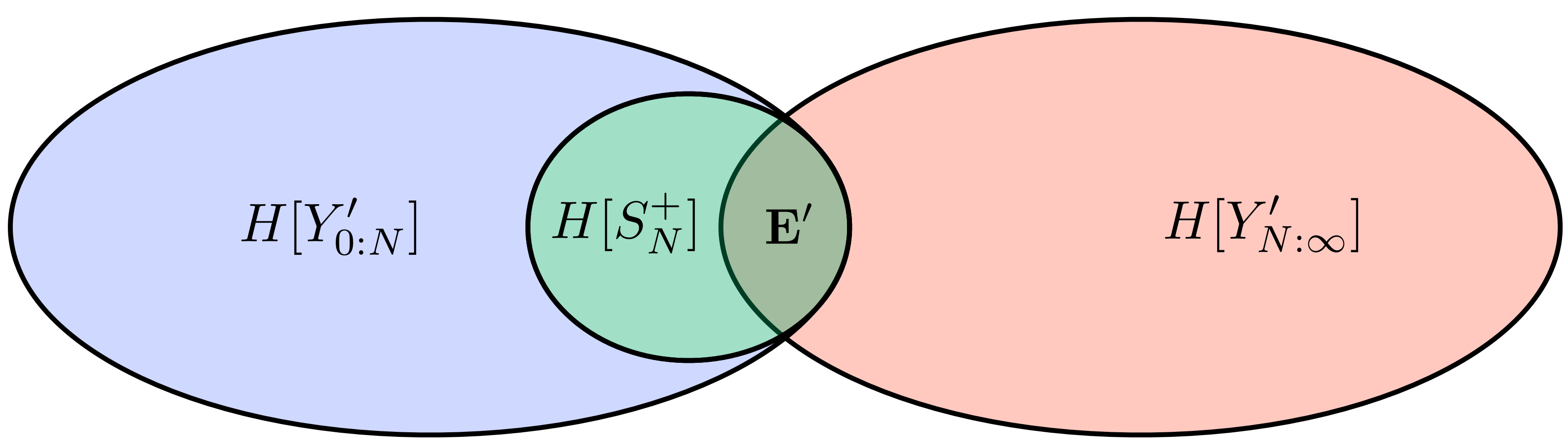


$$\langle \Sigma_N^{\mathrm{mod}} \rangle_{\min} = \frac{2}{3} k_B T \ln 2$$

minimal predictive states: S_N^+



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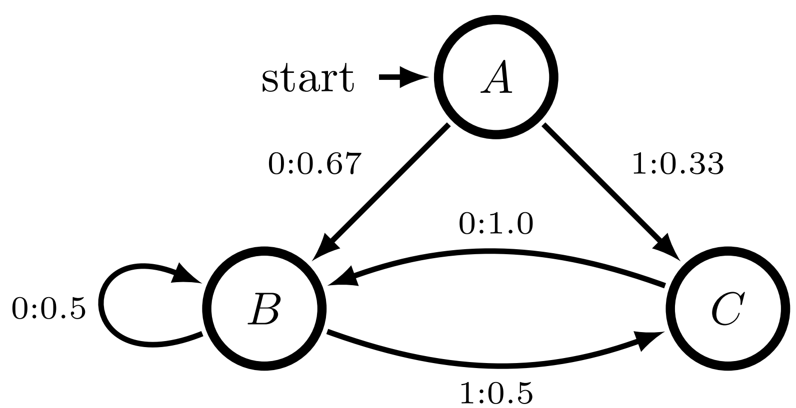
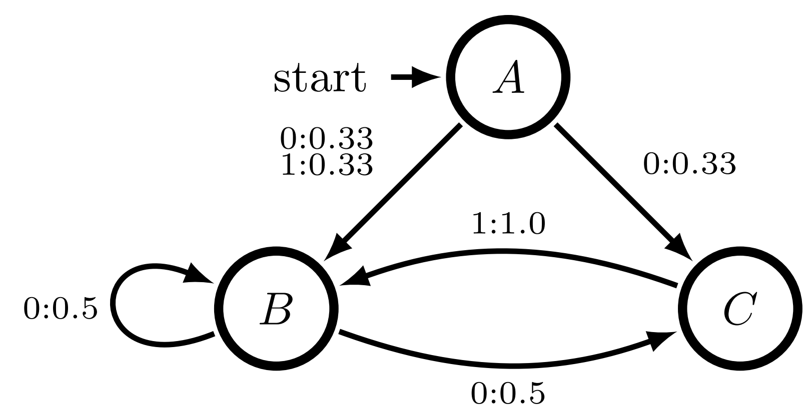
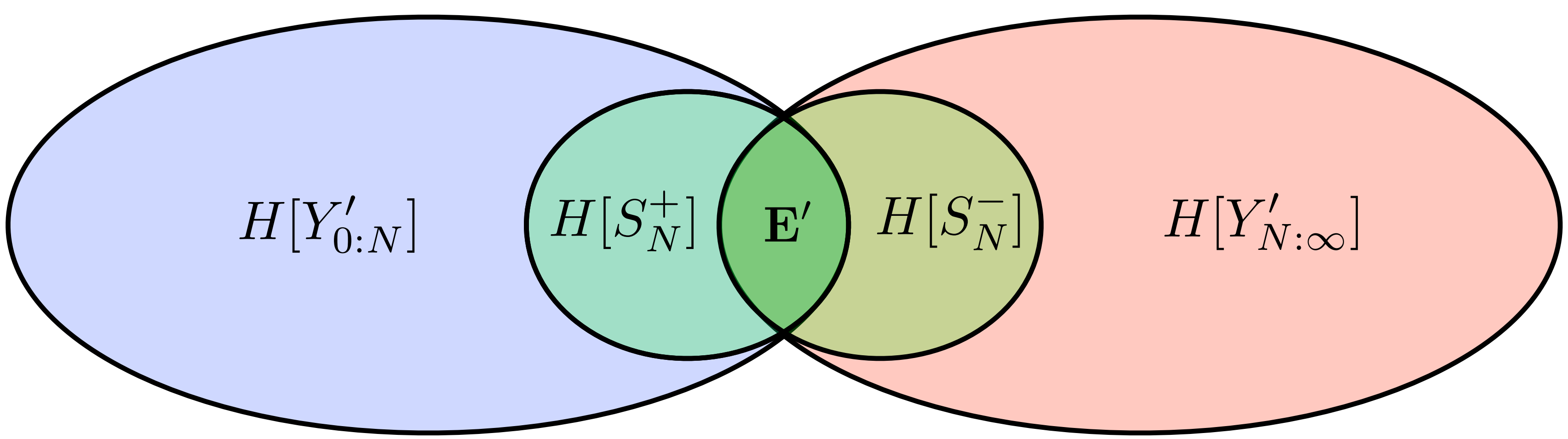


minimal predictive states: S_N^+

time reversal of ϵ-machine



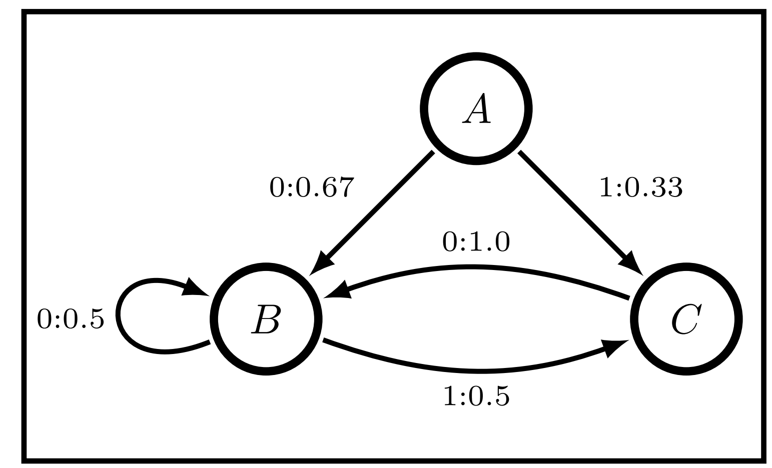
minimal retrodictive states: S_N^-



Golden Mean Generator

Time reversal of reverse time output epsilon-machine gives prescription for designing optimal quasistatic ratchet.

output ϵ-machine



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Make the ratchet input agnostic for any particular HMM

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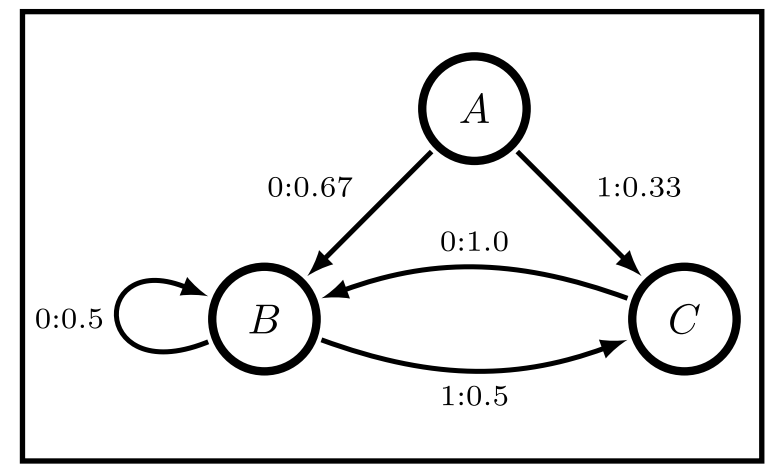

Golden Mean Generator

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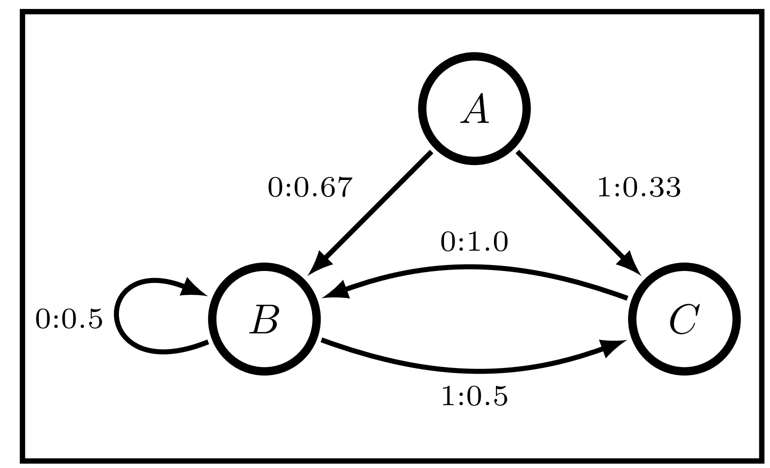
Make the ratchet input agnostic for any particular HMM

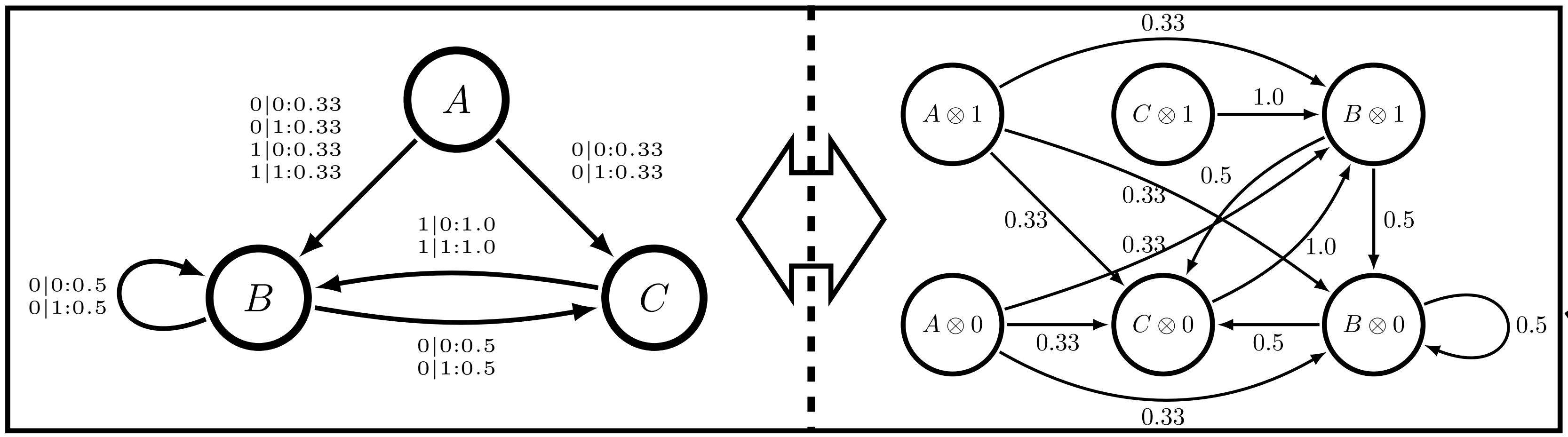
$$M^{\text{local}}_{x,y\to x',y'} = T^{(y')}_{x\to x'}$$

output ϵ-machine



Golden Mean Generator

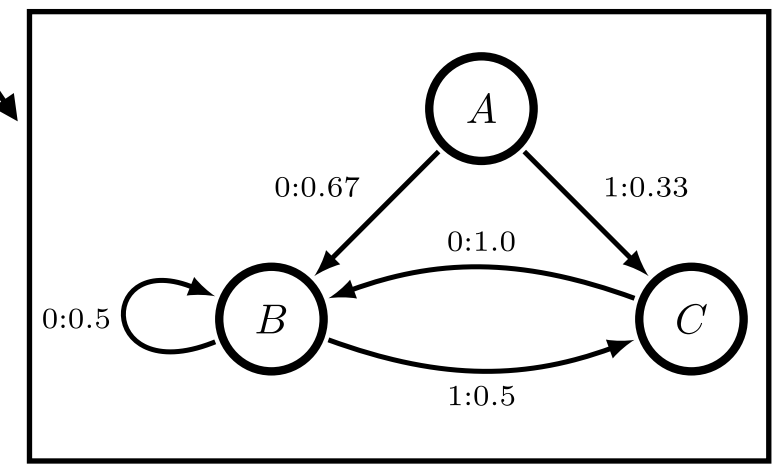
retrodictive efficient generator: $\langle \Sigma^{\mathrm{gen}}_{\infty} \rangle_{\min} = 0$



transducer: $M^{(y'|y)}_{x \to x'}$

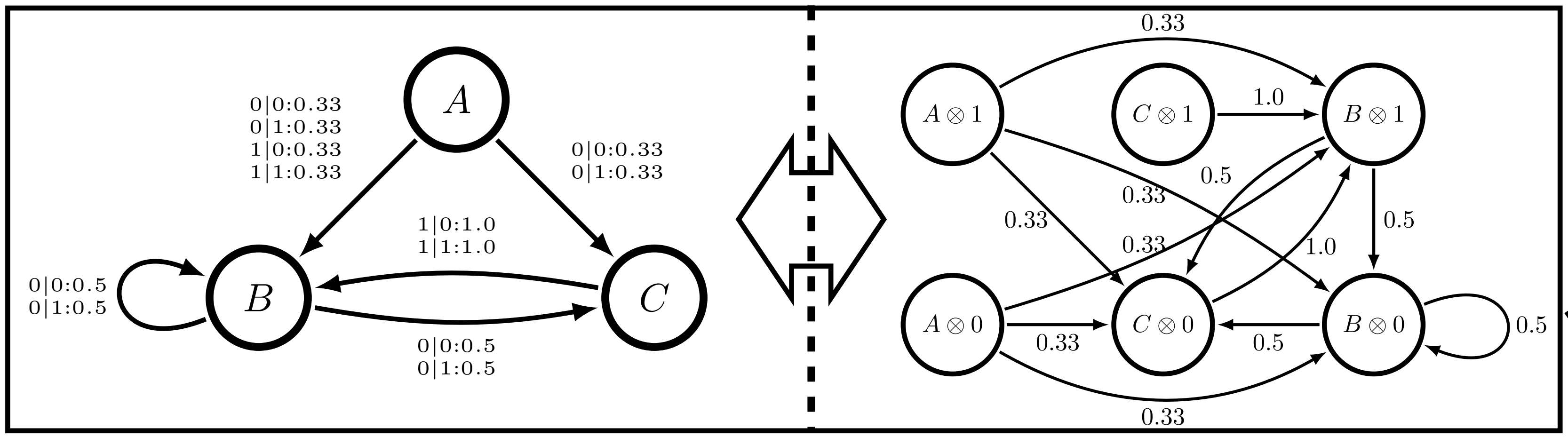
joint Markov: $M^{\mathrm{local}}_{(x,y) \to (x',y')}$

output ϵ-machine



Golden Mean Generator

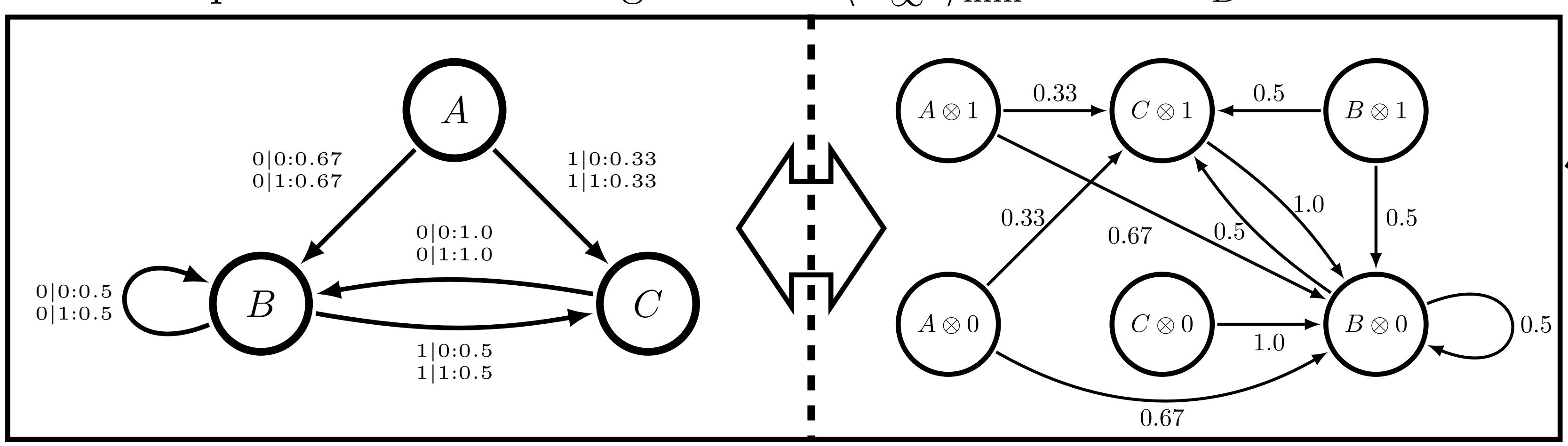
retrodictive efficient generator: $\langle \Sigma_\infty^{\text{gen}} \rangle_{\min} = 0$



transducer: $M_{x \to x'}^{(y'|y)}$

joint Markov: $M_{(x,y)\to(x',y')}^{\text{local}}$

predictive inefficient generator: $\langle \Sigma_\infty^{\text{gen}} \rangle_{\min} = 0.462 k_B$



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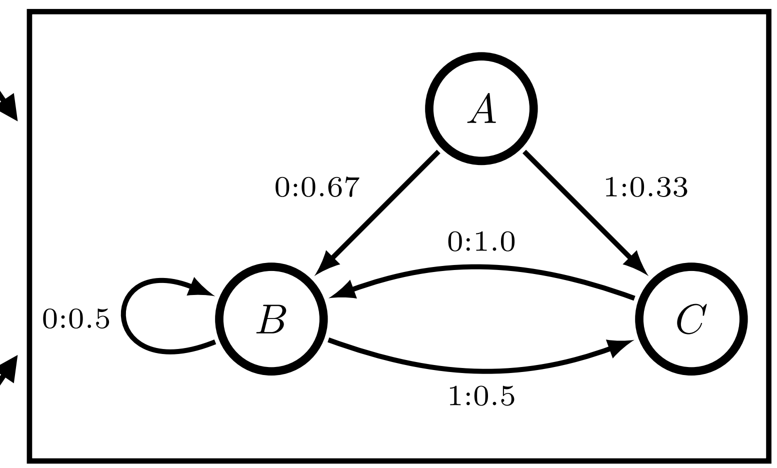

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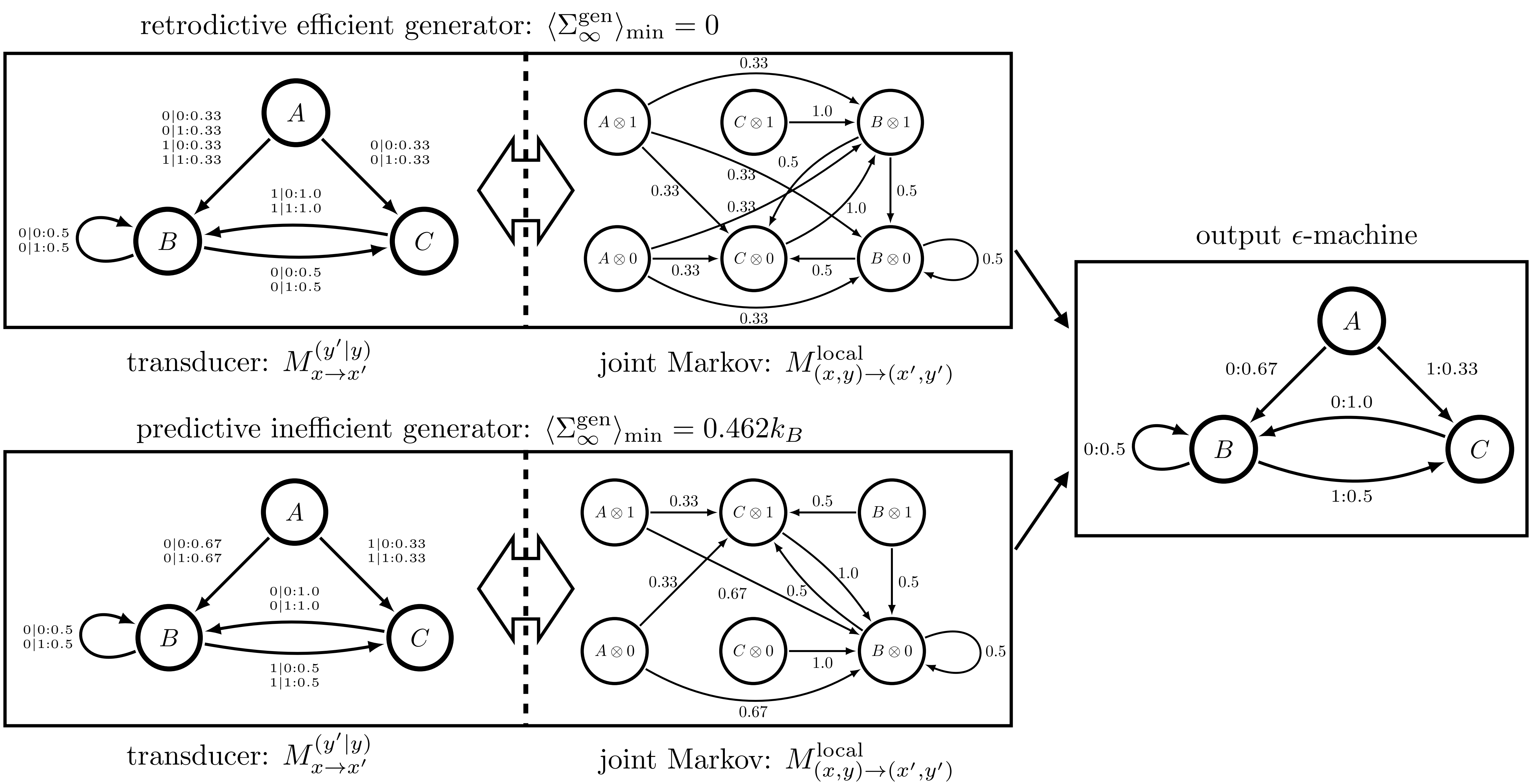
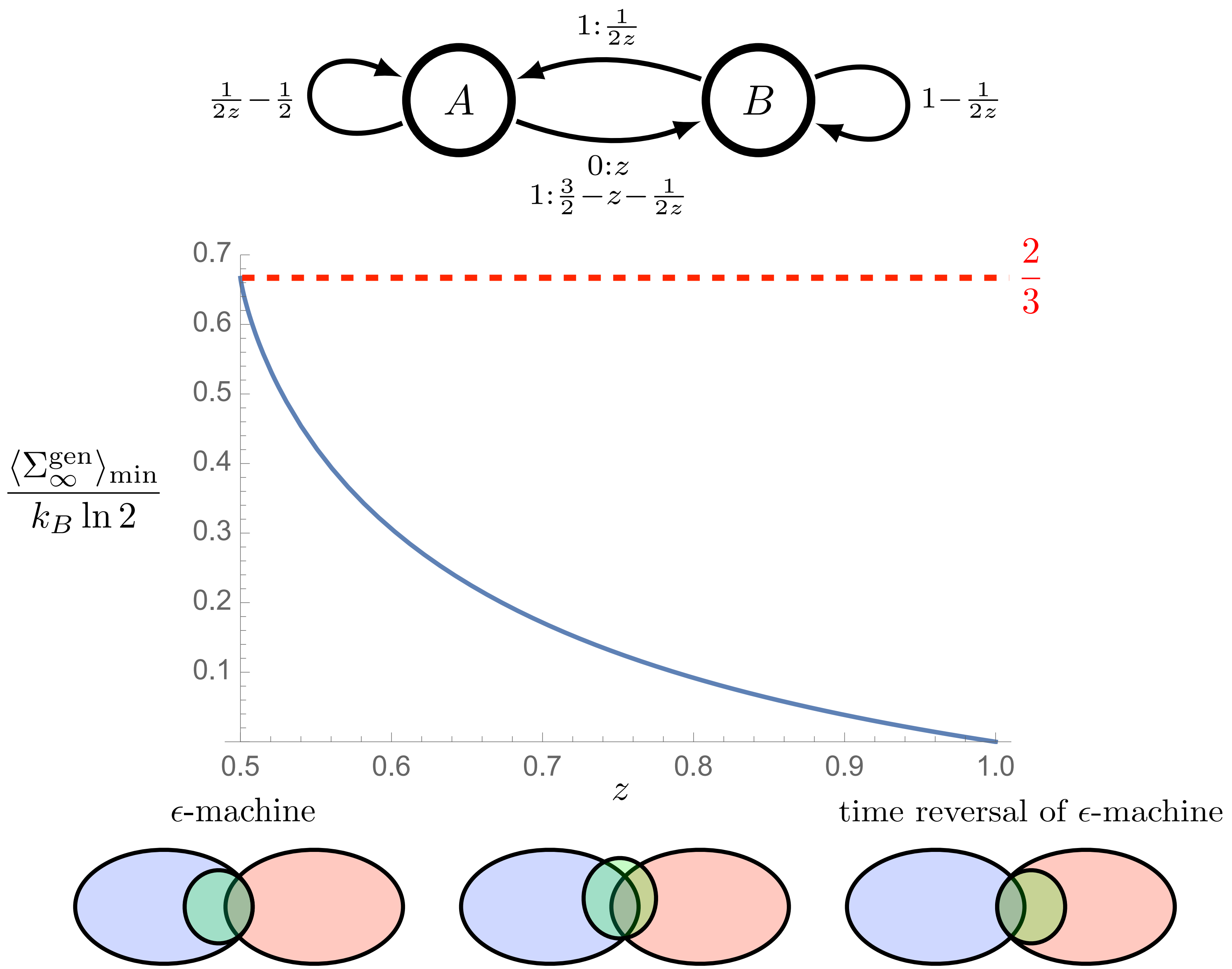


Despite doing the same computation, the predictive ratchet requires an additional amount of work with every time step:

$$\langle W_\infty^{\text{predictive}} \rangle_{\min} = \langle W_\infty^{\text{retrodictive}} \rangle_{\min} + \frac{2}{3} k_B T \ln 2$$

Cost of Unnecessary Memory In Generators



C. J. Ellison, J. R. Mahoney, R. G. James, J. P. Crutchfield, and J. Reichhardt. Information symmetries in irreversible processes. CHAOS (2011)

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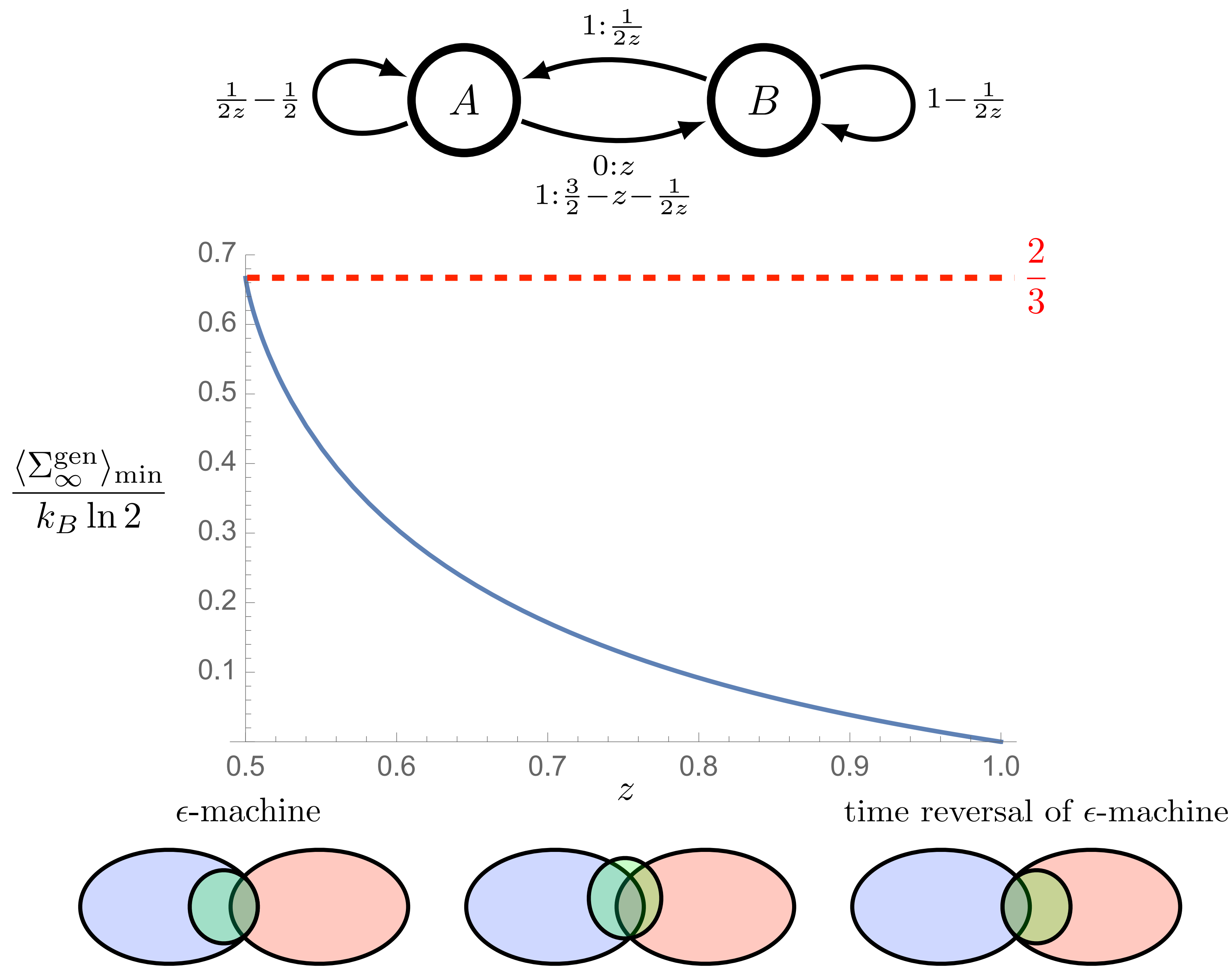

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It appears that the more unnecessary information the ratchet stores about the past, the less efficient the pattern generator is.

Summary

The IPSL (Information Processing Second Law) suggests information is a thermodynamic fuel.

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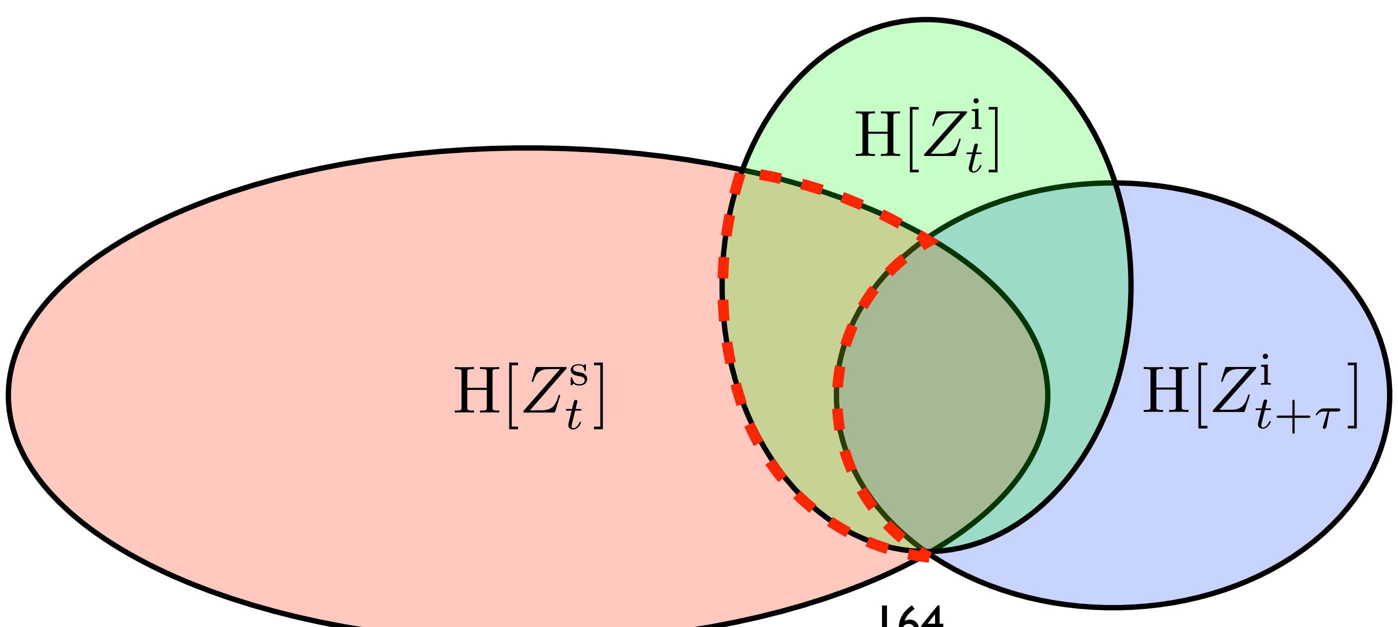
Must minimize modularity dissipation to make full use of information as a thermodynamic fuel.



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Modularity dissipation implies:
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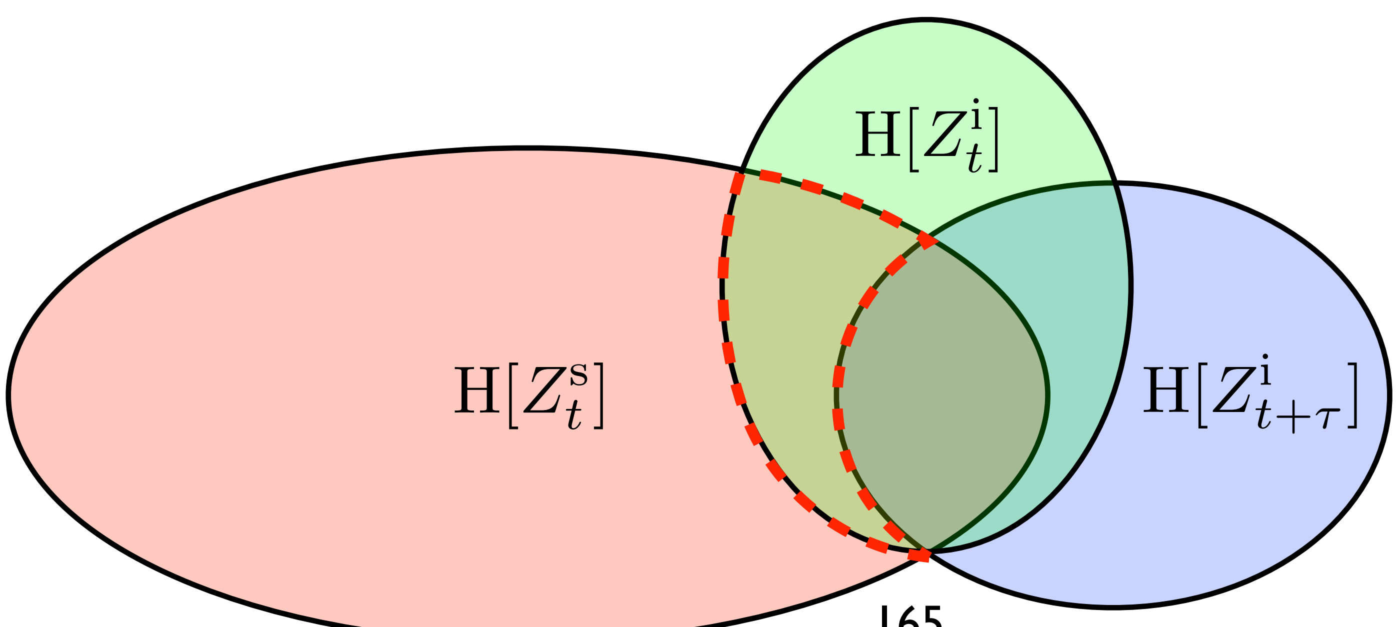

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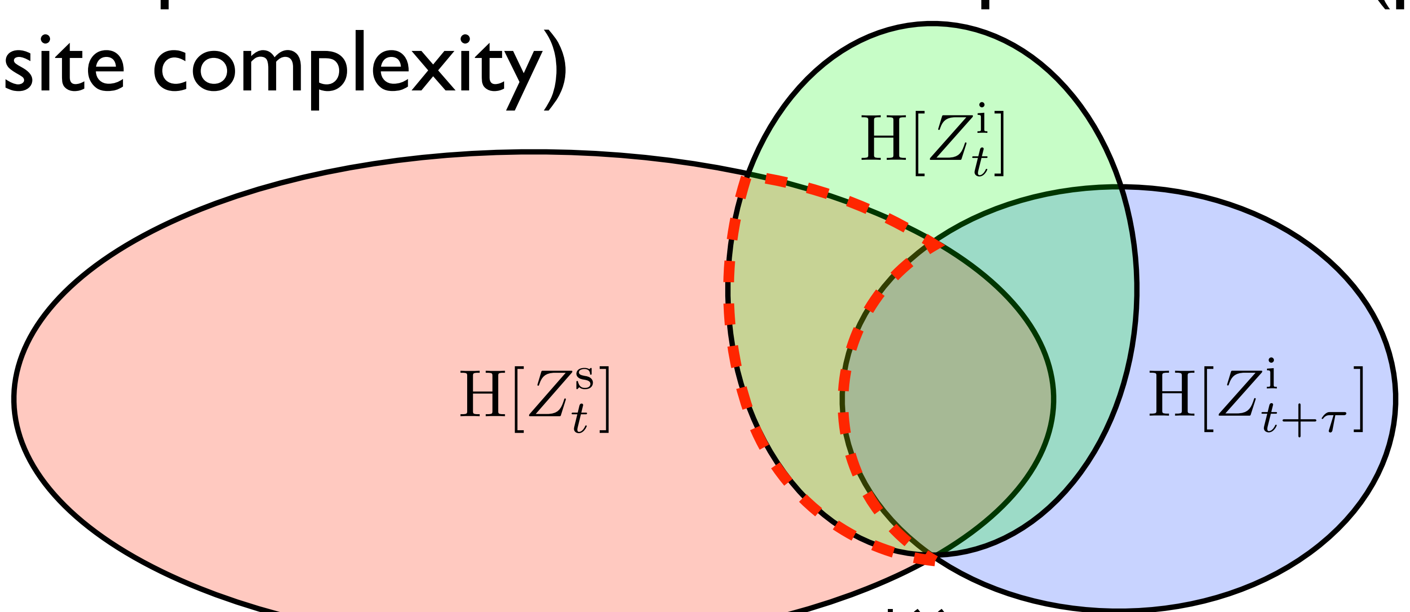
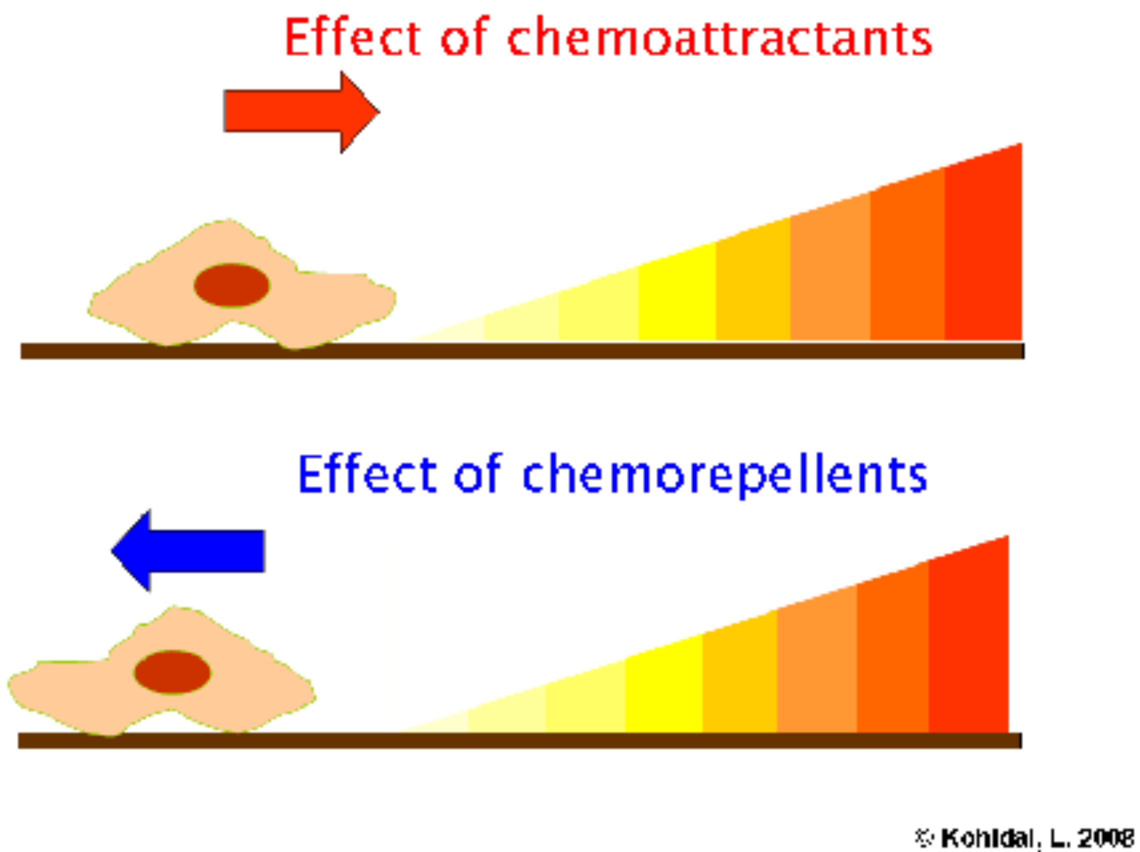
Must minimize modularity dissipation to make full use of information as a thermodynamic fuel.

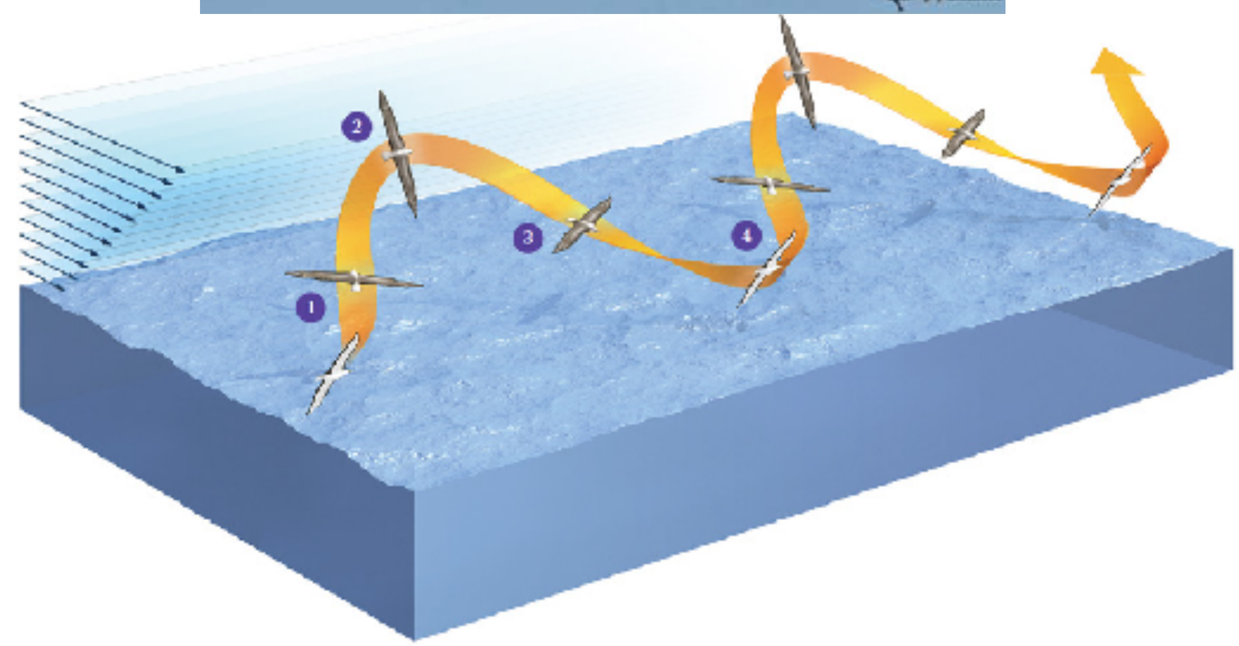
Modularity dissipation implies:
-Efficient pattern generators are retrodictive. (cost to excess memory of past)
-Efficient pattern extractors are predictive. (principle of requisite complexity)



Broader Picture

Principle of Requisite Complexity





Acknowledgements

Collaborators

-James P. Crutchfield (UC Davis)
-Dibyendu Mandal (UC Berkeley)

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