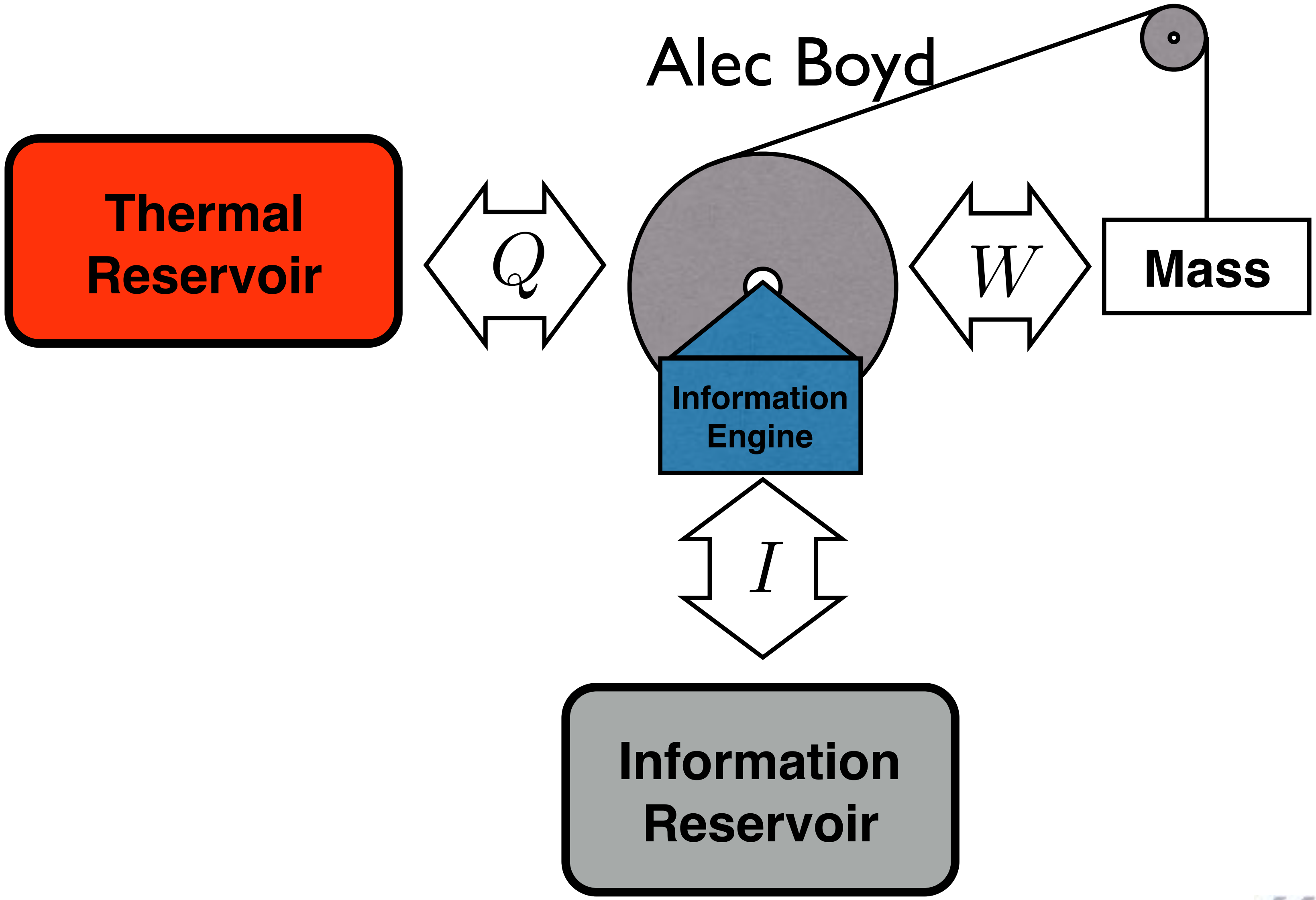
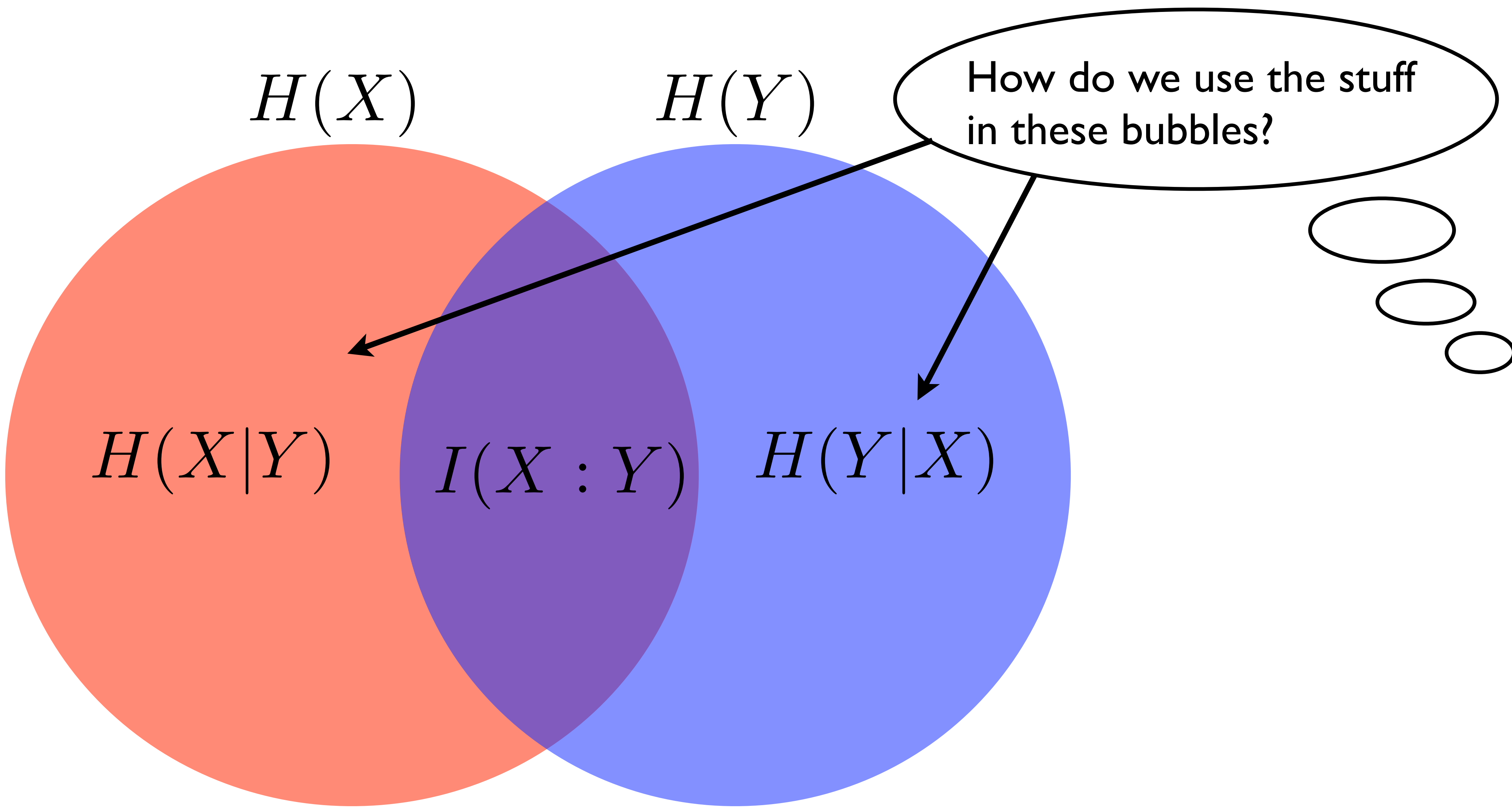
Thermodynamics of Information
Part I

Alec Boyd



What does information do?

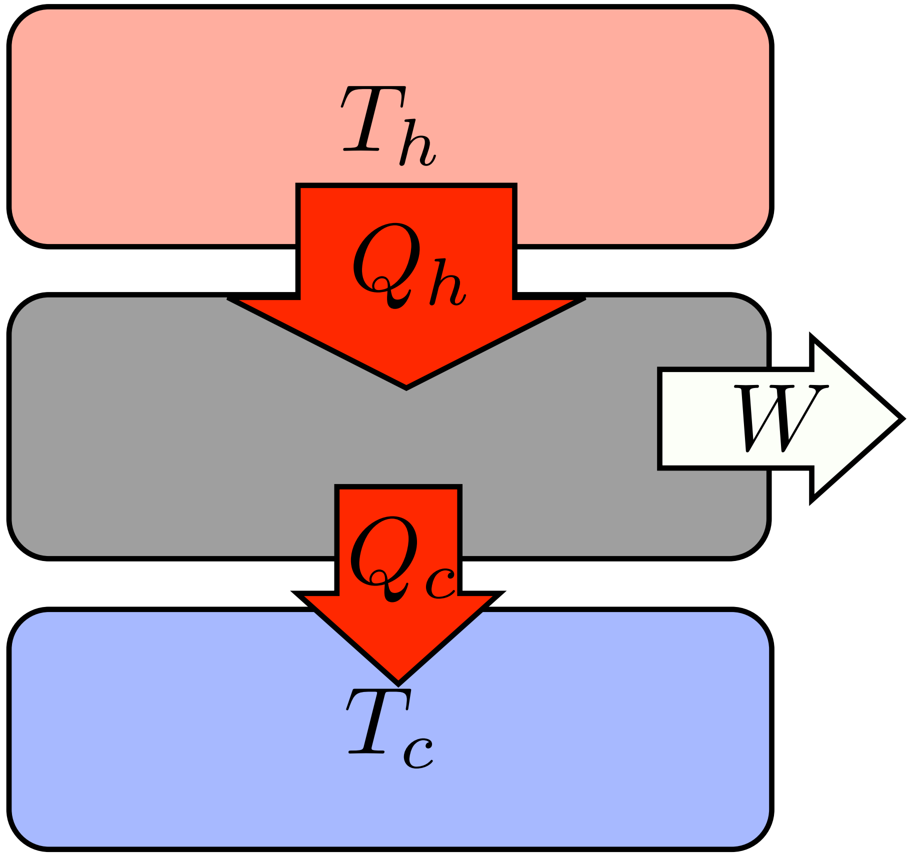


Thermodynamics

Thermodynamics treats physical systems many elements.

Relevant quantities:

1) Energy



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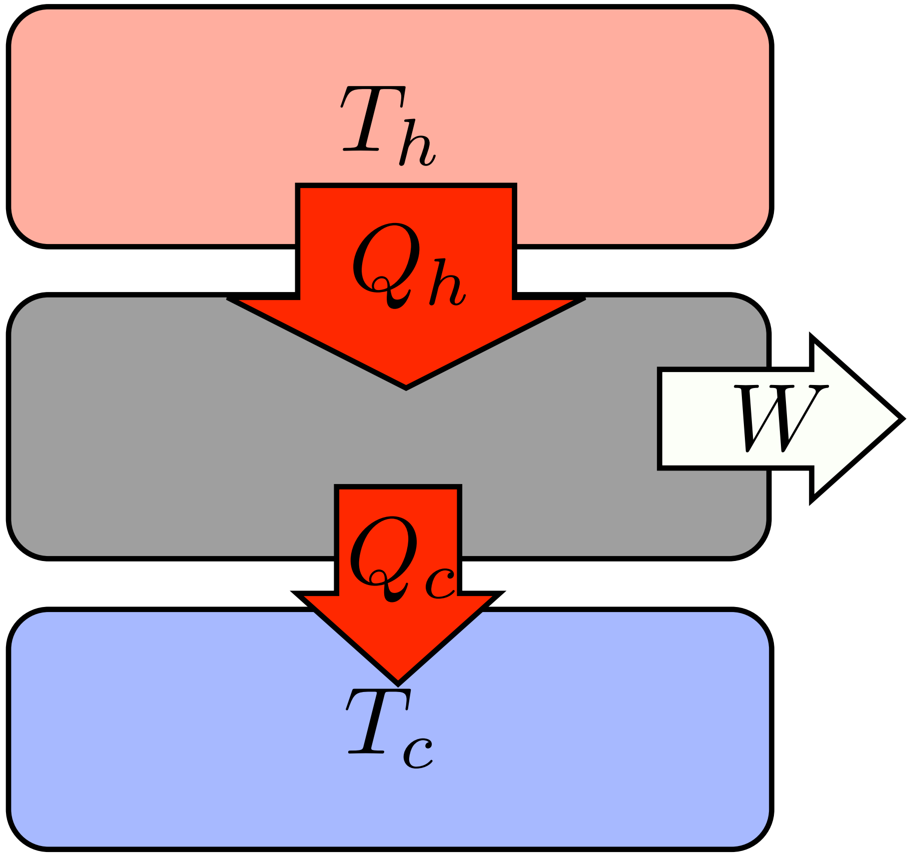

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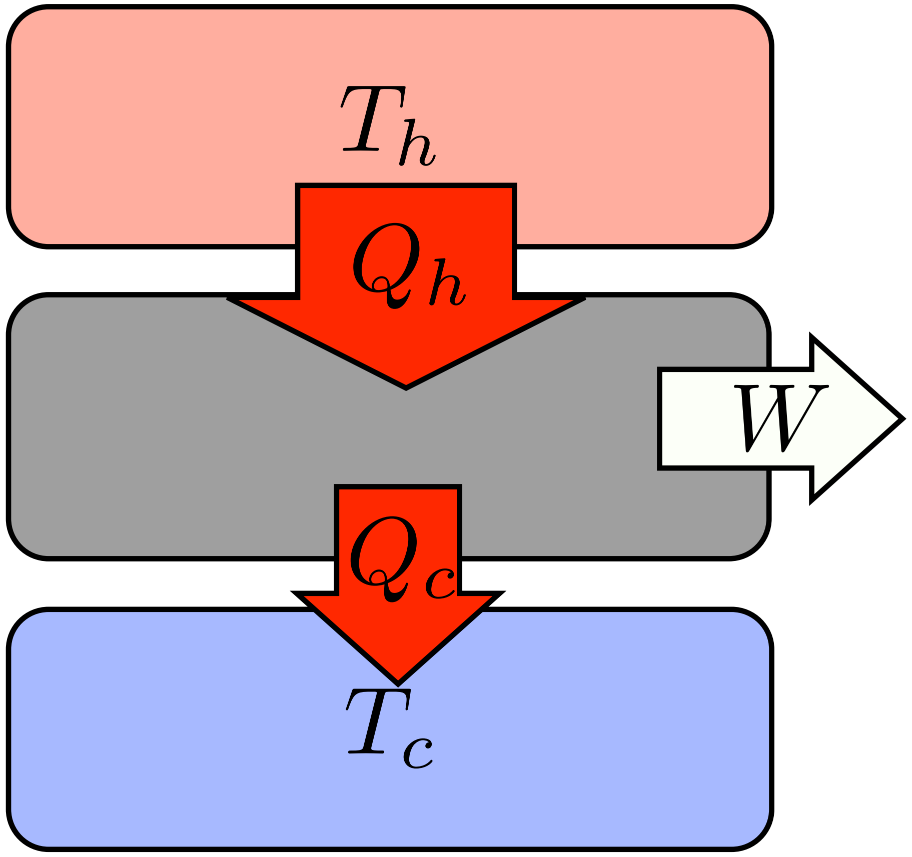

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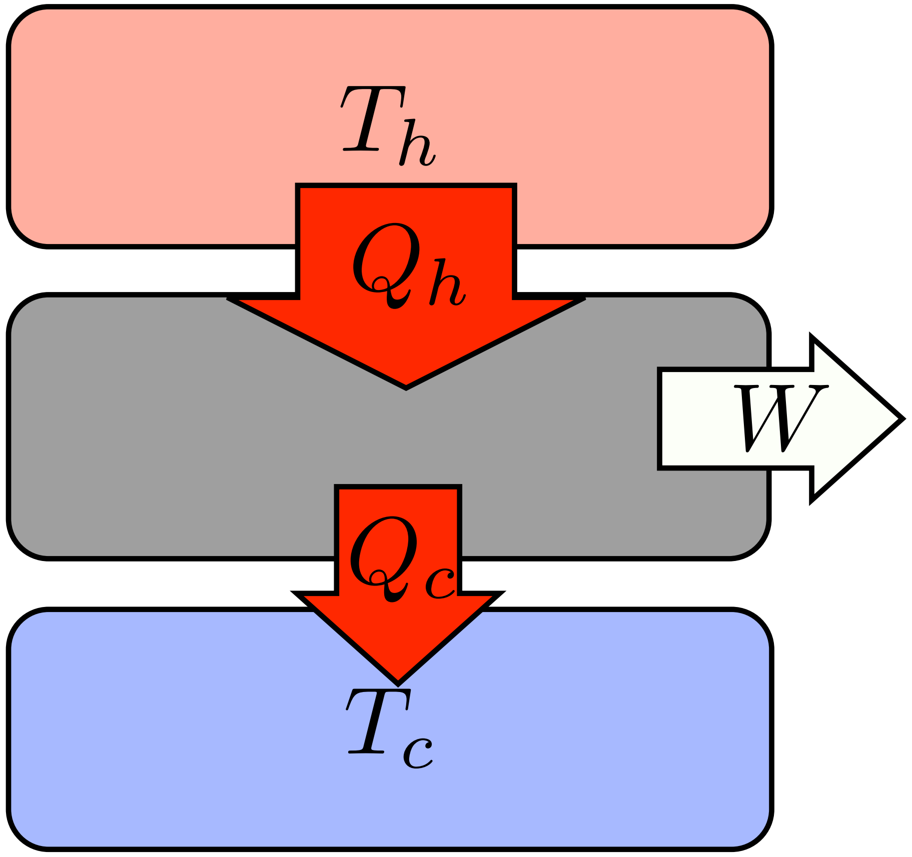

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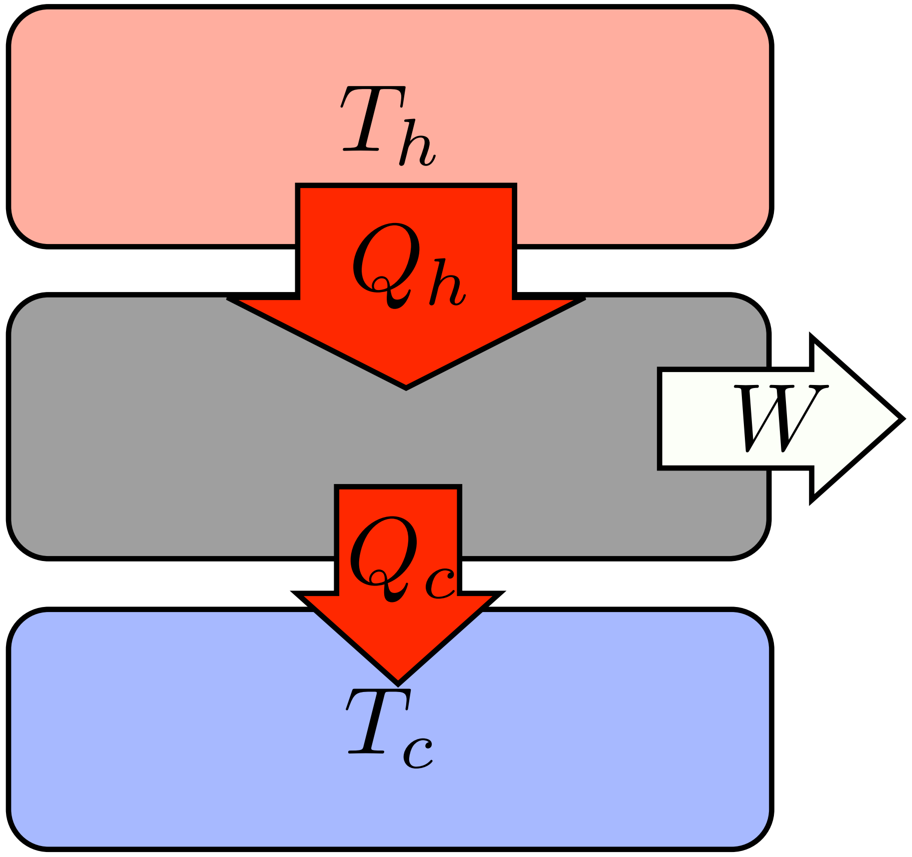

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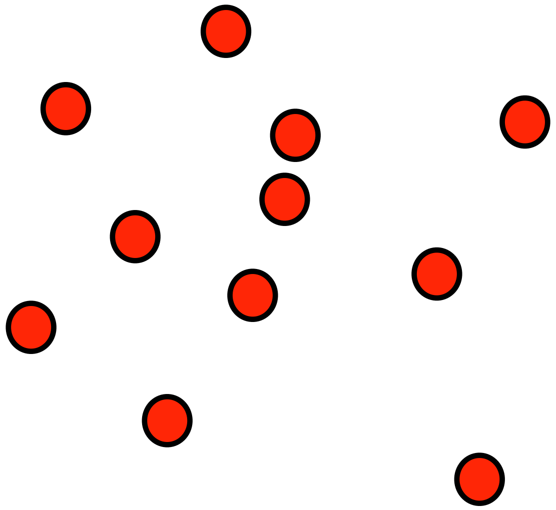
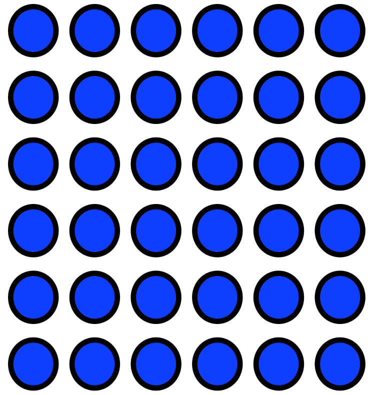
Entropy Comparison

Thermodynamic entropy as a measure of disorder.

Disordered (Hot Gas)



Ordered (Cold Solid)



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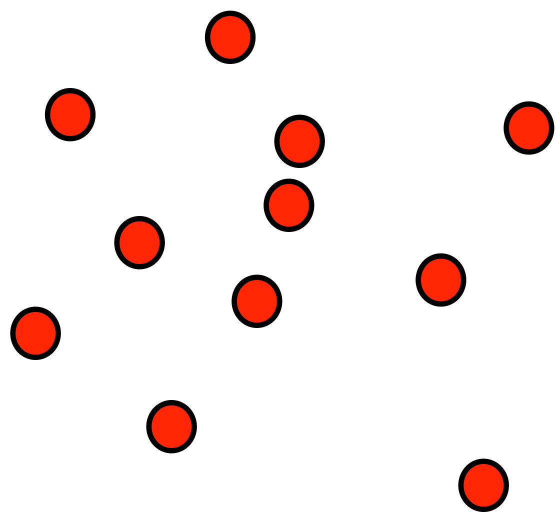

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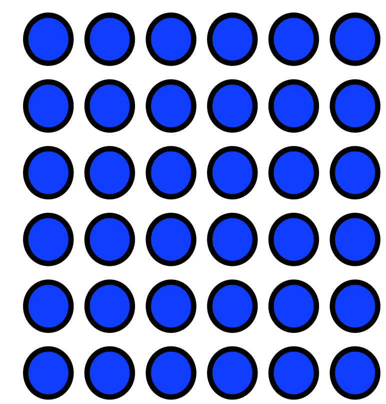

Shannon entropy as a measure of uncertainty or unpredictability.

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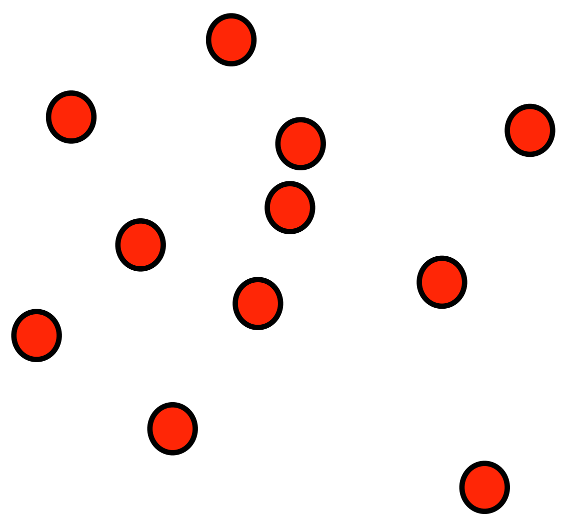

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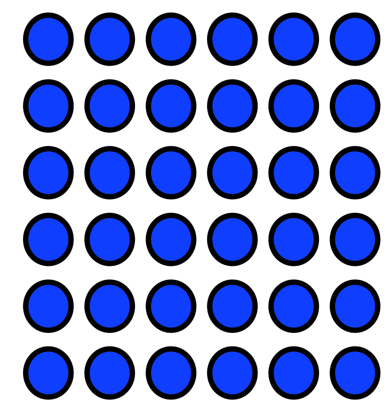

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String of measurements of unpredictable variable

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String of measurements of predictable variable

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Classical Physics

Physical states described by positions q_i and their conjugate momenta p_i :

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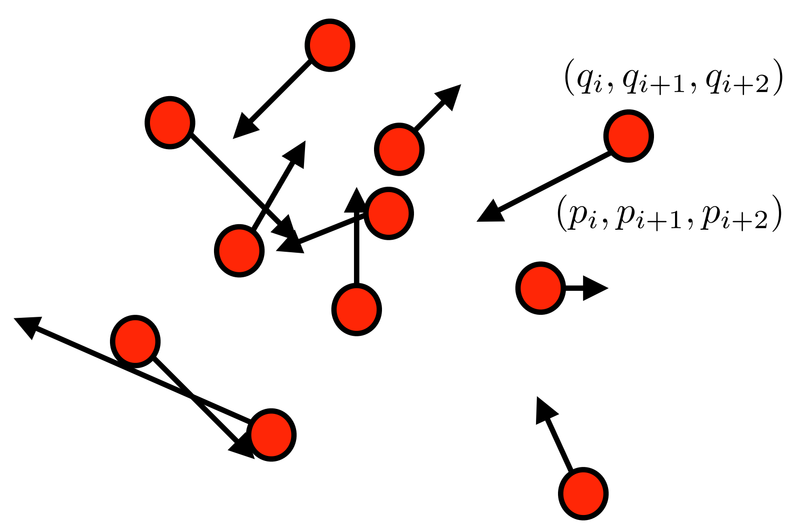

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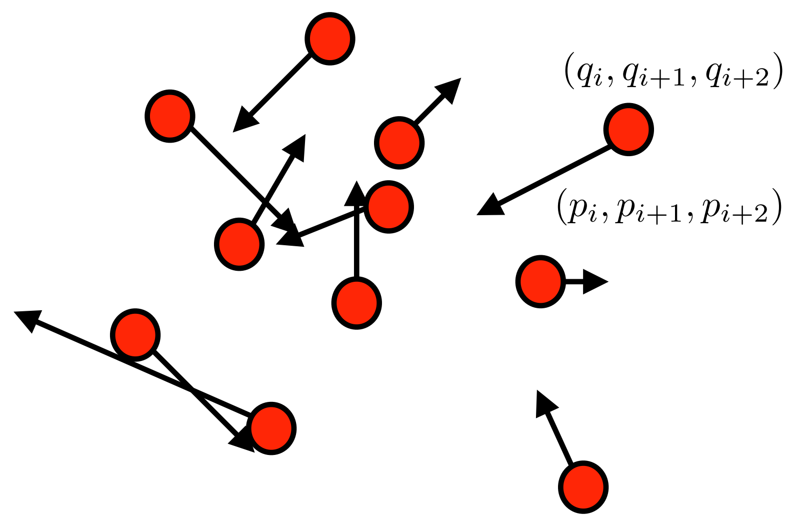

How does this change with time?

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Energy of each state specified by Hamiltonian function of state

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Equations of motion conserve energy

$$\Delta_{t \to t+\tau} E = E(x(t+\tau)) - E(x(t)) = 0$$

$$\frac{dE(x(t))}{dt} = 0$$

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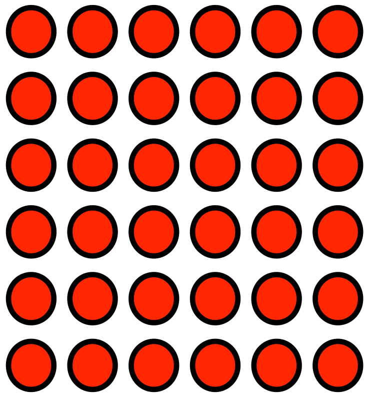

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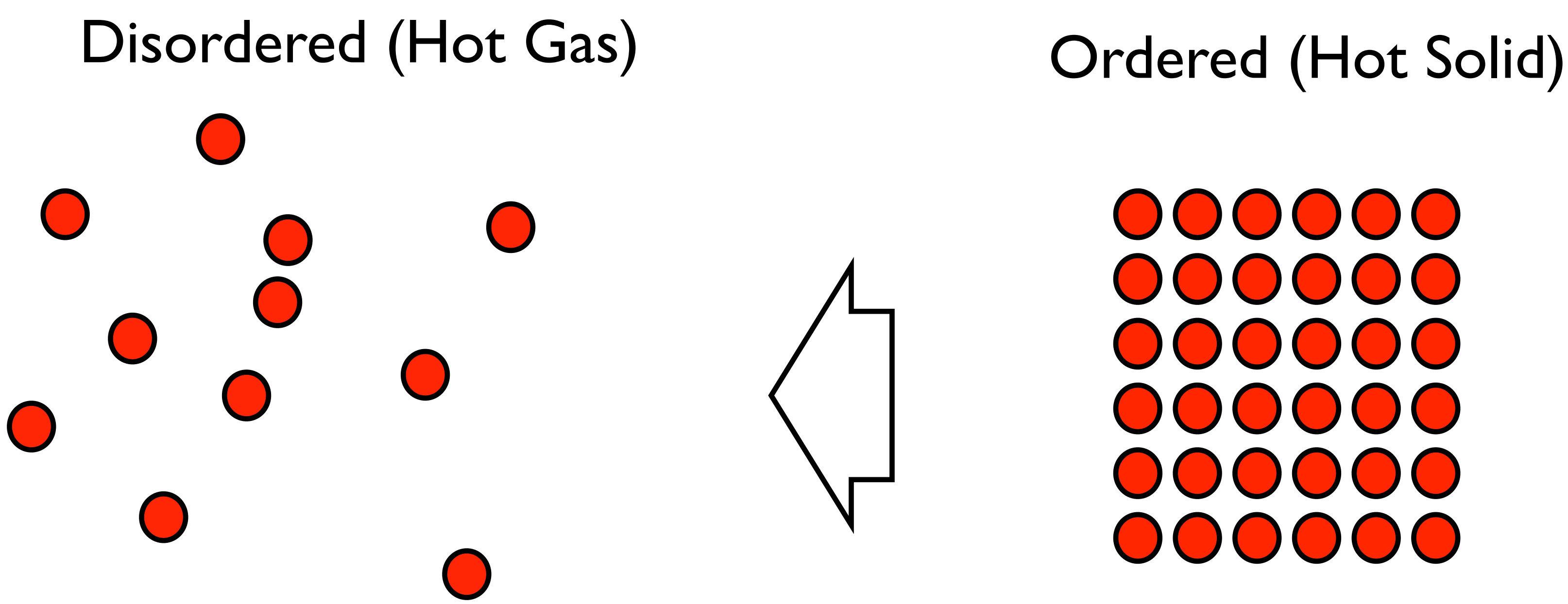

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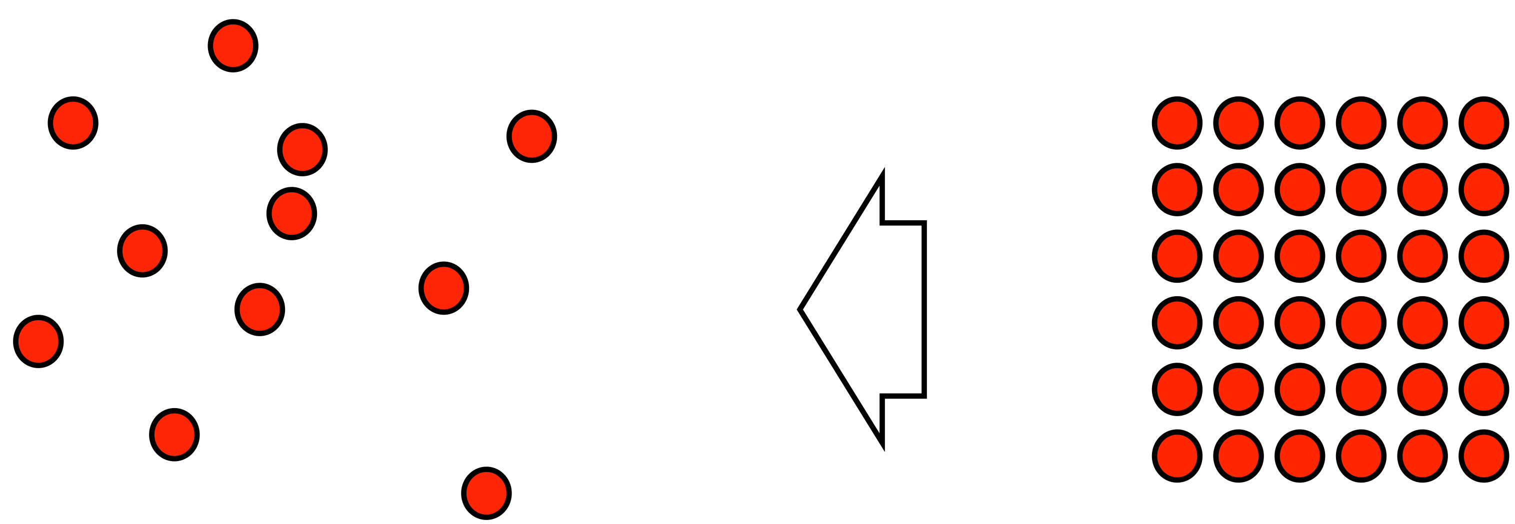

Example another and another and another

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This leads to an equilibrium estimate of entropy: $S[X^{\mathrm{eq}}|E(x) = E'] = k_B \ln \Omega(E')$

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What is temperature?

-Average energy?

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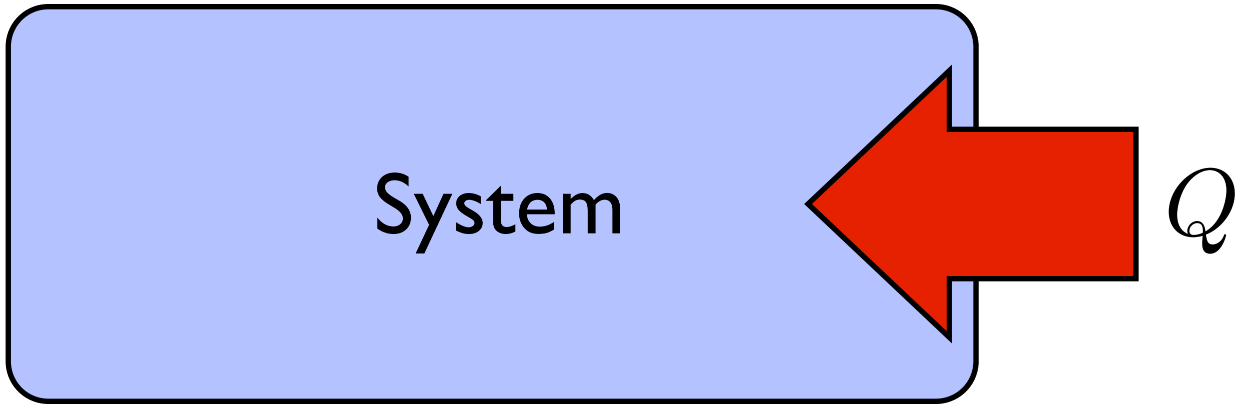

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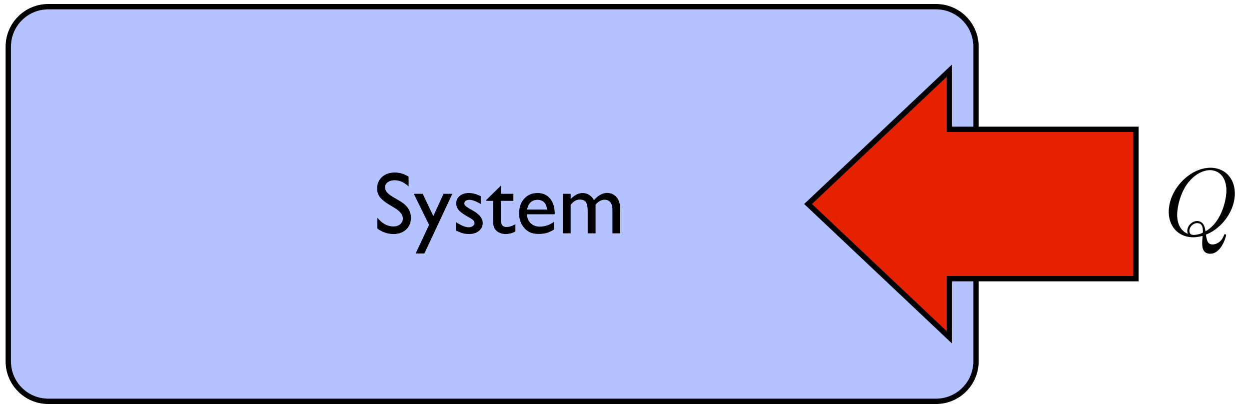


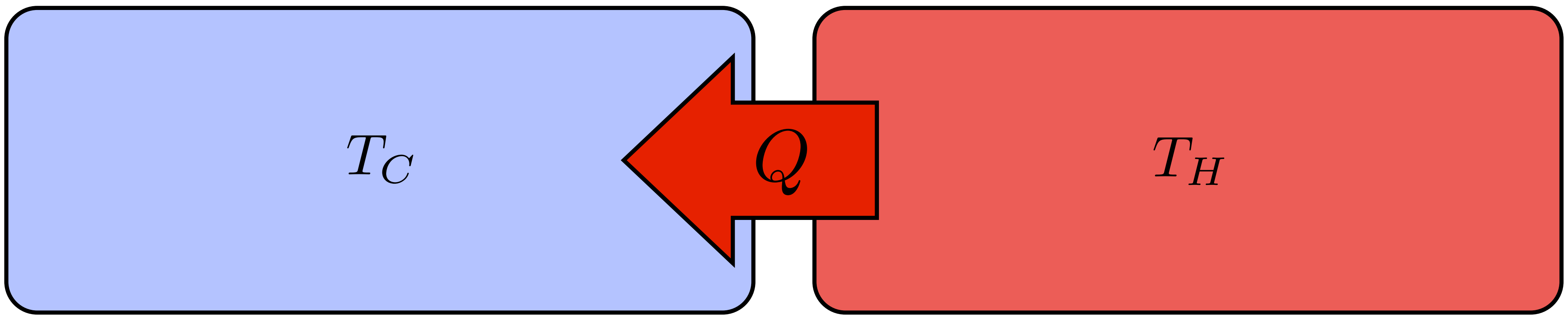
In most systems, every degree of freedom stores energy proportional to the temperature:

$$\langle E_{q_i} \rangle \approx \langle E_{p_i} \rangle \approx \frac{k_B T}{2}$$

Heat Flow

Entropy explains why heat diffuses from hot to cold:

$$T_H > T_C$$



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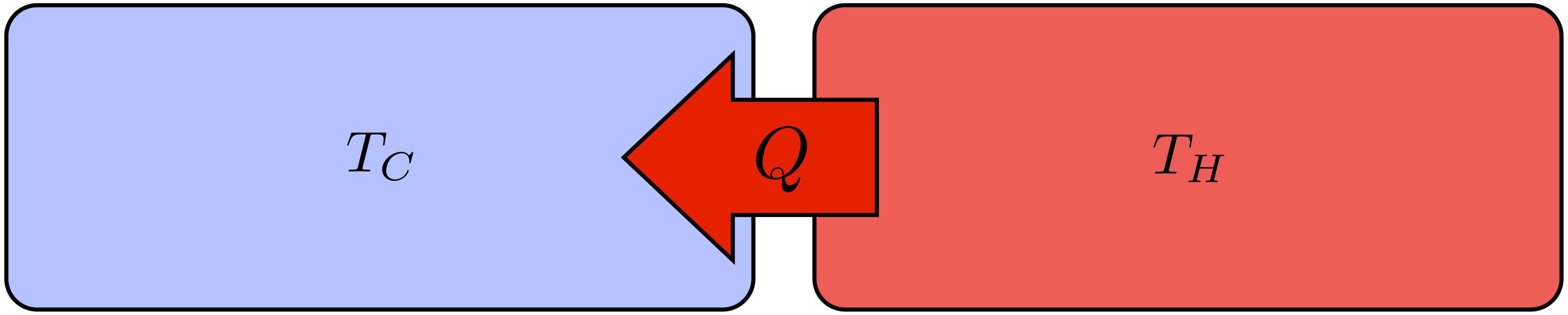

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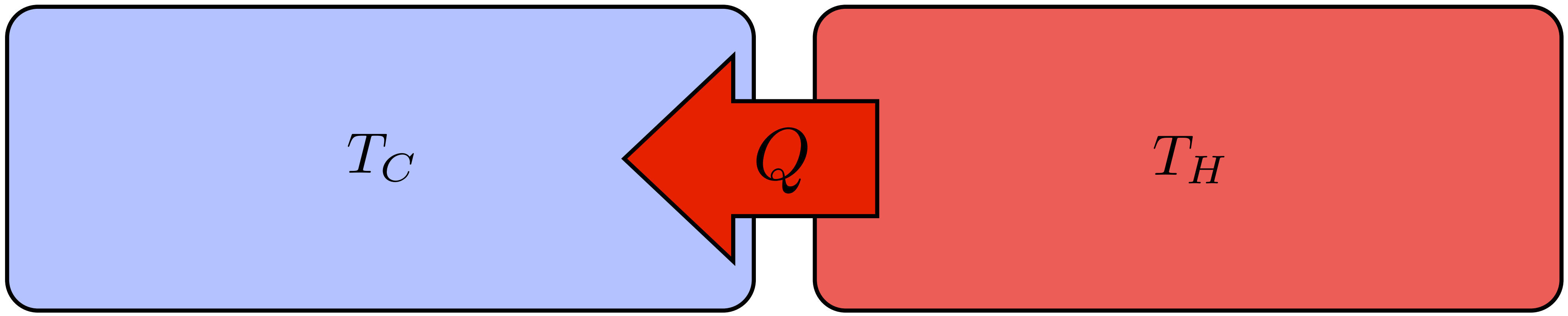

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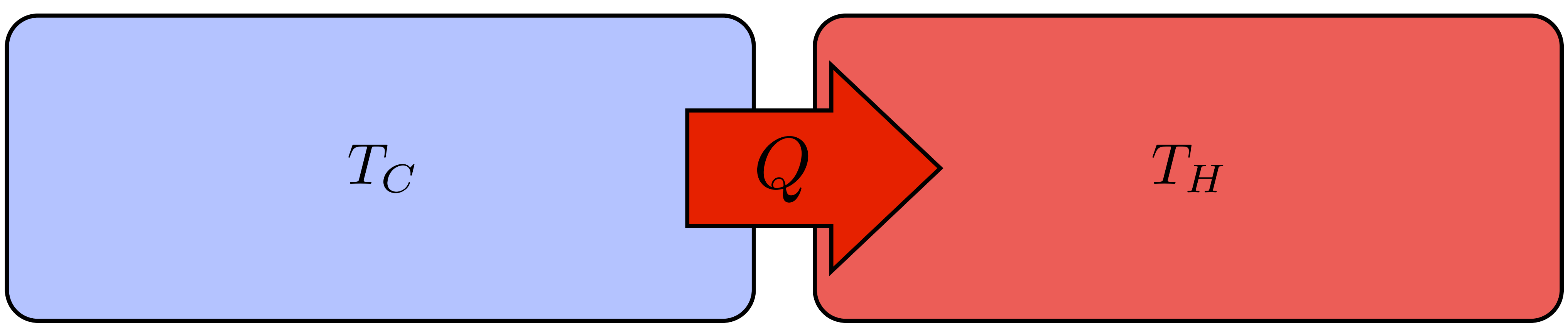

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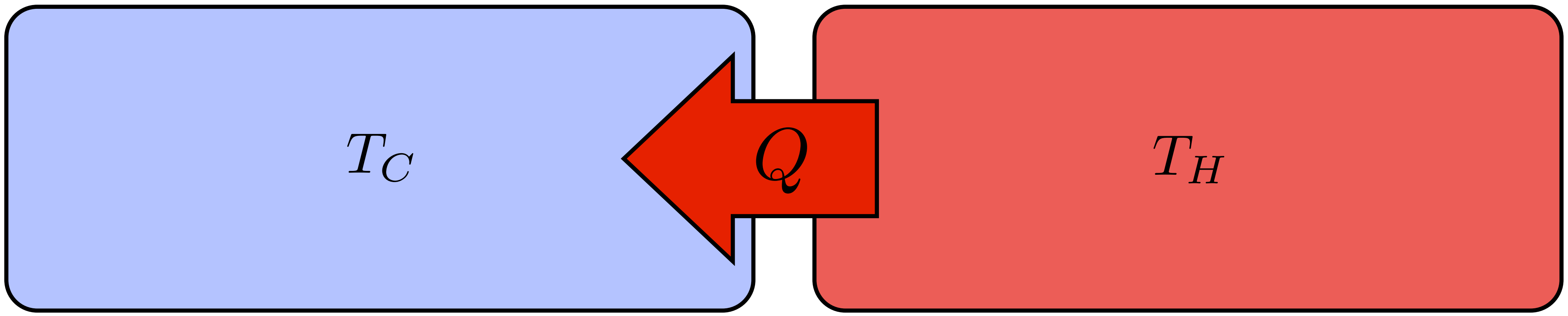

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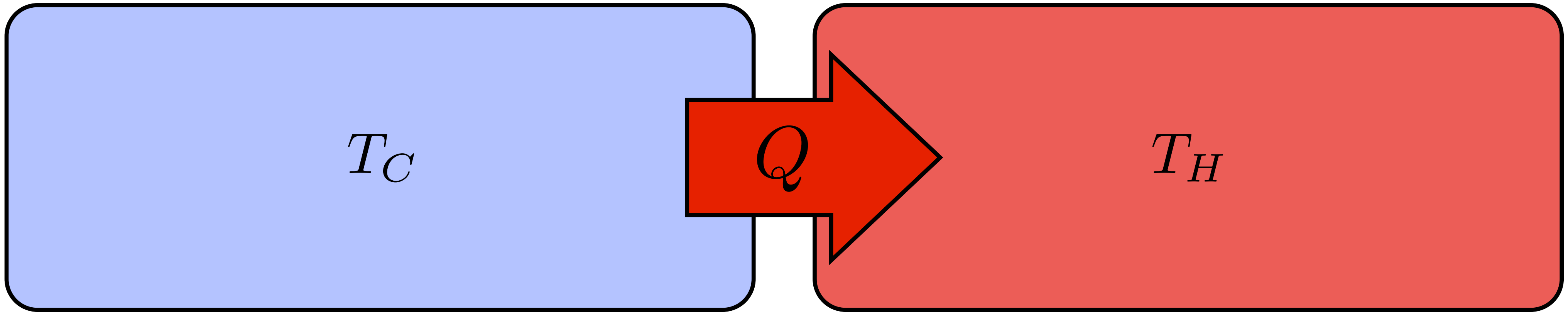

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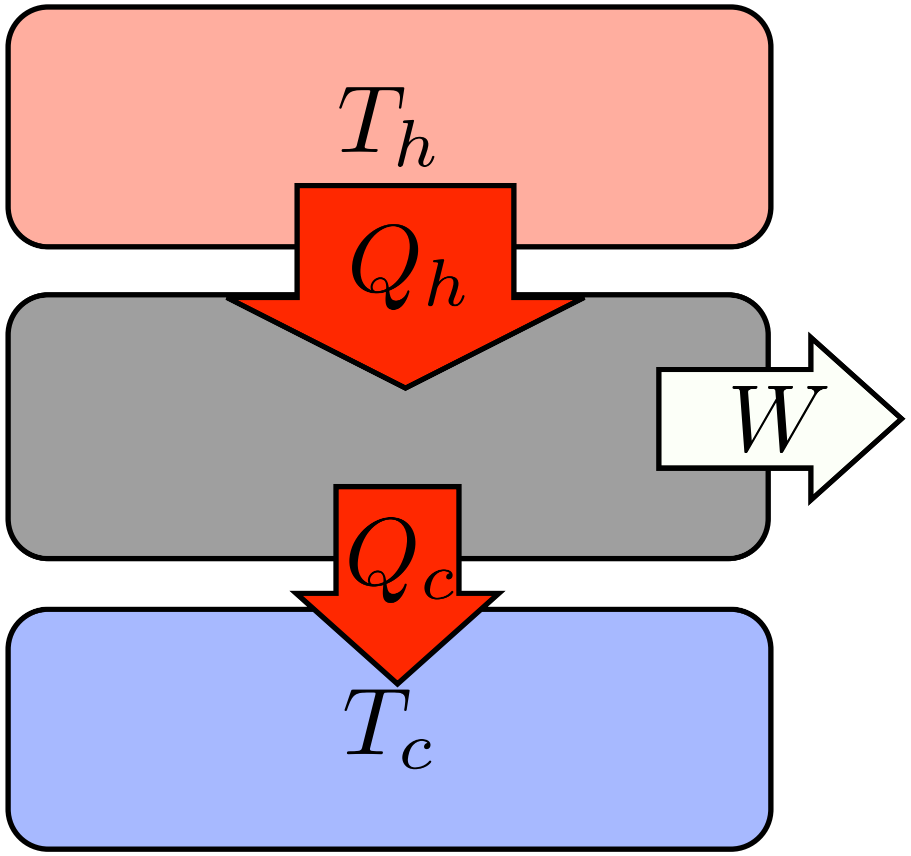

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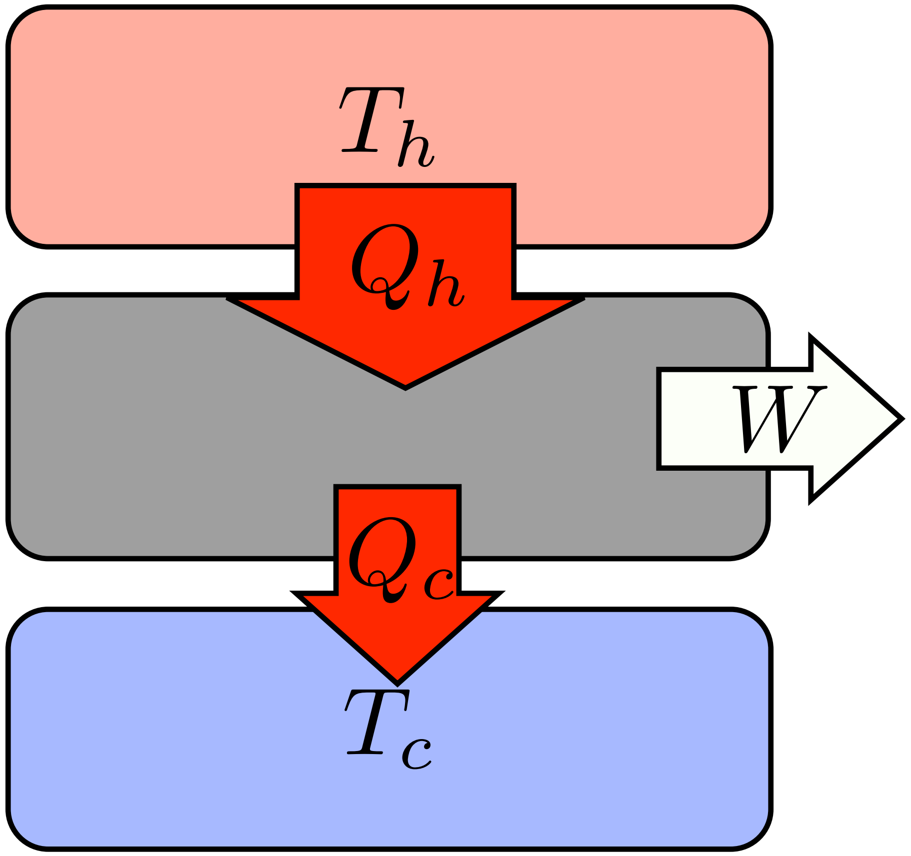

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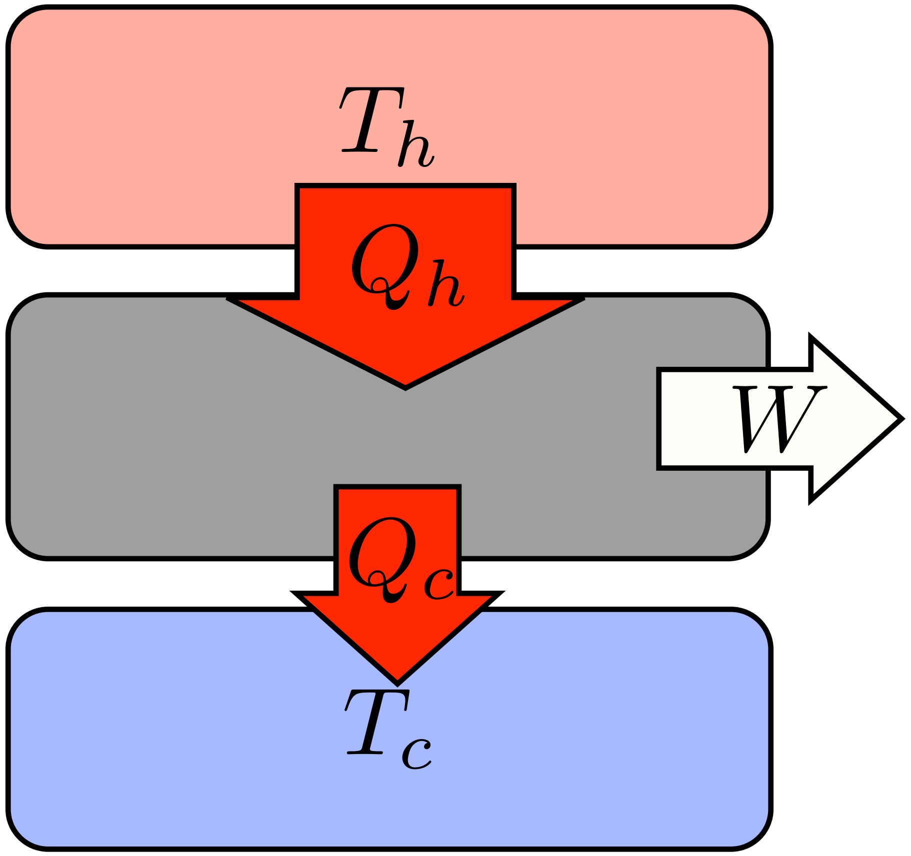

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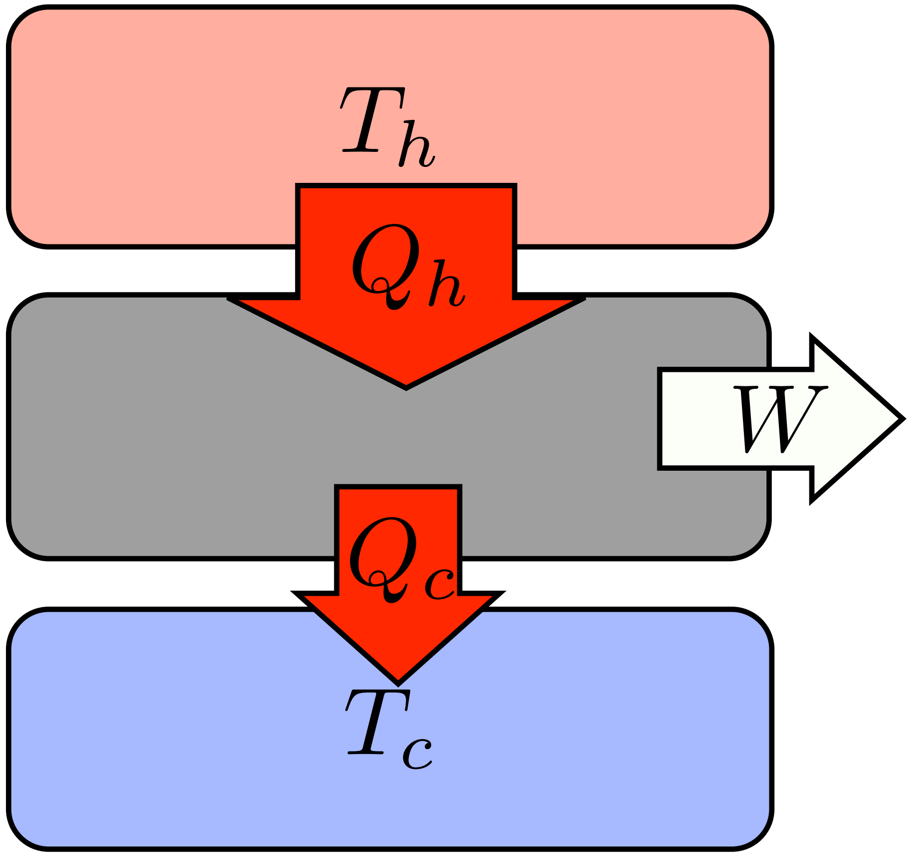

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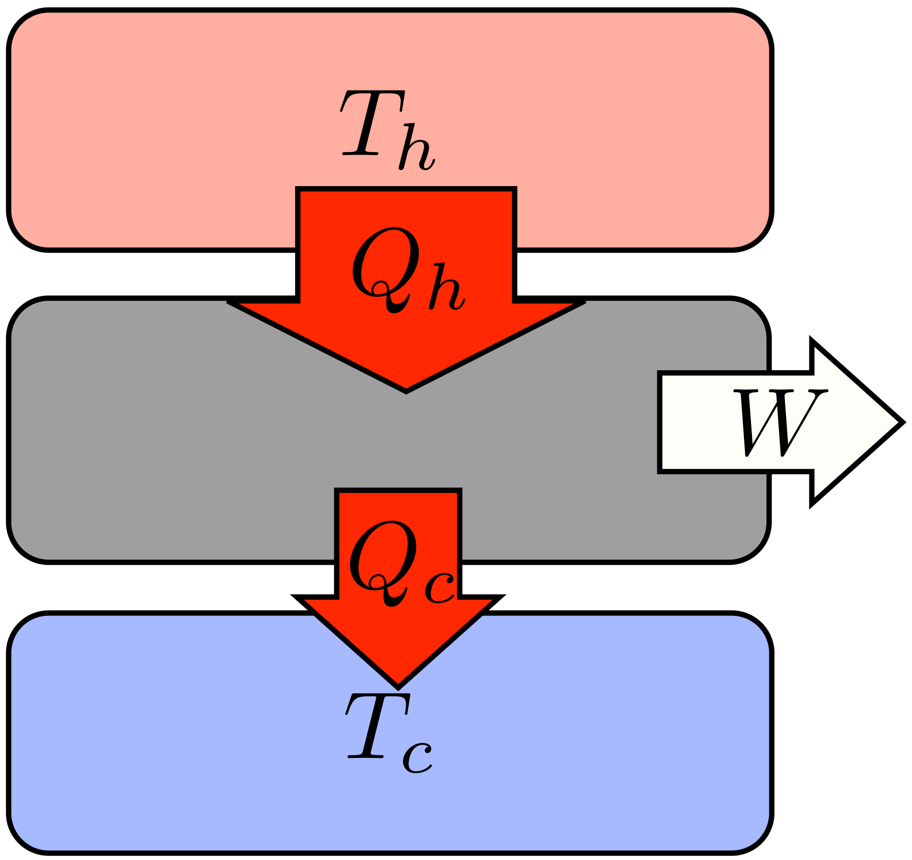

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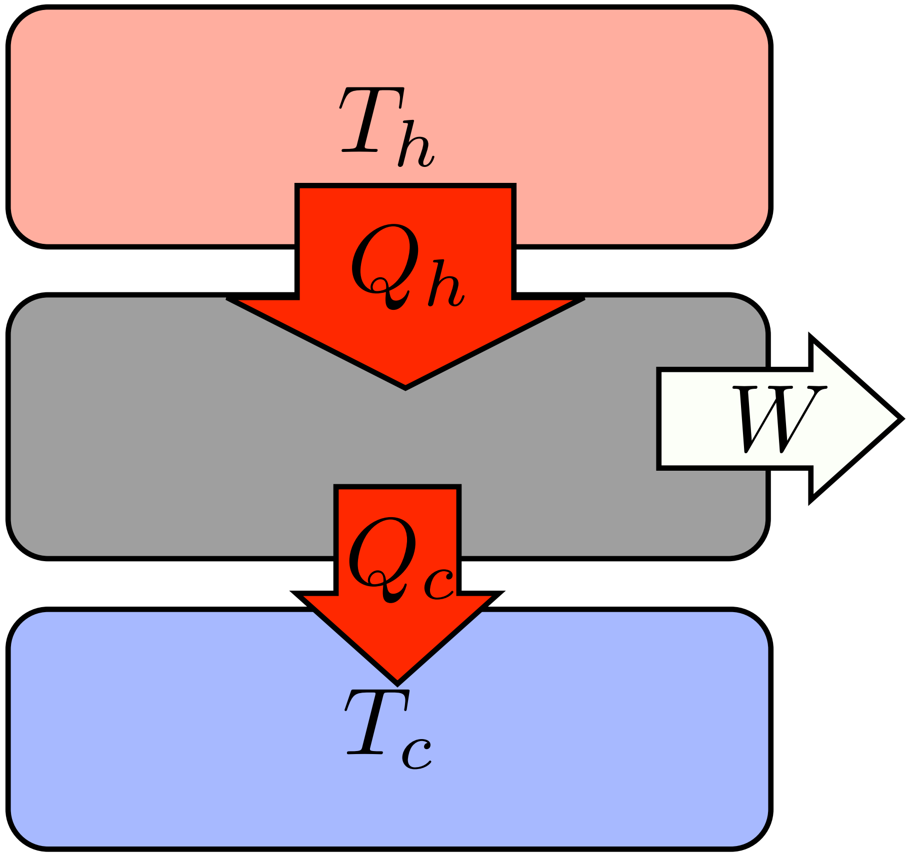

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$$e \leq 1 - \frac{T_c}{T_h}$$

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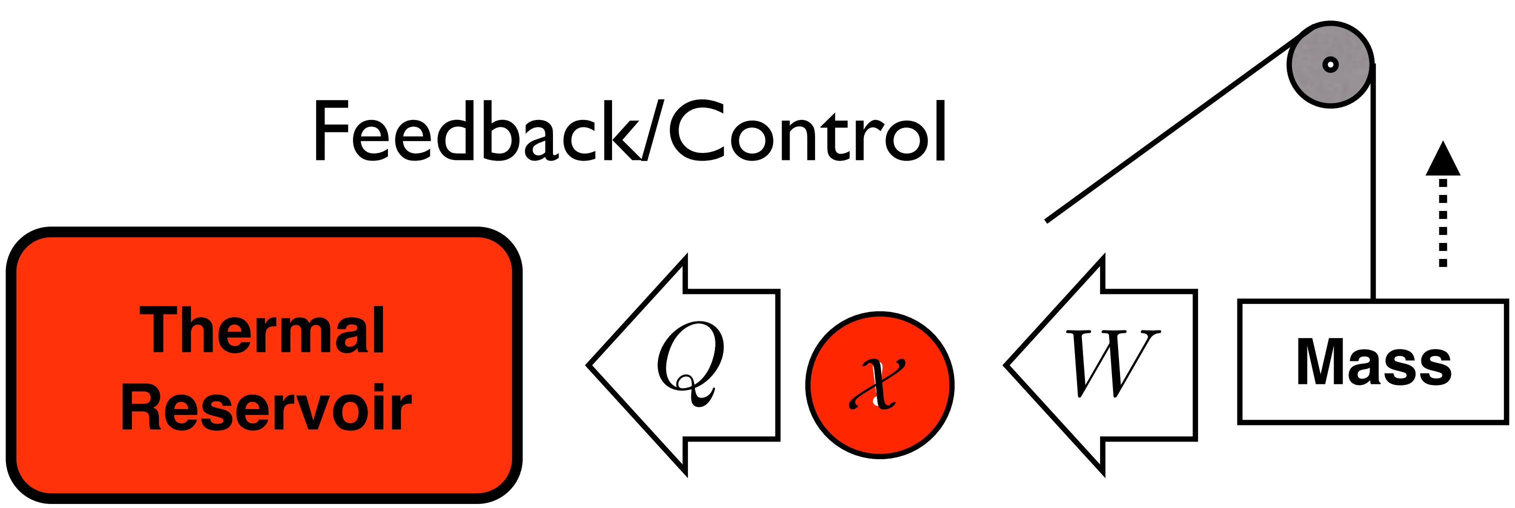
Feedback/Control



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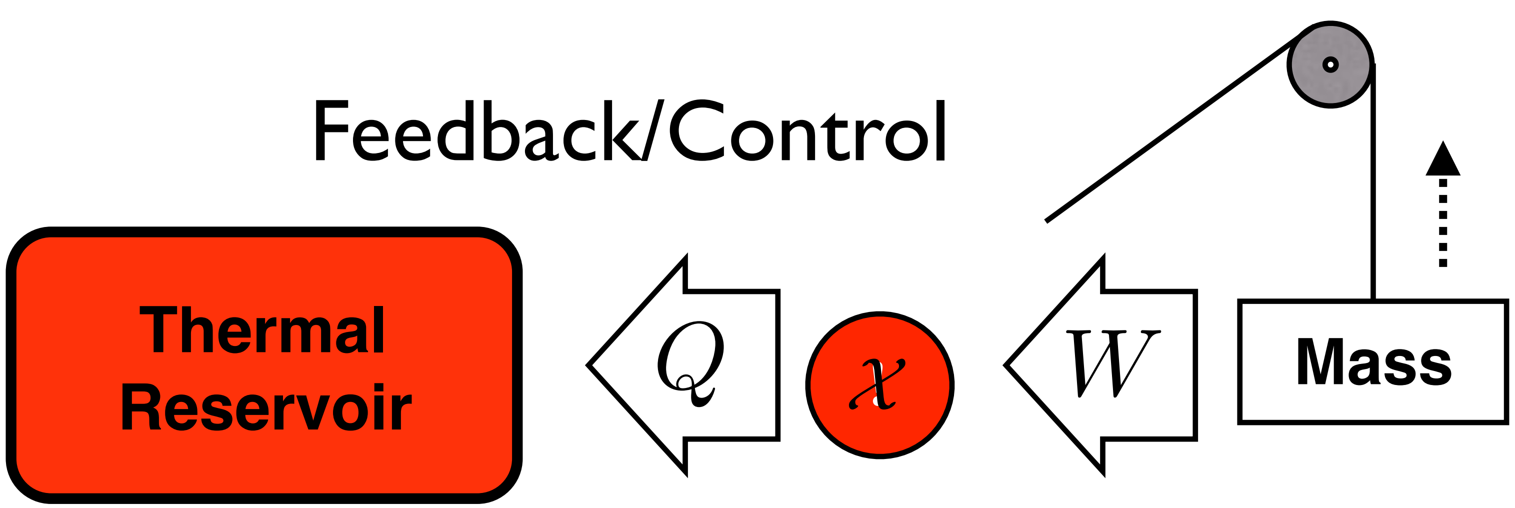
Through control of the energy landscape

$$d\langle E\rangle = \sum_x \Pr(X_t = x)\, dE(x) + \sum_x E(x)\, d\Pr(X_t = x)$$



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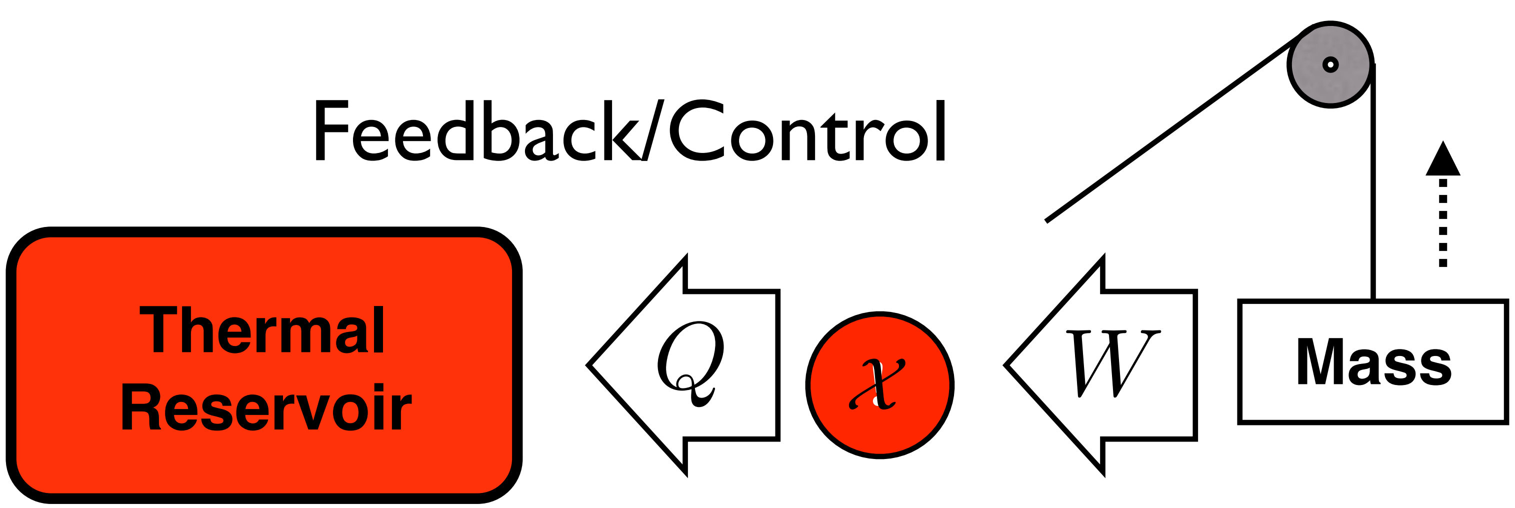

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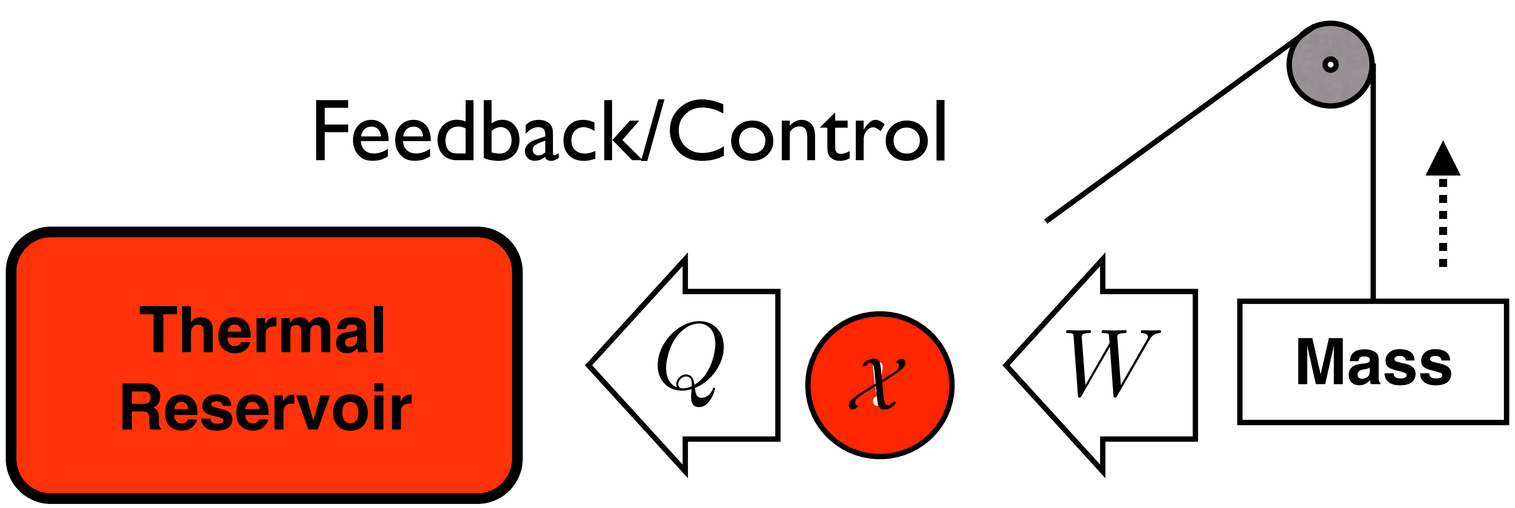
We consider control of a system $\mathcal{X}$ which interacts with an ideal thermal reservoir $\mathcal{R}$ at temperature T to produce work



Entropy of Control

The total entropy production of the joint reservoir system

$$\Delta S_{\text{total}} = \Delta S[R] + \Delta S[X] - \Delta I[X;R]$$

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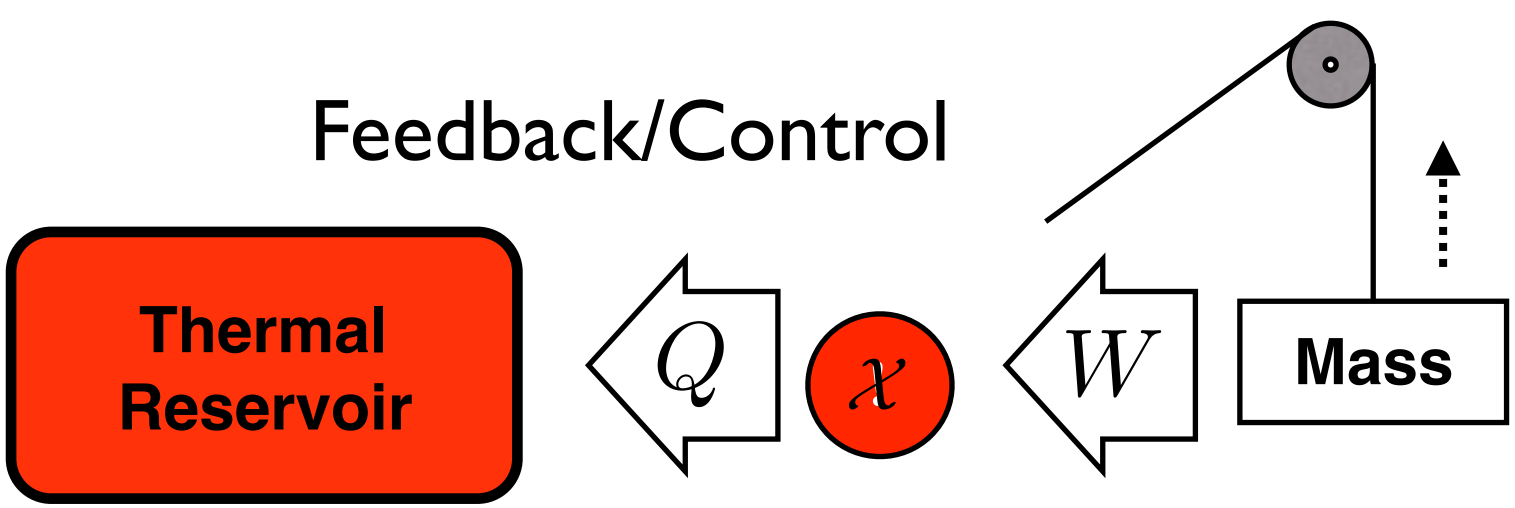

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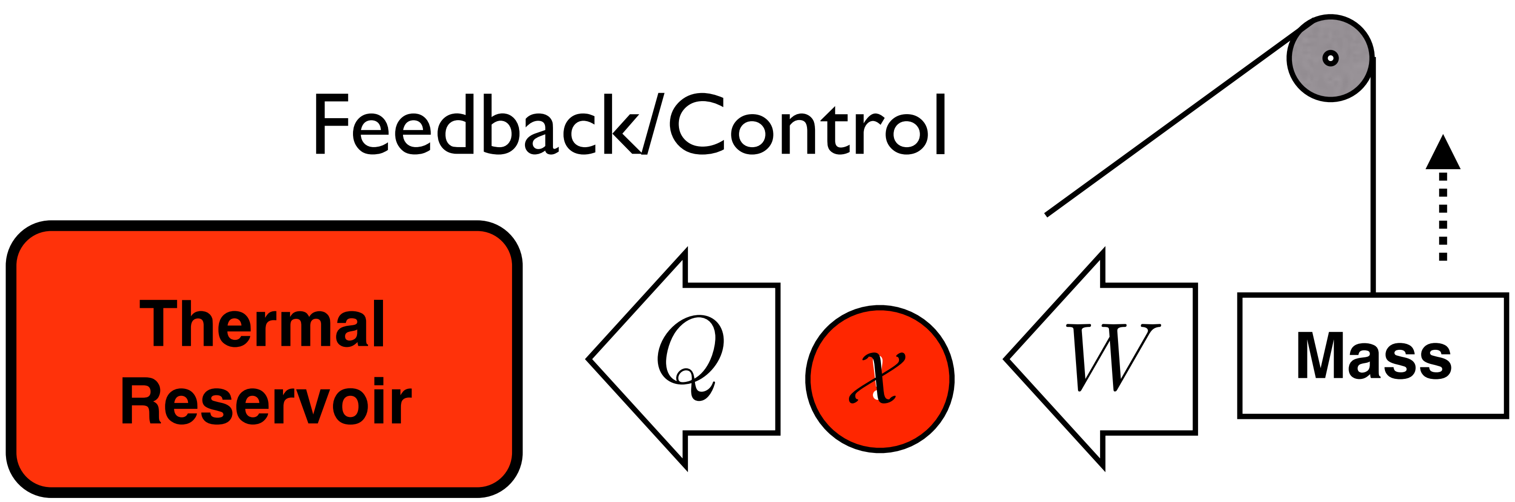

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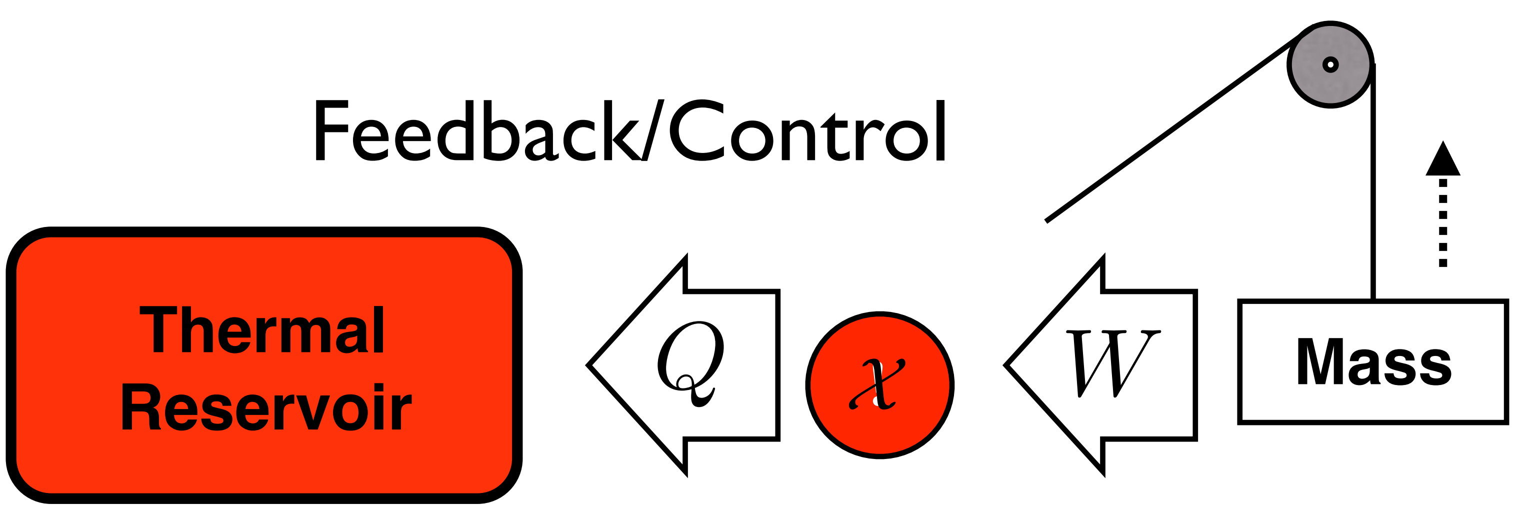

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$$\langle Q \rangle \geq -T \Delta S[X]$$

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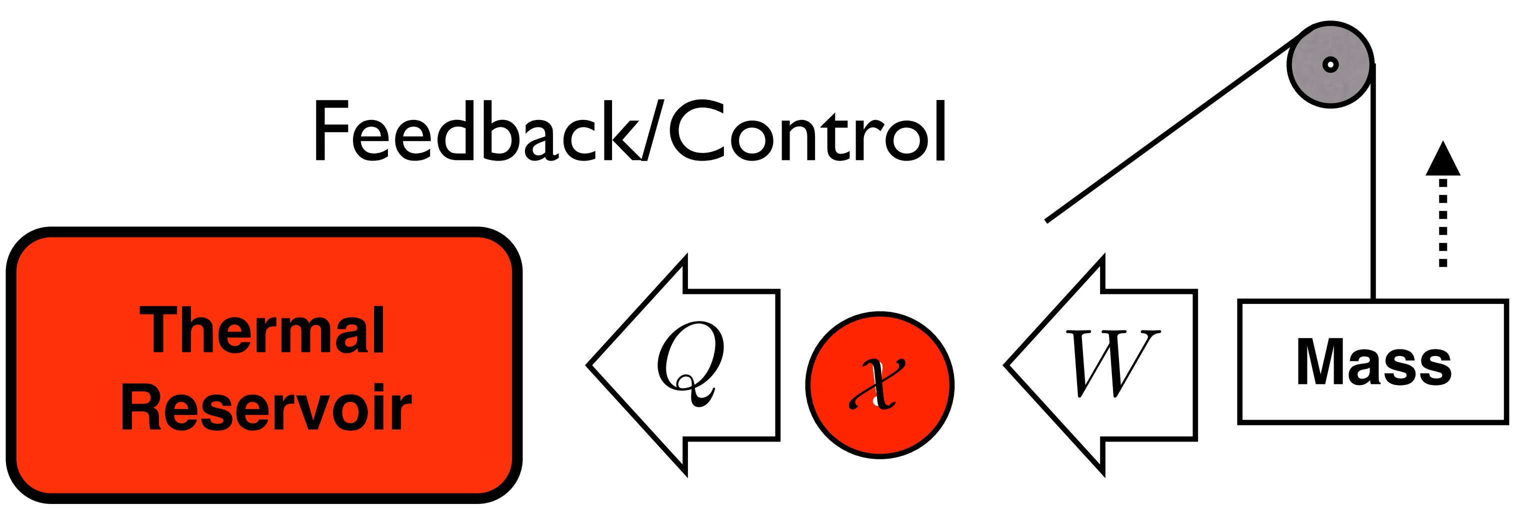

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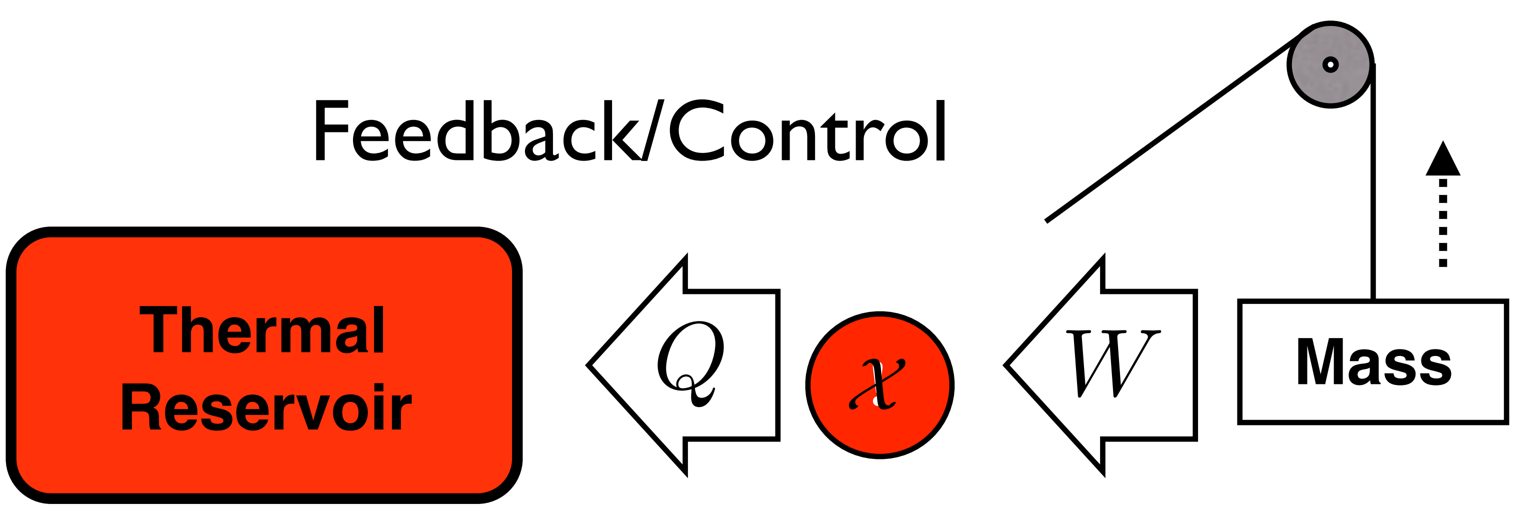

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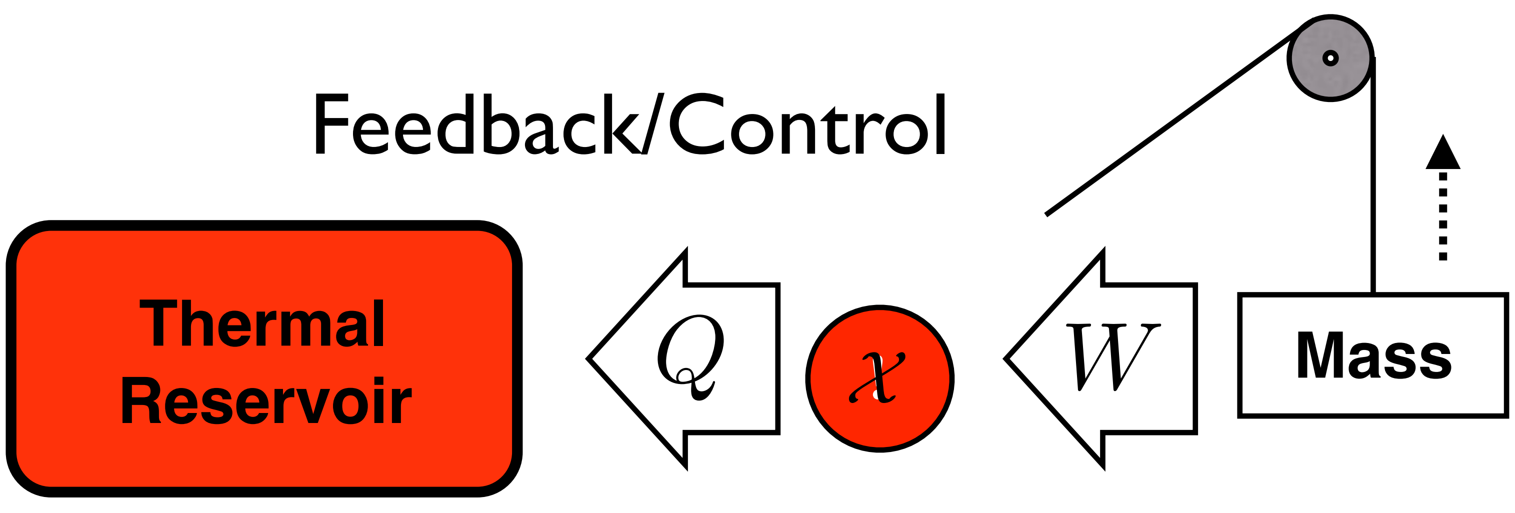
This can be interpreted as dissipated work

$$\langle W_{\text{diss}} \rangle = \langle W \rangle - \Delta F^{\text{NEQ}}$$



Entropy of Control

This can be re-expressed with work

$$T\Delta S_{\mathrm{total}} = \langle W \rangle - \Delta \langle E[X] \rangle + T\Delta S[X]$$

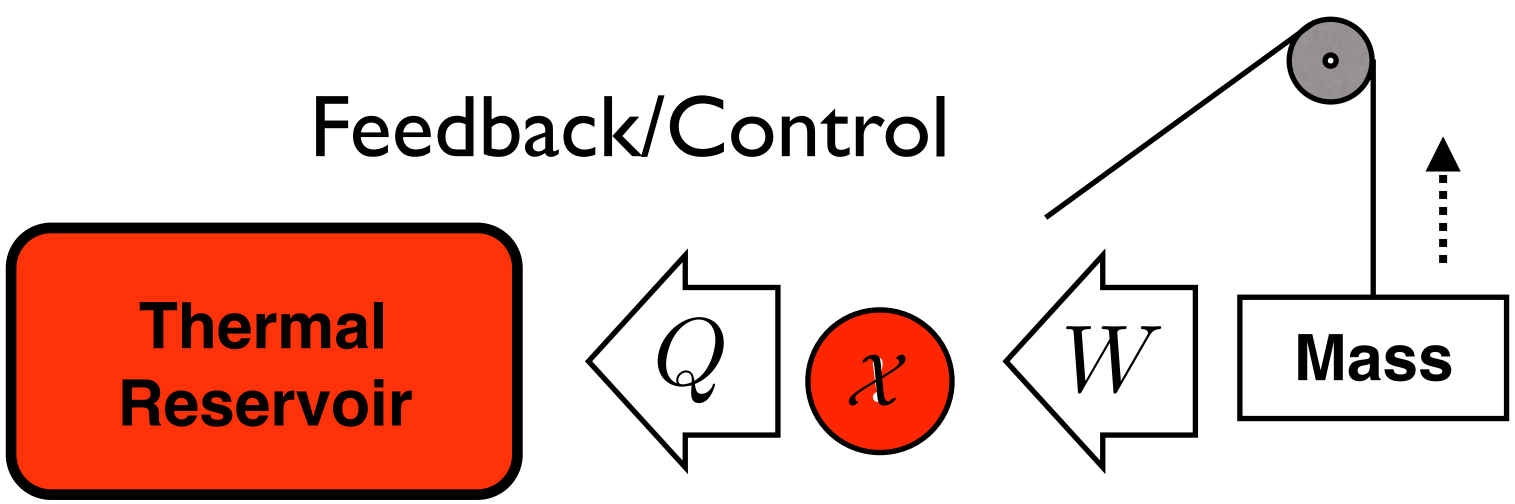
This can be interpreted as dissipated work

$$\langle W_{\mathrm{diss}} \rangle = \langle W \rangle - \Delta F^{\mathrm{NEQ}}$$

This includes a new quantity: the non-equilibrium free energy, which specifies the amount of work that could have been harvested:

$$F^{\mathrm{NEQ}} = \langle E[X] \rangle - TS[X]$$

Feedback/Control



Stepping Back

What does entropy do?

Stepping Back

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1) Bounds function of heat engine

$$e = \frac{\text{output}}{\text{input}} = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

$$e \leq 1 - \frac{T_c}{T_h}$$

Stepping Back

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$$\langle Q \rangle \geq -k_B T \ln 2 \, \Delta H[X]$$

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$$\langle Q \rangle \geq -k_B T \ln 2 \, \Delta H[X]$$

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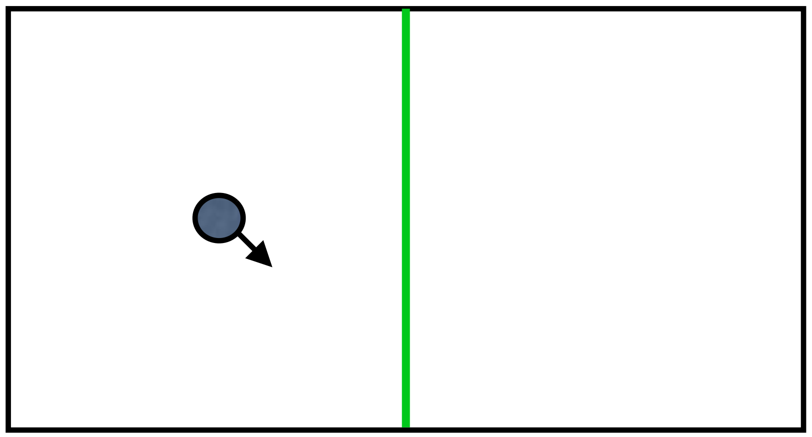
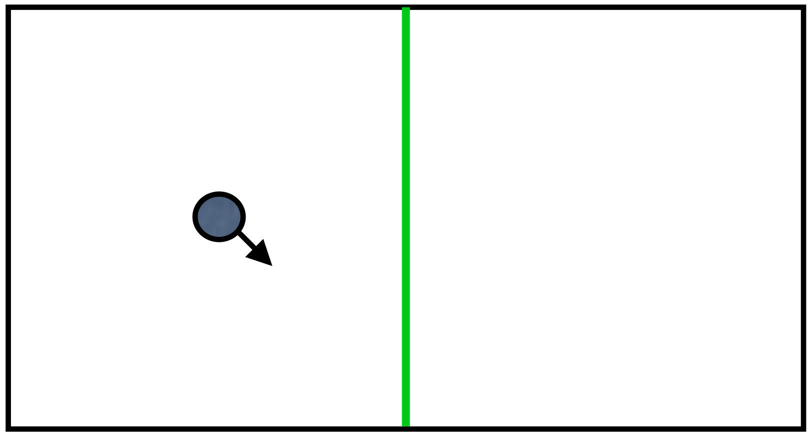
Control Example

Single particle in a box



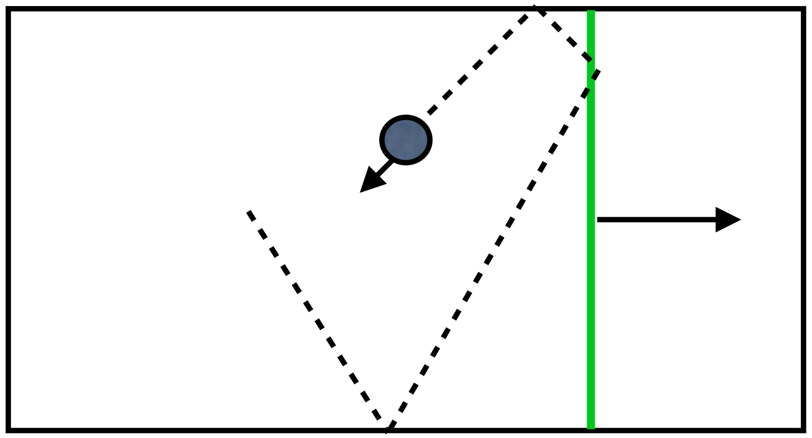
Control Example

Single particle in a box, insert barrier



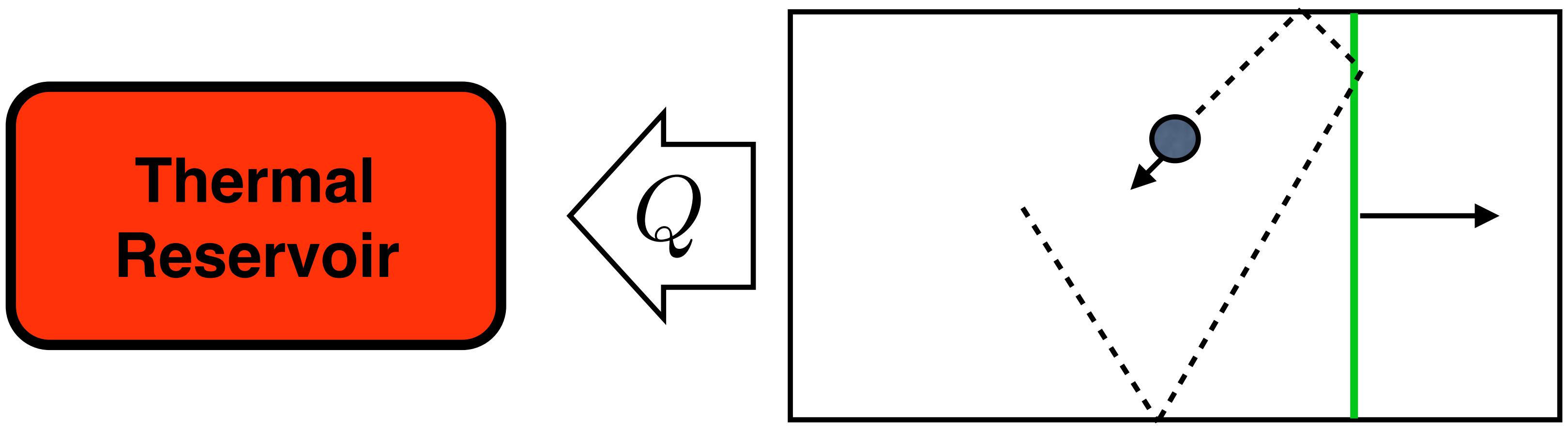
Control Example

Single particle in a box, insert barrier, slide barrier away from particle



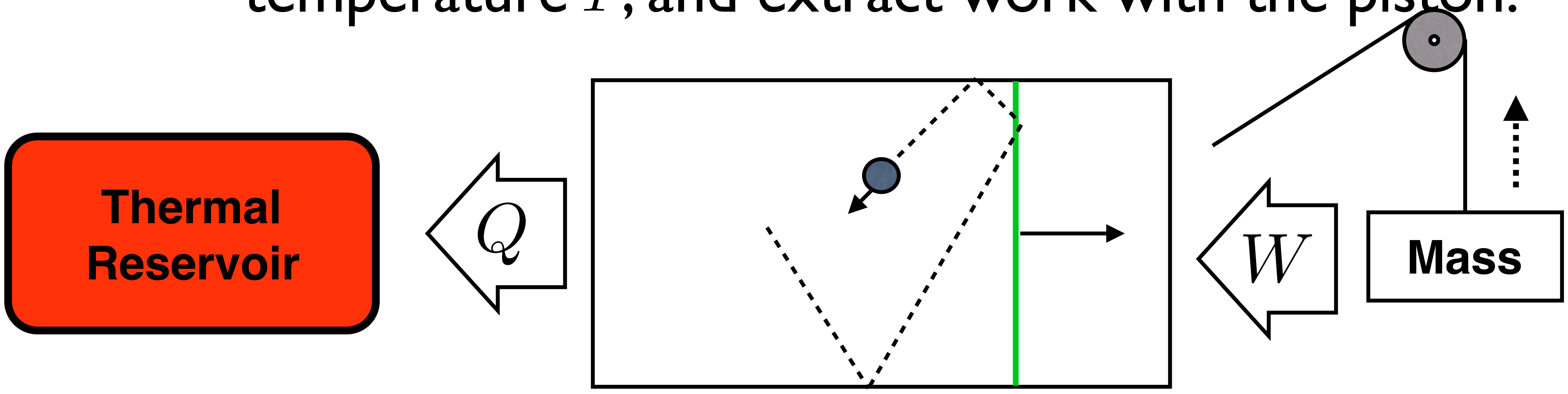
Control Example

Single particle in a box, insert barrier, slide barrier away from particle, while in contact with heat reservoir at temperature T



Control Example

Single particle in a box, insert barrier, slide barrier away from particle, while in contact with heat reservoir at temperature T, and extract work with the piston.



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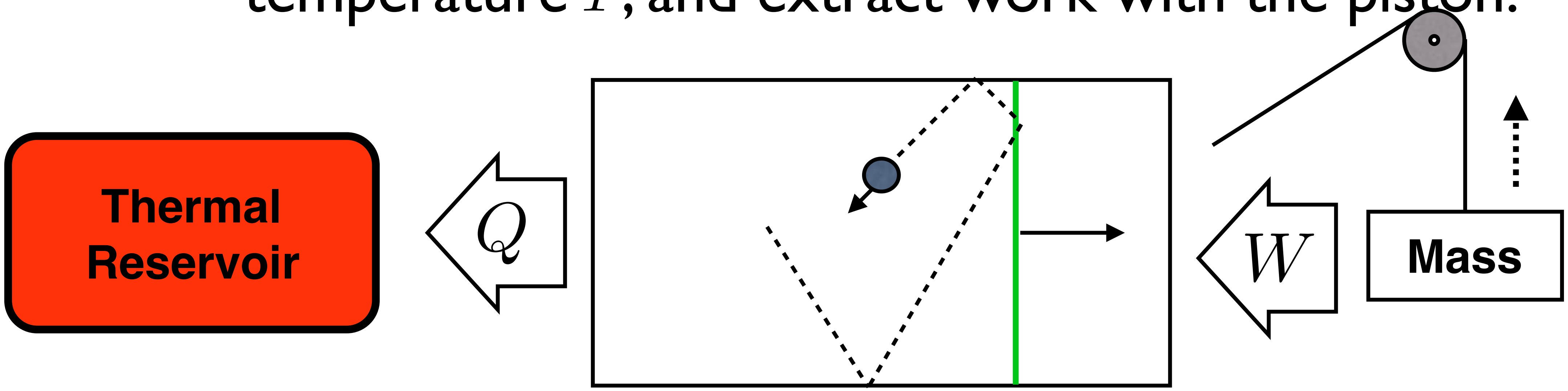


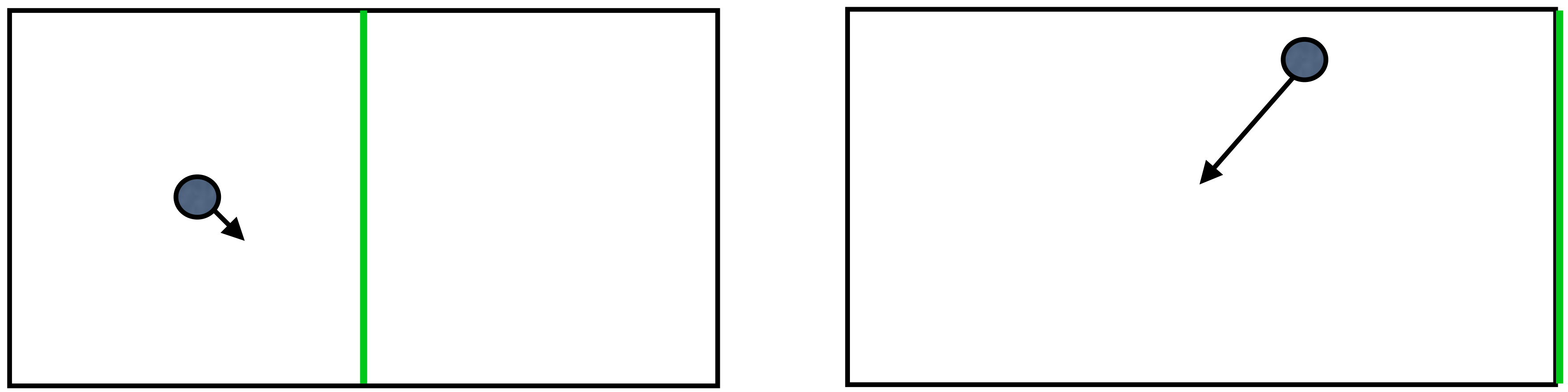
How much much work and heat flow?

$$\langle Q \rangle \geq -k_B T \ln 2 \, \Delta H[X]$$

$$\langle W \rangle \geq \langle \Delta E[X] \rangle - k_B T \ln 2 \, \Delta H[X]$$

Control Example

Initial vs. final energies and entropies:



Control Example

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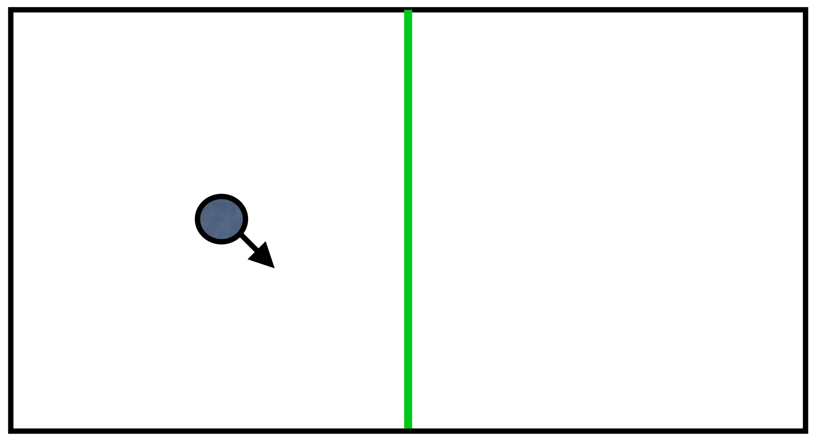 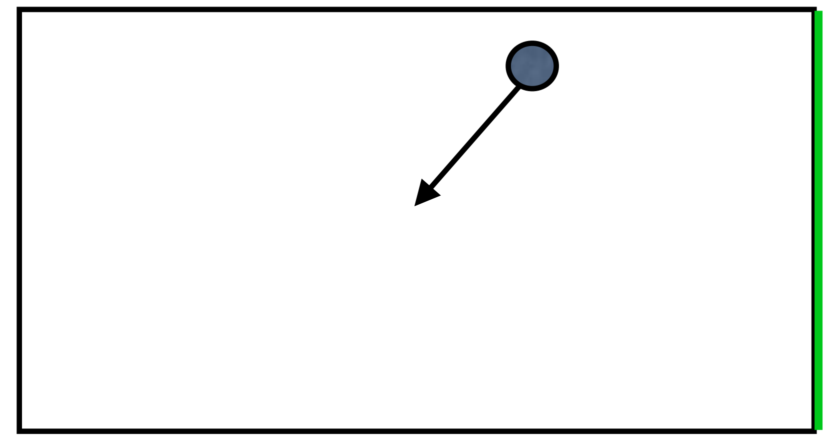

Energy of each position is the same, so average energy of positions is the same: $\langle E_{\vec{q}} \rangle_{\text{initial}} = \langle E_{\vec{q}} \rangle_{\text{final}}$

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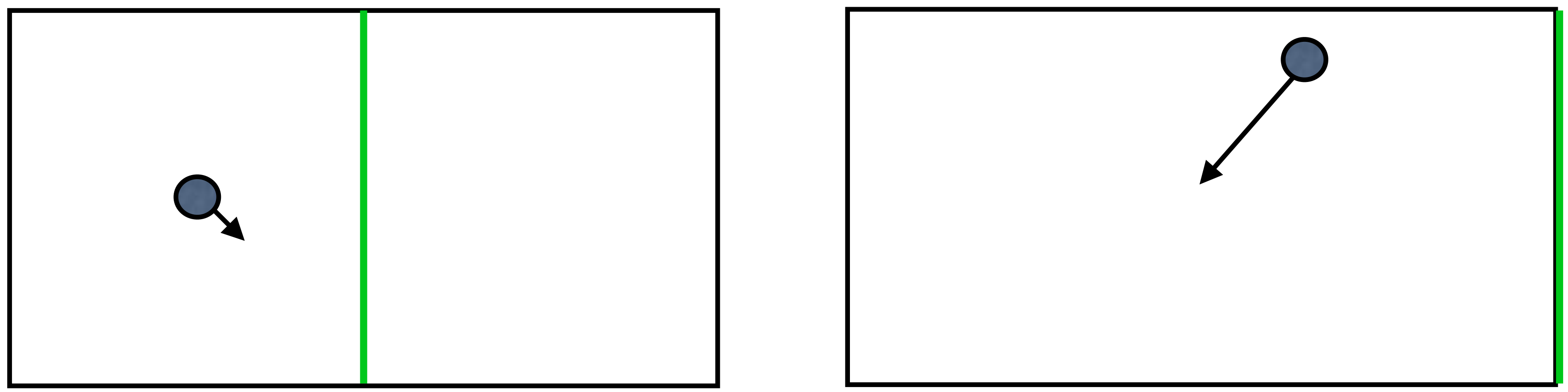

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If in equilibrium, average kinetic energy of momentum variable is the same:

$$\langle E_{\vec{p}} \rangle_{\text{initial}} = \langle E_{\vec{p}} \rangle_{\text{final}} = \frac{k_B T}{2} = \langle \text{Kinetic Energy} \rangle$$

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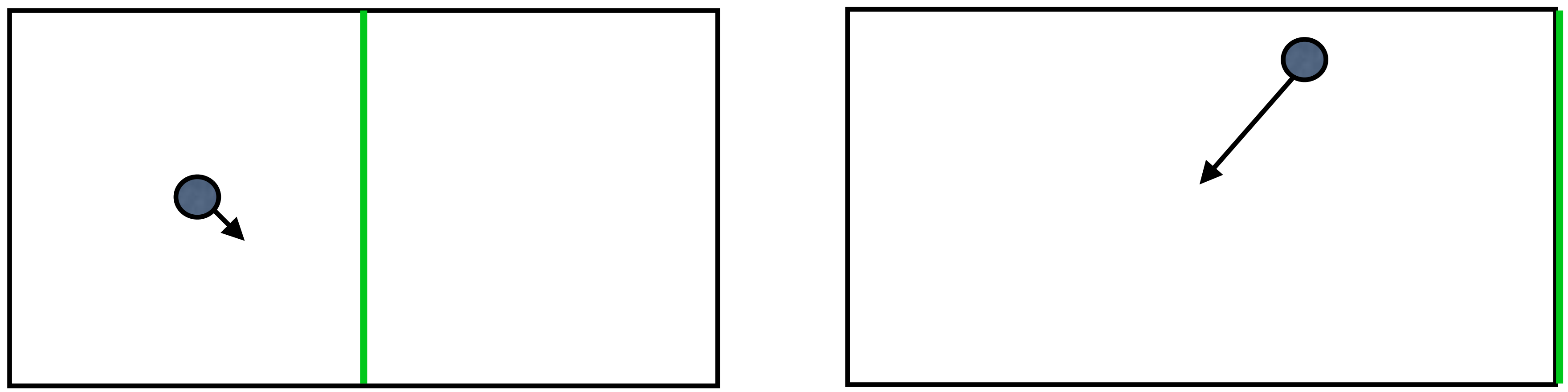

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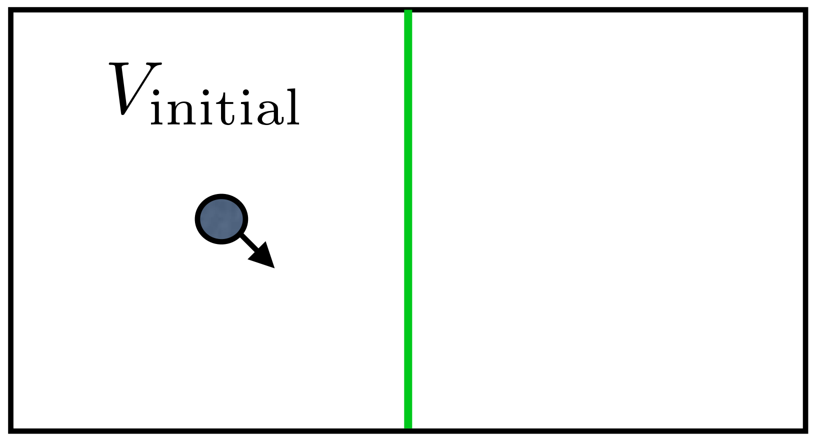
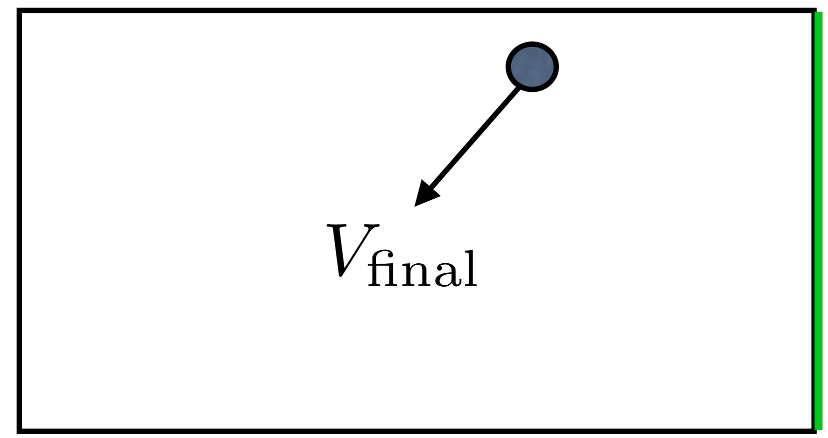
Average energy doesn't change: $\langle \Delta E[X] \rangle = 0$

Control Example

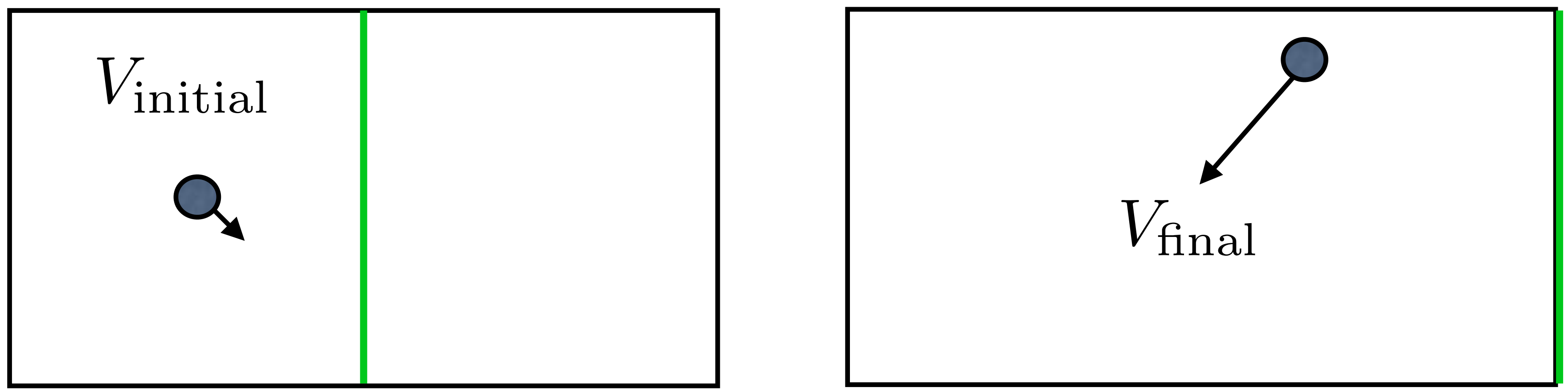
Initial vs. final energies and entropies:





Control Example

Initial vs. final energies and entropies:

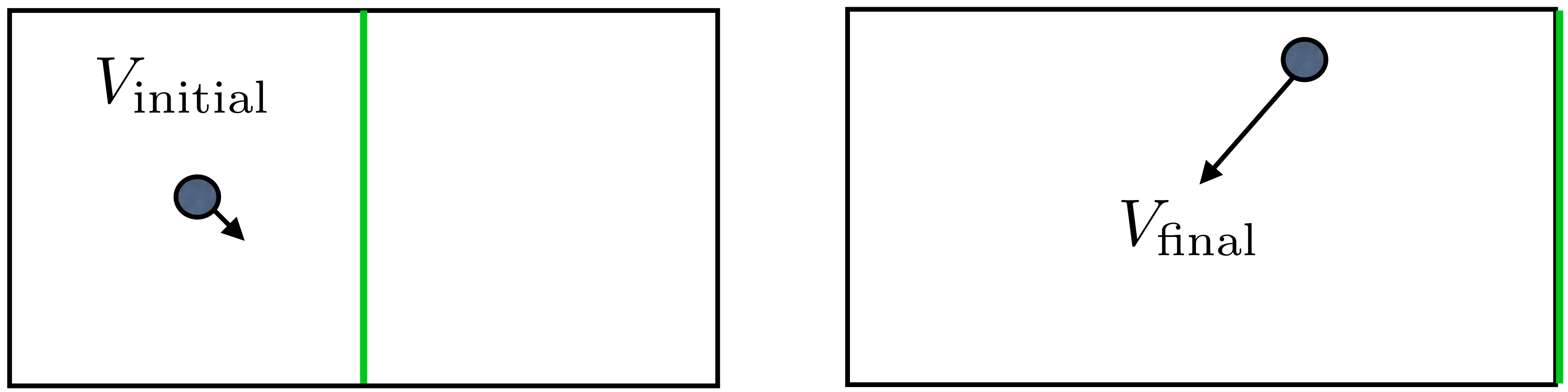


Probability of each position is uniform and inversely proportional to the available volume:

$$\Pr(\vec{Q}_{\text{initial}} = \vec{q}) = \frac{c}{V_{\text{initial}}} \qquad \Pr(\vec{Q}_{\text{final}} = \vec{q}) = \frac{c}{V_{\text{final}}}$$

Control Example

Initial vs. final energies and entropies:



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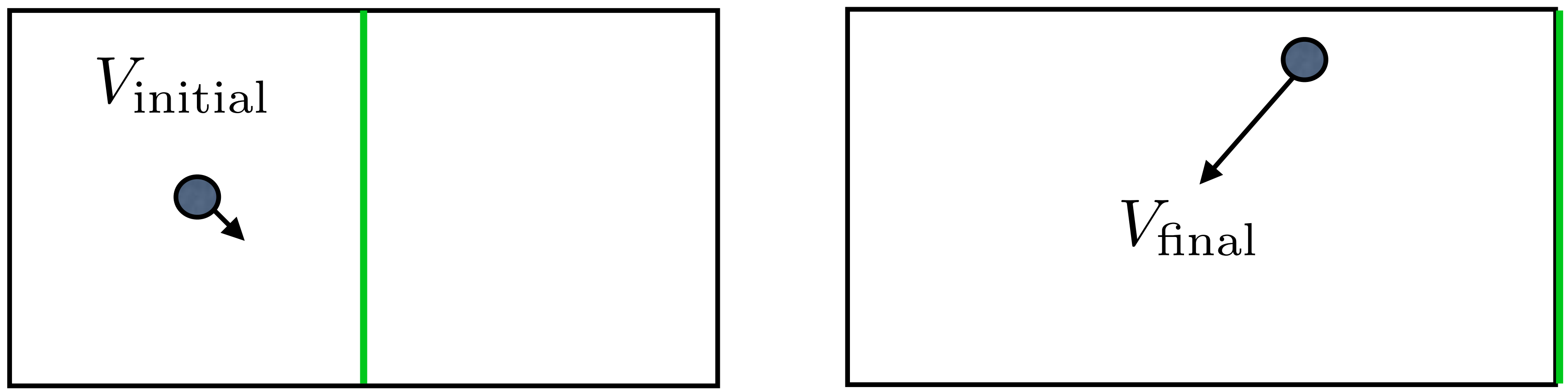
At equilibrium, the probability of momentum is the same initially and finally

$$\Pr(\vec{P}_{\text{initial}} = \vec{p}) = \Pr(\vec{P}_{\text{final}} = \vec{p}) = \Pr(\vec{P}^{\text{eq}} = \vec{p})$$

Control Example

Initial vs. final energies and entropies:



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$$\Pr(\vec{Q}_{\mathrm{initial}} = \vec{q}) = \frac{c}{V_{\mathrm{initial}}} \qquad \Pr(\vec{Q}_{\mathrm{final}} = \vec{q}) = \frac{c}{V_{\mathrm{final}}}$$

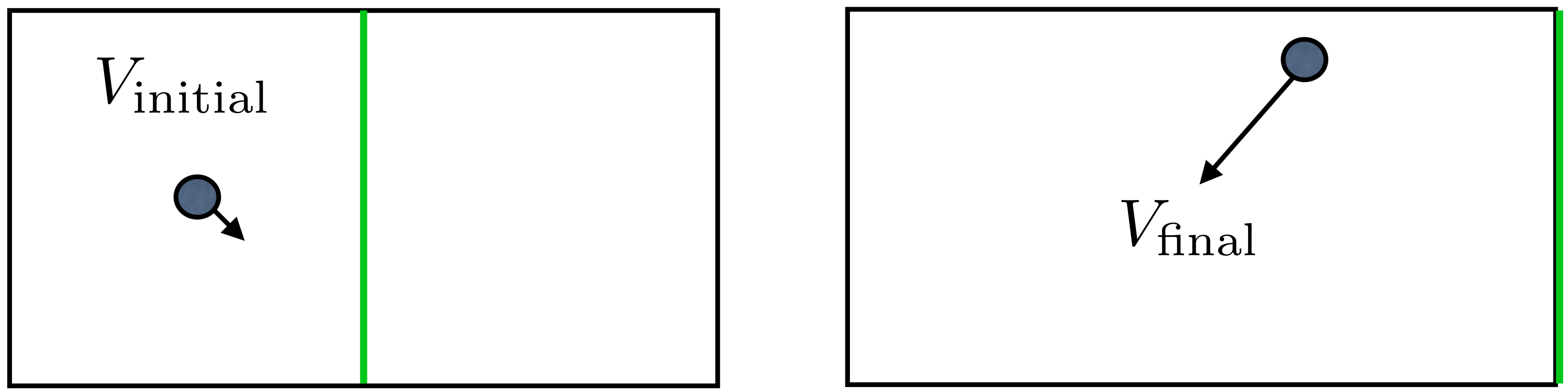
At equilibrium, the probability of momentum is the same initially and finally, and independent of position distribution.

$$\Pr(\vec{P}_{\mathrm{initial}} = \vec{p}) = \Pr(\vec{P}_{\mathrm{final}} = \vec{p}) = \Pr(\vec{P}^{\mathrm{eq}} = \vec{p})$$

$$\Pr(X_{\mathrm{initial}} = (\vec{q}, \vec{p})) = \frac{c\,\Pr(\vec{P}_{\mathrm{initial}} = \vec{p})}{V_{\mathrm{initial}}} \quad \Pr(X_{\mathrm{final}} = (\vec{q}, \vec{p})) = \frac{c\,\Pr(\vec{P}_{\mathrm{final}} = \vec{p})}{V_{\mathrm{final}}}$$

Control Example

Initial vs. final energies and entropies:

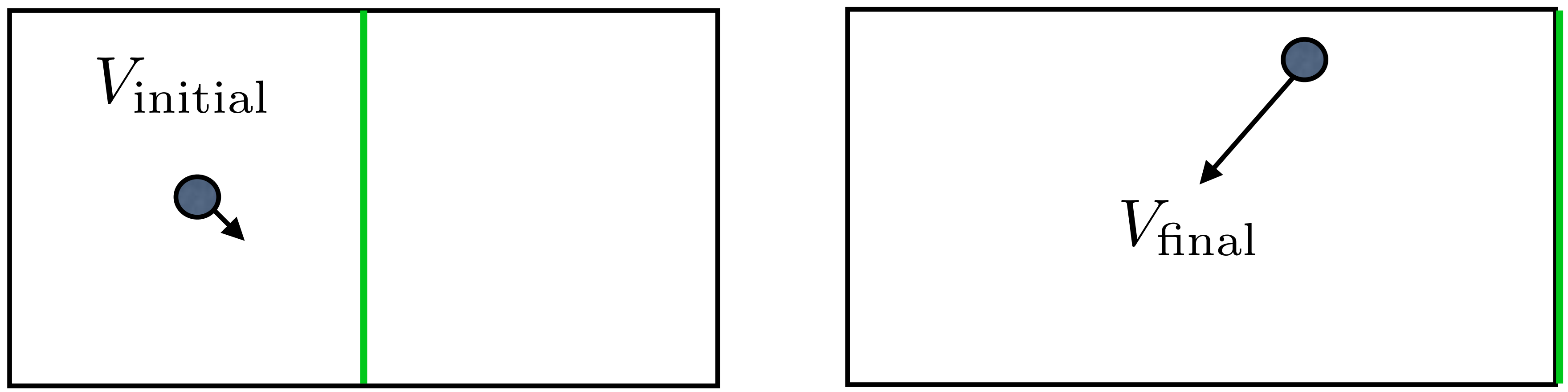


Thus, the change in Shannon entropy is

$$\Delta H[X] = H[X_{\text{final}}] - H[X_{\text{initial}}]$$

Control Example

Initial vs. final energies and entropies:



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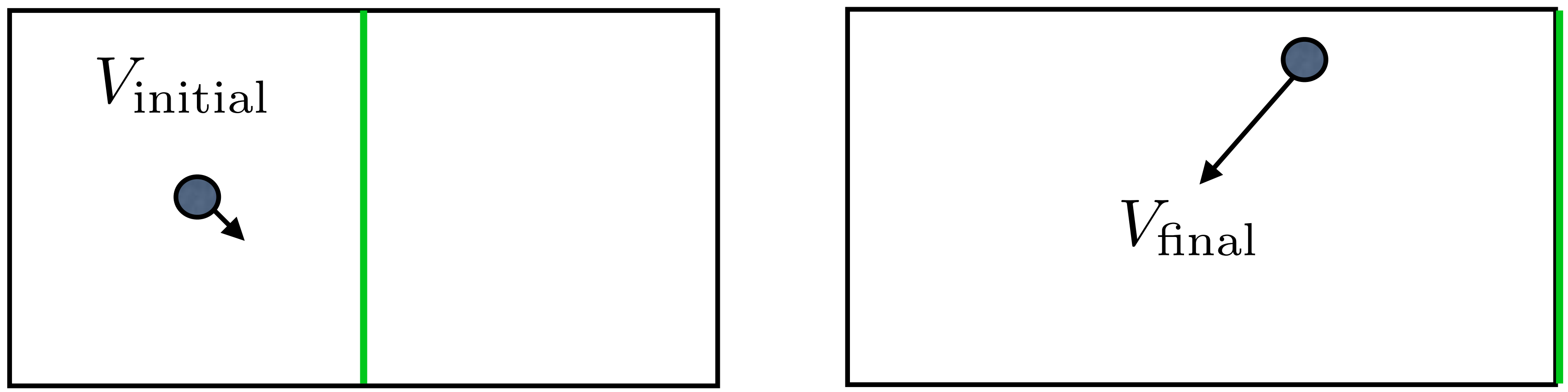
$$= H[\vec{P}^{\mathrm{eq}}] - \sum_{x \in V_{\mathrm{final}}} \frac{c}{V_{\mathrm{final}}} \log_2 \frac{c}{V_{\mathrm{final}}} - H[\vec{P}^{\mathrm{eq}}] + \sum_{x \in V_{\mathrm{initial}}} \frac{c}{V_{\mathrm{initial}}} \log_2 \frac{c}{V_{\mathrm{initial}}}$$

Control Example

Initial vs. final energies and entropies:



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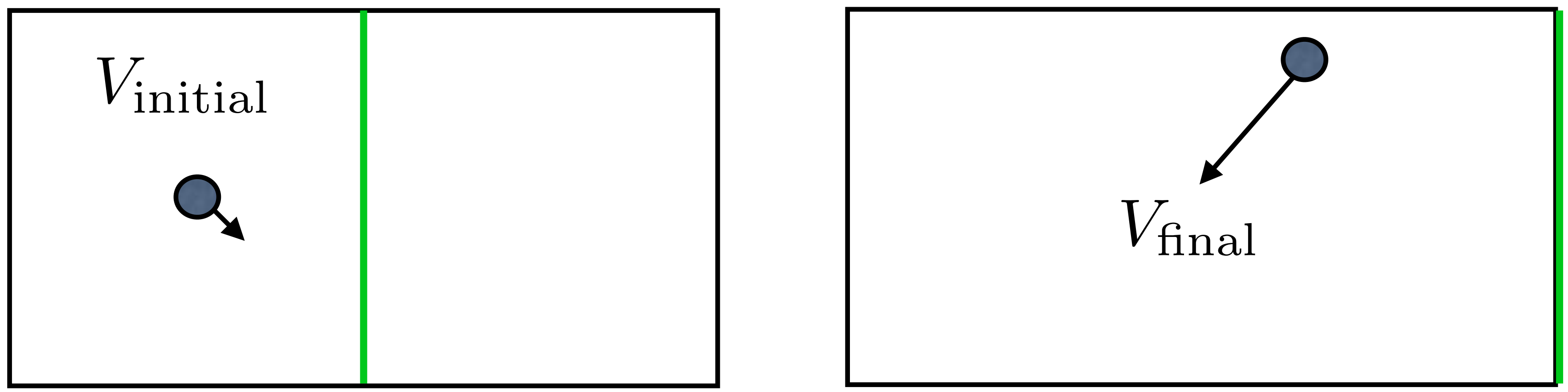
&= \log_2 \frac{V_{\text{final}}}{V_{\text{initial}}}
\end{aligned}$$

Control Example

Initial vs. final energies and entropies:



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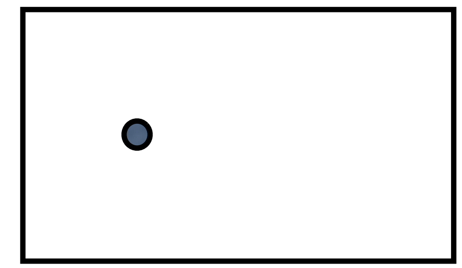
And the work and heat are bounded by the log ratio of the volumes (achievable if done very slowly):

$$\langle W \rangle \geq -k_B T \ln 2 \, \log_2 \frac{V_{\text{final}}}{V_{\text{initial}}} \qquad \langle Q \rangle \geq -k_B T \ln 2 \, \log_2 \frac{V_{\text{final}}}{V_{\text{initial}}}$$

Szilard's Information Engine

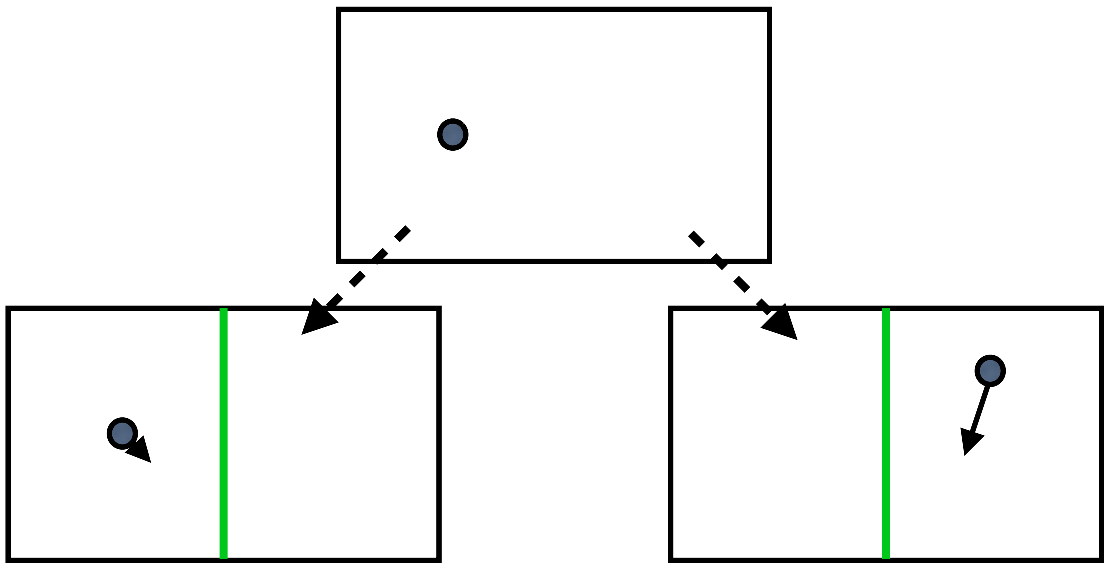
Take a particle in a box:



Szilard's Information Engine

Take a particle in a box:

Insert a barrier at halfway:

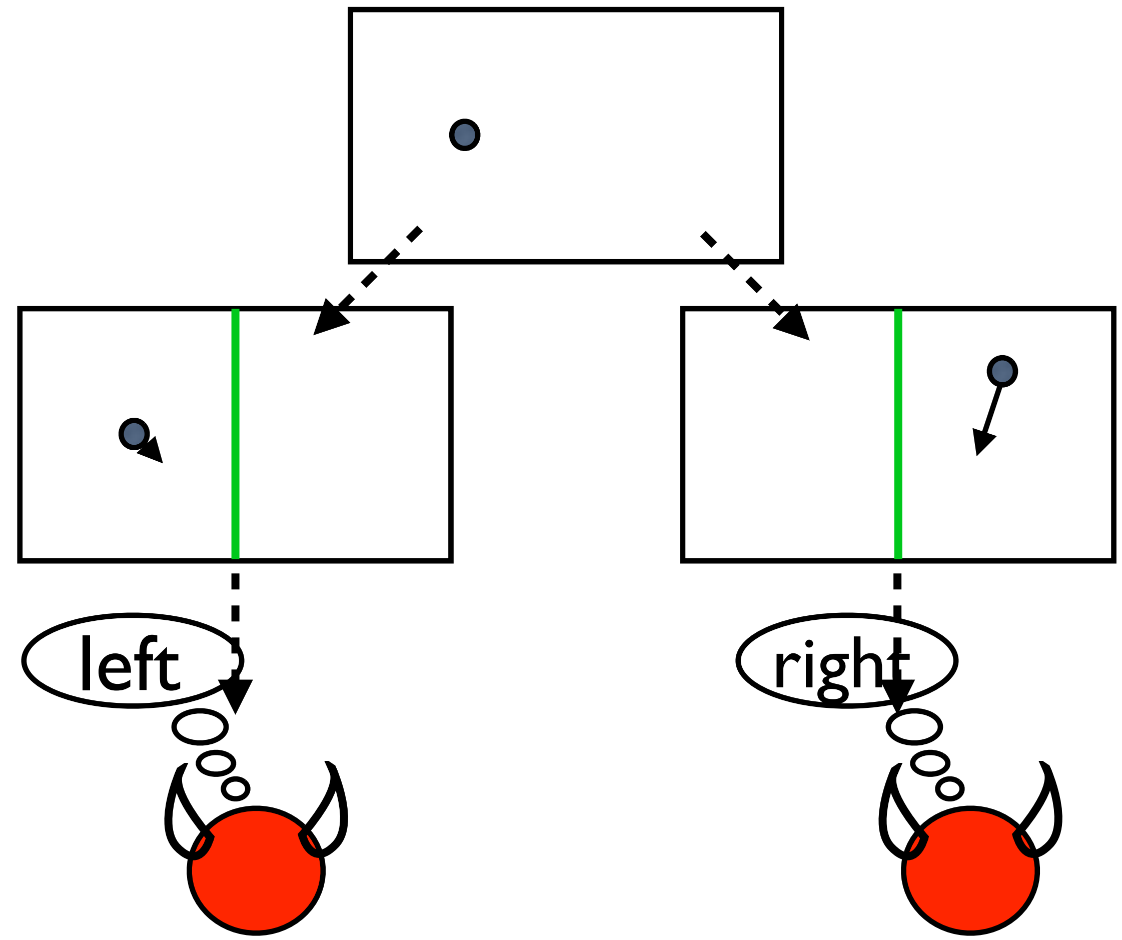


Szilard's Information Engine

Take a particle in a box:

Insert a barrier at halfway:

Measure which side the particle lands on:



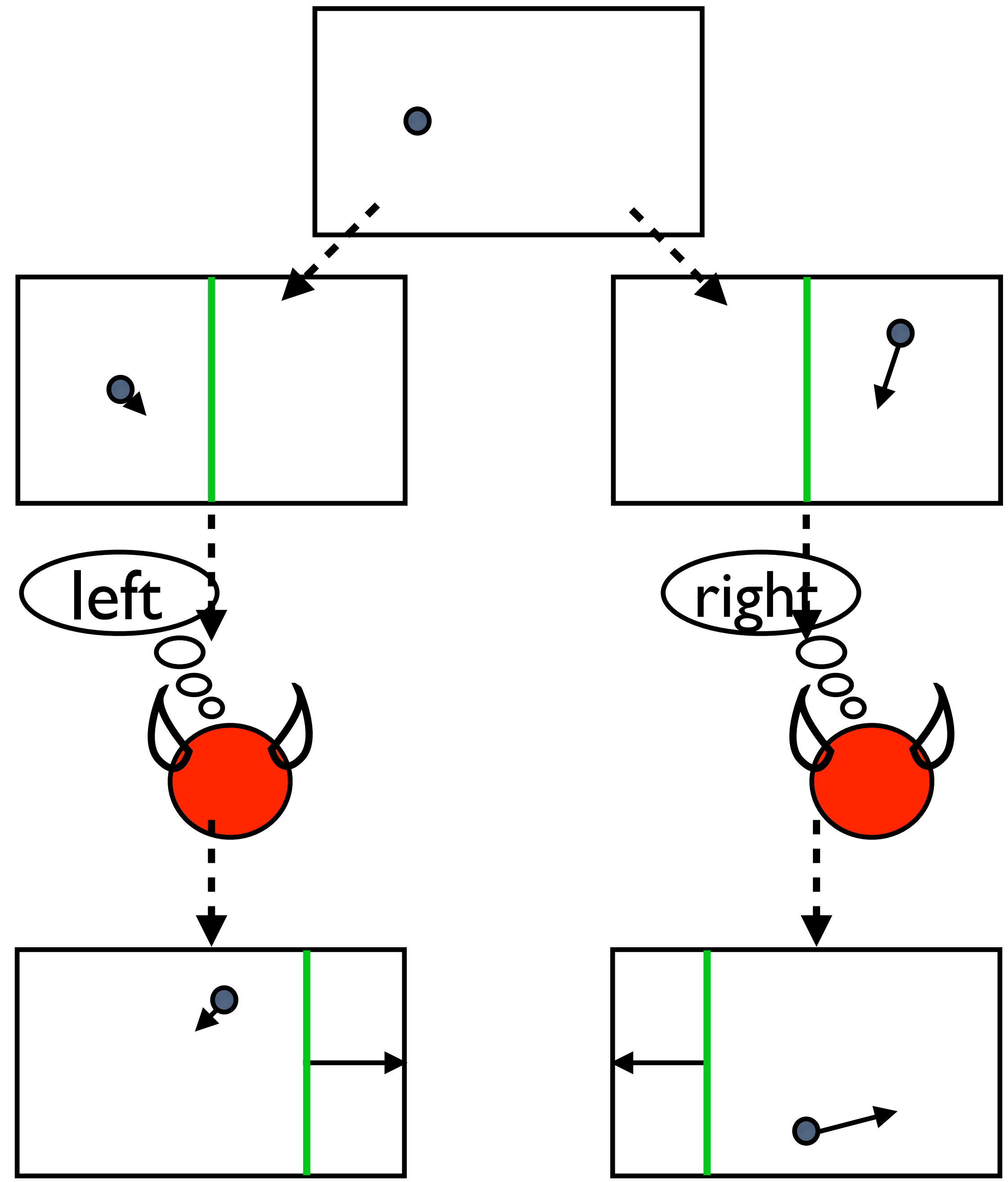
Szilard's Information Engine

Take a particle in a box:

Insert a barrier at halfway:

Measure which side the particle lands on:

Use measurement to slowly slide barrier away from particle, extracting work:



Szilard's Information Engine

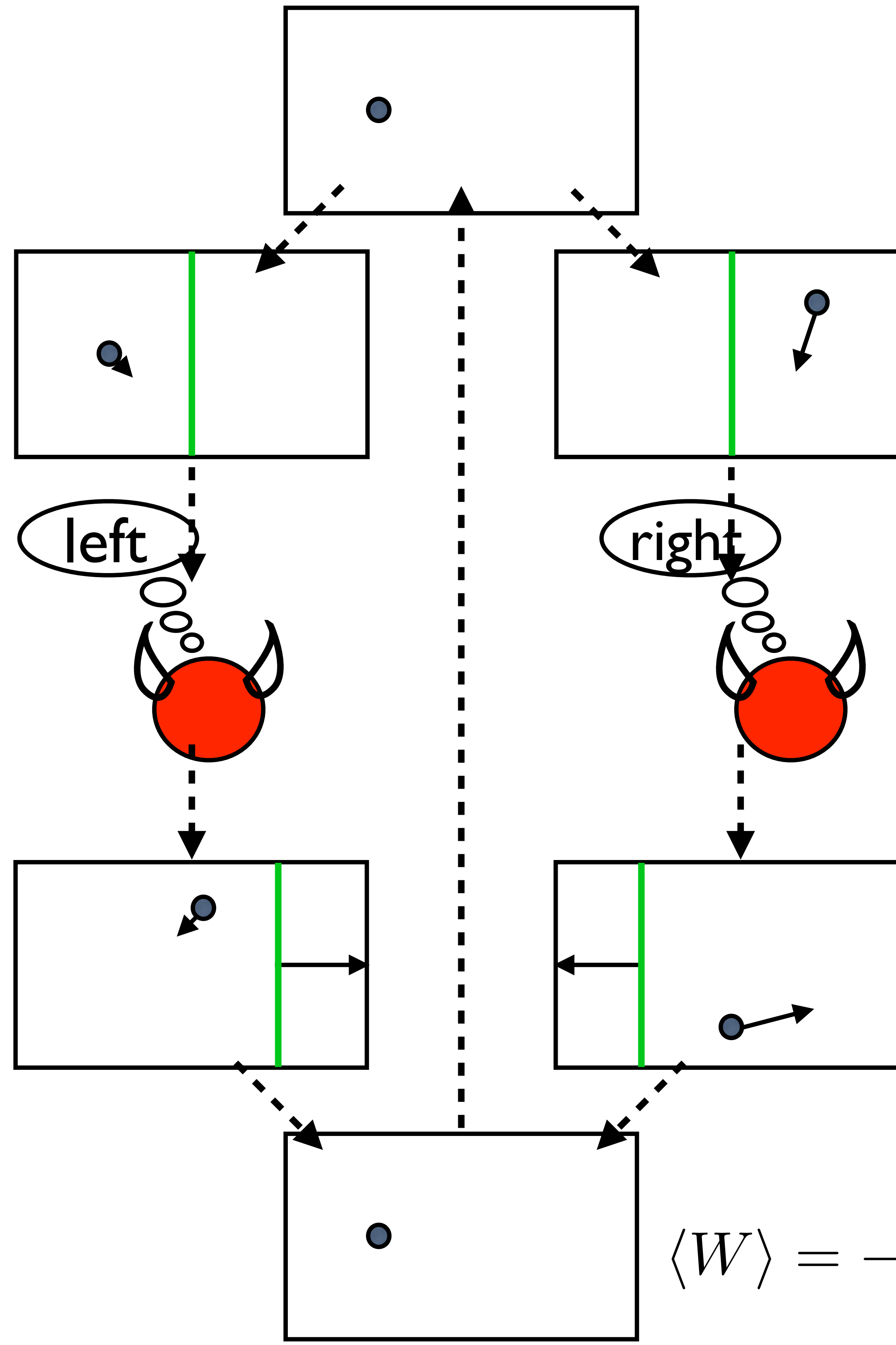
Take a particle in a box:

Insert a barrier at halfway:

Measure which side the particle lands on:

Use measurement to slowly slide barrier away from particle, extracting work:

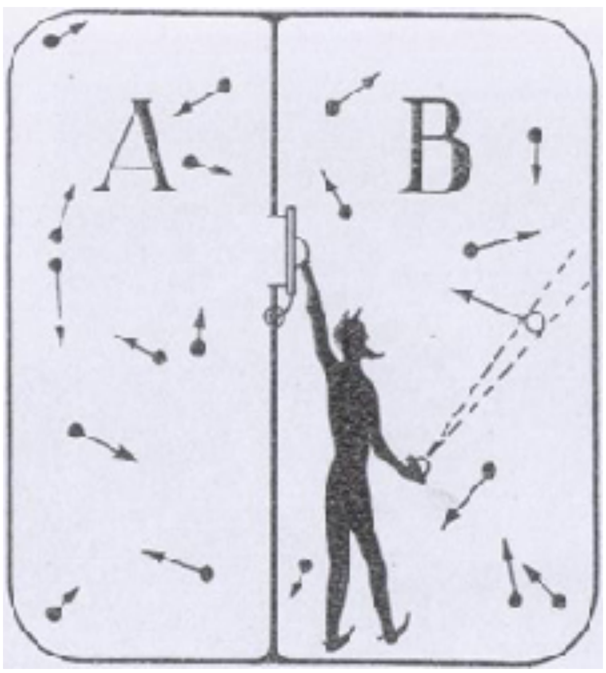
Store work, repeat:



$$\langle W \rangle = -k_B T \ln 2$$

Maxwell’s Demon

Maxwell’s Demon



http://www.eoht.info/page/Maxwell's+demon

Maxwell's Demon

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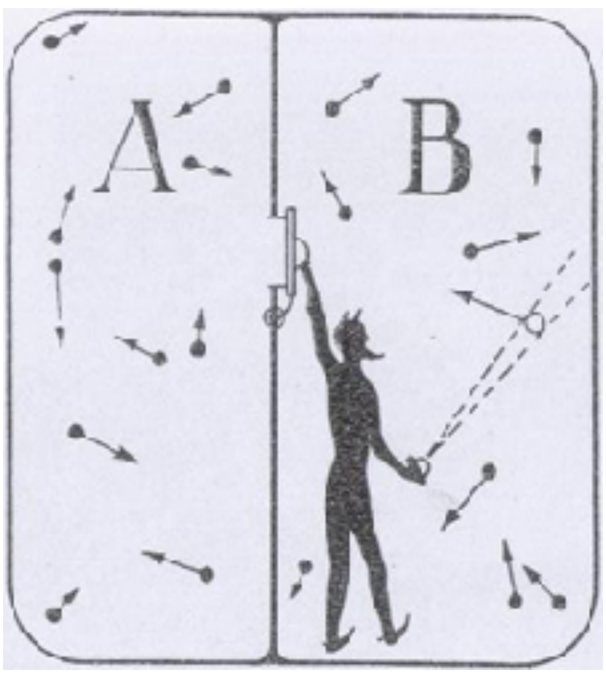

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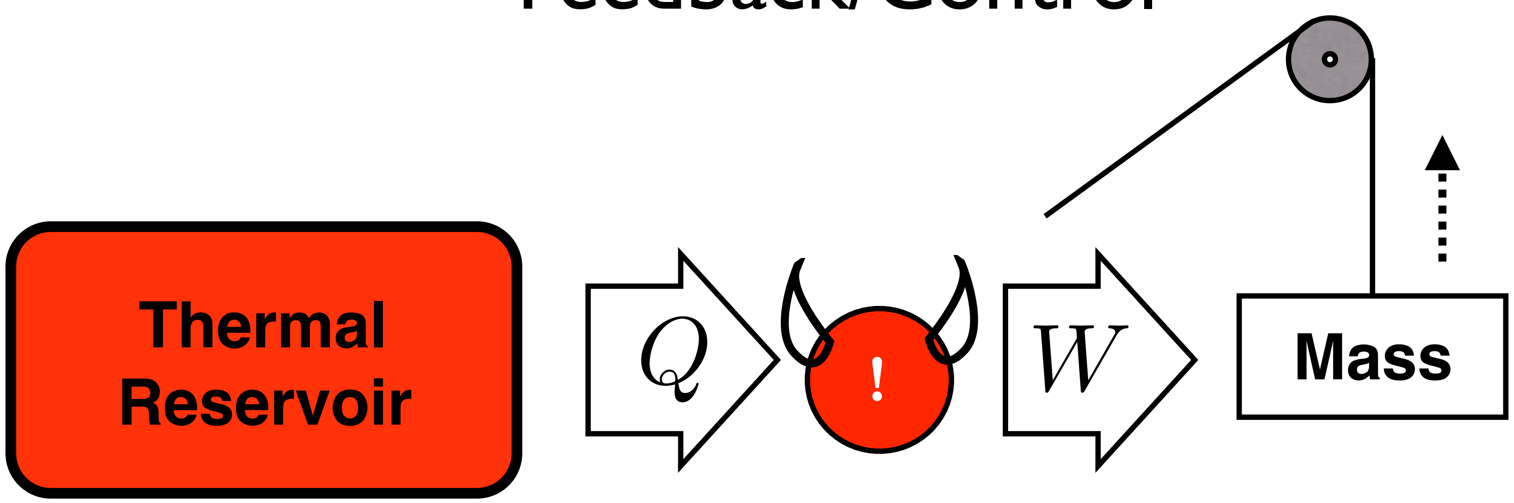
Feedback/Control



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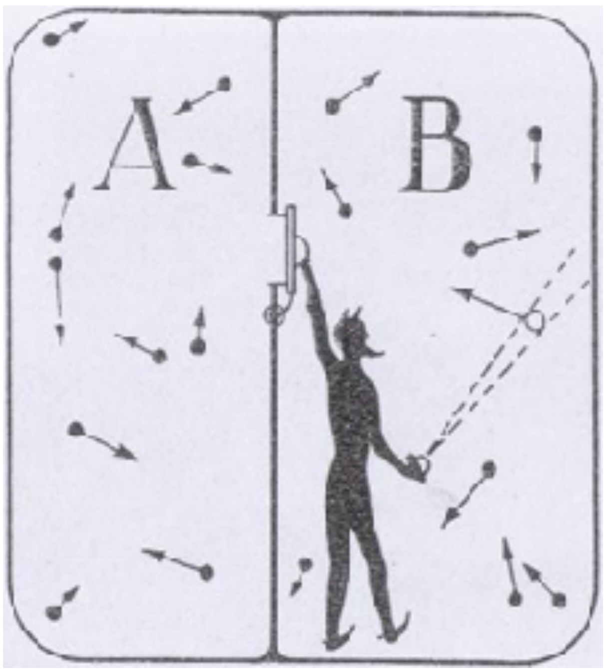

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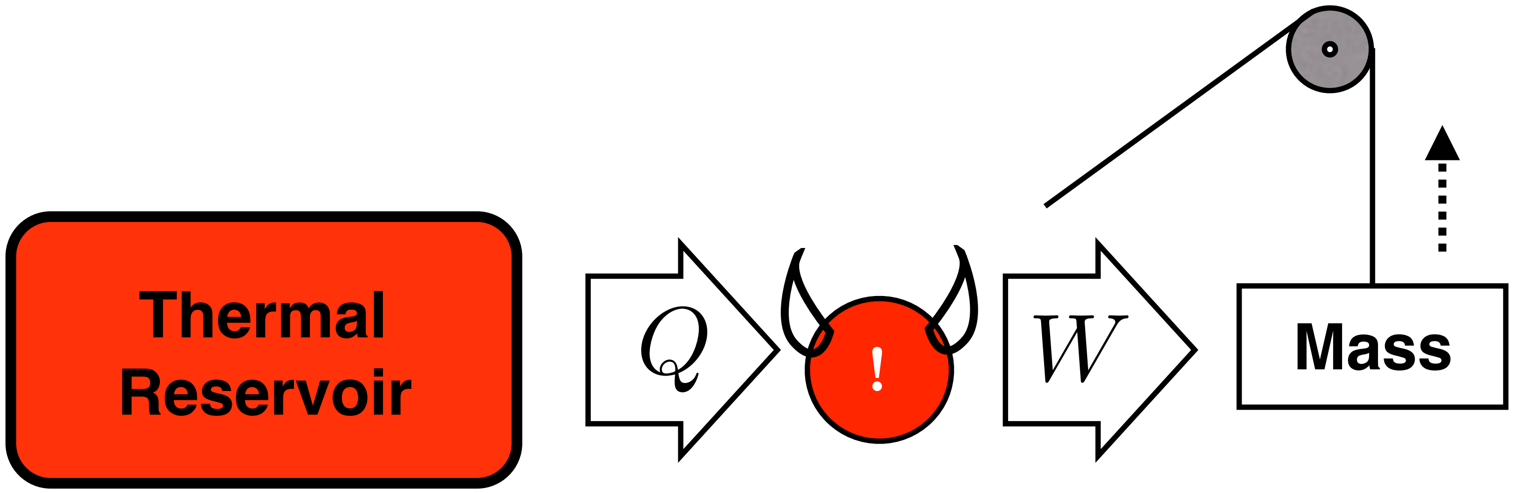

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Maxwell's Demon

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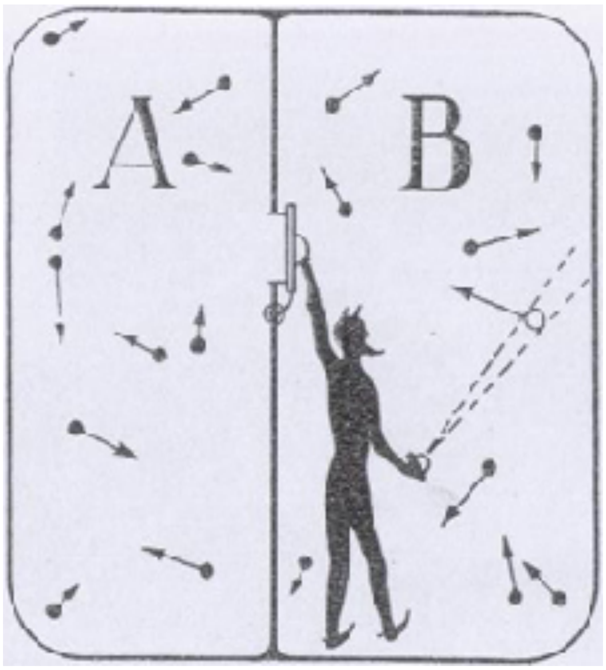

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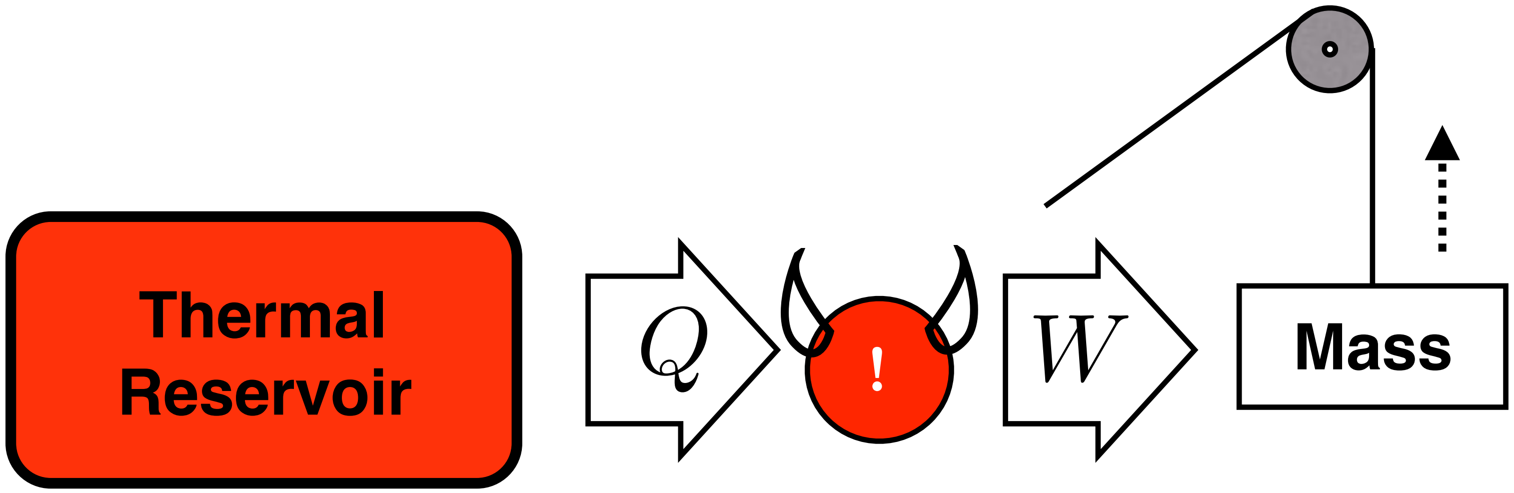

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Maxwell’s Demon

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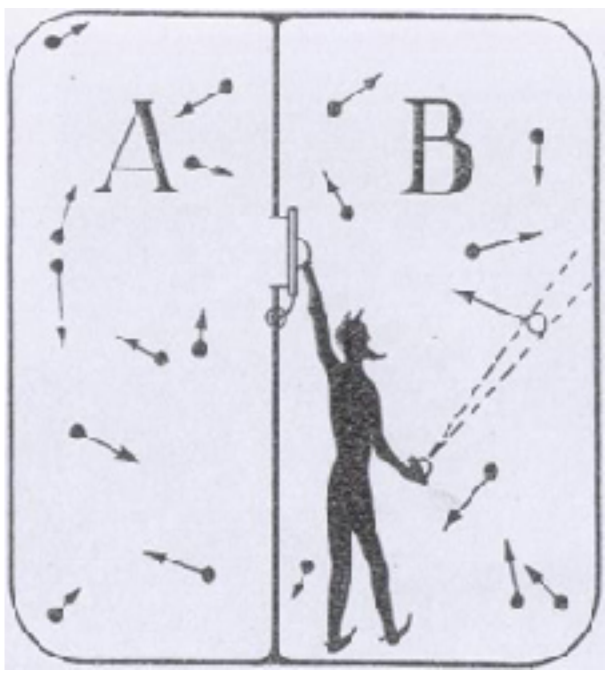

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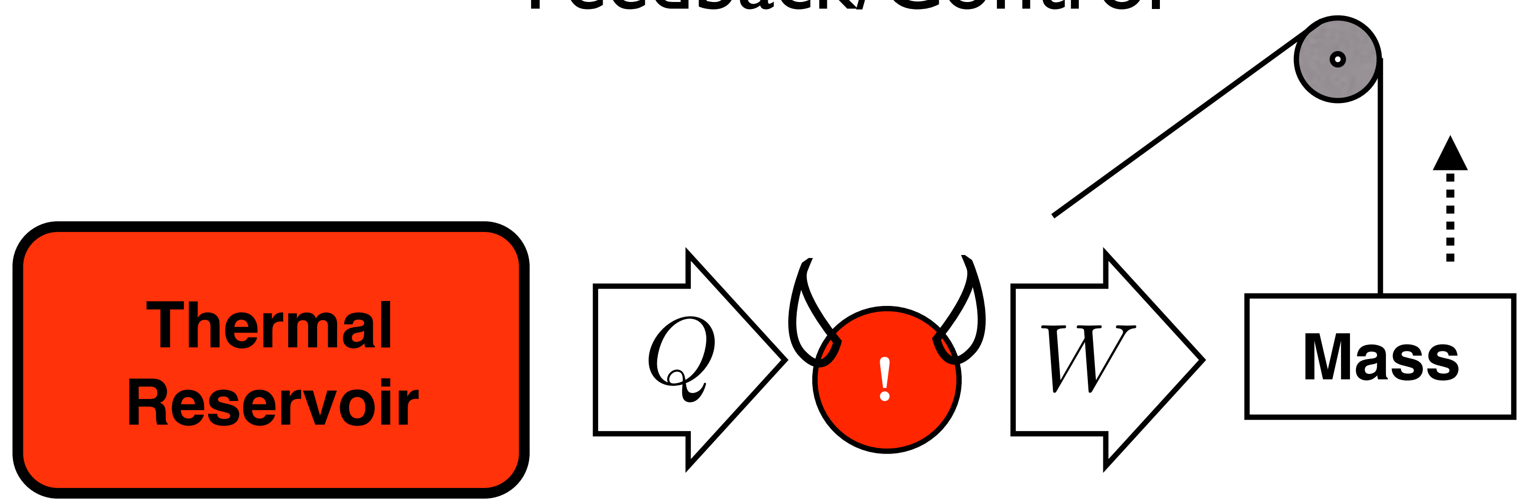

$$\langle W \rangle = \langle Q \rangle = -k_B T \ln 2$$

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$$\Delta S_{\text{total}} = \frac{\langle Q \rangle}{T} + k_B \ln 2 \, \Delta H[X]$$

Maxwell’s Demon

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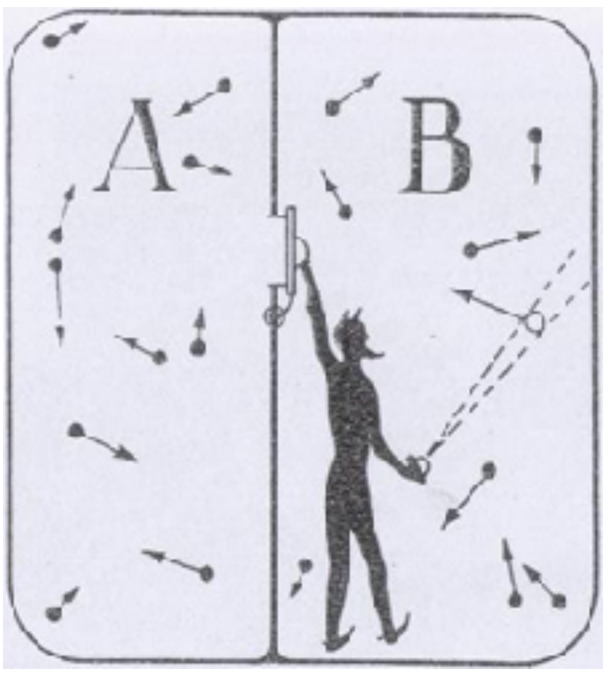

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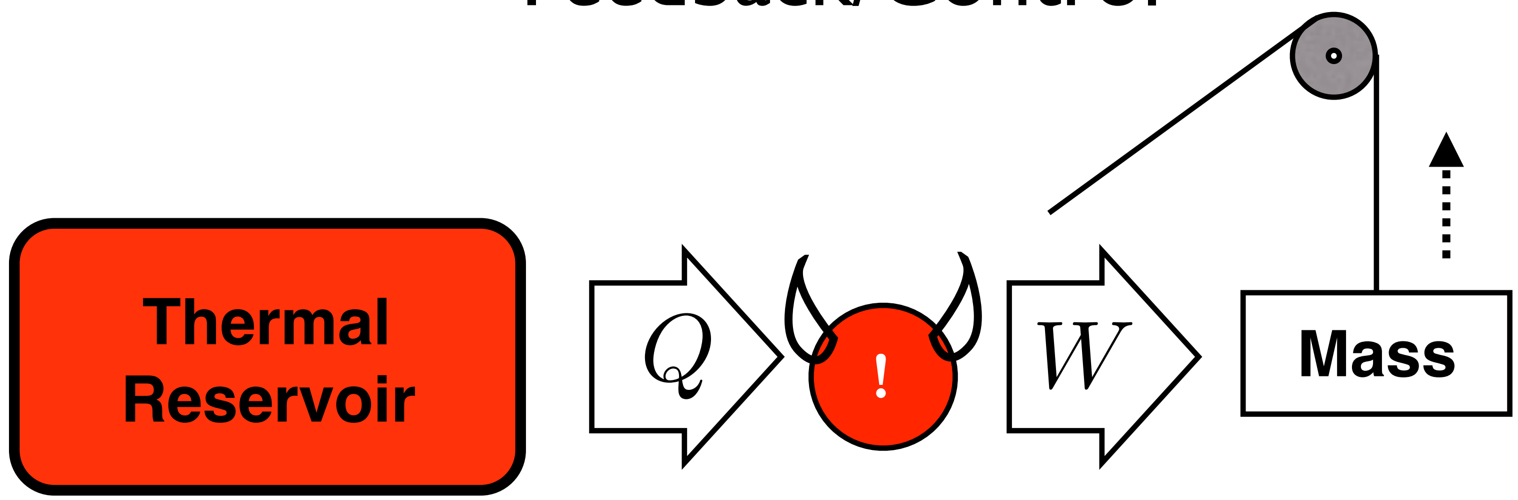

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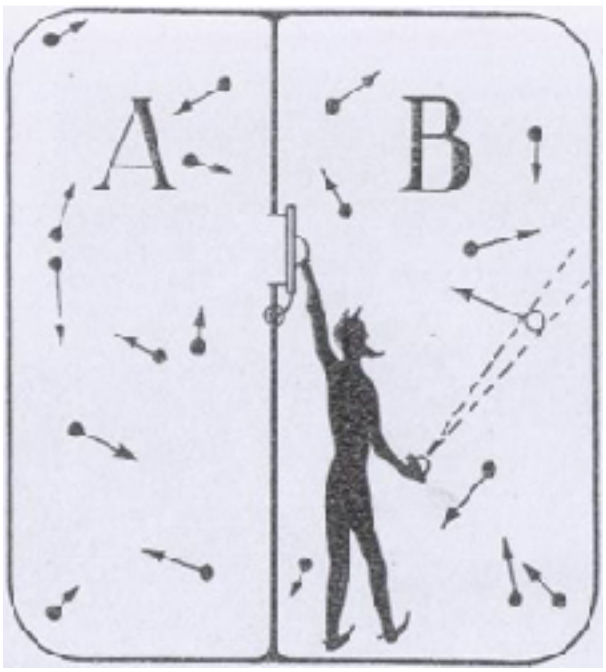
$$= -k_B \ln 2 < 0$$

Maxwell's Demon

Maxwell's Demon



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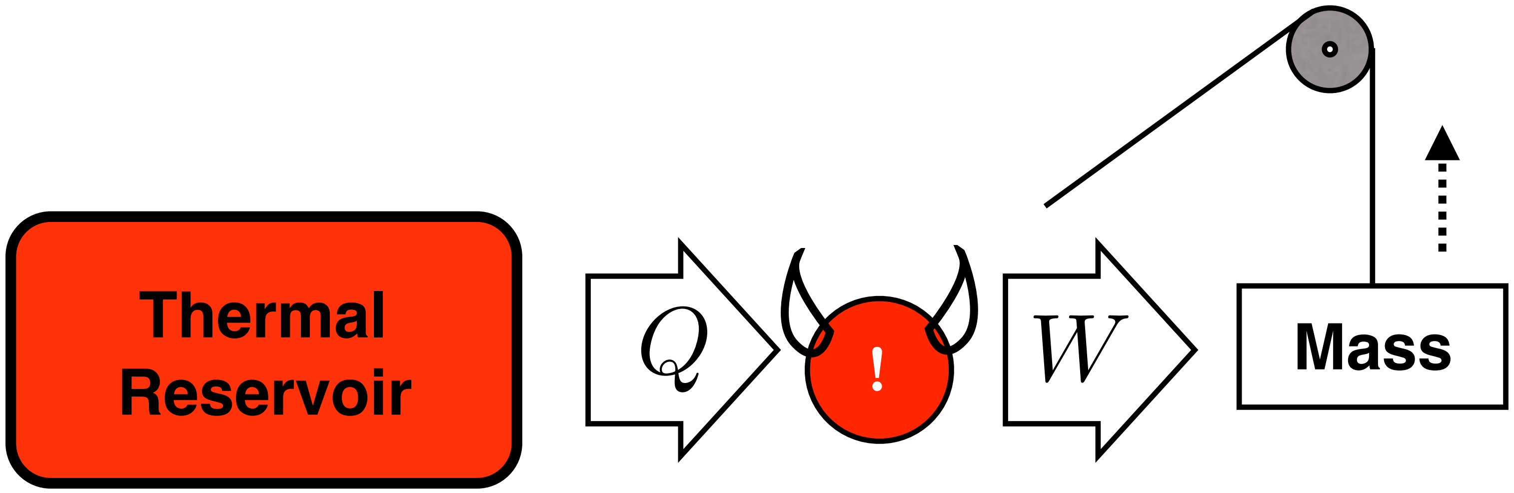

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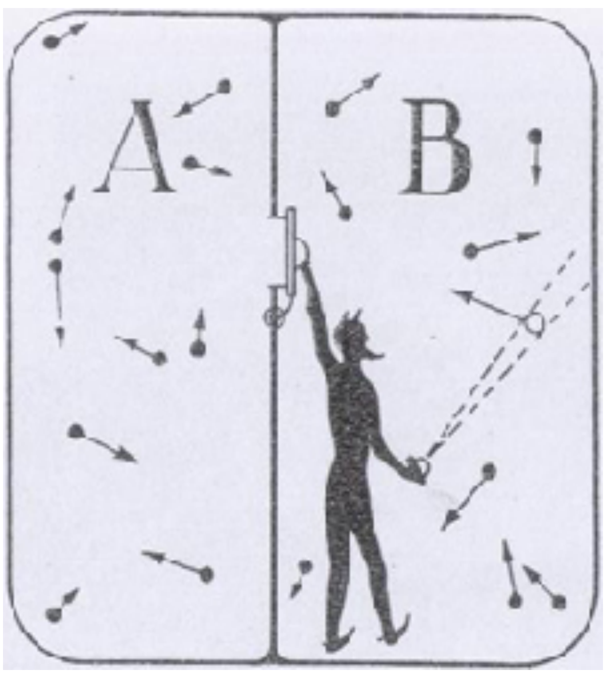
Maxwell's Demon

Maxwell's Demon



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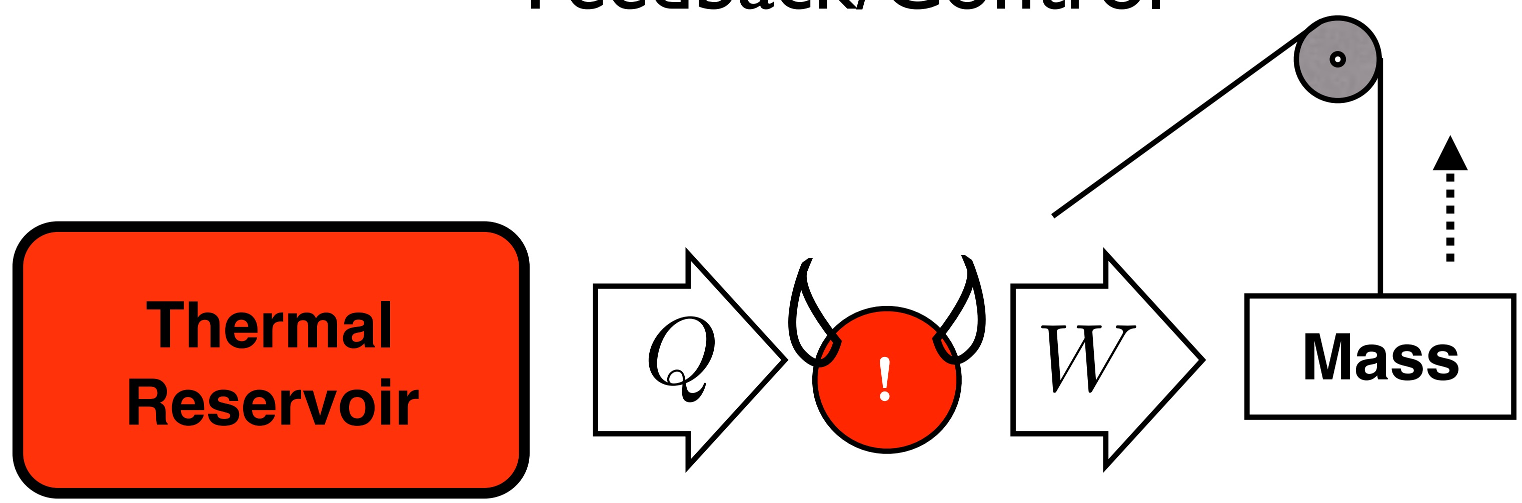
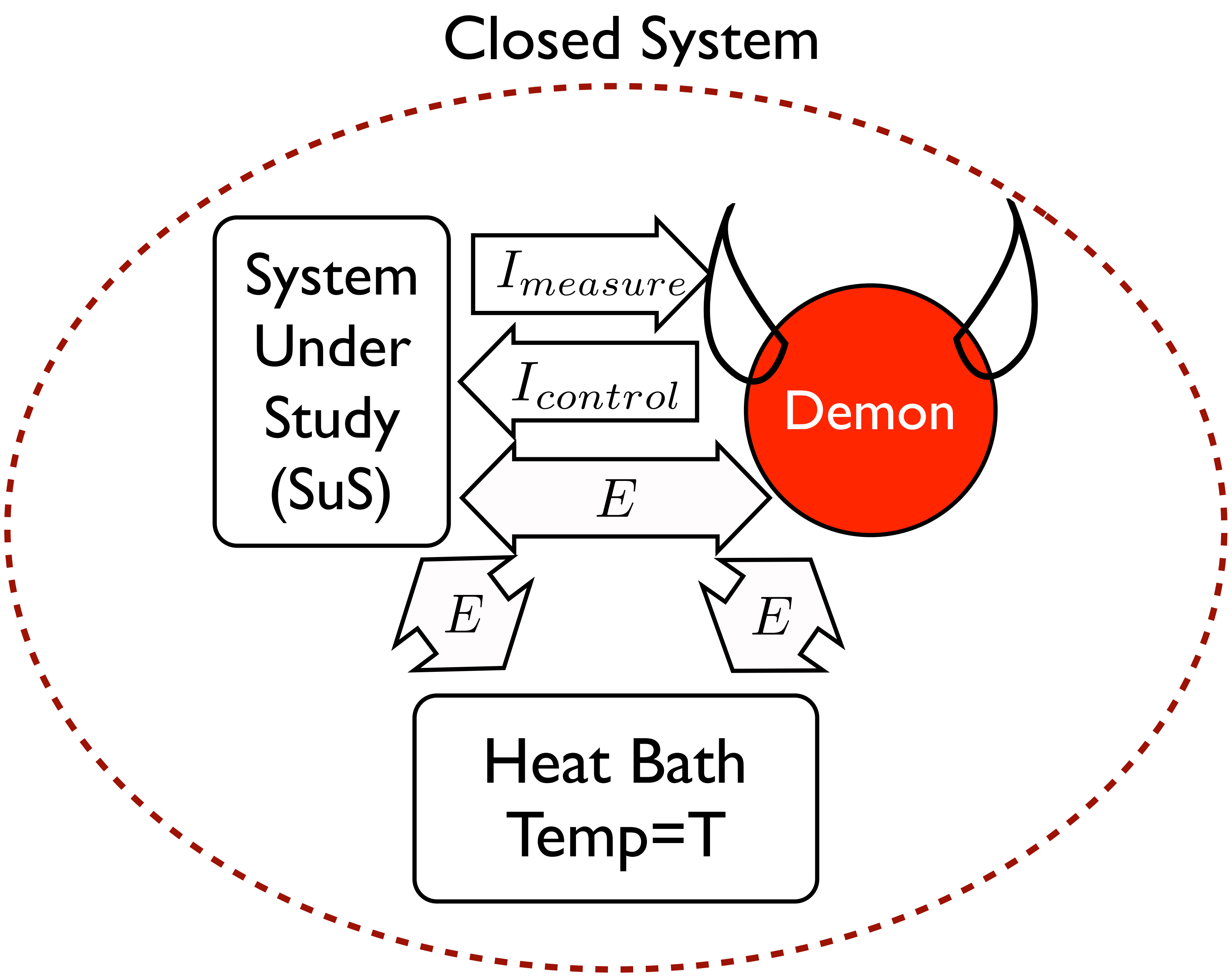
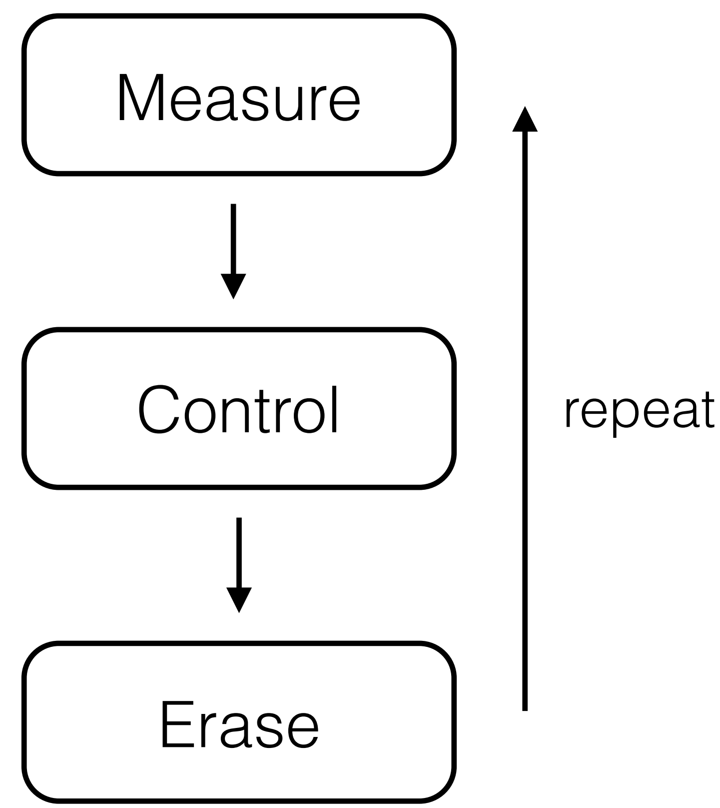


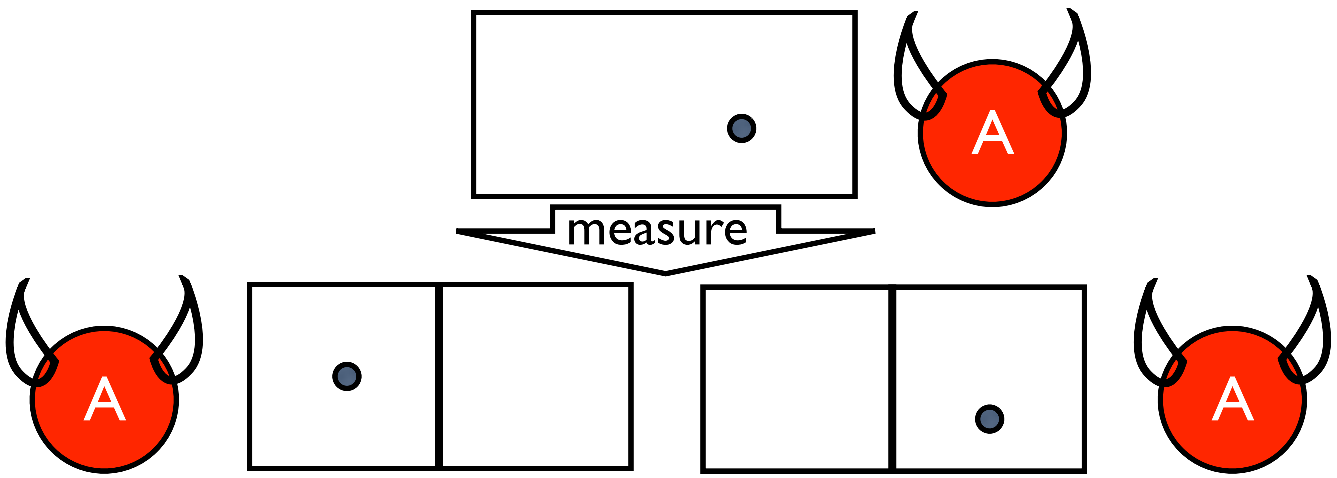
$$\Delta S_{\text{total}} < 0?$$

Maxwellian Demons

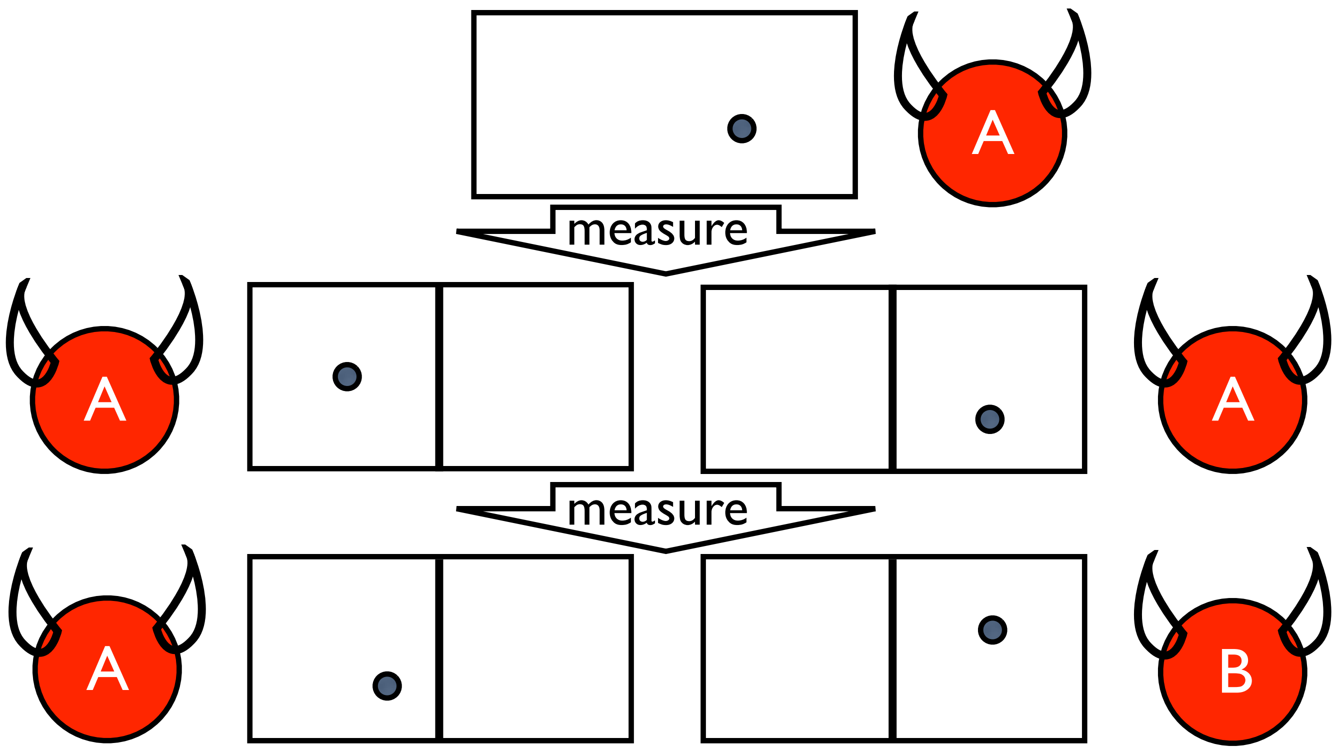




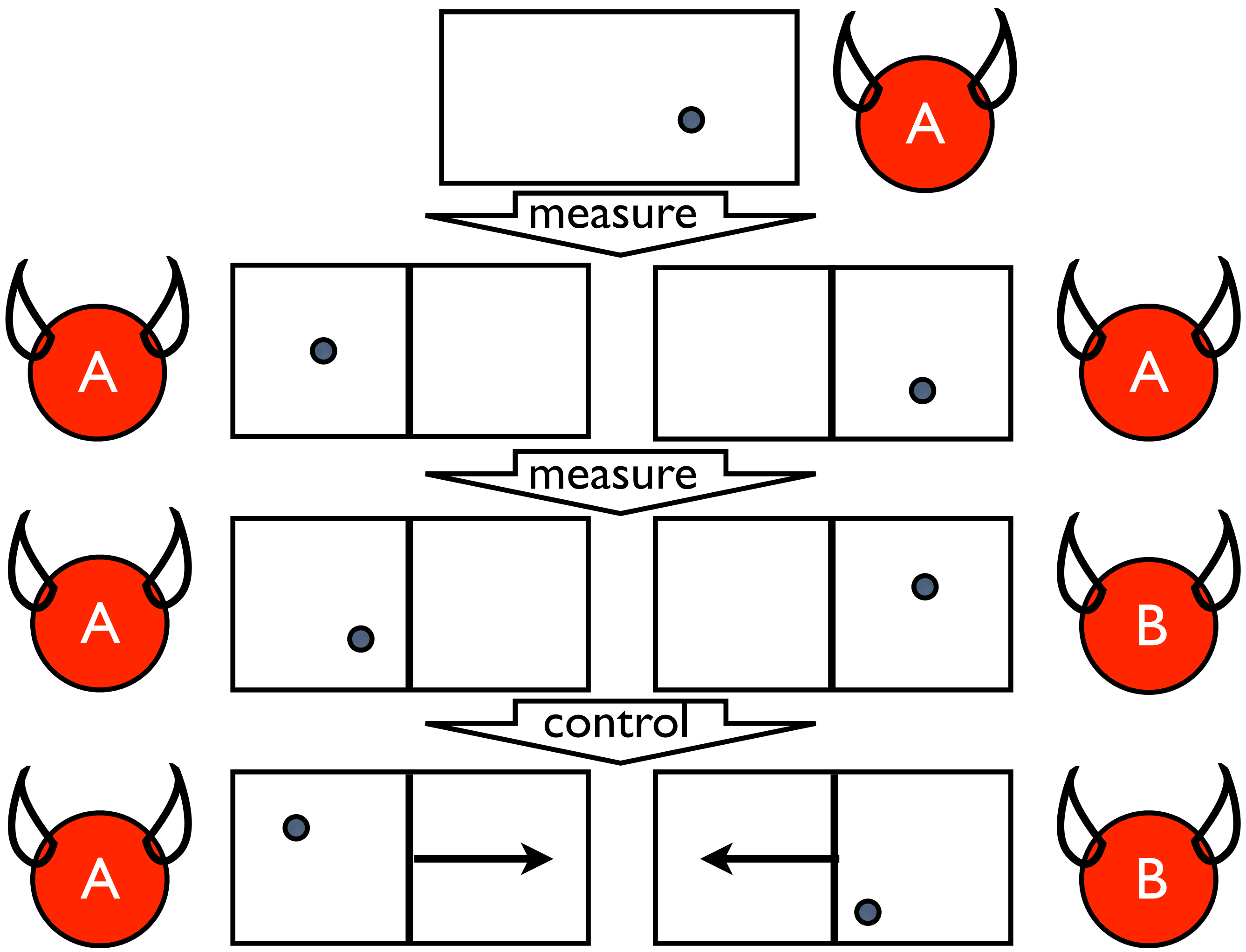
Szilard Engine Cycle



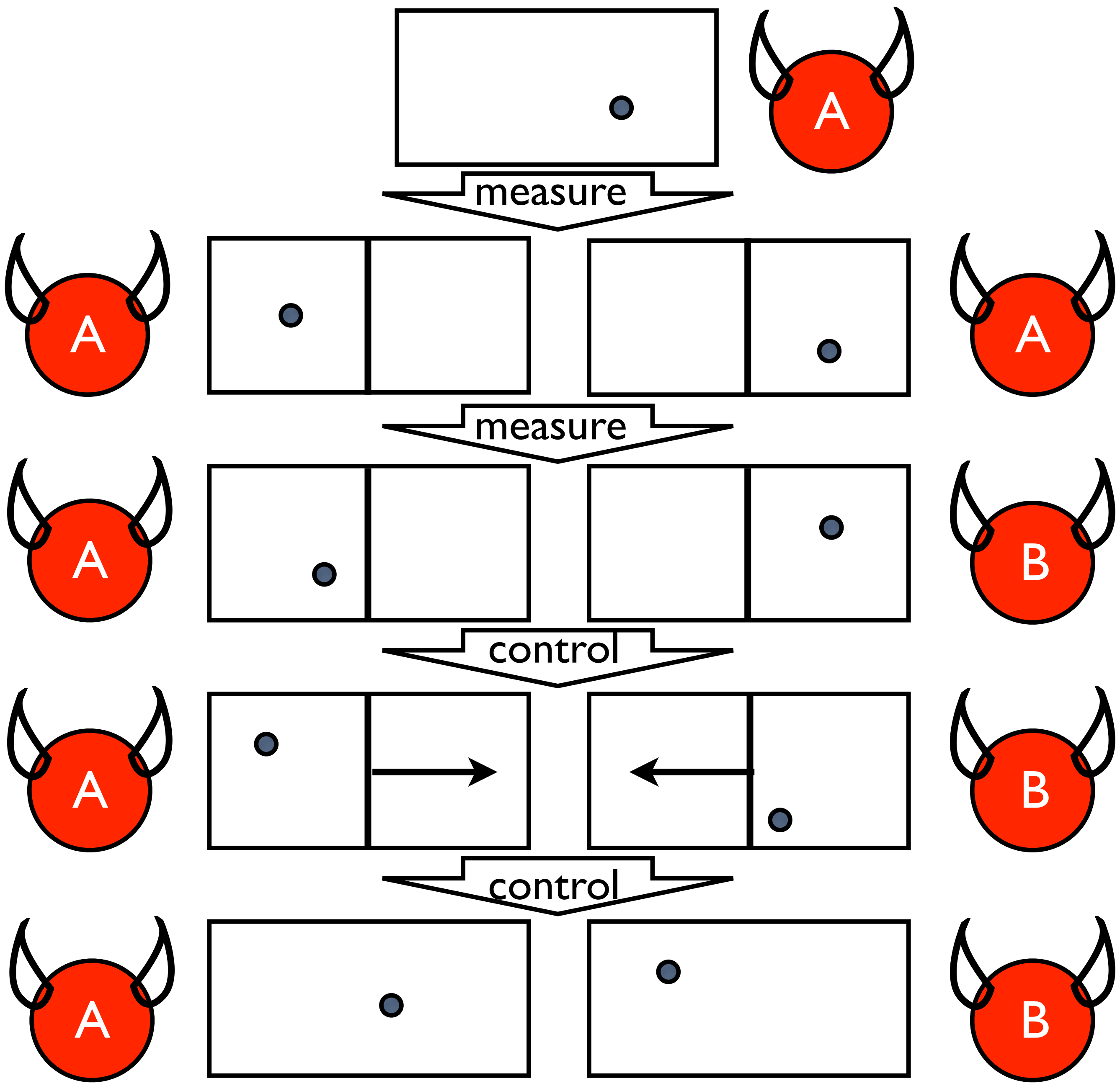
Szilard Engine Cycle



Szilard Engine Cycle

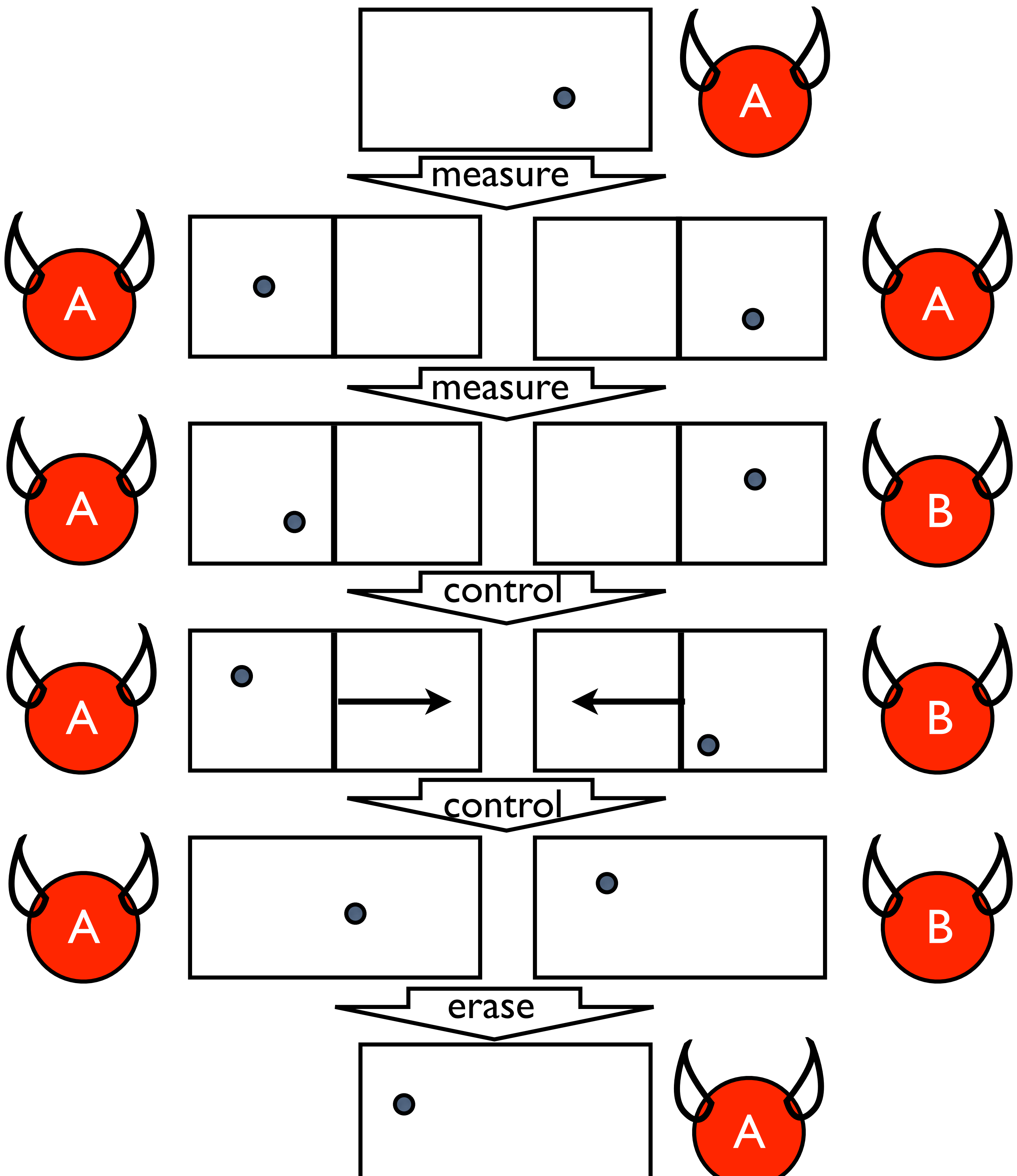


Szilard Engine Cycle



Demon is able to extract and store $k_B T \ln 2$ work every cycle.

Szilard Engine Cycle



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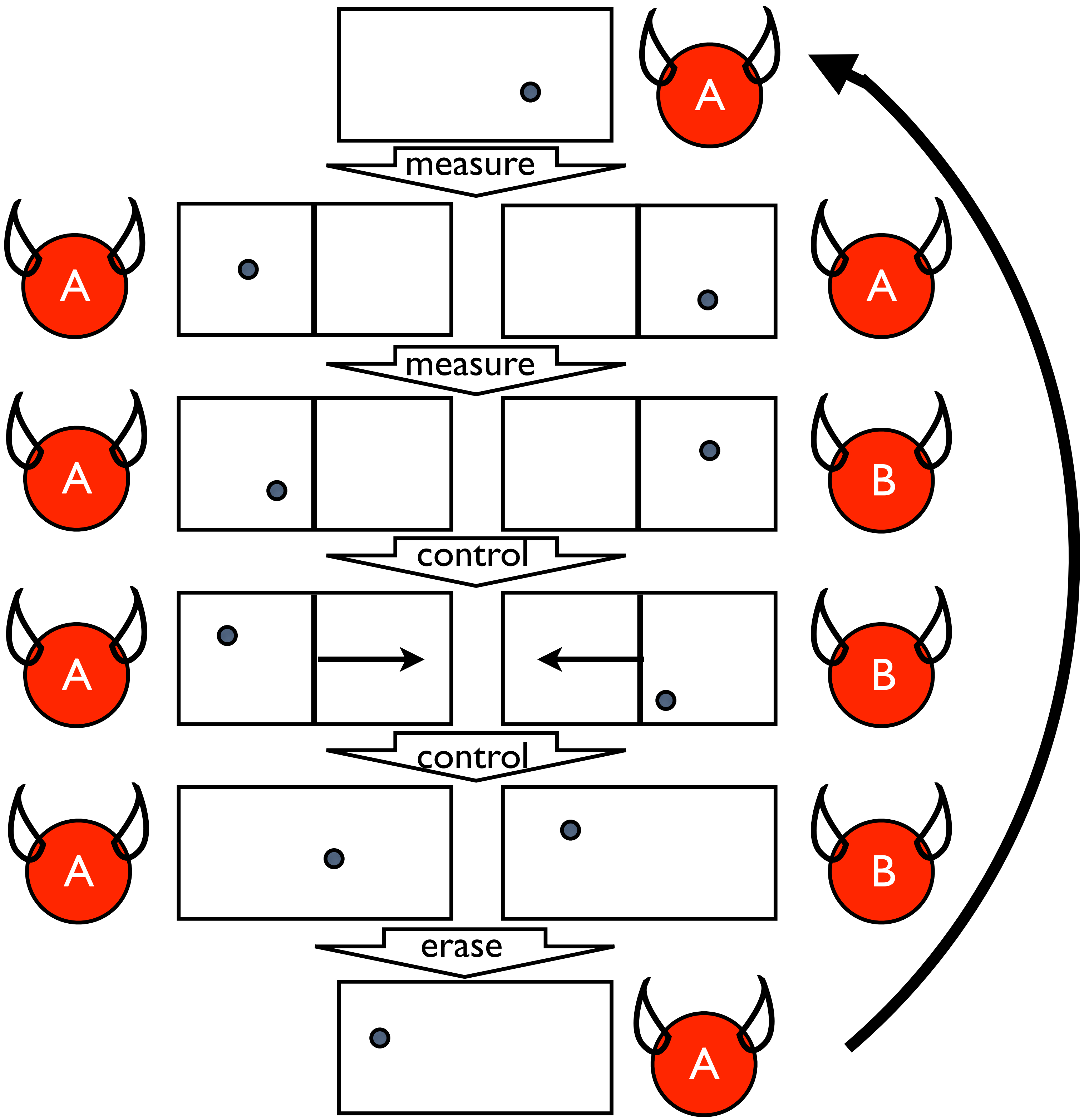
Szilard Engine Cycle

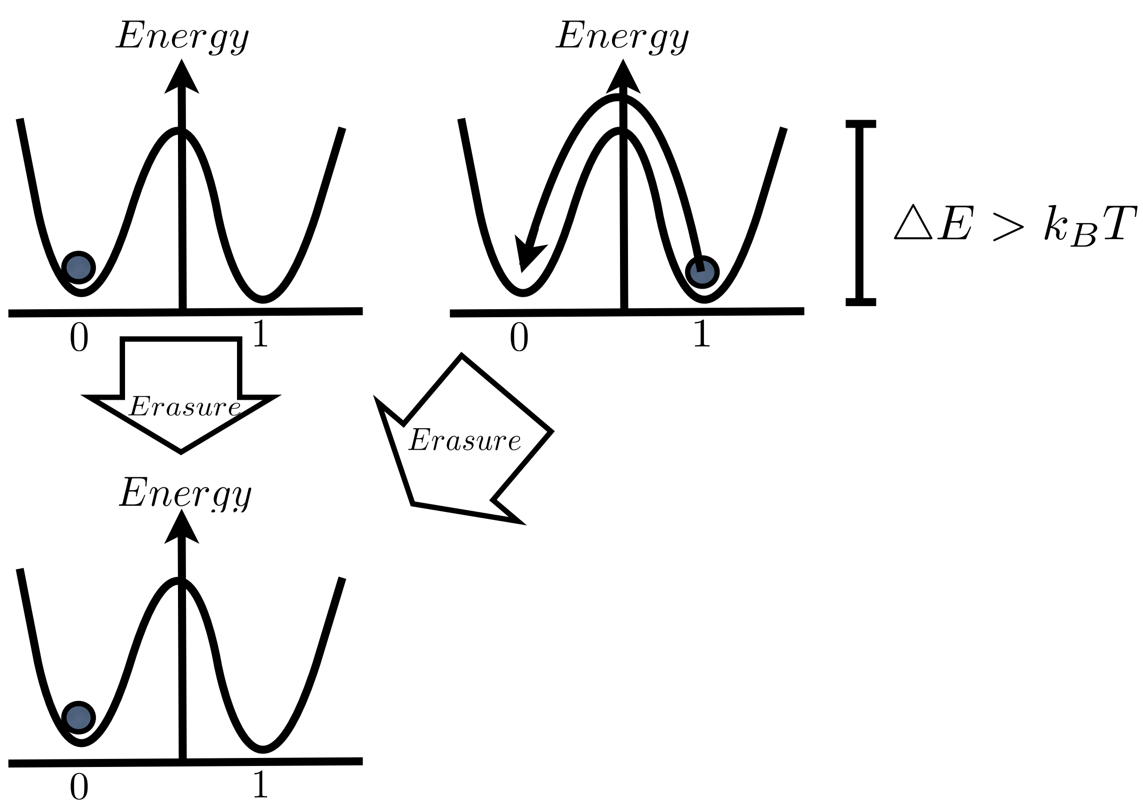


Demon is able to extract and store $k_B T \ln 2$ work every cycle.

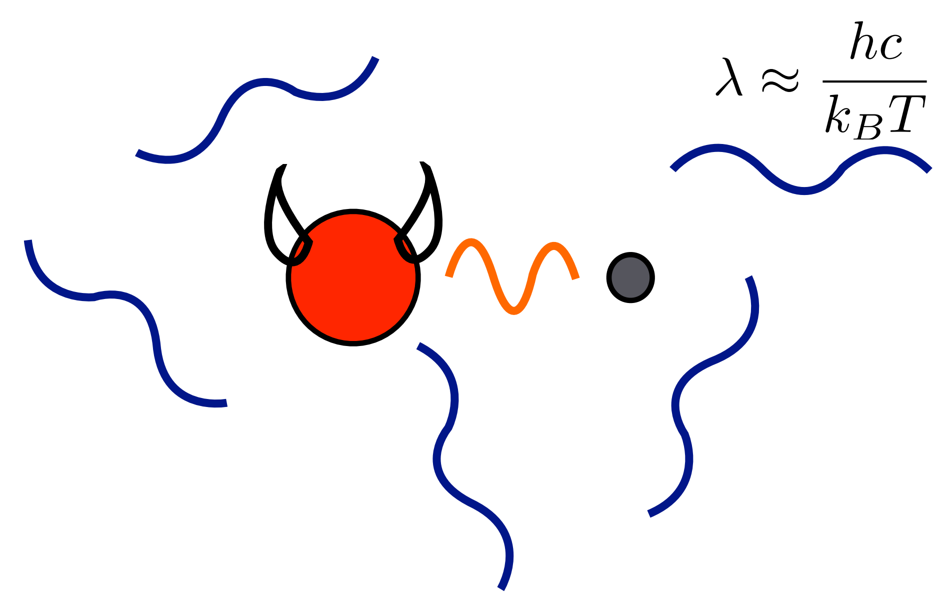
Violates the second law?

Energetic Costs

Erasure:
(resetting memory)



Measurement:
(writing to memory)



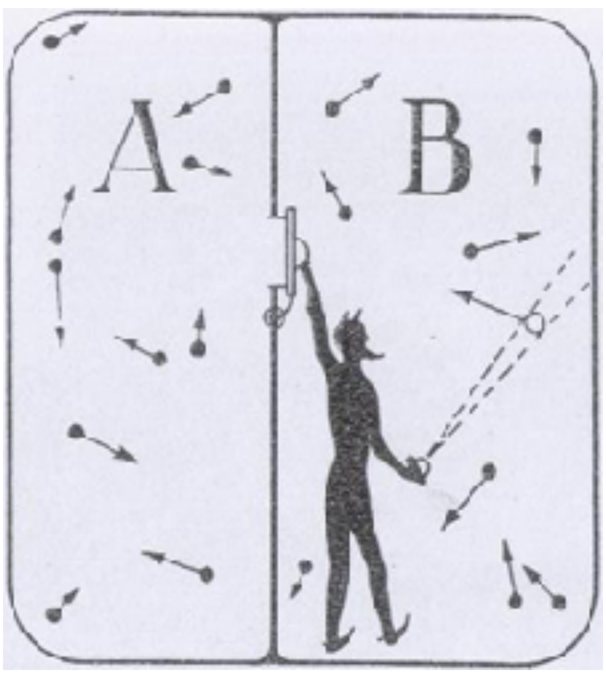
L. Brillouin "Maxwell's Demon Cannot Operate", (1951).

$$\boxed{\text{Landauer's principle:} \quad Q_{\text{erase}} \geq k_B T \ln 2}$$

R. Landauer, "Irreversibility and heat generation in the computing process", (1961).

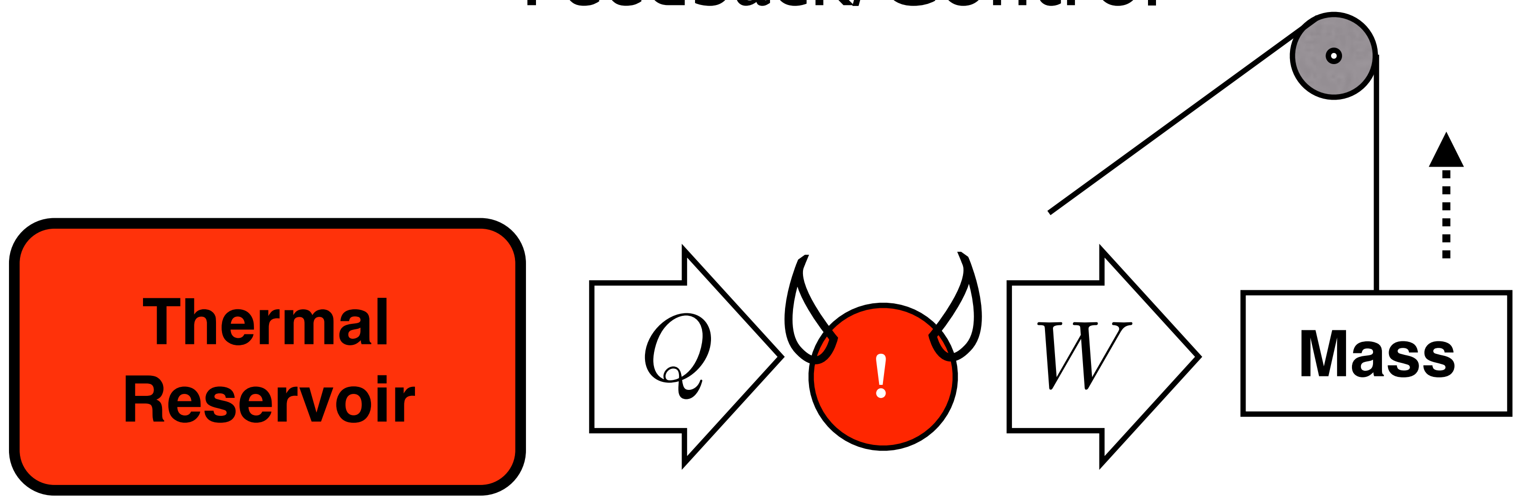
Information Bearing Degrees of Freedom

Maxwell's Demon



http://www.eoht.info/page/Maxwell's+demon

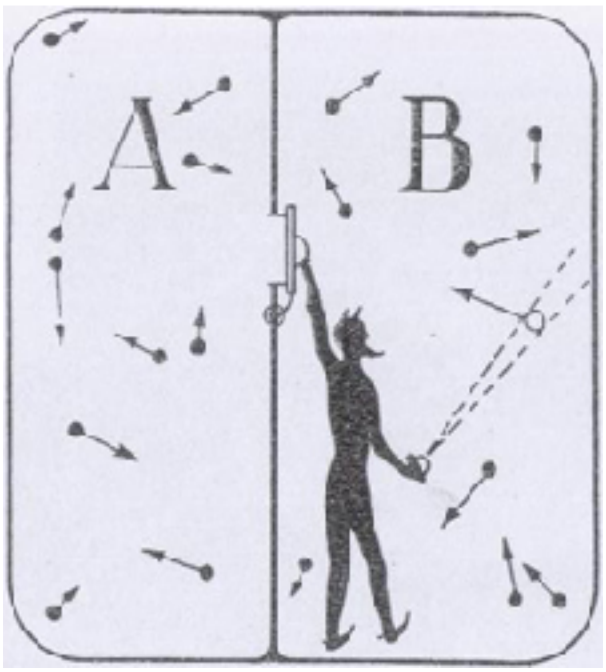
Feedback/Control



$$\Delta S_{\text{total}} < 0?$$

Information Bearing Degrees of Freedom

Maxwell's Demon



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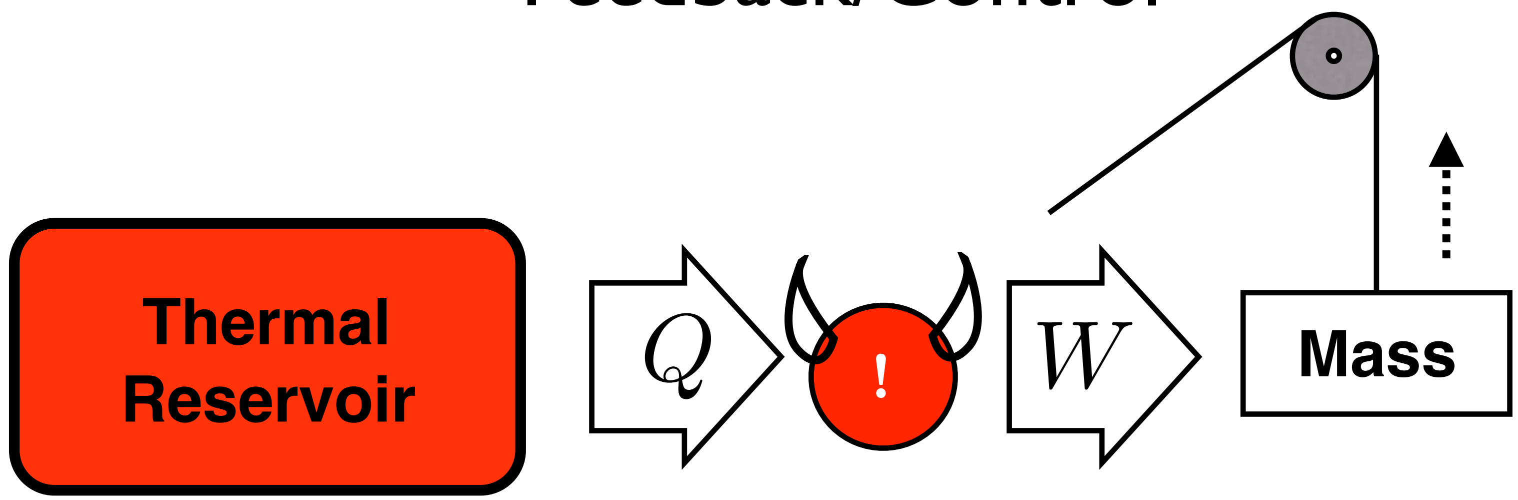
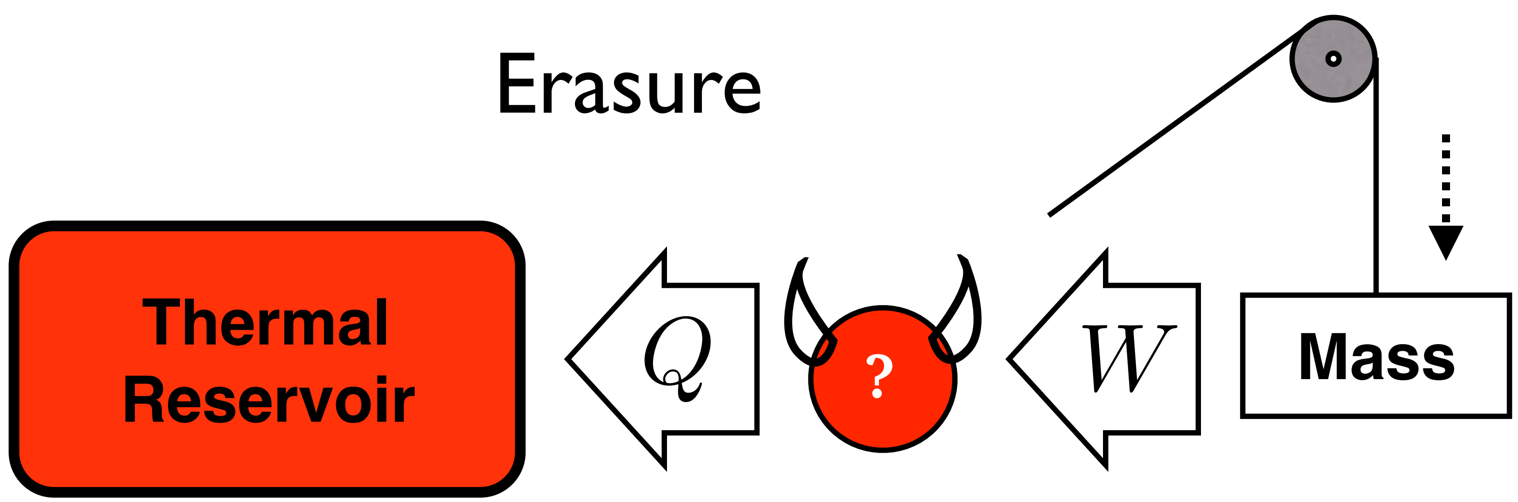
Feedback/Control



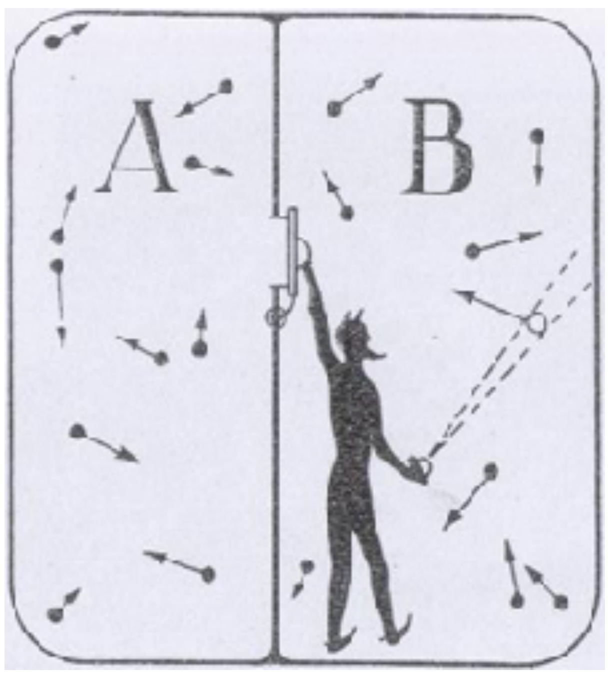
$$\Delta S_{\text{total}} < 0?$$

Erasure



Information Bearing Degrees of Freedom

Maxwell's Demon



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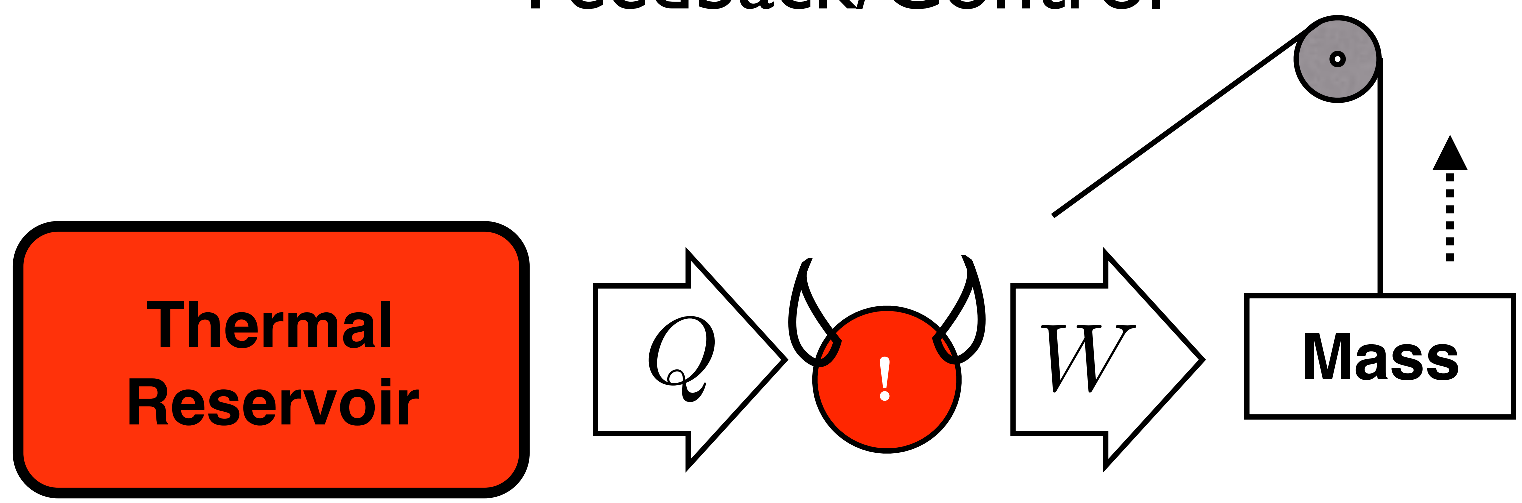
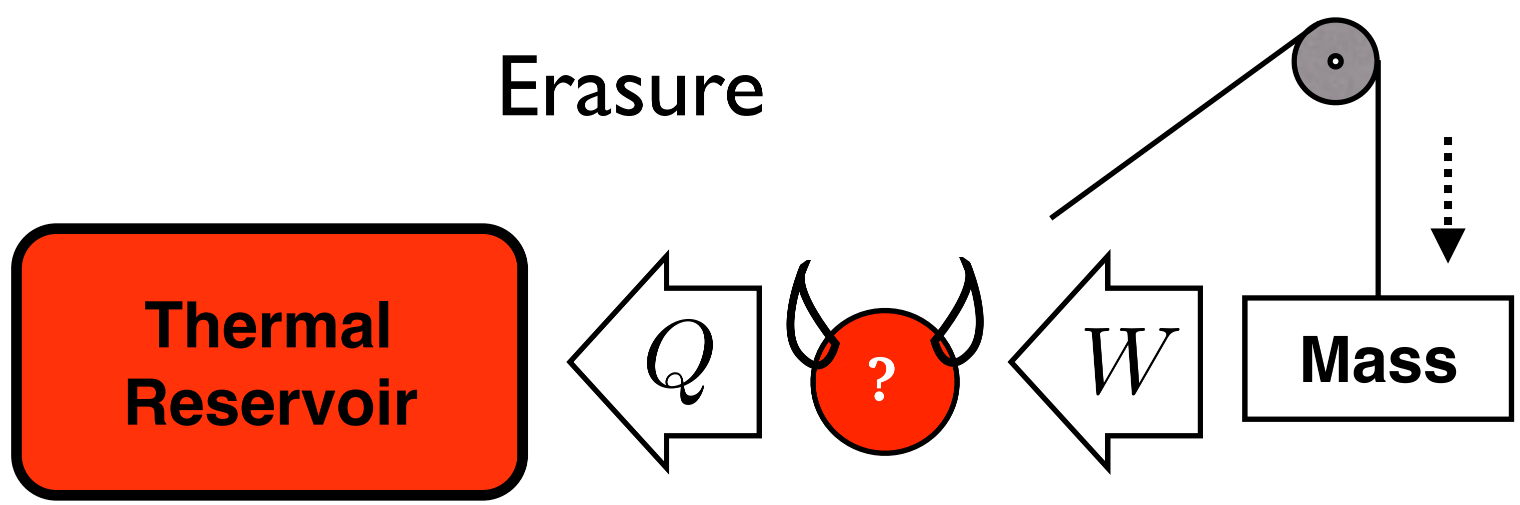
Feedback/Control



$\cancel{\Delta S_{\text{total}} < 0?}$

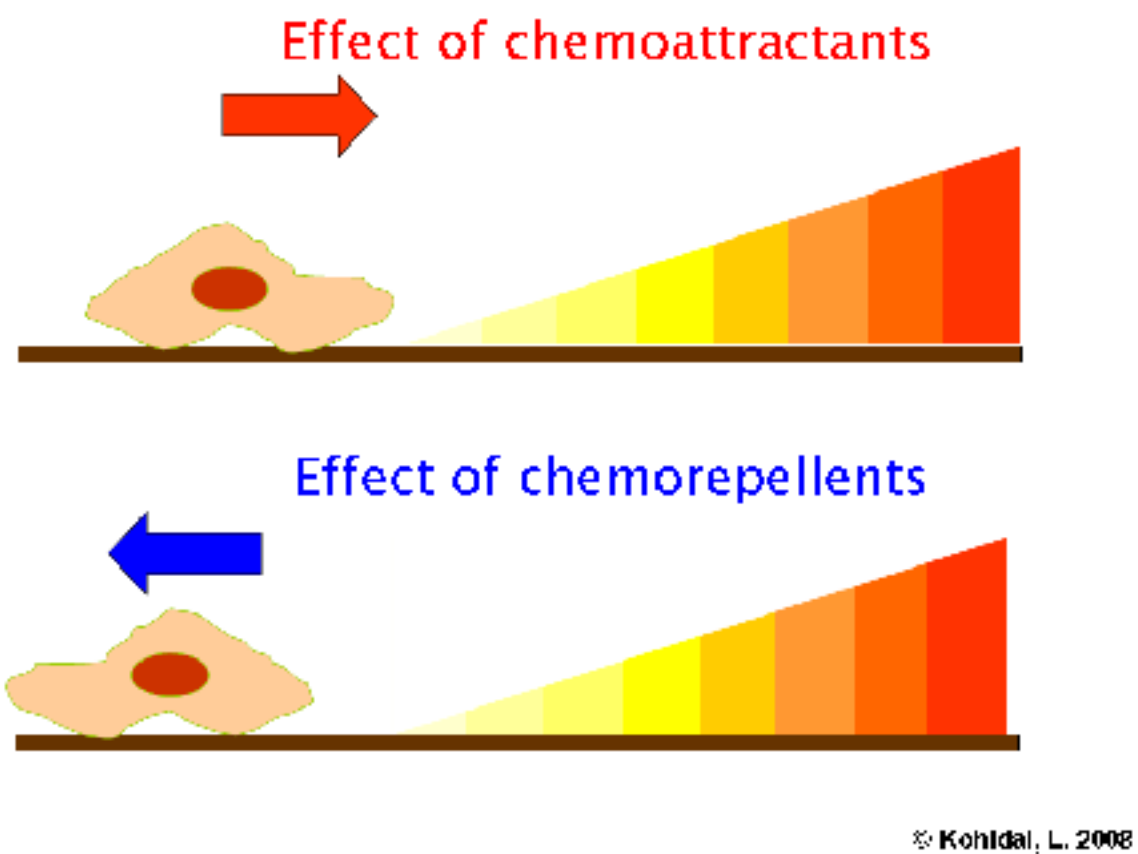
Erasure

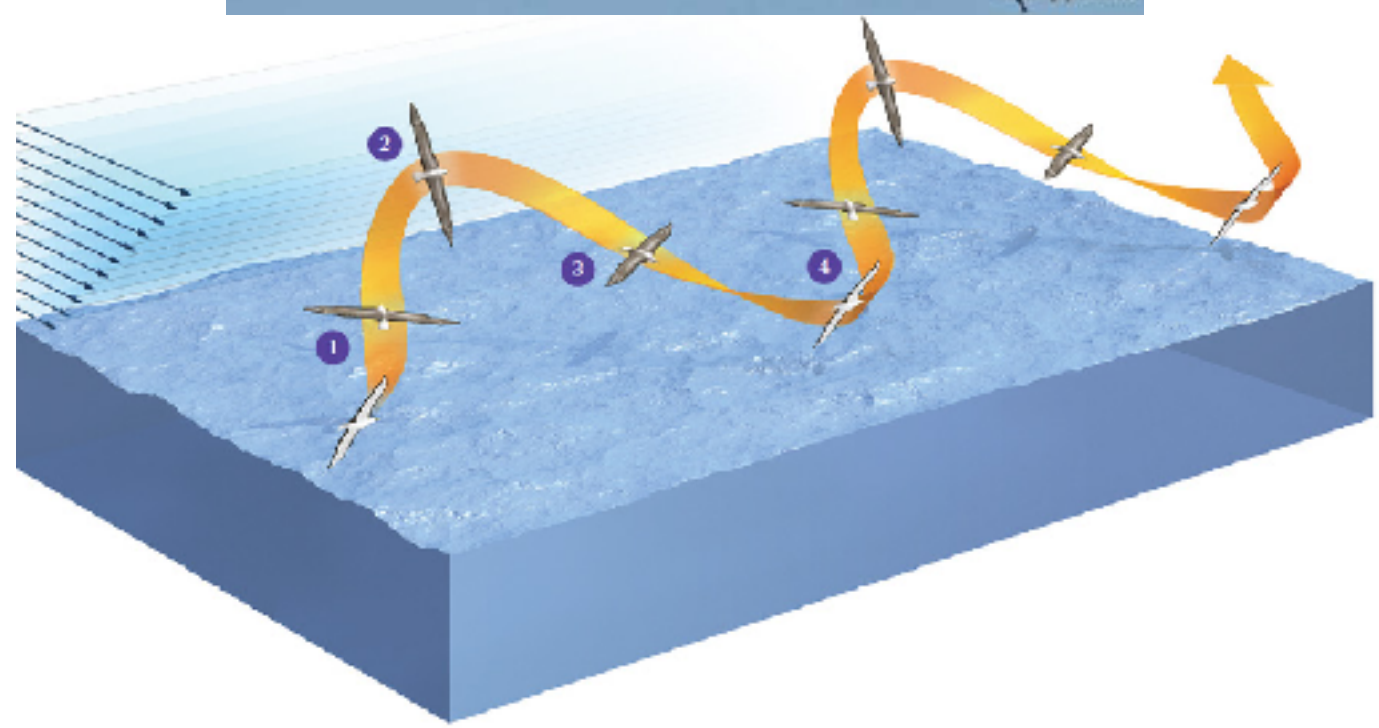


Landauer's Principle: $W_{\text{erase}} \geq k_B T \ln 2 \rightarrow \Delta S_{\text{total}} \geq 0$

Benefit of Information Processing

Information processing allows systems to leverage structure of environment





http://spectrum.ieee.org/aerospace/robotic-exploration/the-nearly-effortless-flight-of-the-albatross

Cost of Information Processing

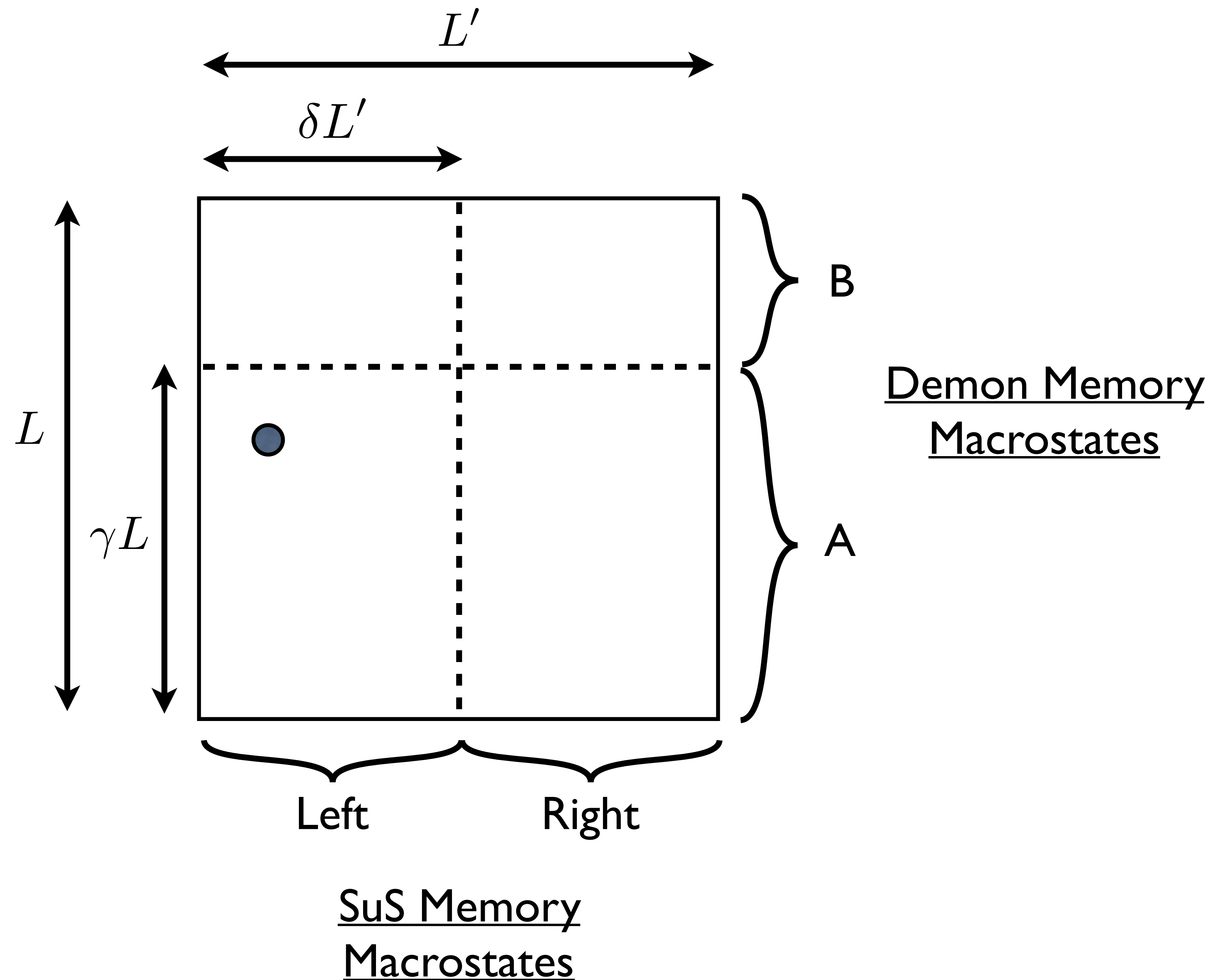
Processing information costs work and dissipates heat.

http://spectrum.ieee.org/computing/hardware/new-tech-keeps-data-centers-cool-in-warm-climates

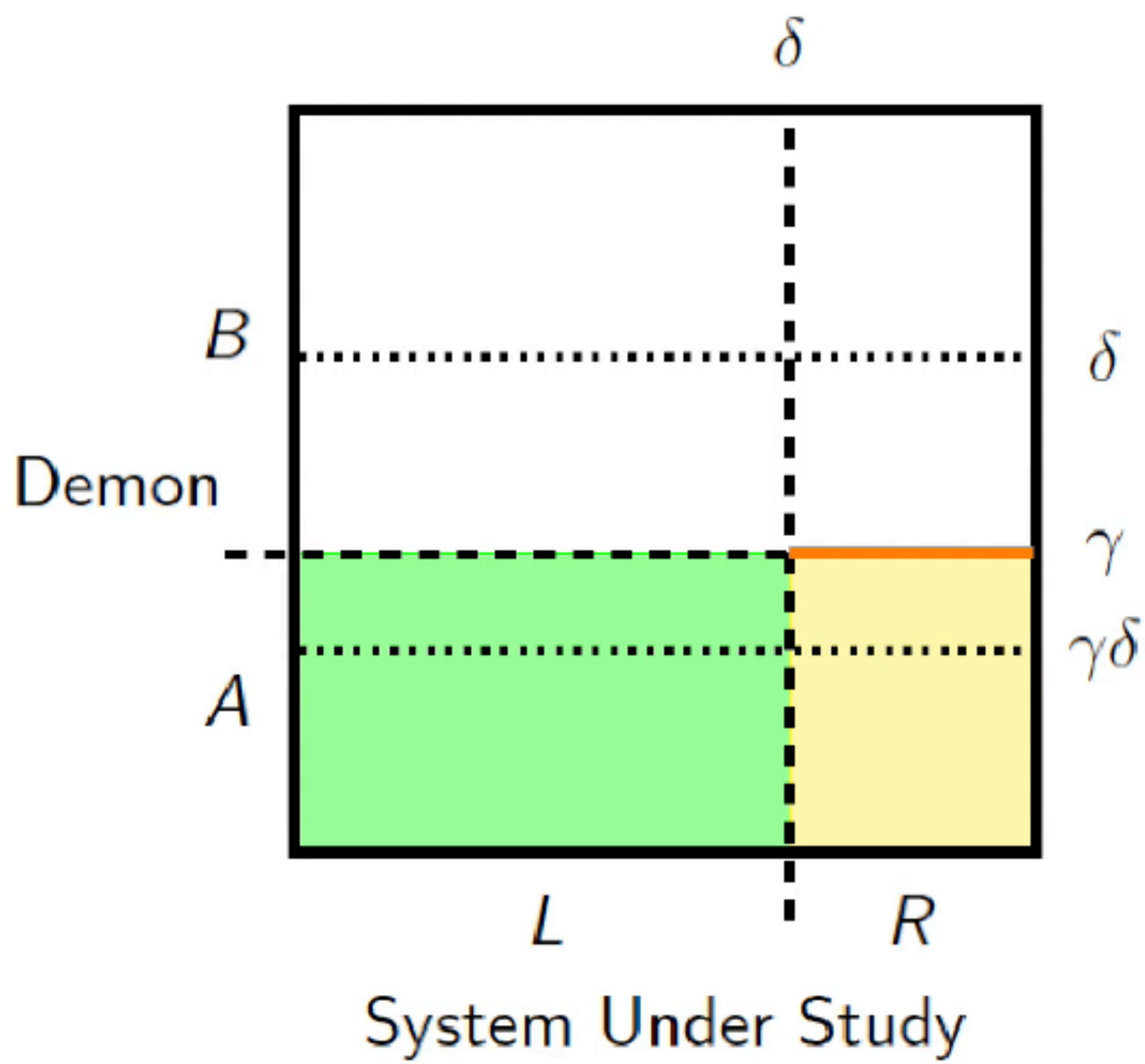
http://pooh.wikia.com/wiki/Think,_Think,_Think

Szilard Example

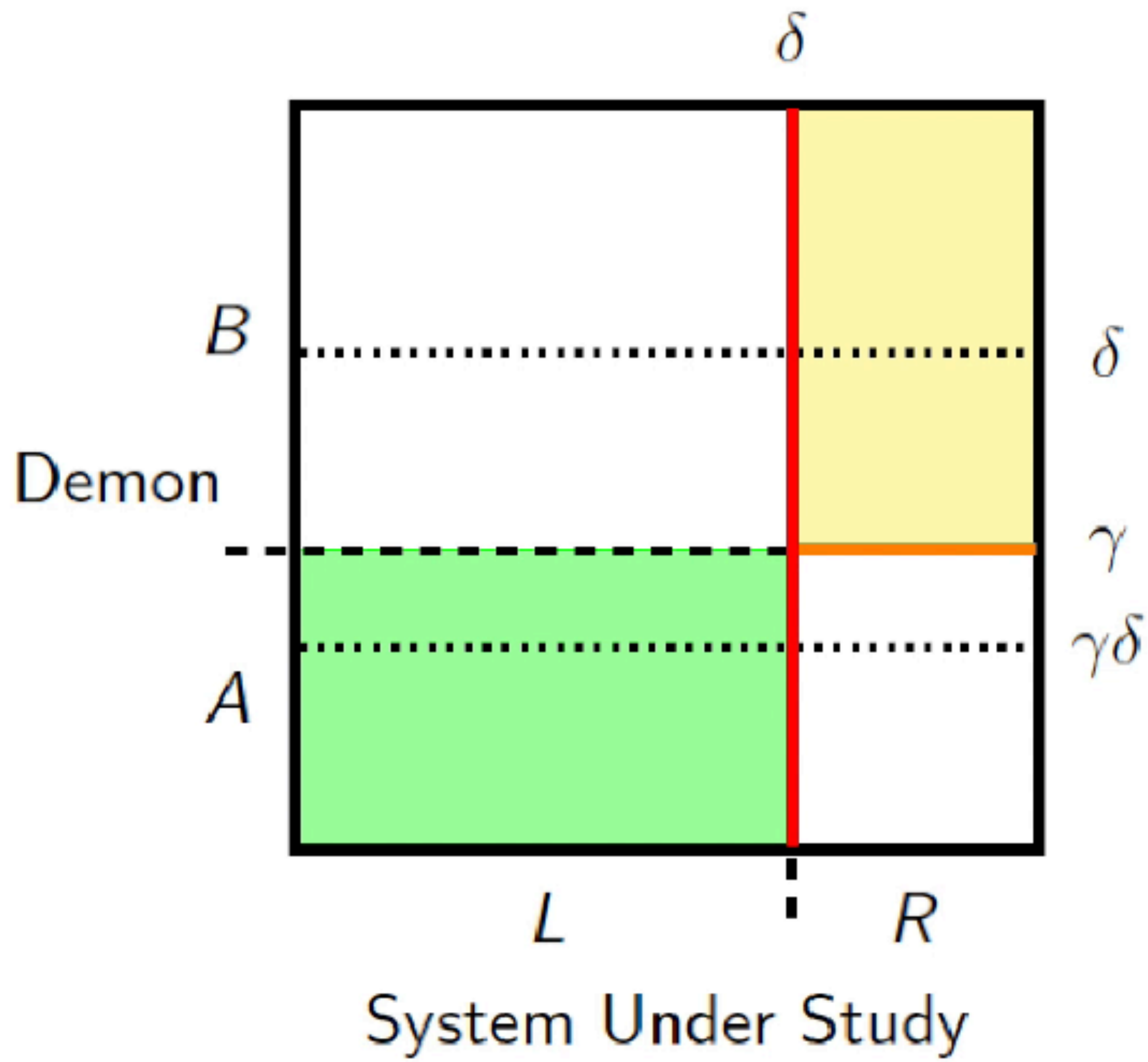
The Szilard system can be reformulated as a 2D box of gas.



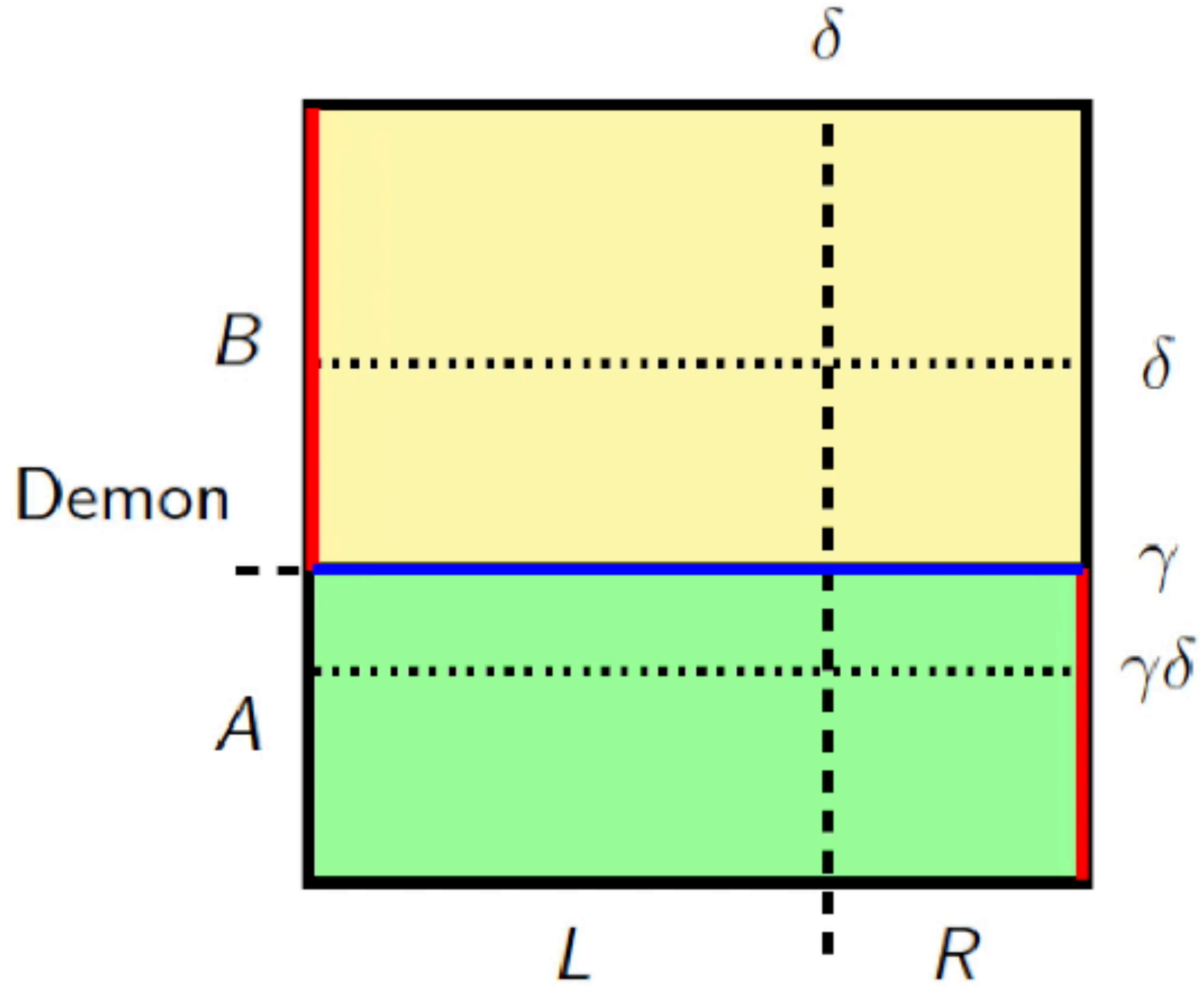
C. Bennet, "Thermodynamics of Computation-A Review", (1981) Measure Control Erase



Measure Control Erase



Measure **Control** Erase



System Under Study

Szilard Engine Dissipation

The average dissipated heat is exactly calculable as the work done to slide the barriers:

$$Q = W = -\int P dV$$

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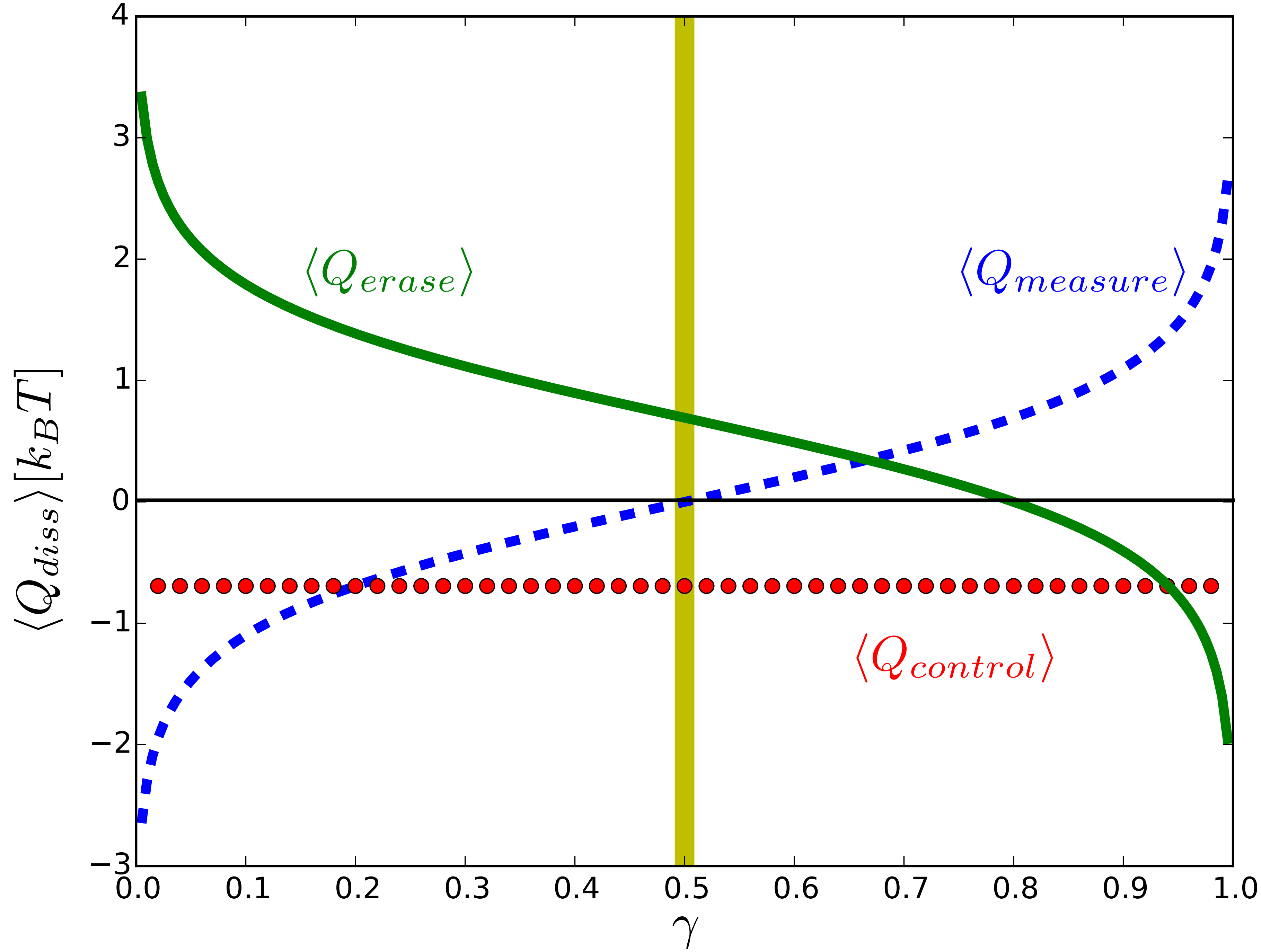
$$Q = W = -\int PdV$$

$$= -T\Delta S[X]$$

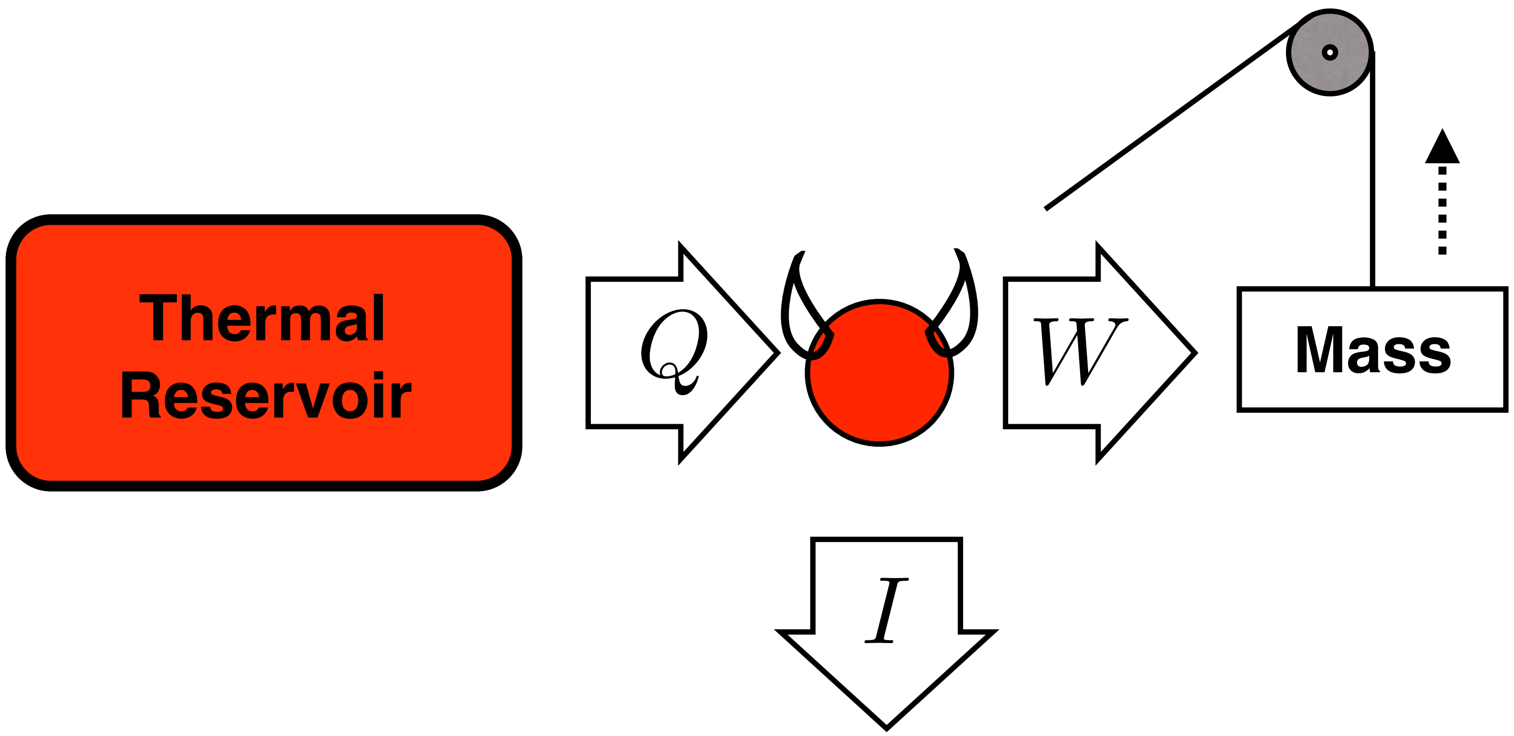
$$\Delta S_{\text{total}} = 0!$$

Szilard Engine Dissipation



A. Boyd, J. Crutchfield, "Maxwell Demon Dynamics: Deterministic Chaos, the Szilard Map, and the Intelligence of Thermodynamic Systems", PRL, (2016)

Information is a Thermodynamic Fuel



Instead of erasing, write to a hard drive