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Length Scales in Complex Time Series: Markov and Cryptic Orders: A Rope of Sand

Ryan G. James

Complexity Sciences Center Department of Physics University of California, Davis One Shields Avenue, Davis, CA 95616

May 7^{th} 2013



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Background			
Natural Con	nputation: Processe	S	

$\mathcal{P} = (\mathbf{X}, \mu) : \mathbf{X} \subseteq \mathcal{A}^{\mathbb{Z}}, \sigma(\mathbf{X}) = \mathbf{X}$

$\cdots X_{-3} X_{-2} X_{-1} X_0 X_1 X_2 X_3 \cdots$

Additional properties:

- Stationary
- Ergodic
- Discrete



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 $X_{-1:2}$
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Natural Con	nputation: Process	es	

$$\mathcal{P} = (\mathbf{X}, \mu) : \quad \mathbf{X} \subseteq \mathcal{A}^{\mathbb{Z}}, \sigma(\mathbf{X}) = \mathbf{X}$$
$$X_{:-1} \qquad X_{-1:2}$$
$$\cdots \qquad X_{-3} X_{-2} X_{-1} X_0 X_1 X_2 X_3 \cdots$$

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Natural Con	nputation: Processe	S	

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 $X_{:-1} \qquad X_{-1:2} \qquad X_{2:}$
 $\cdots \qquad X_{-3} \qquad X_{-2} \qquad X_{-1} \qquad X_0 \qquad X_1 \qquad X_2 \qquad X_3 \qquad \cdots$

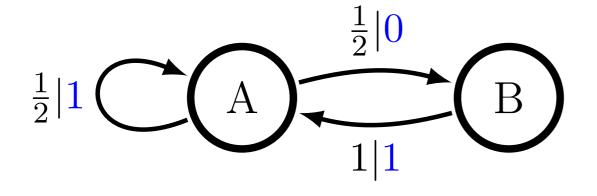
Additional properties:

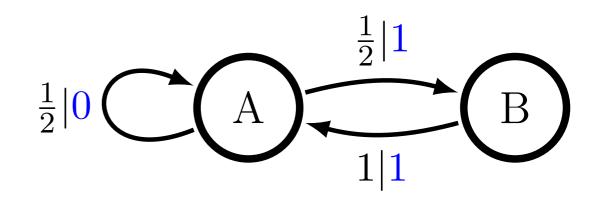
- Stationary
- Ergodic
- Discrete



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Natural Computation: ϵ -Machines





Properties:

- Minimal unifilar
- States are causal
- Causal states are sufficient statistics



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Definitions			

Markov and Cryptic Orders

Definition

The Markov order of a process is:

$$R = \underset{\ell}{\operatorname{argmin}} \left\{ \Pr(X_0 | X_{-\ell:0}) = \Pr(X_0 | X_{:0}) \right\}$$



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Markov and Cryptic Orders

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$$= \underset{\ell}{\operatorname{argmin}} \{ H[\mathcal{S}_{\ell} | X_{0:\ell}] = 0 \}$$



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Markov and Cryptic Orders

Definition

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$$= \underset{\ell}{\operatorname{argmin}} \left\{ H[\mathcal{S}_{\ell} | X_{0:\ell}] = 0 \right\}$$

Definition

The *cryptic order* of a process is:

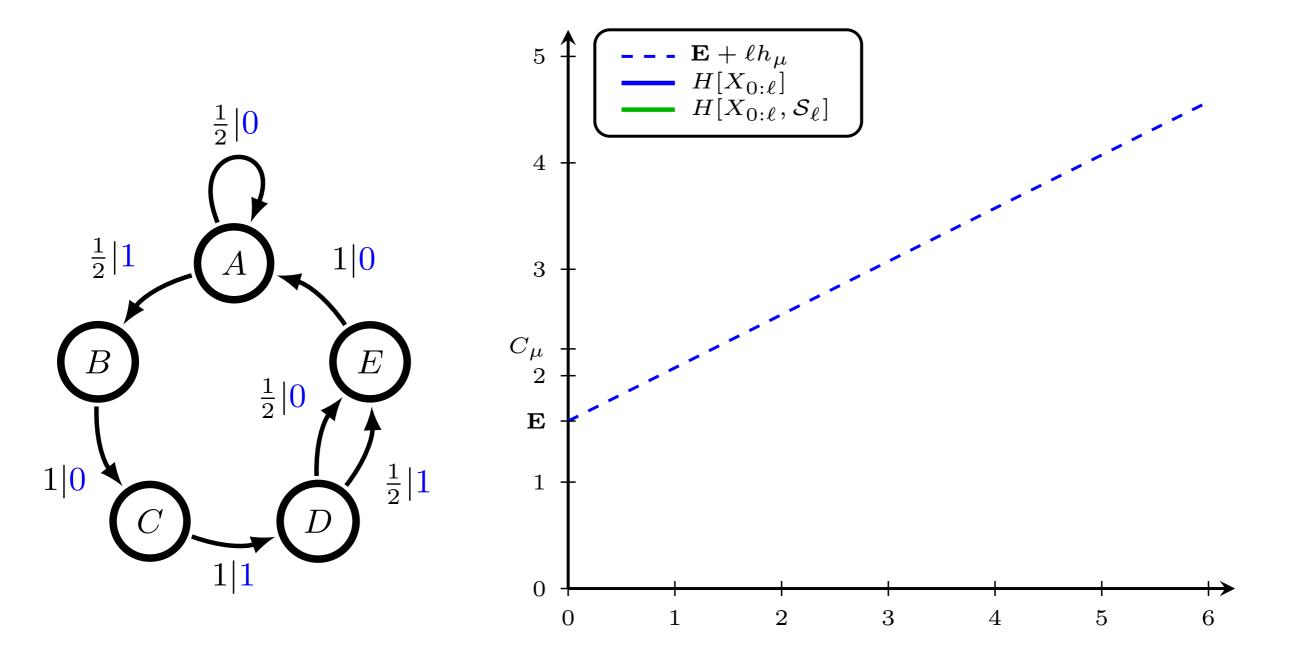
$$k_{\chi} = \underset{\ell}{\operatorname{argmin}} \left\{ H[\mathcal{S}_{\ell} | X_{0:}] = 0 \right\}$$

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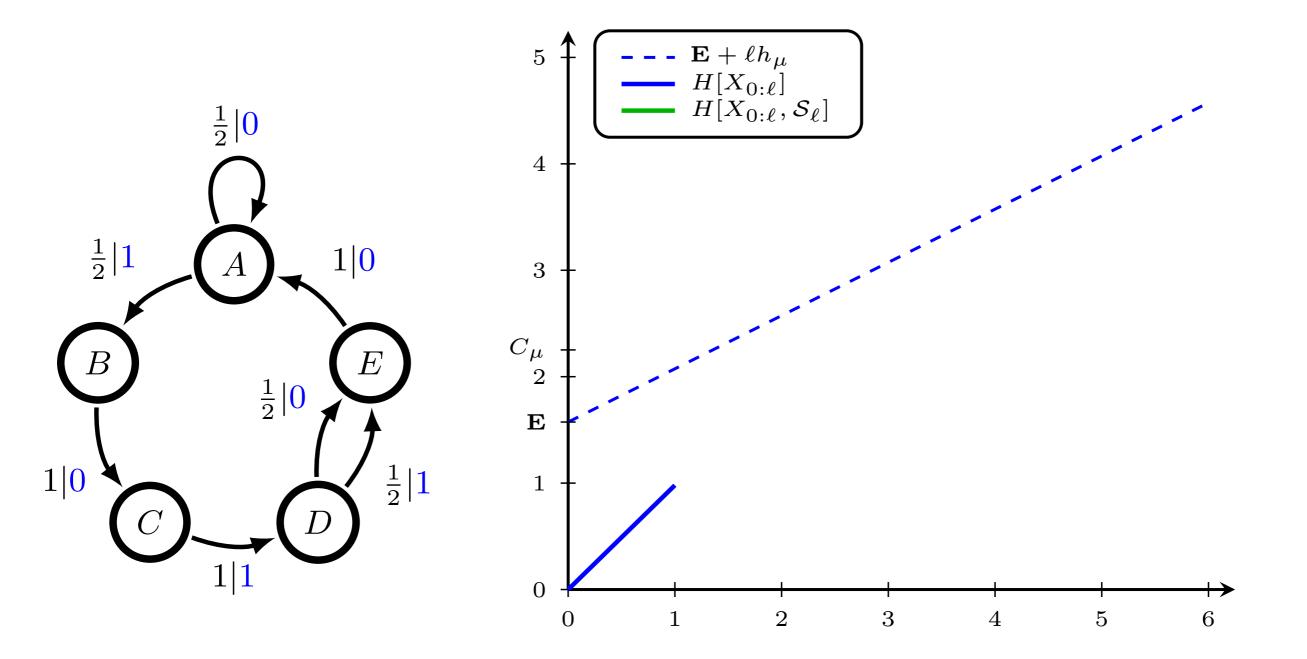
Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

Sunday, May 4, 14

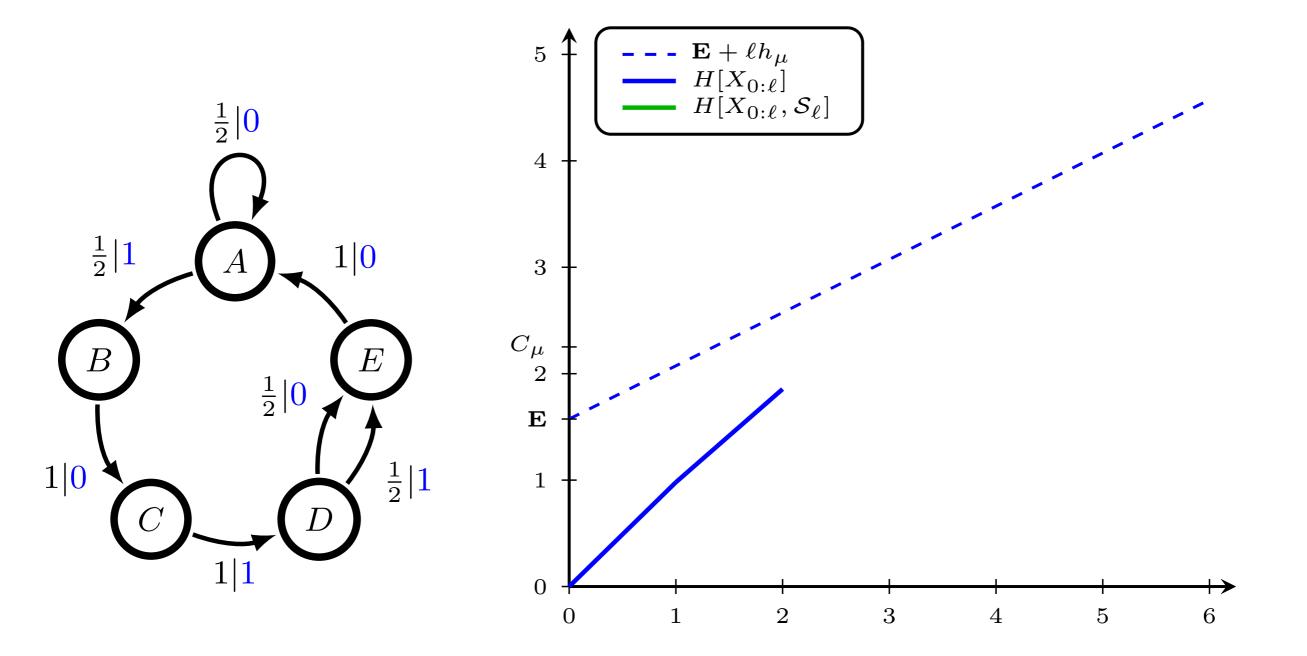
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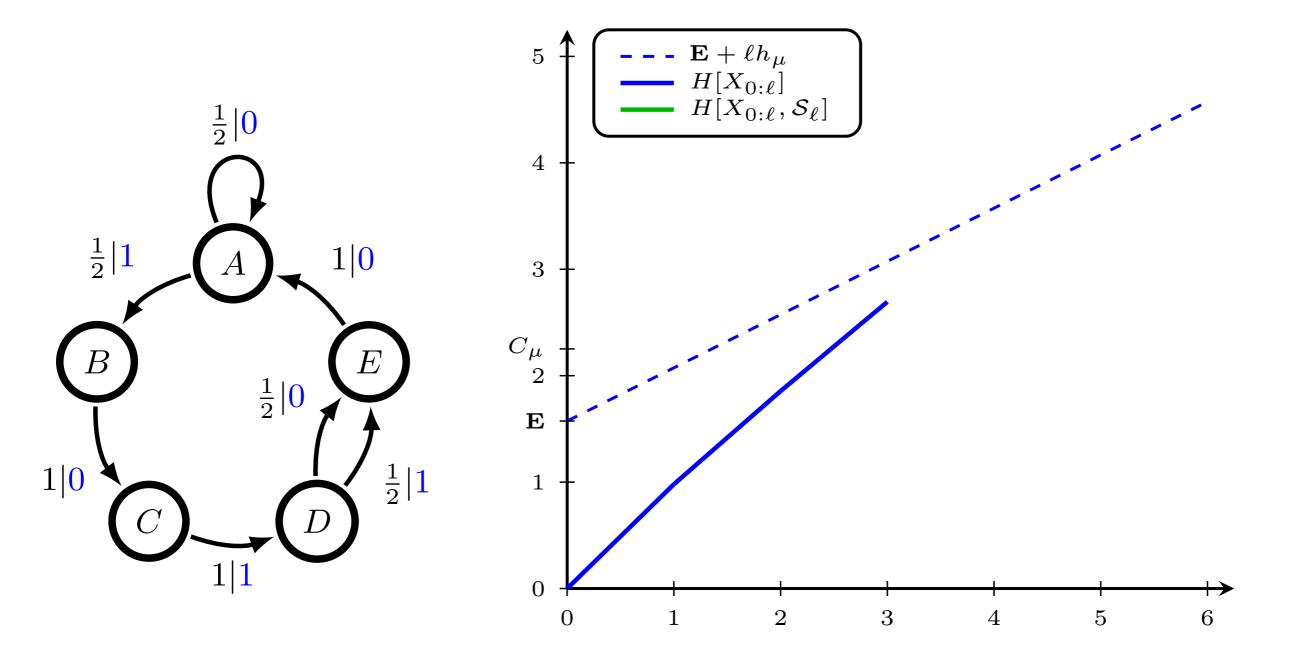


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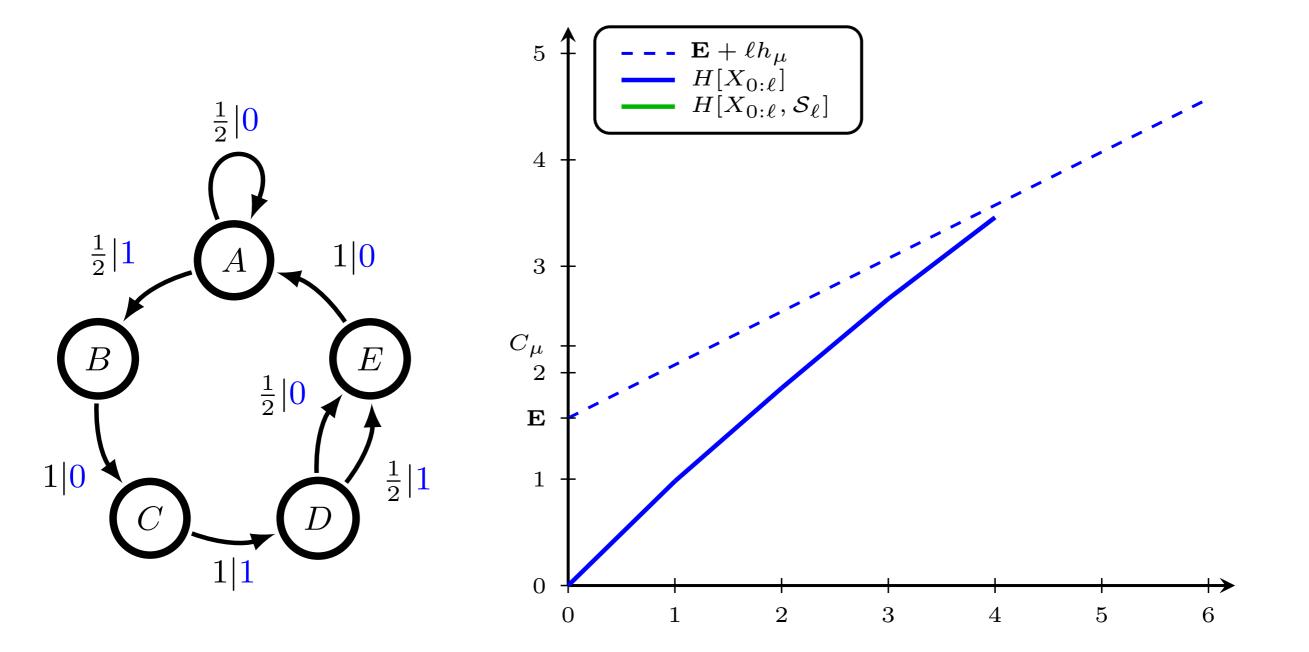




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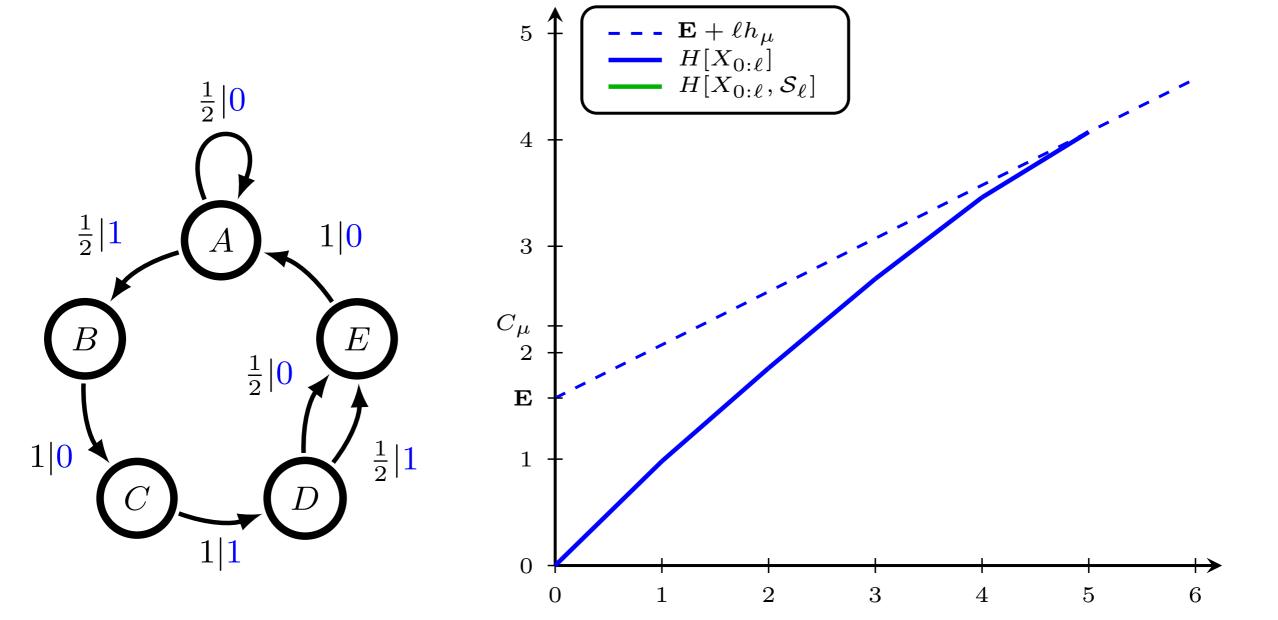


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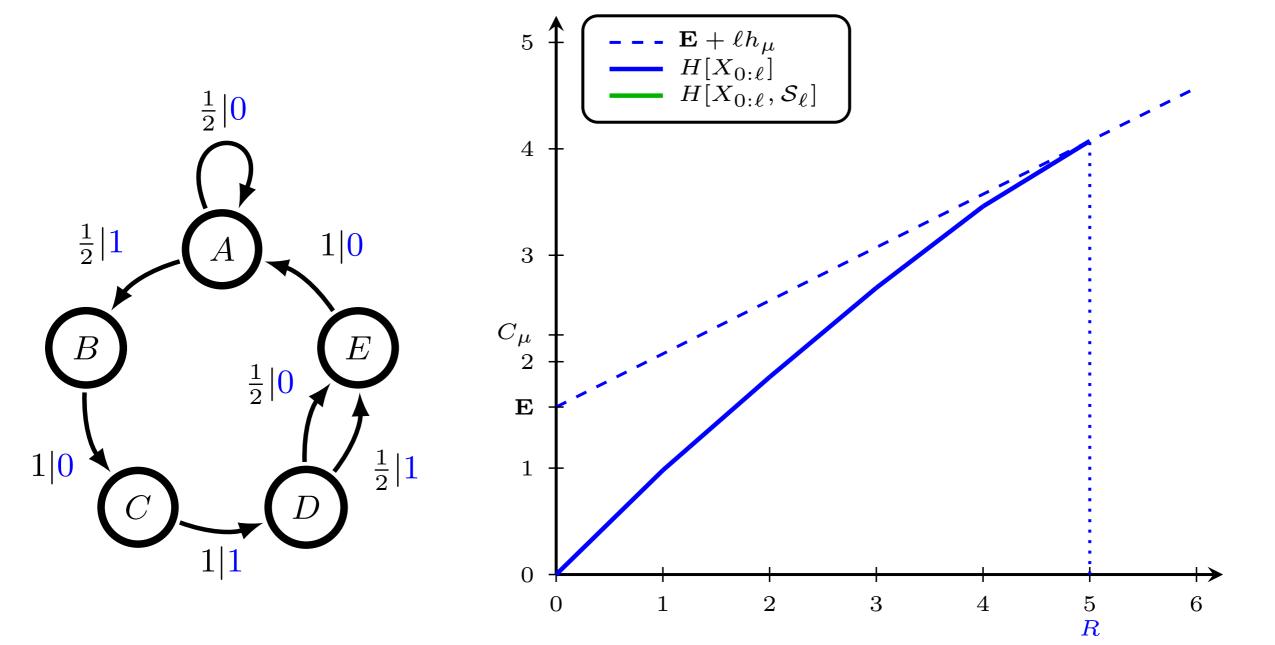


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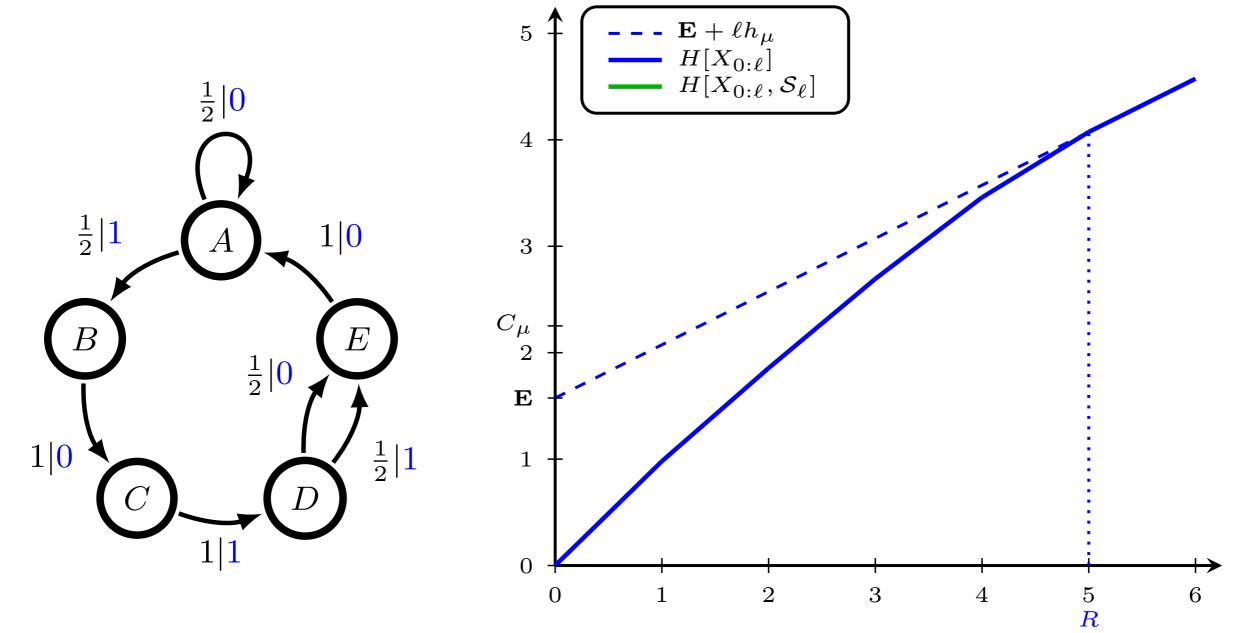




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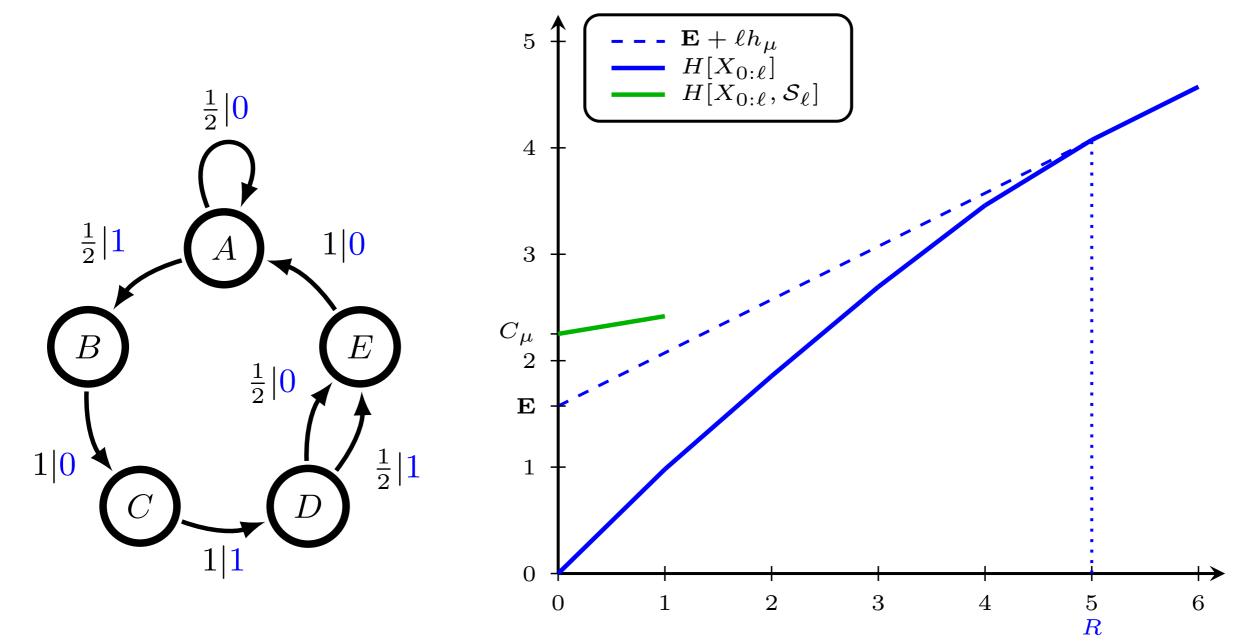


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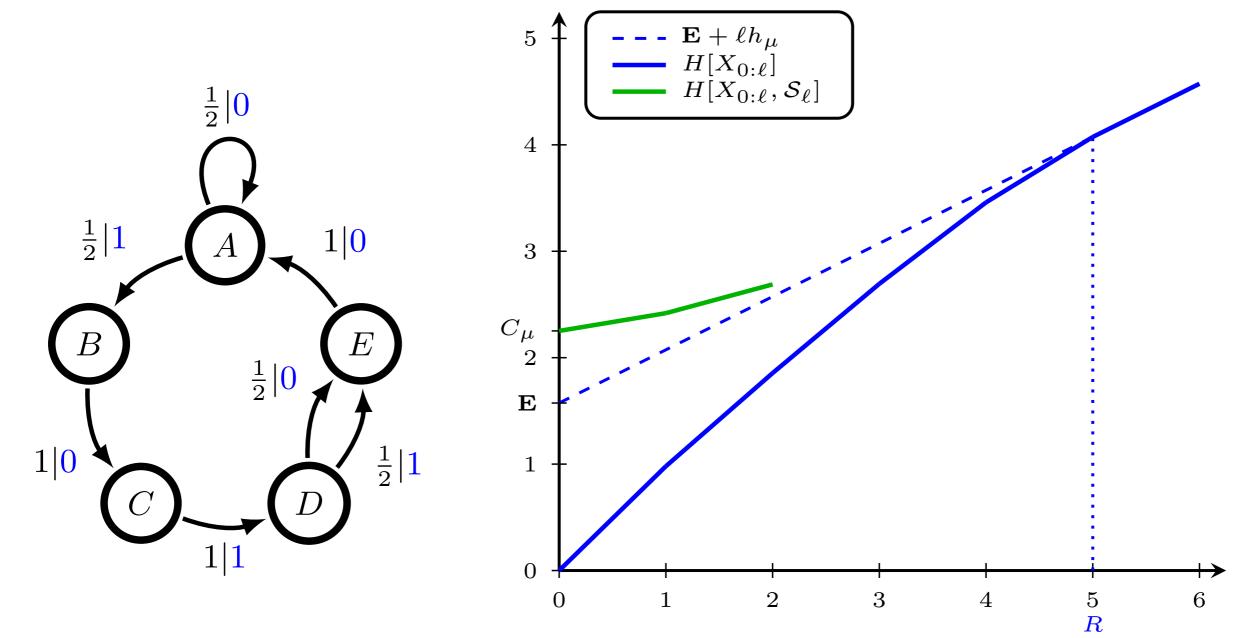




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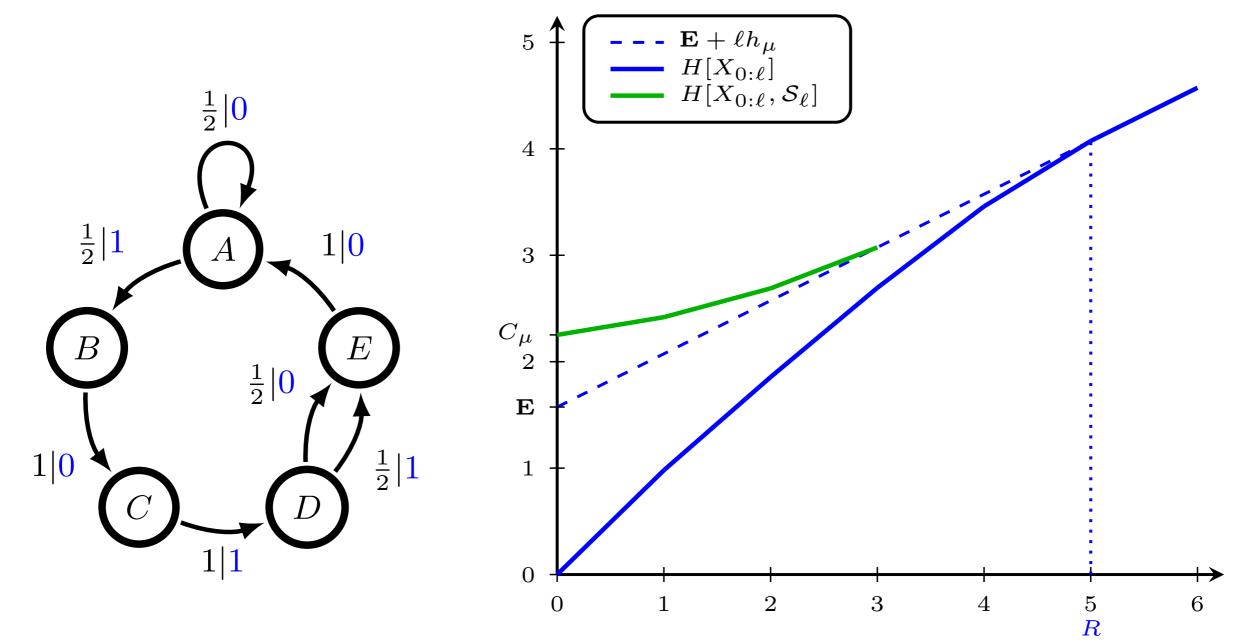


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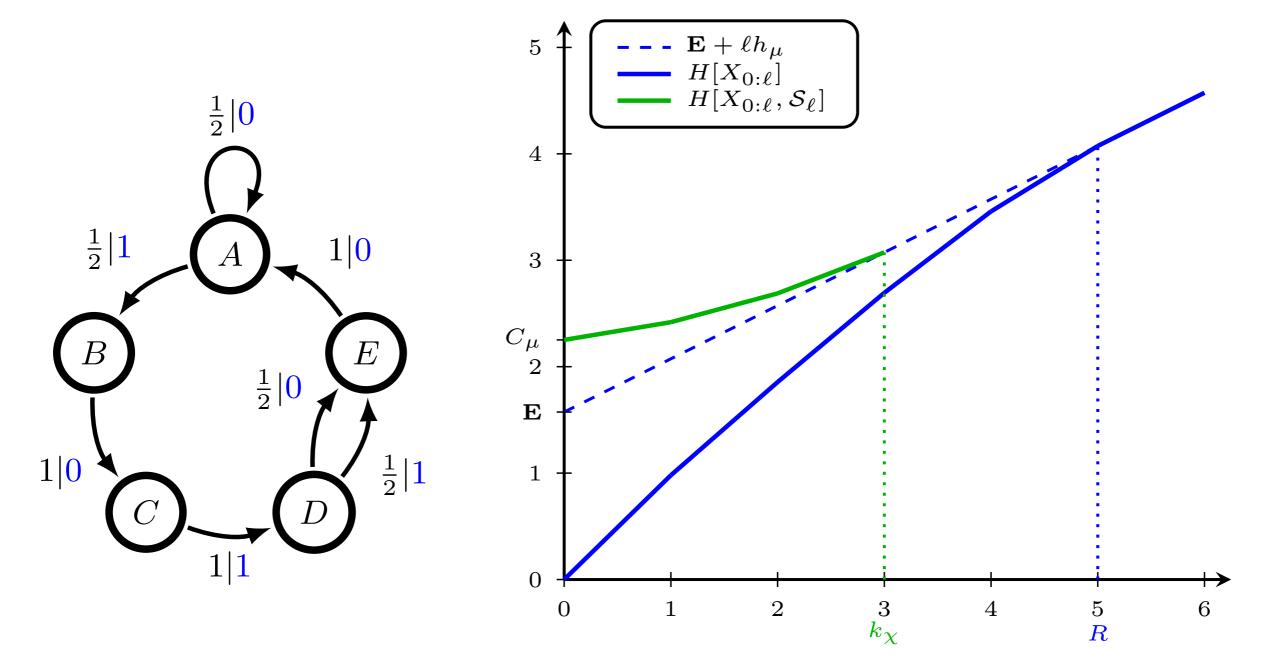




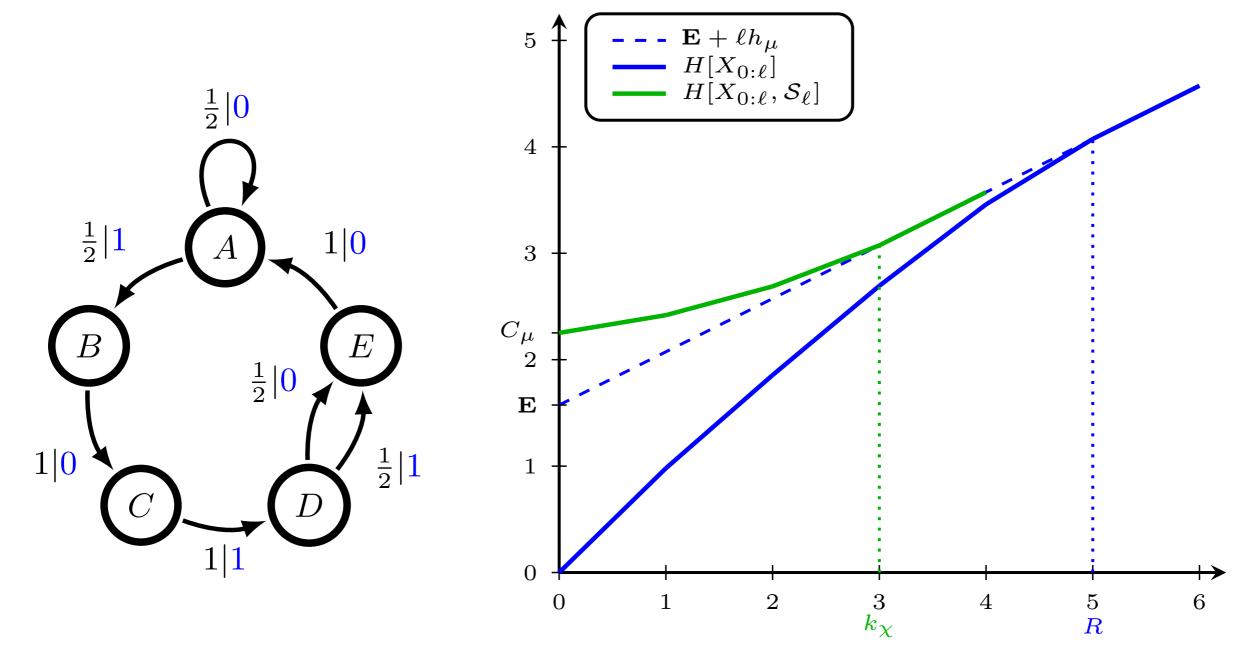
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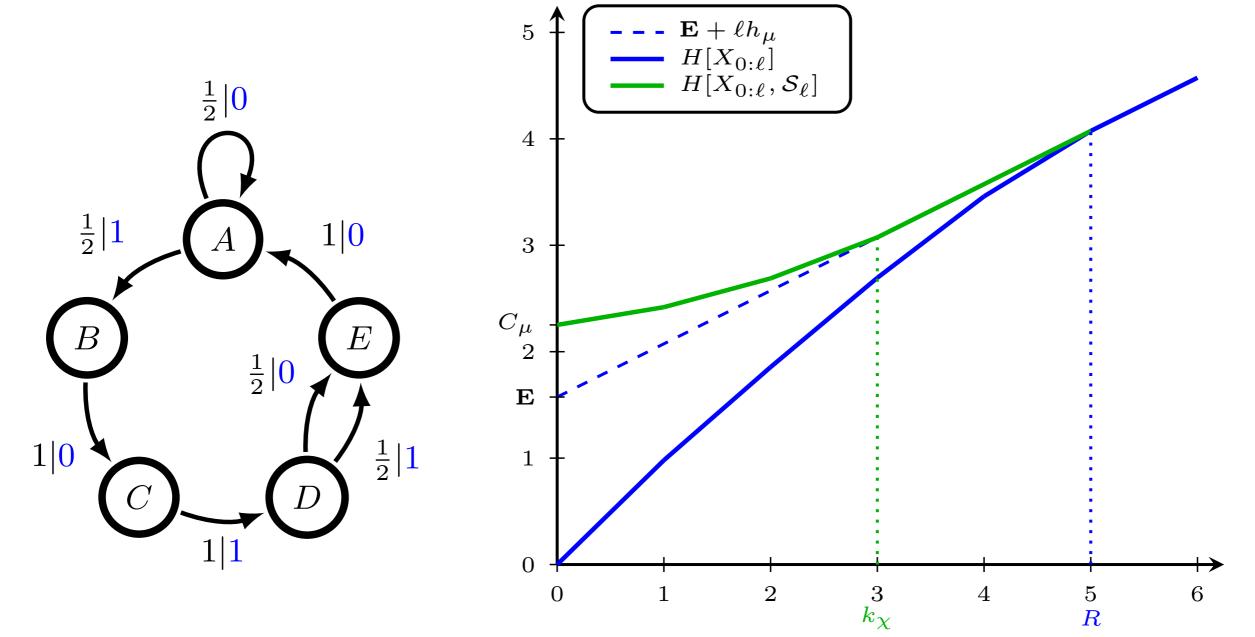


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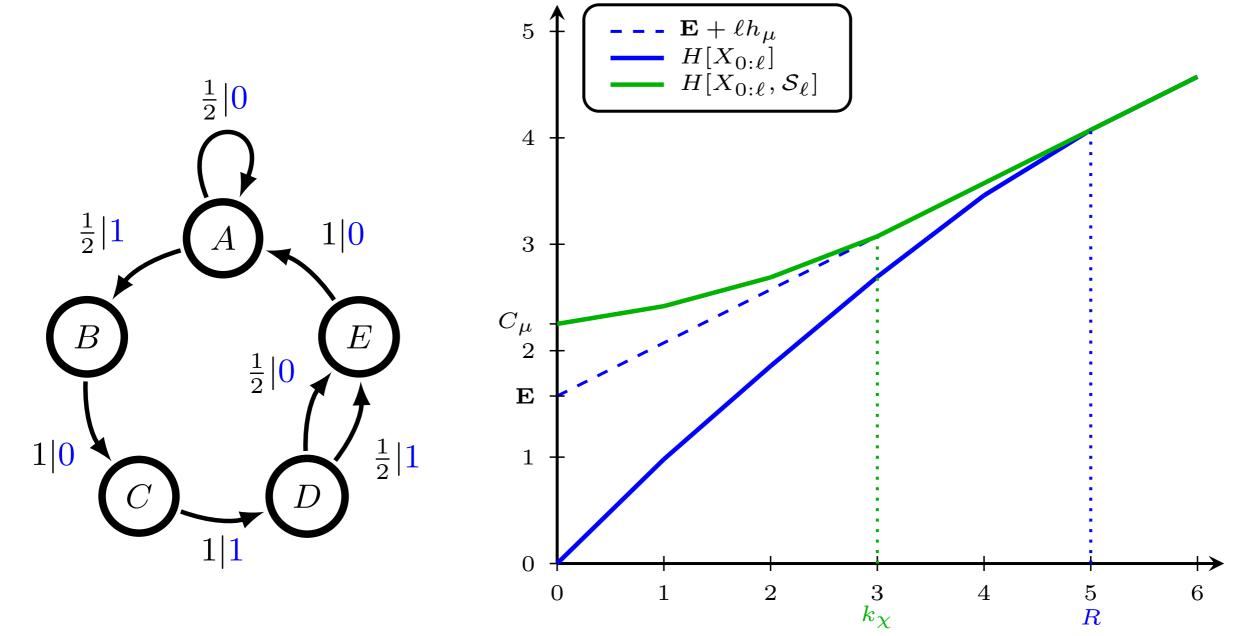




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Discussion			
Problems W	ith This Approach		



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• Knew **E** exactly



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- $\bullet~{\rm Knew}~{\bf E}~{\rm exactly}$
- Knew h_{μ} exactly



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Problems	With This Approacl	h	

- $\bullet~{\rm Knew}~{\bf E}$ exactly
- Knew h_{μ} exactly
- Could differentiate *exactly on* the asymptote from *less than machine precision away from* the asymptote



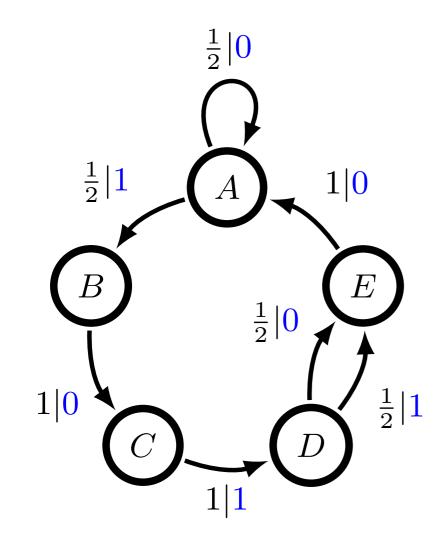
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Problems W	ith This Approach		

• Knew **E** exactly

- Knew h_{μ} exactly
- Could differentiate *exactly on* the asymptote from *less than* machine precision away from the asymptote
- Could "guess" when R or k_{χ} were infinite, else we'd be computing block entropies indefinitely

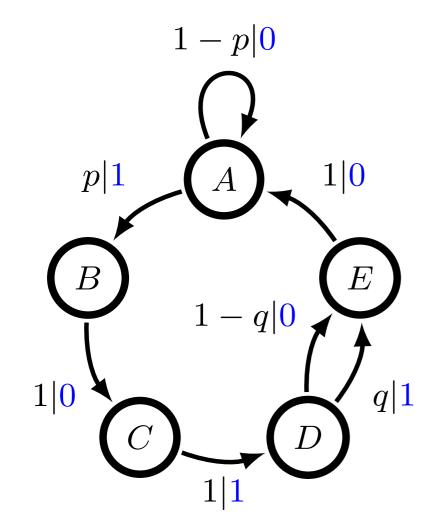


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A Hint of	a Solution		



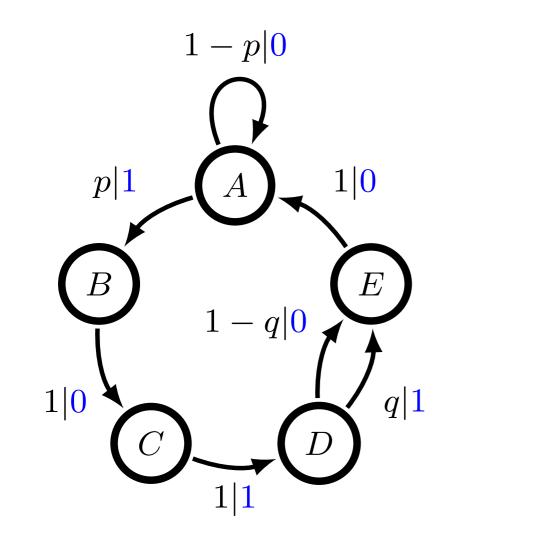


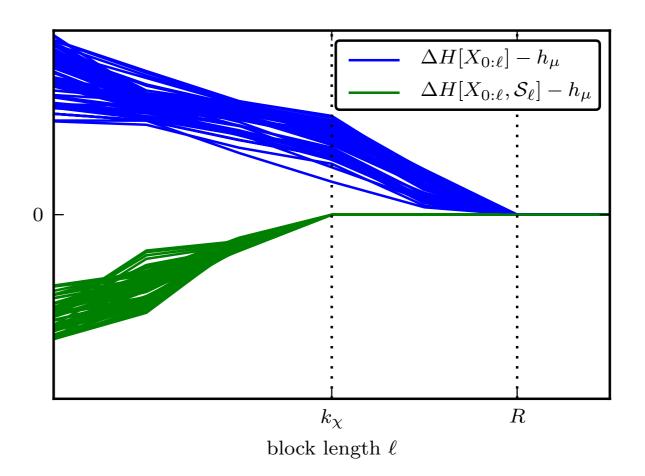
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A Hint of a Solution

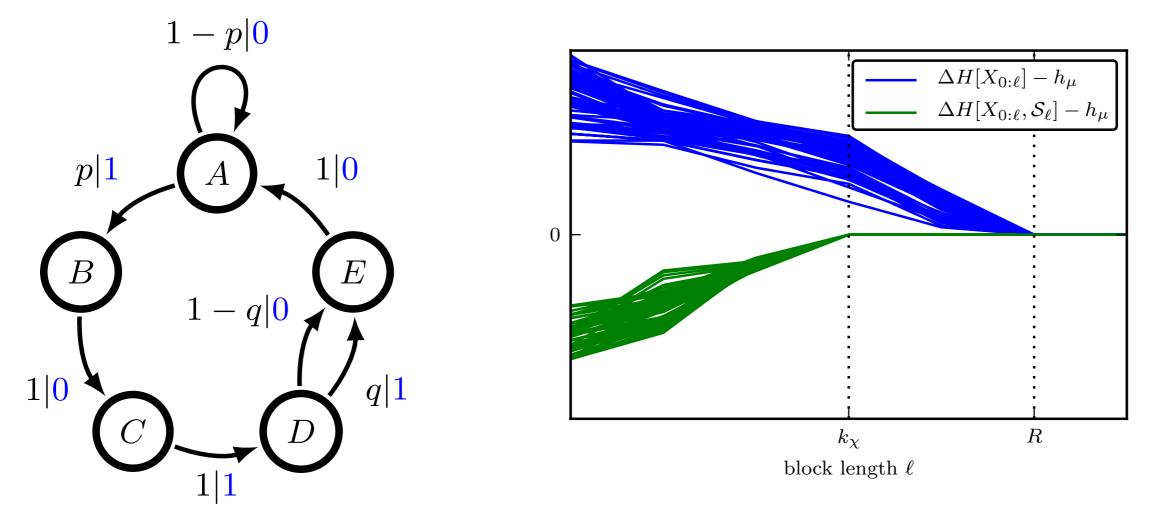






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A Hint of a Solution

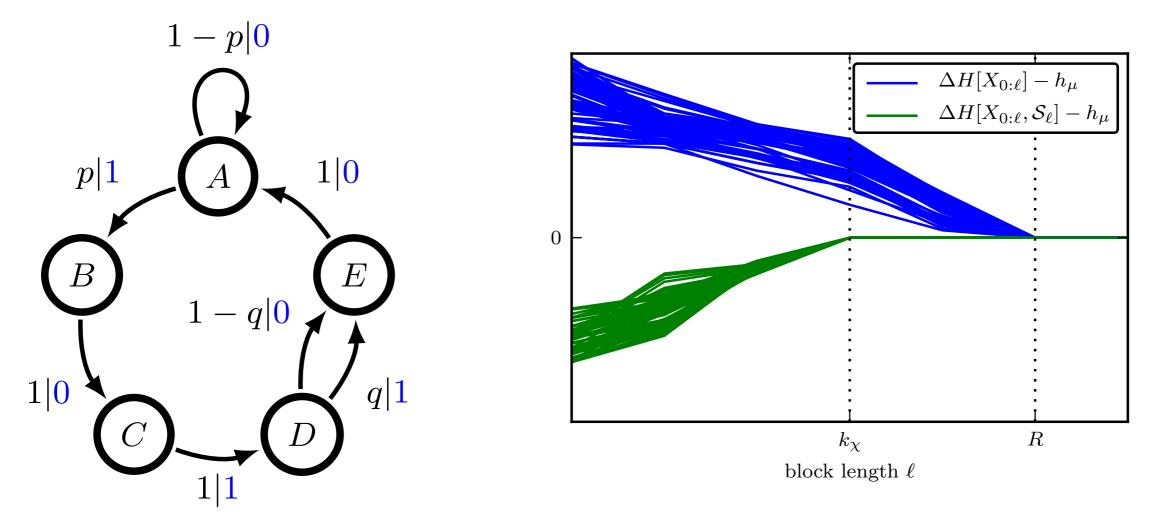


Markov and cryptic orders are *independent* of the probabilities!

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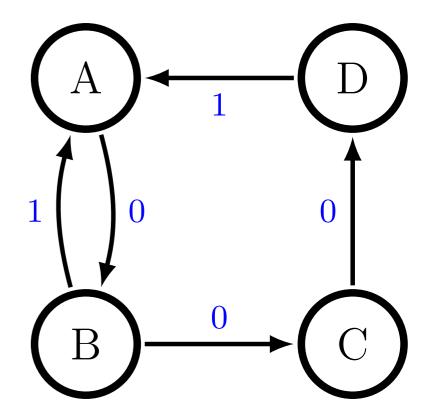
A Hint of a Solution



Markov and cryptic orders are *independent* of the probabilities! They depend only on the *topology* of the ϵ -machine!

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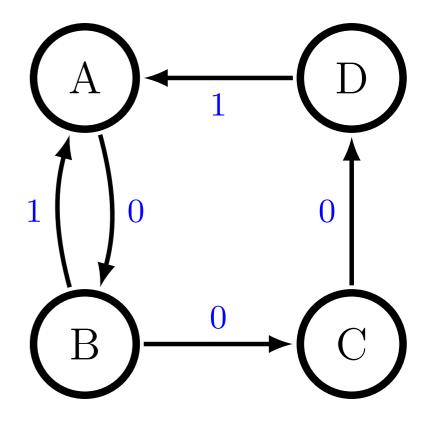
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Walking Pa	aths		





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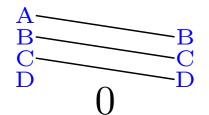


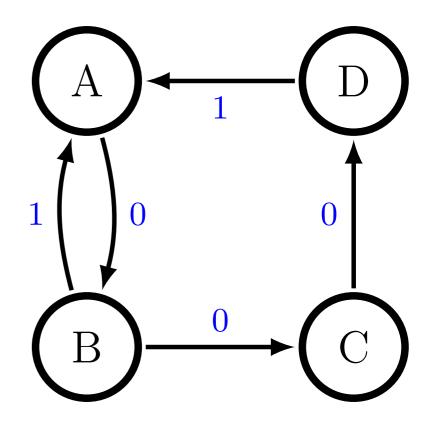


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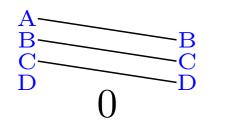
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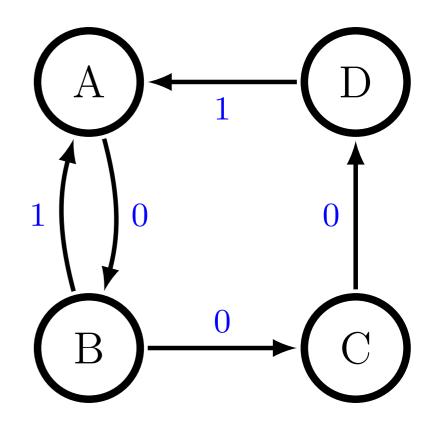




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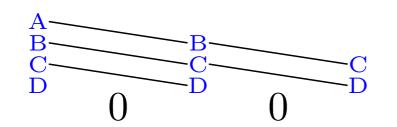


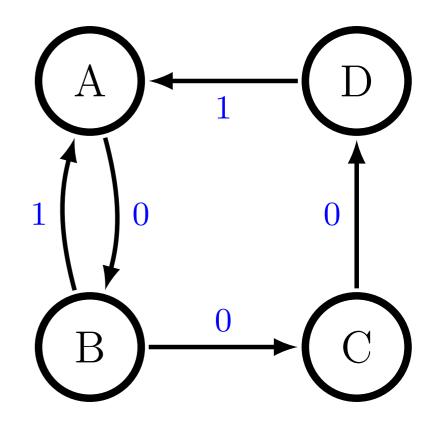
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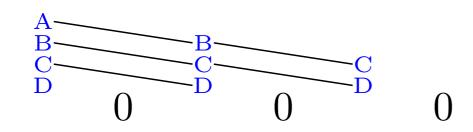
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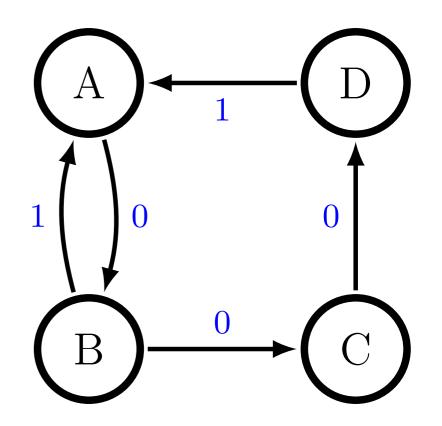






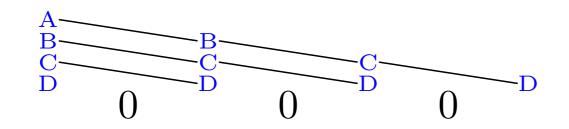
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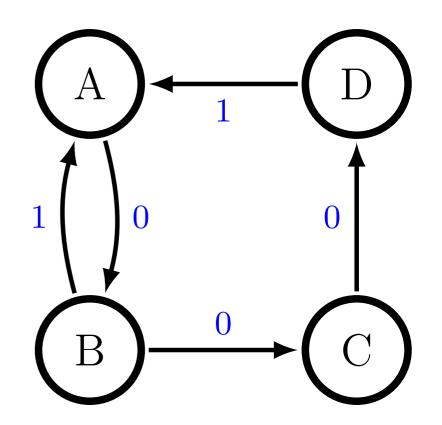






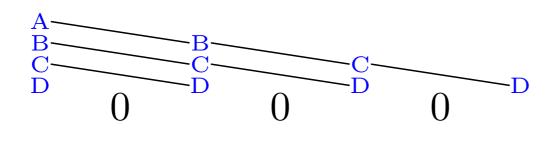
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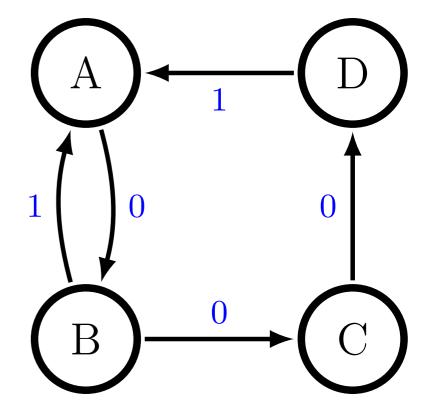




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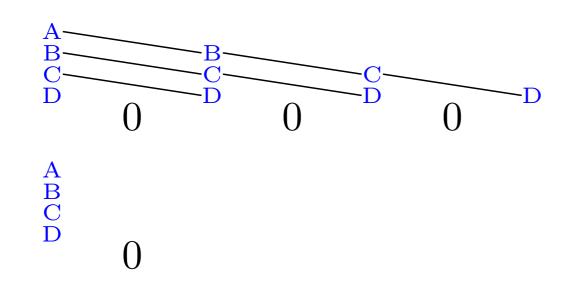


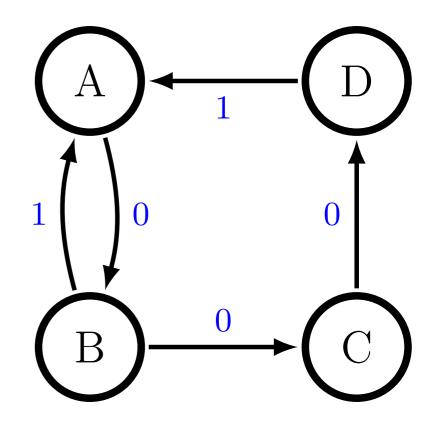






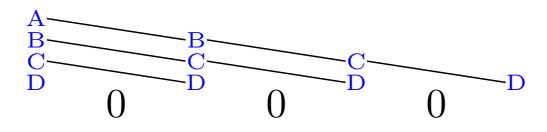
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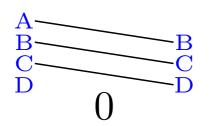


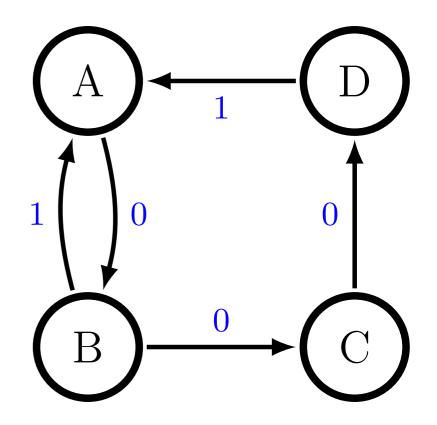




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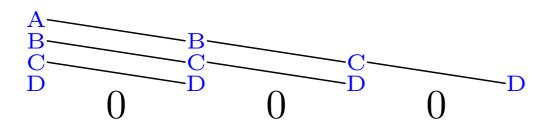


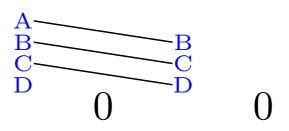


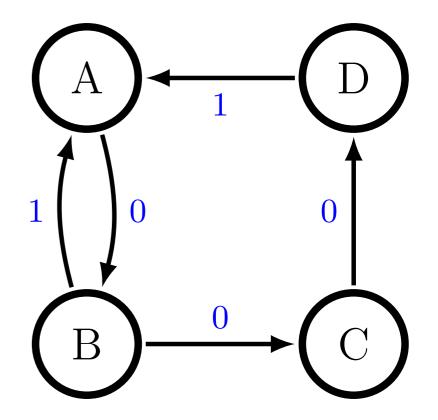




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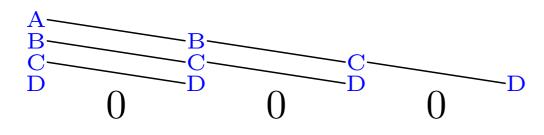


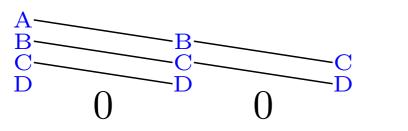


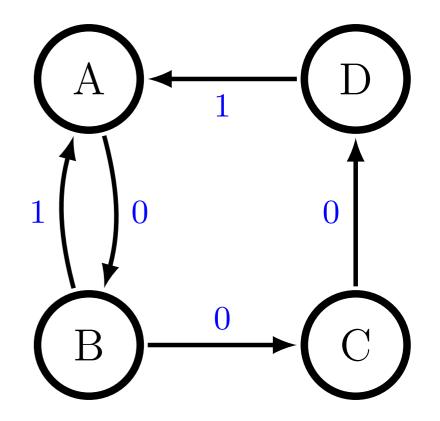




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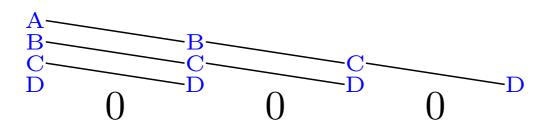


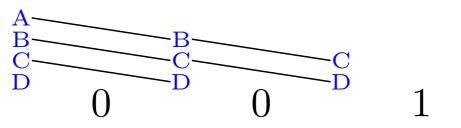


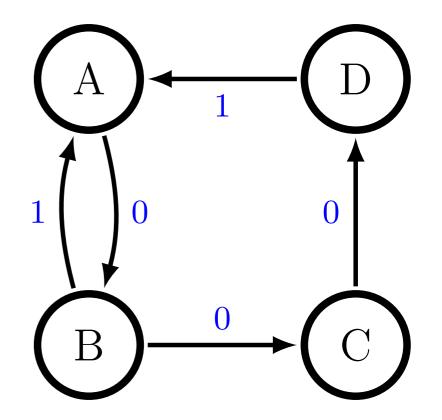




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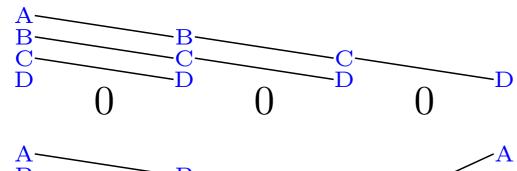


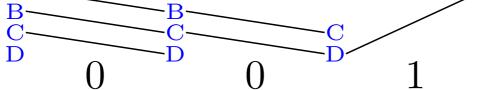


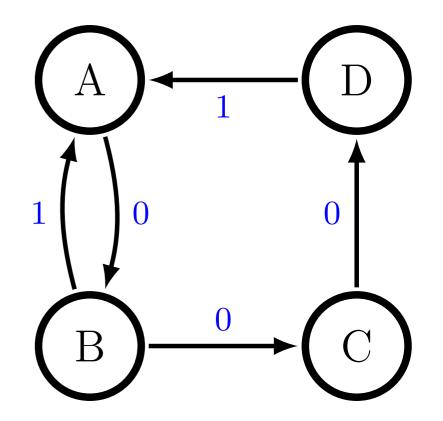




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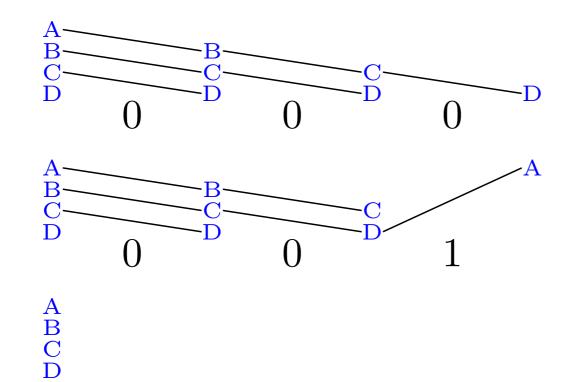


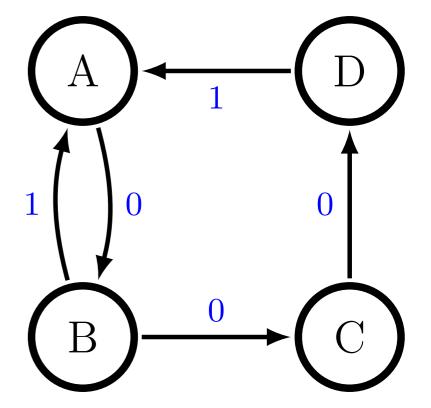






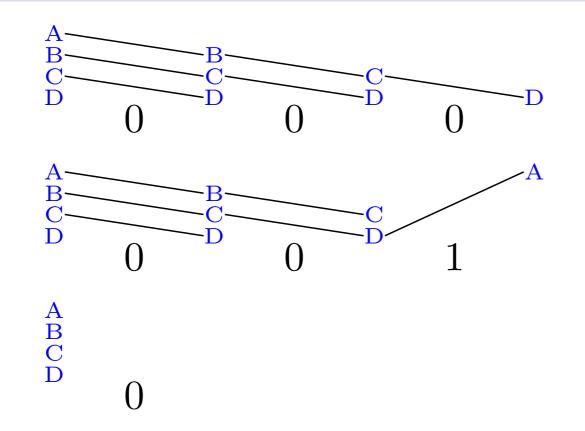
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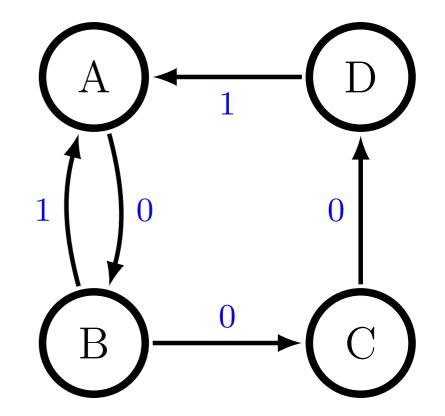






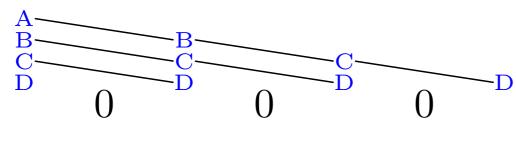
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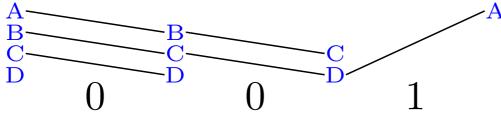


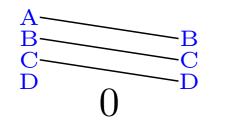


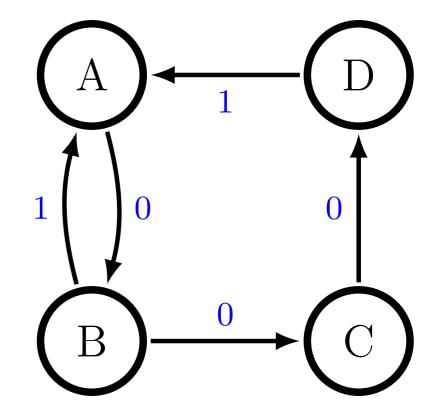


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Synchronization			



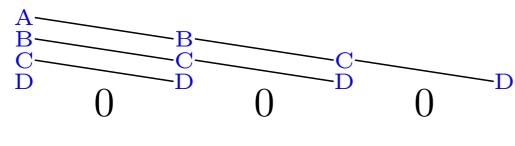


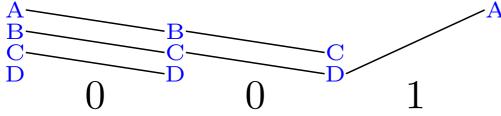


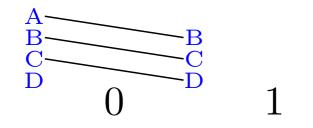


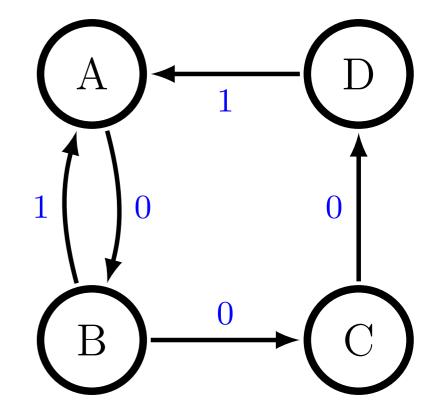


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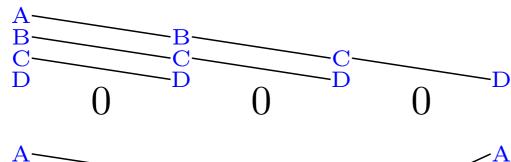


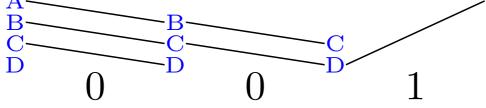


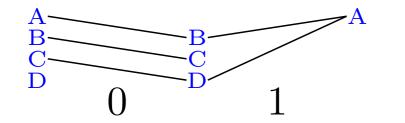


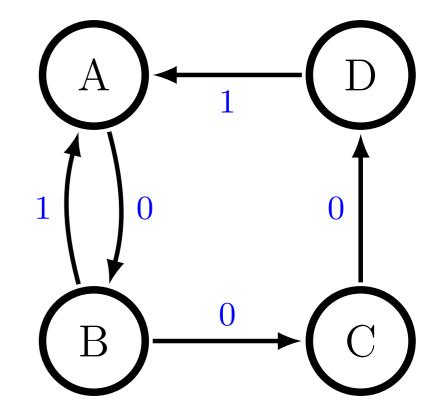


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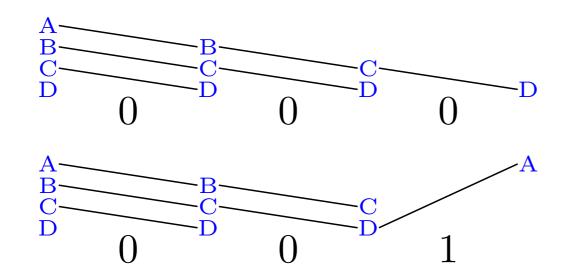


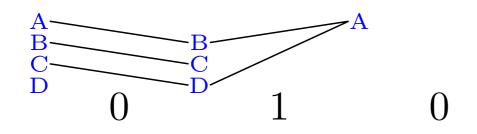


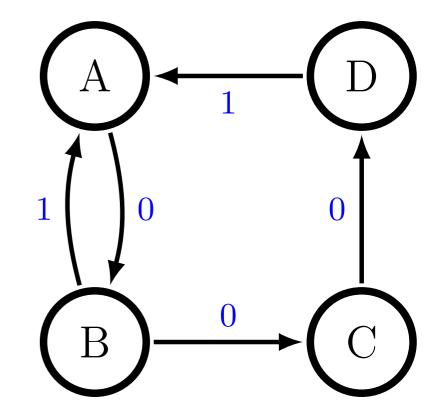




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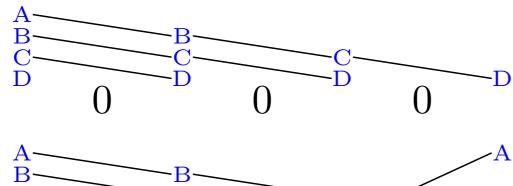


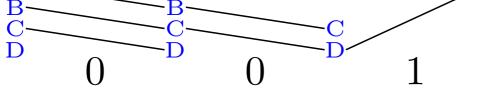


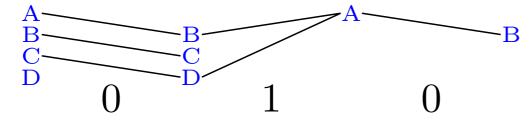


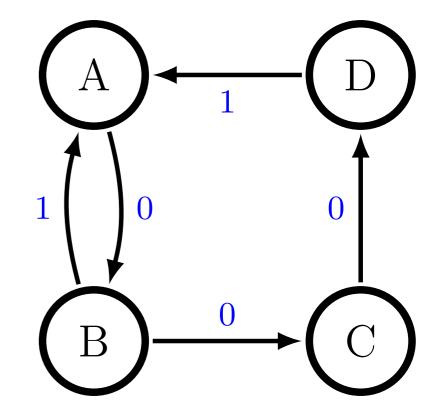


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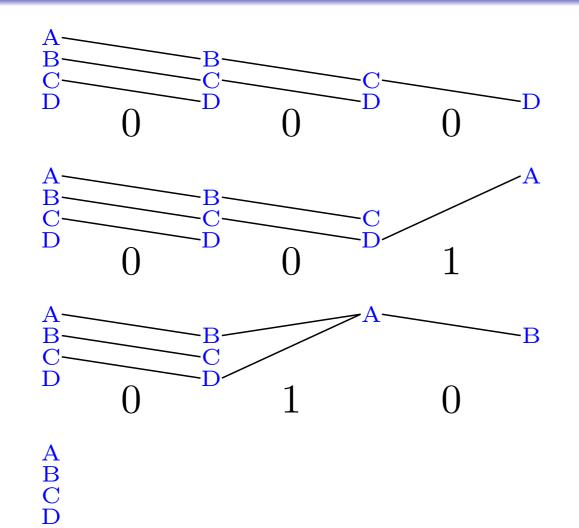


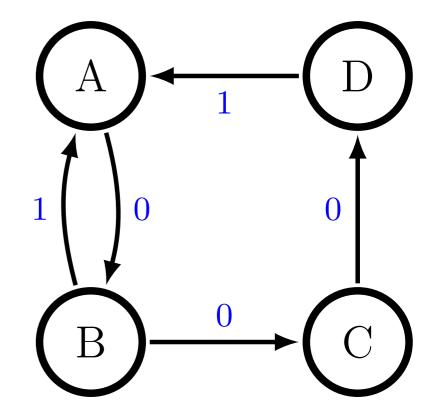






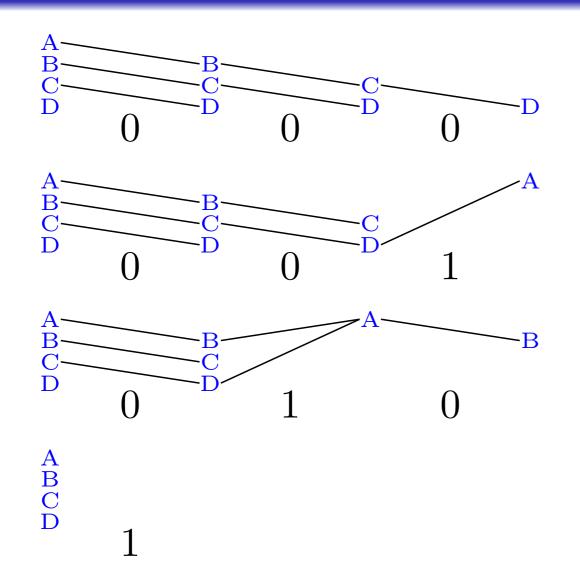
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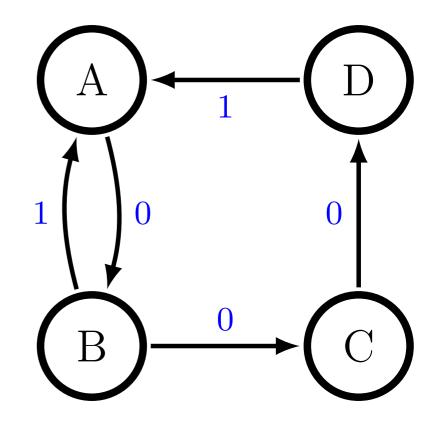






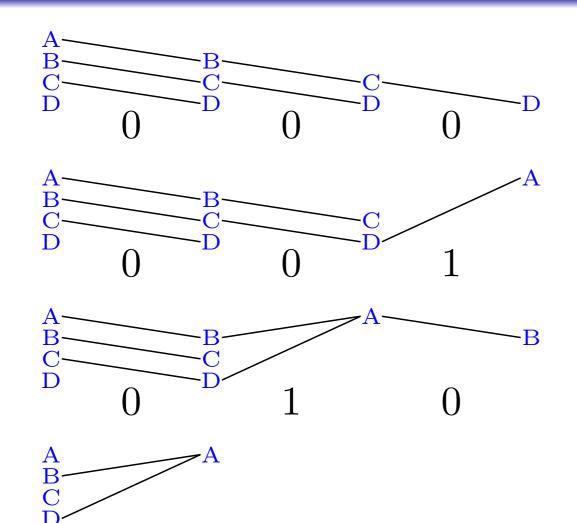
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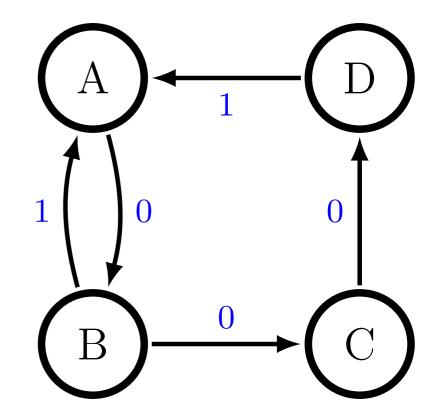






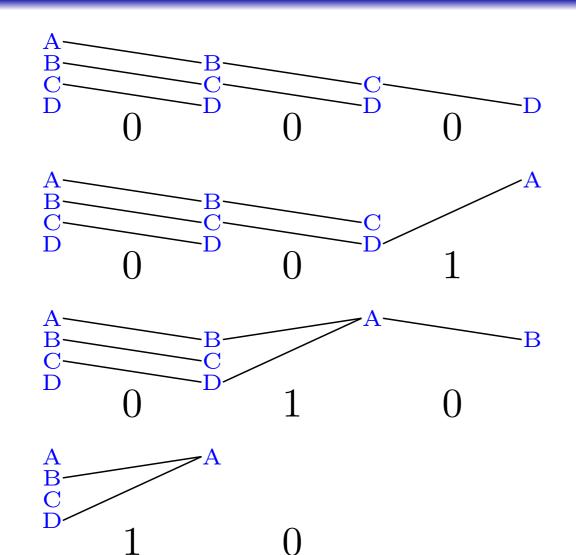
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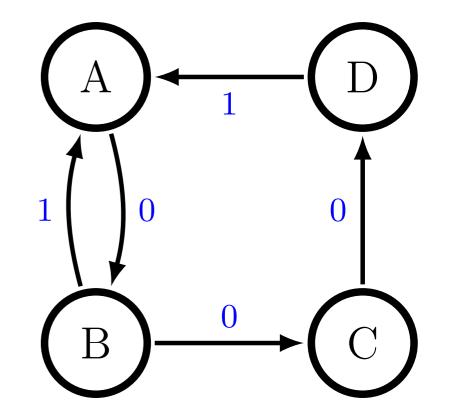






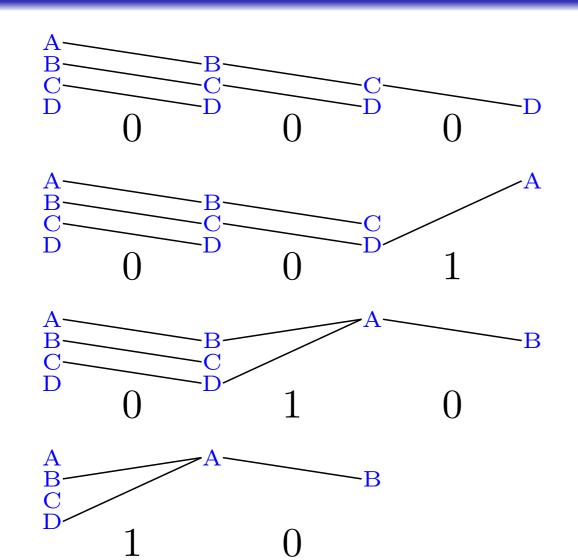
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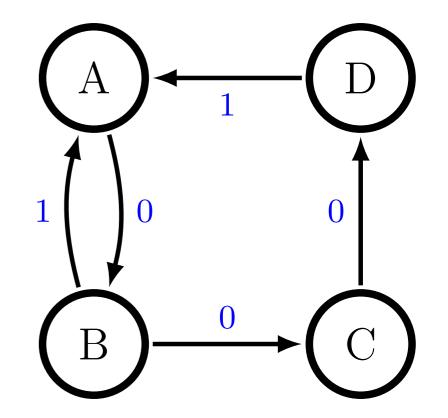






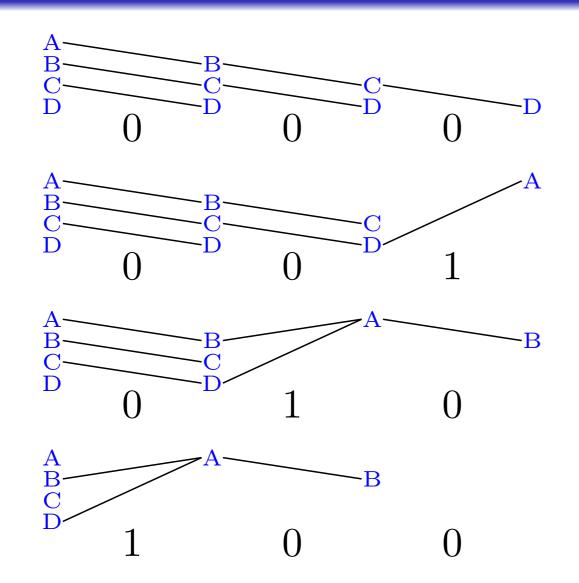
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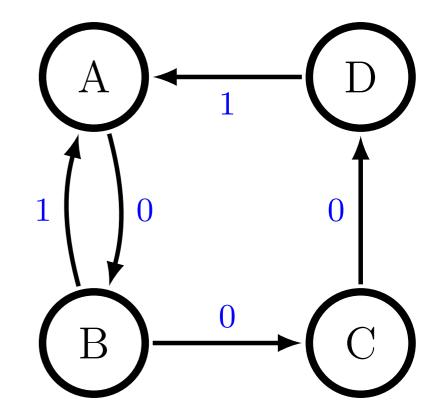






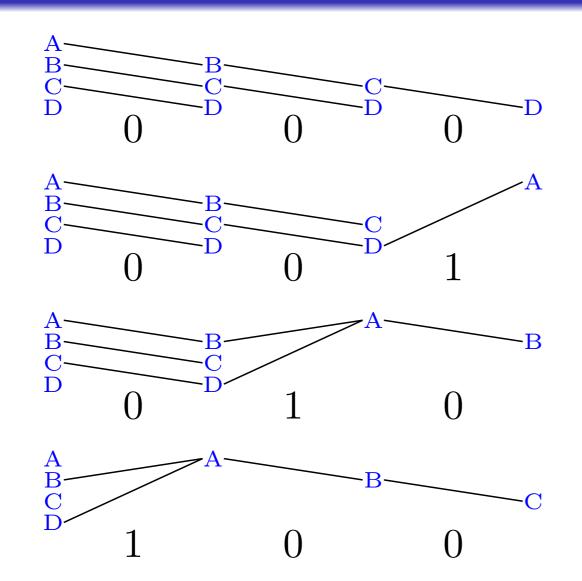
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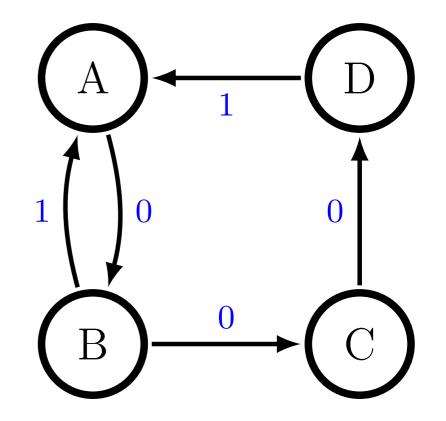






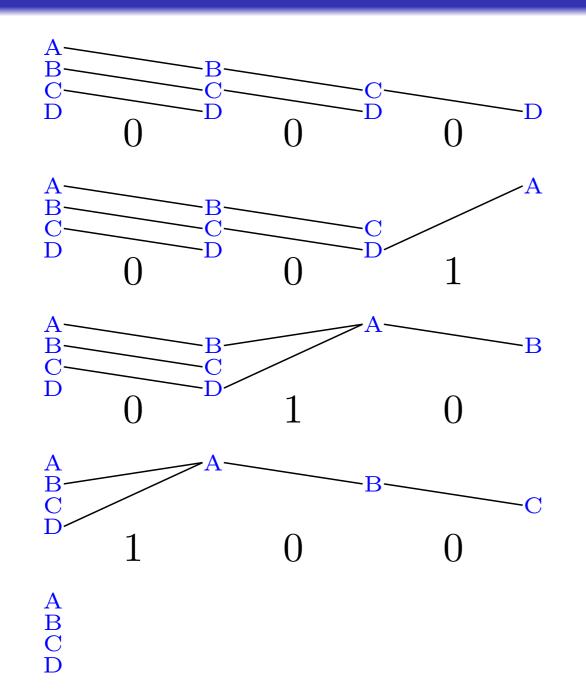
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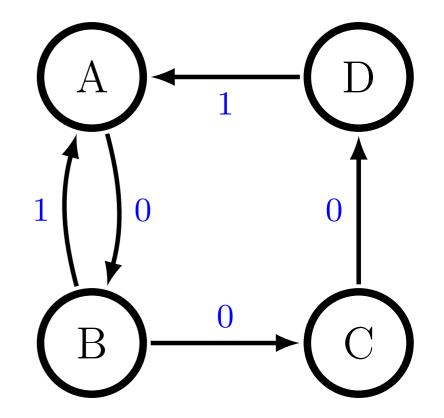






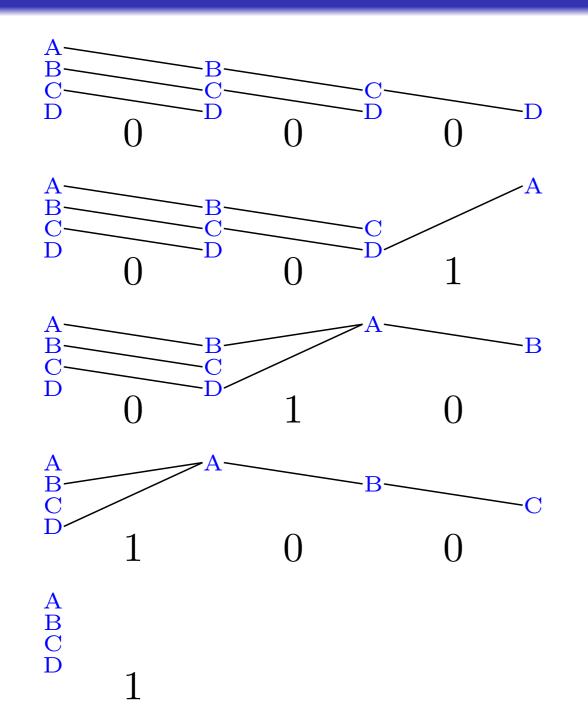
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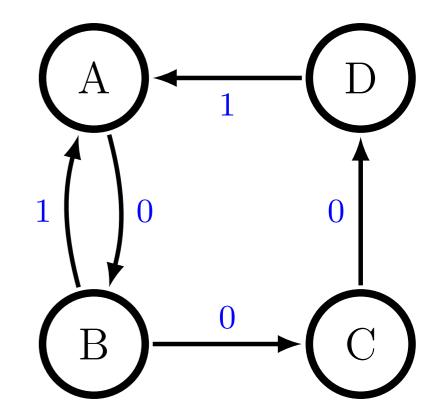






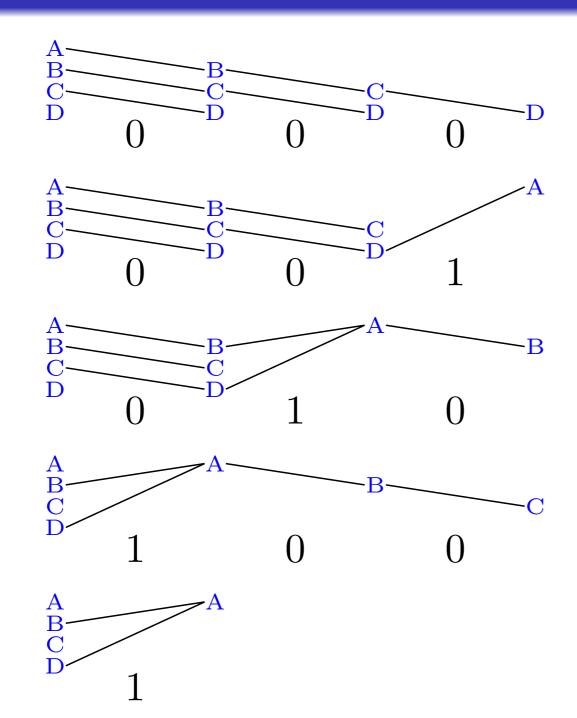
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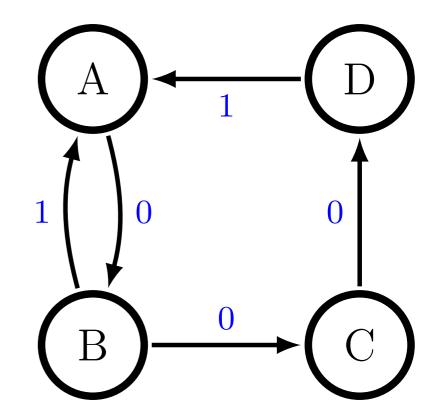






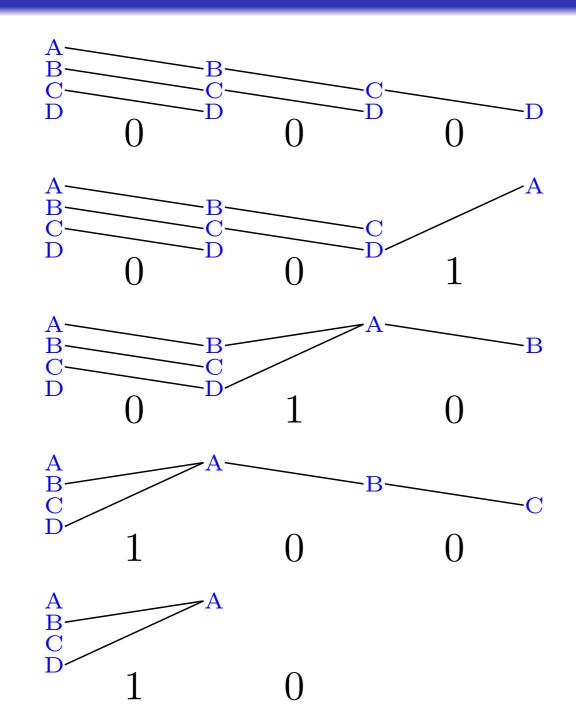
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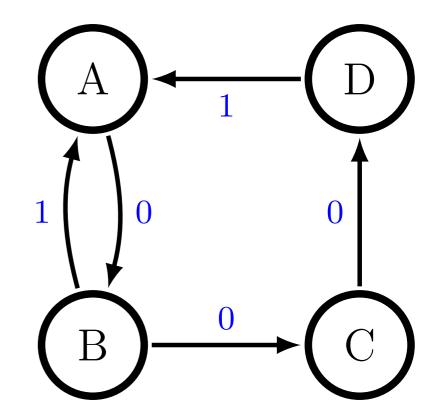






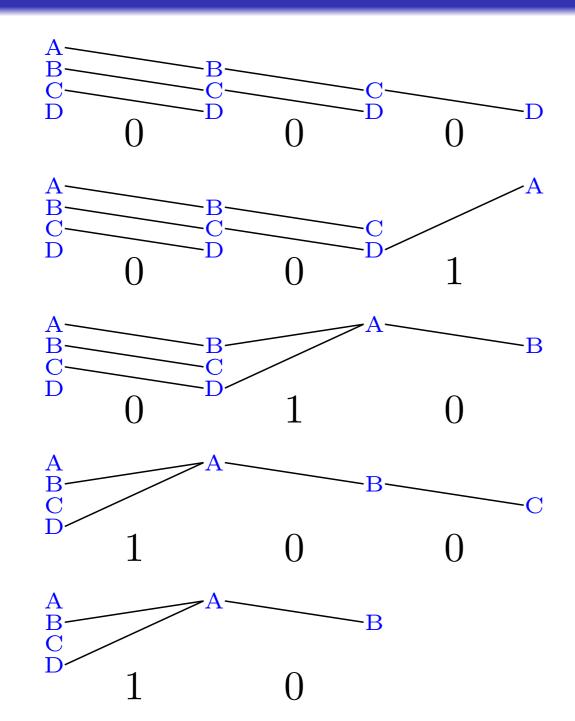
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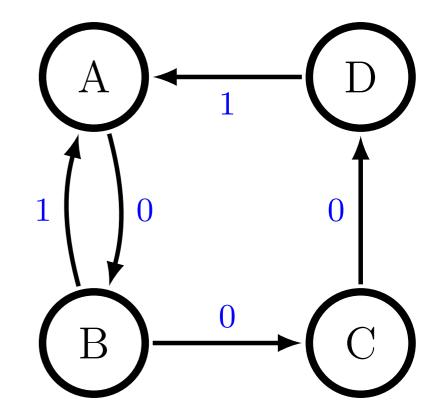






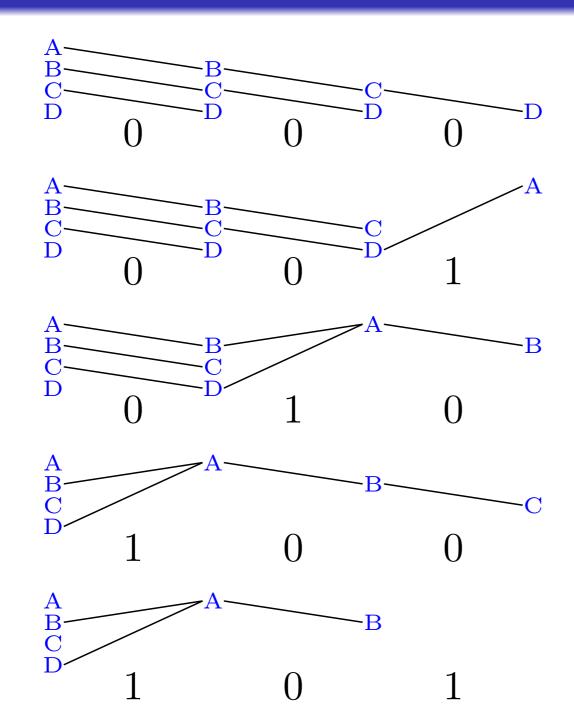
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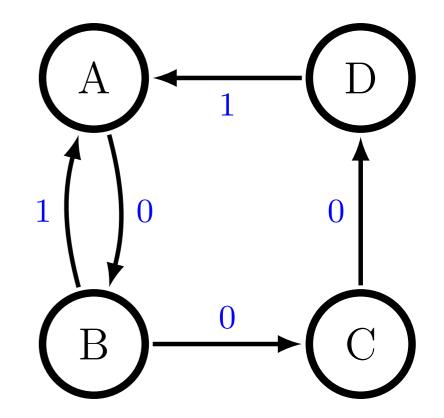






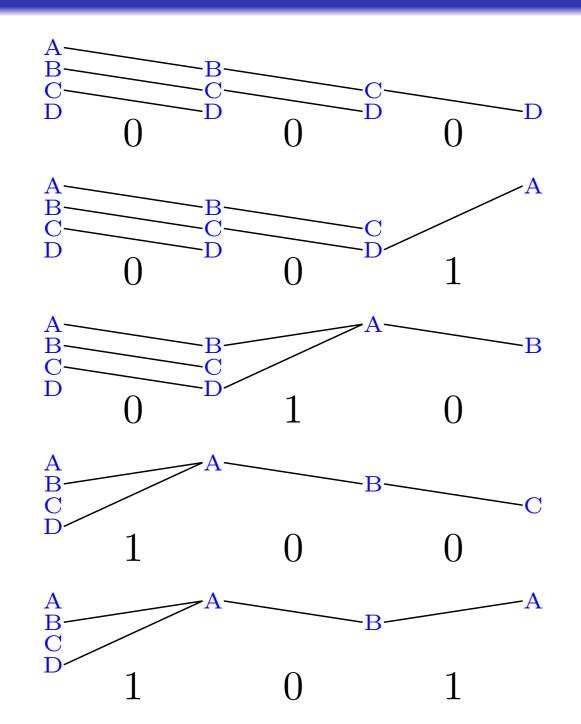
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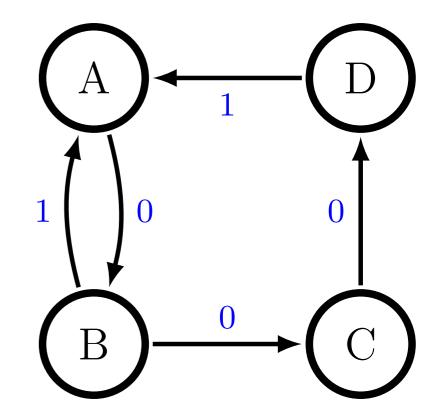






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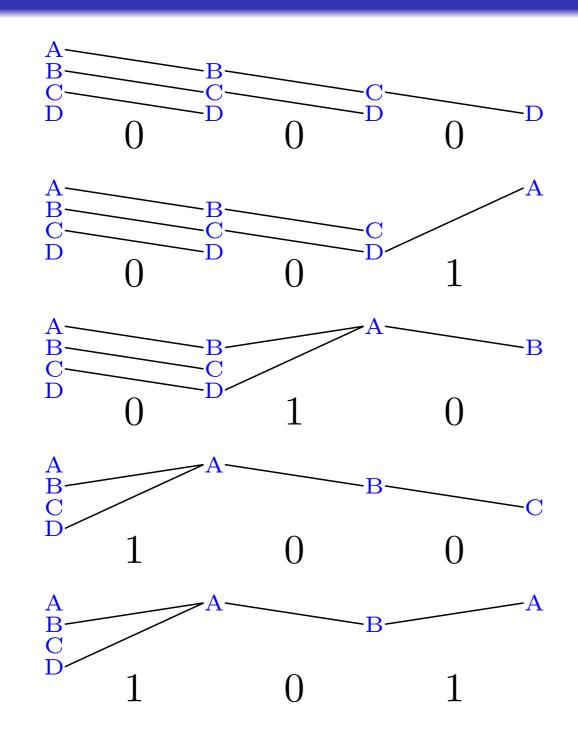




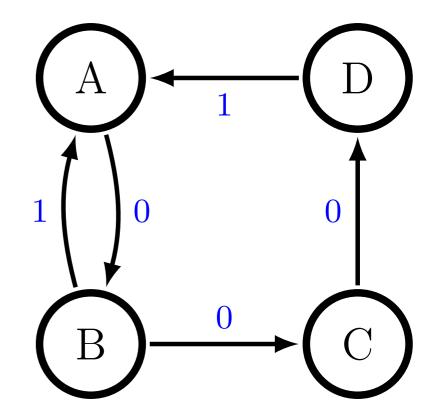


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Markov Order			

Are We There Yet?



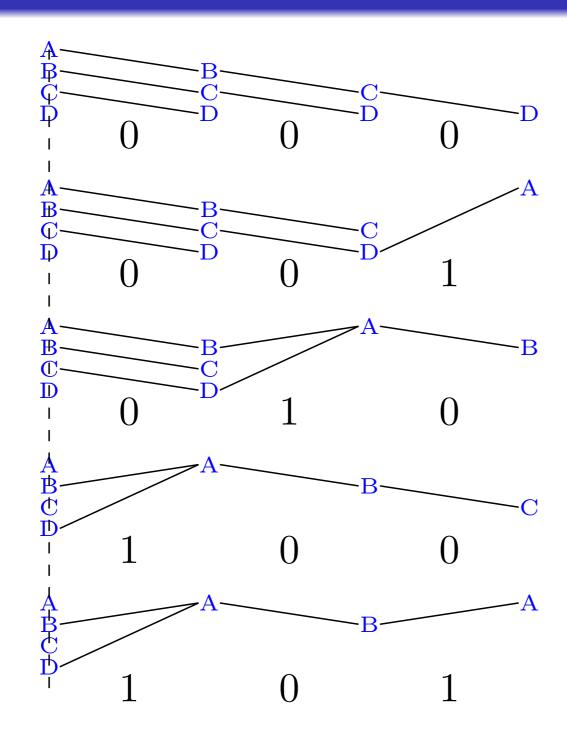
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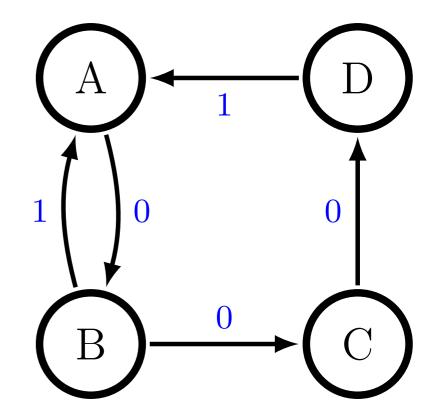


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Markov Order			

Are We There Yet?

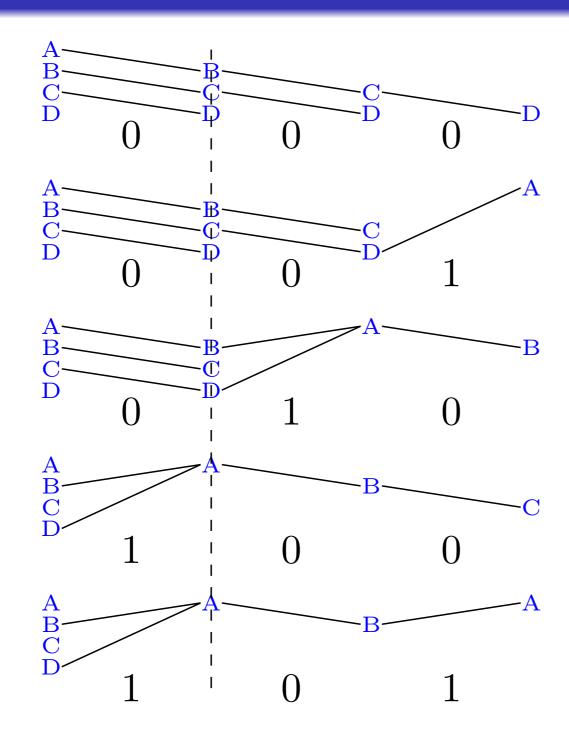


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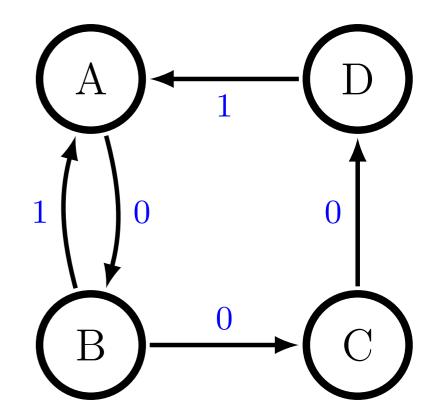




Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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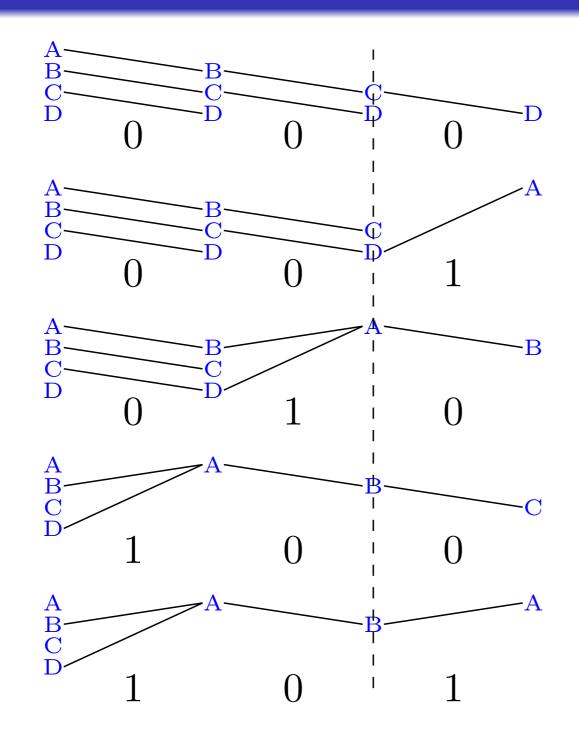


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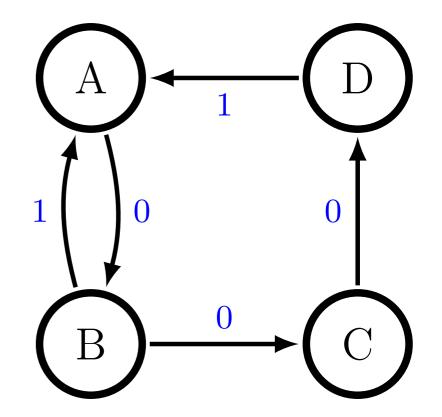




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Markov Order			

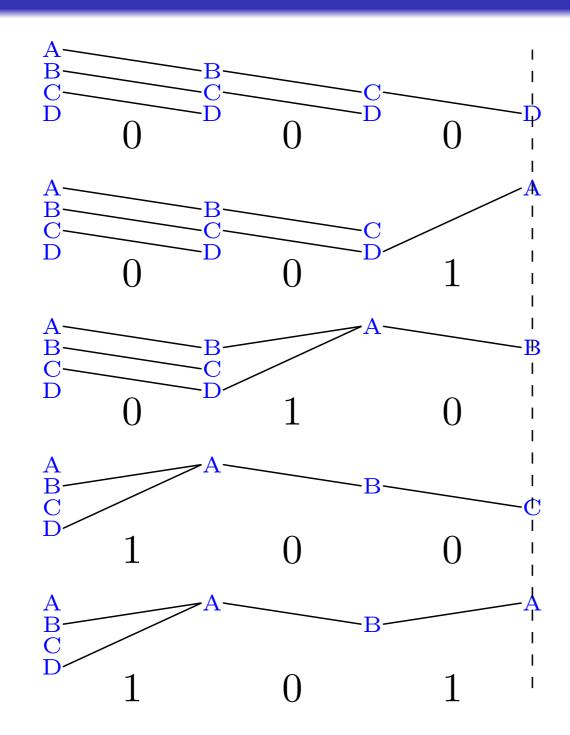


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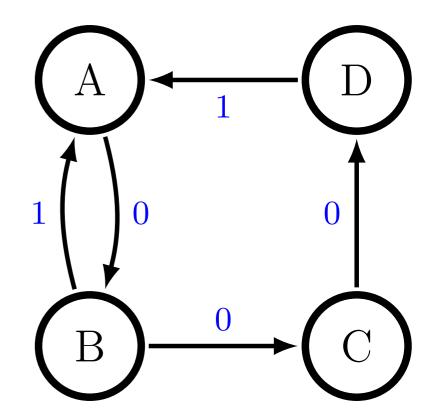




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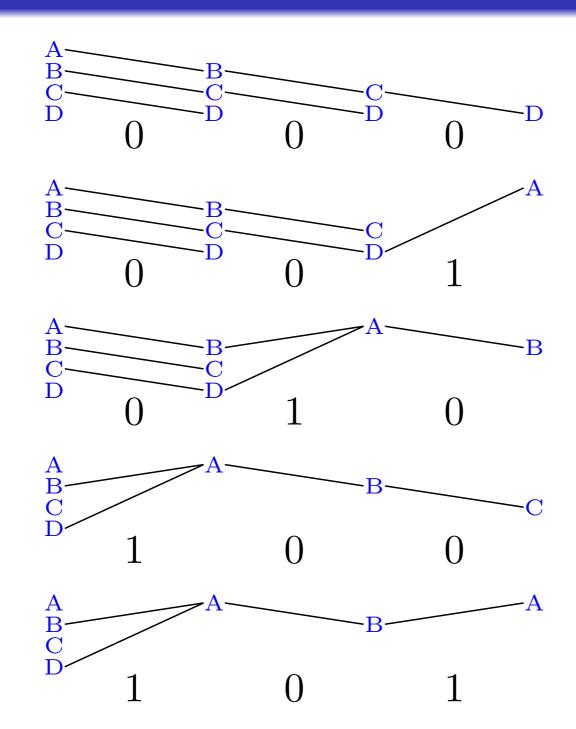


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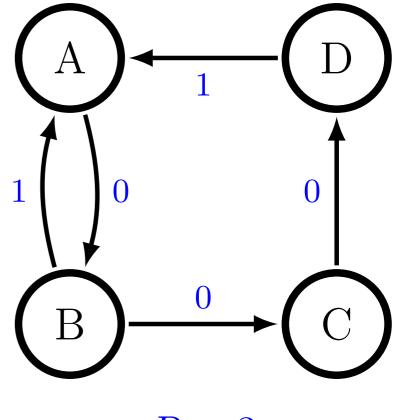




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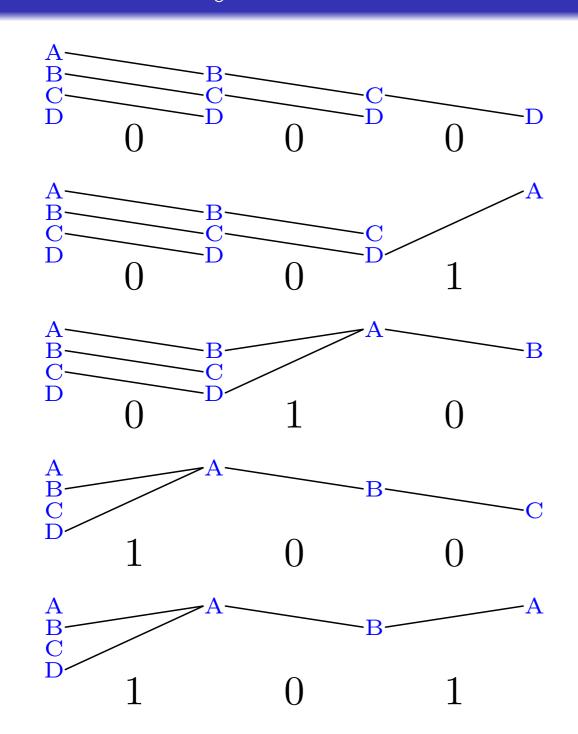
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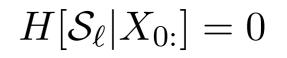


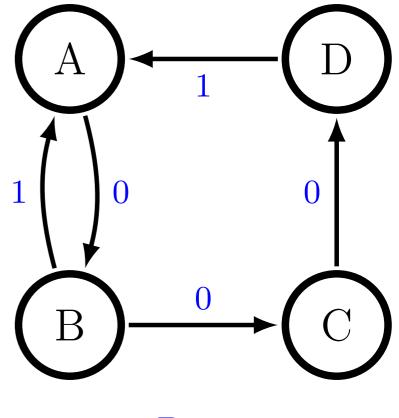
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Cryptic Order			



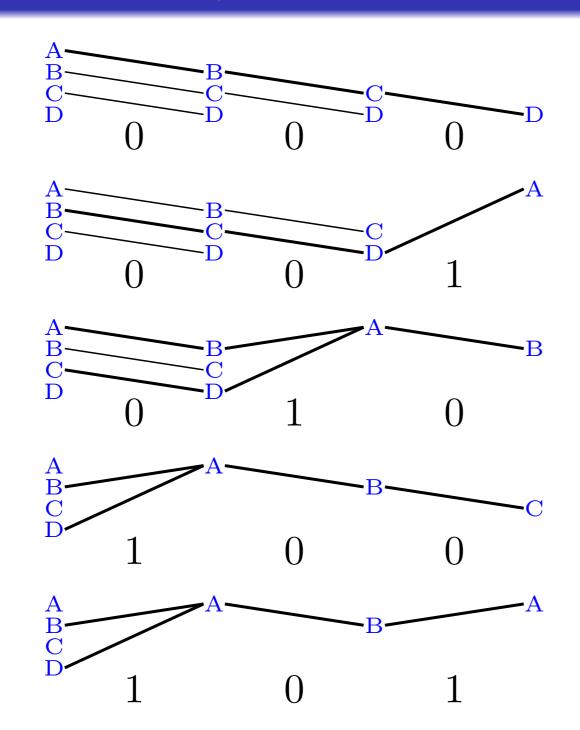


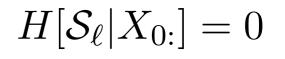


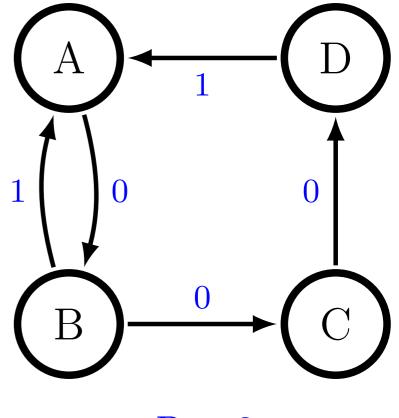
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Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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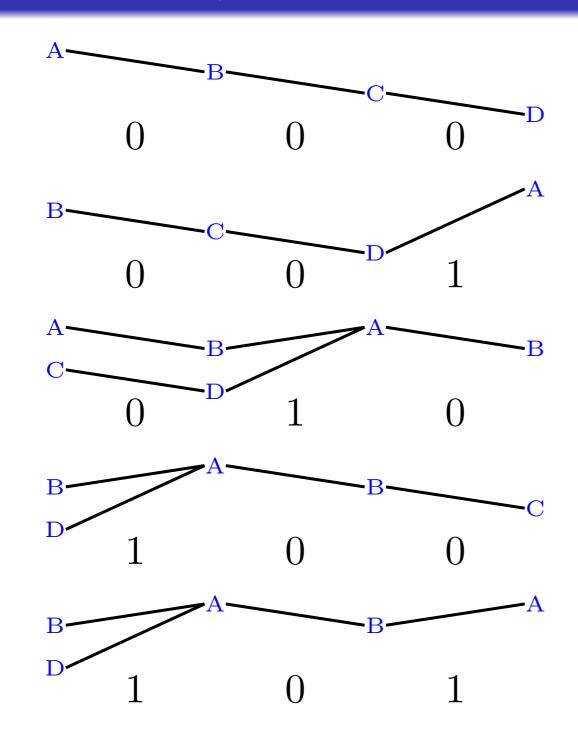




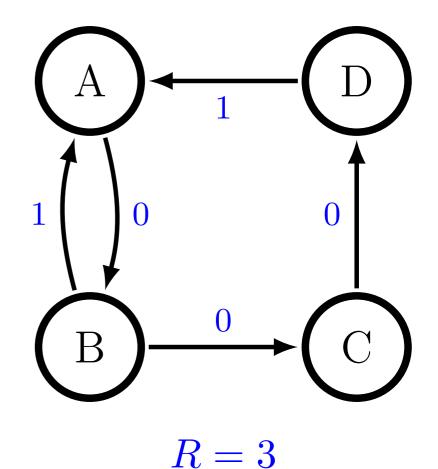
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Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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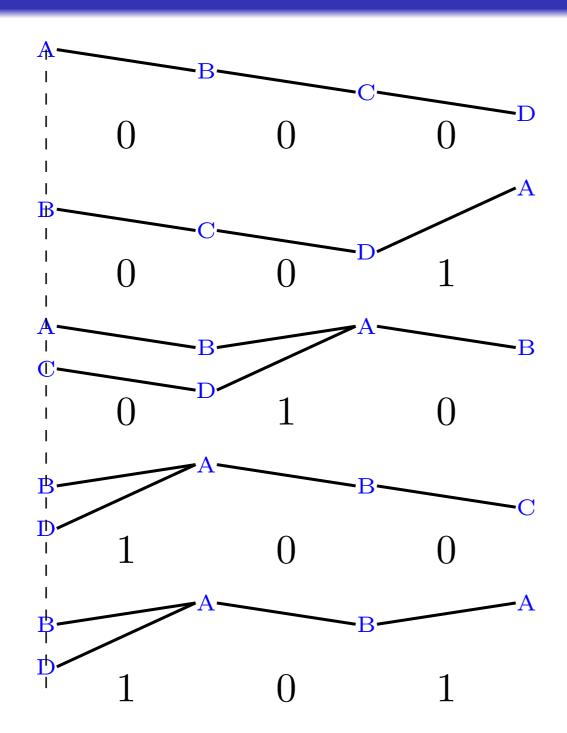


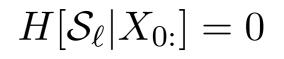
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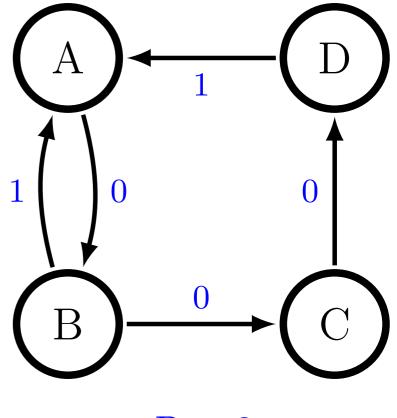




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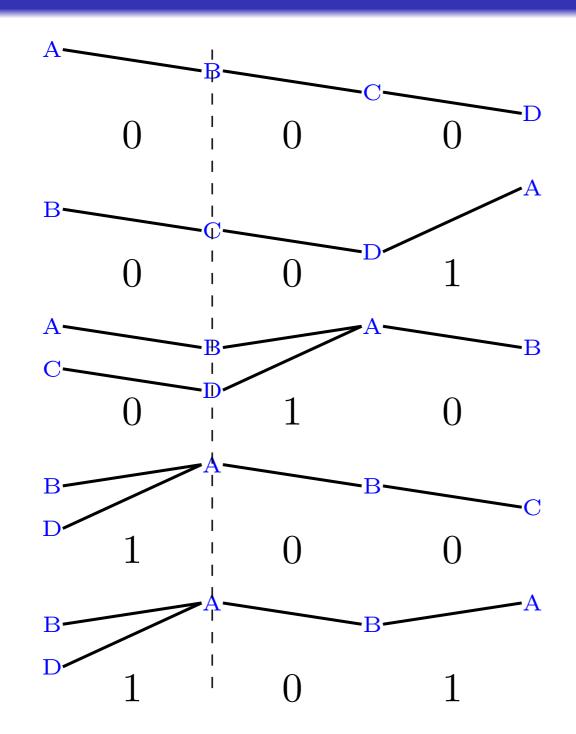




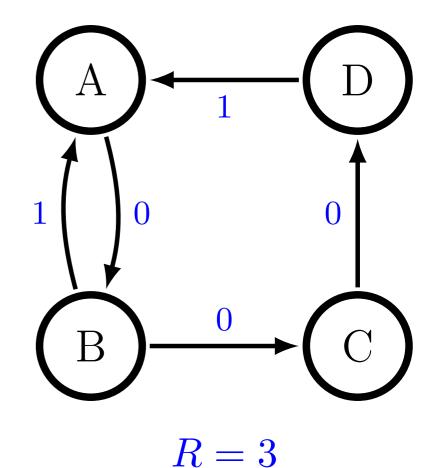
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Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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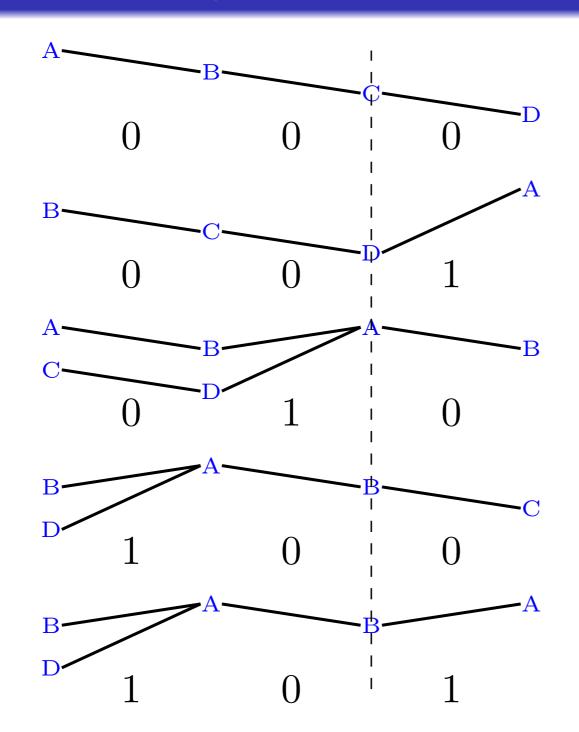


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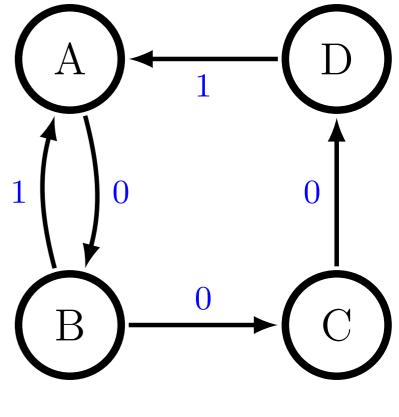




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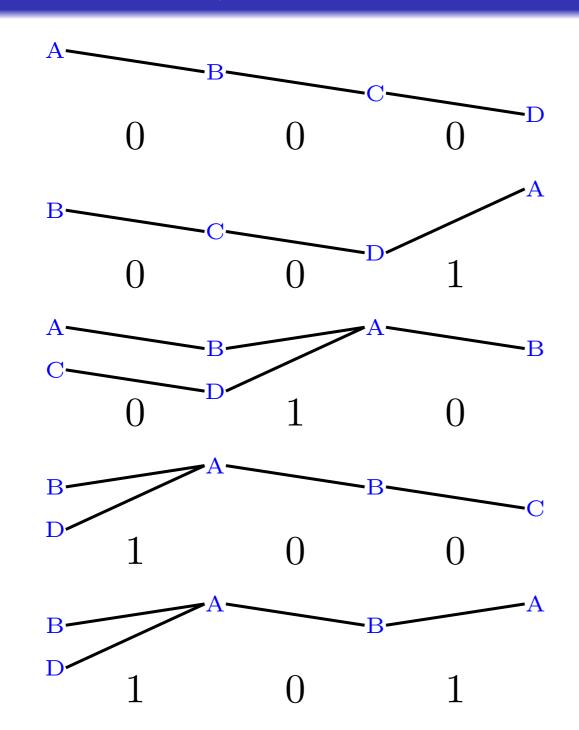
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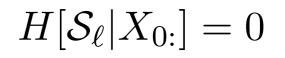


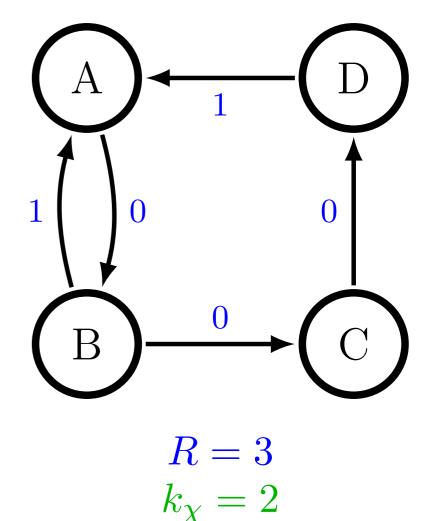
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Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Discussion			
Improvemen	ts		



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Discussion			
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• Don't need \mathbf{E} or h_{μ}



Introduction 000000	Via Synchronizing Words 000	Via Topology 00000000	Results & Survey 00
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- Don't need \mathbf{E} or h_{μ}
- Integer based, so don't need to worry about machine precision



Introduction 000000	Via Synchronizing Words $000 \bullet$	Via Topology 00000000	Results & Survey 00
Discussion			
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- Don't need **E** or h_{μ}
- Integer based, so don't need to worry about machine precision
- But ... There could be an arbitrary number of synchronizing words, each of arbitrary length

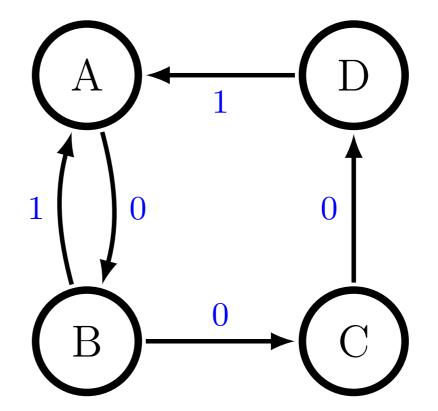


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Improvemen	ts		

- Don't need **E** or h_{μ}
- Integer based, so don't need to worry about machine precision
- But ... There could be an arbitrary number of synchronizing words, each of arbitrary length
- Can we do better?

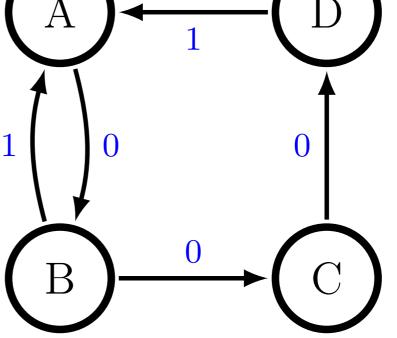


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Synchronizing Word	ls		
Powersets			



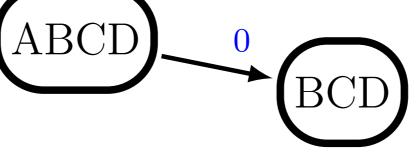


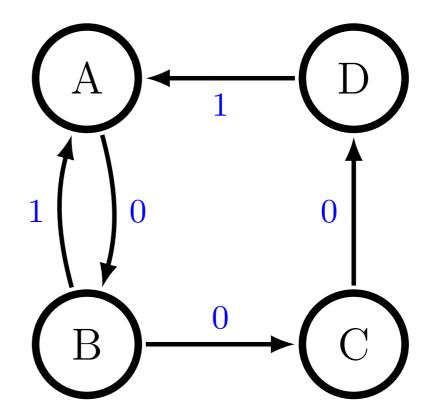
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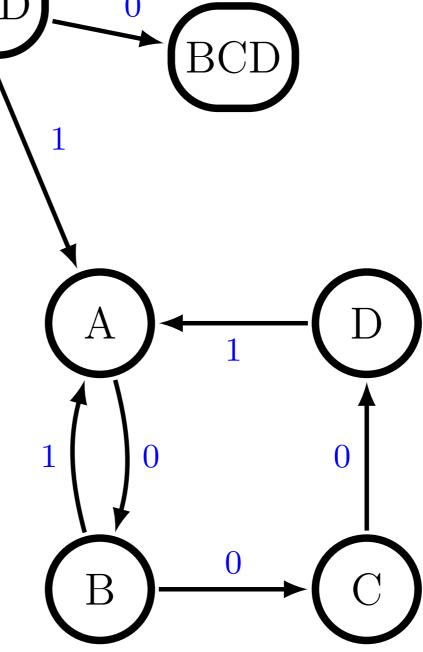
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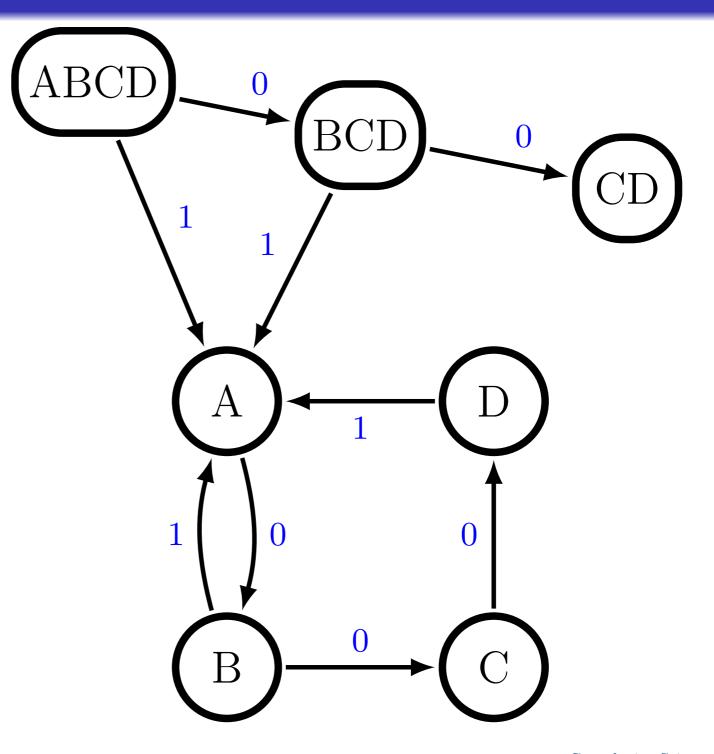
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Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

Complexity Sciences Center

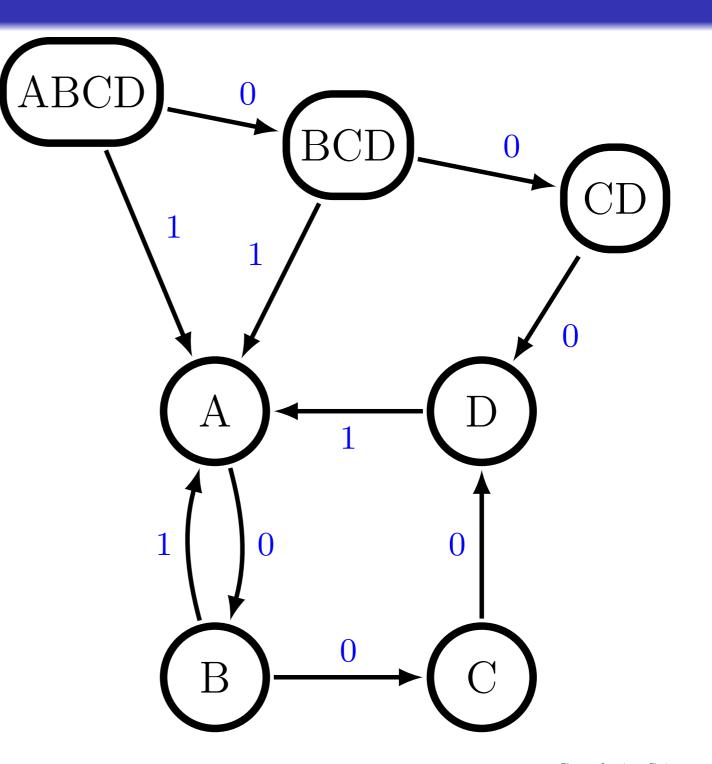
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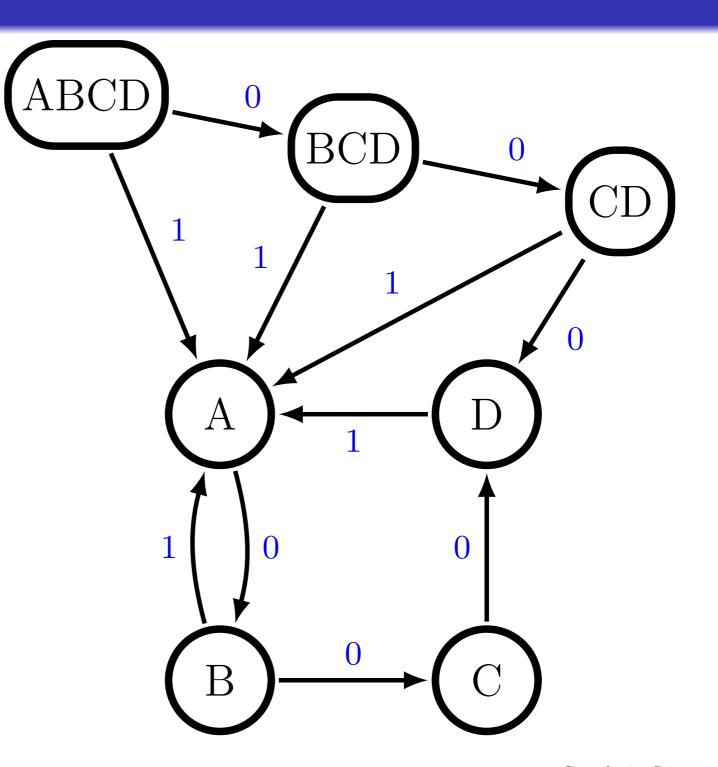


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Synchronizing Words			
Powersets			



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Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

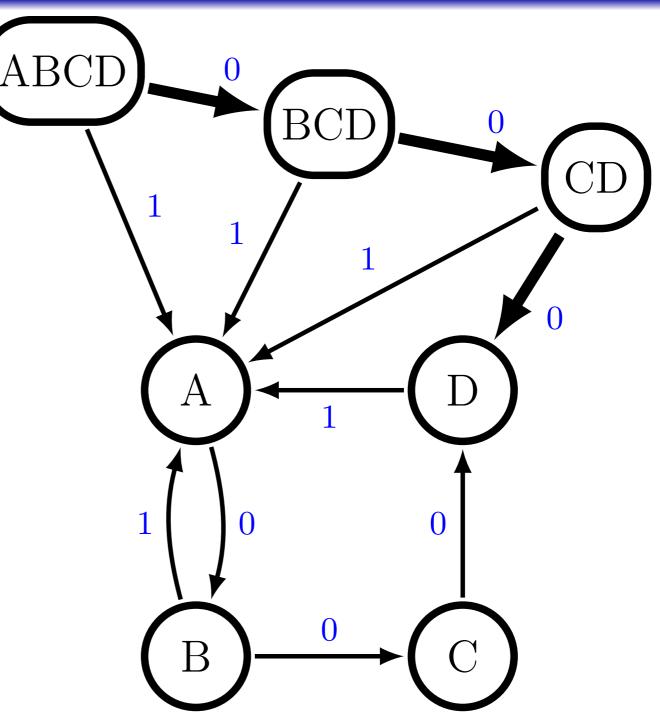
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Synchronizing Words:

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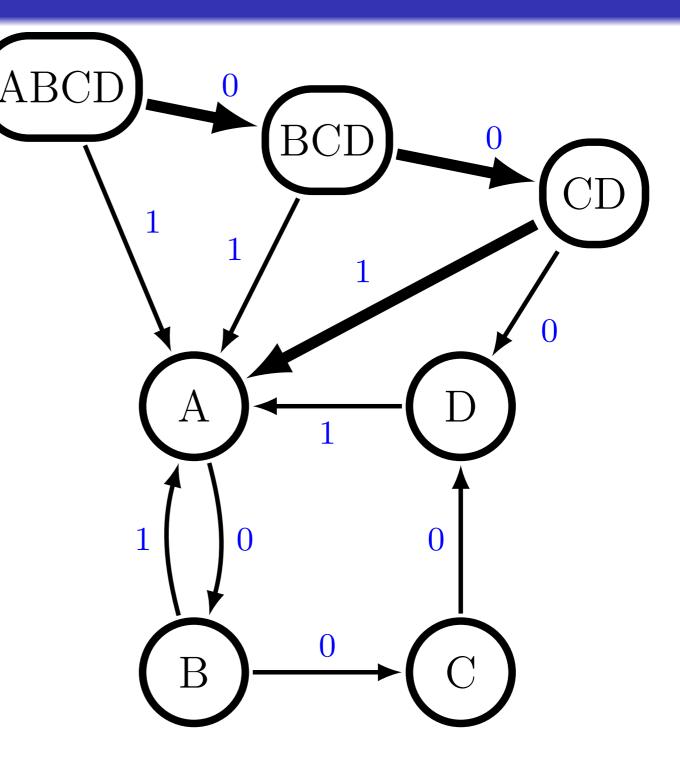




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Synchronizing Words:

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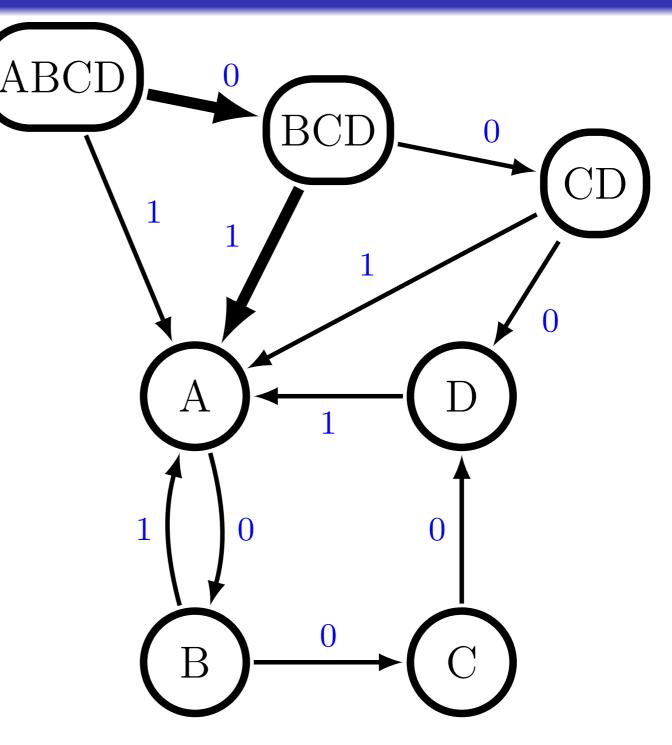


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Synchronizing Words			

Powersets

Synchronizing Words:

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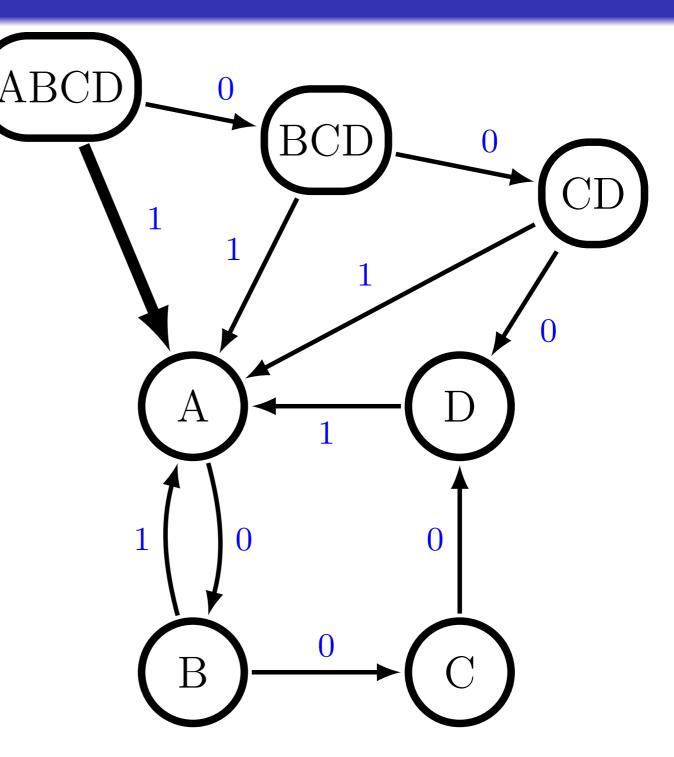




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Synchronizing Words			
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Synchronizing Words:

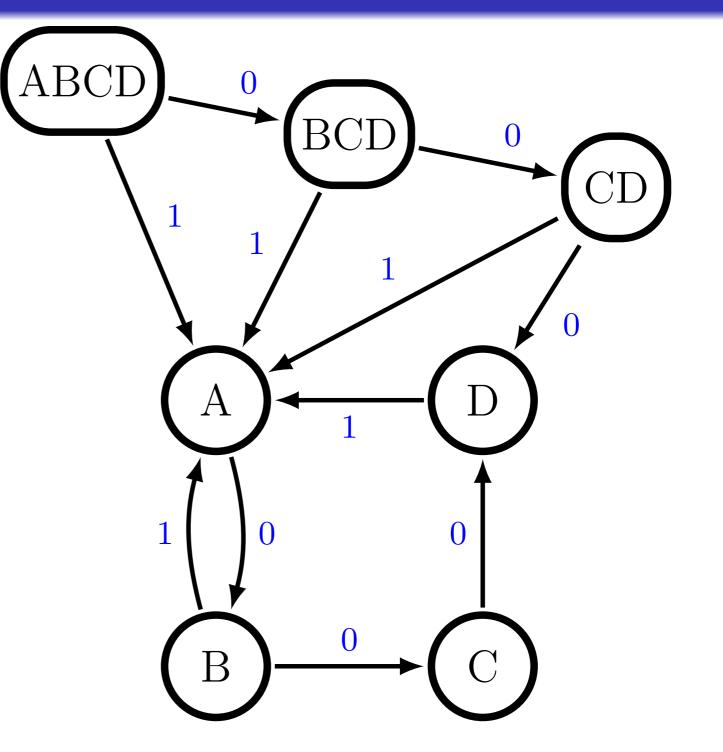
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Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

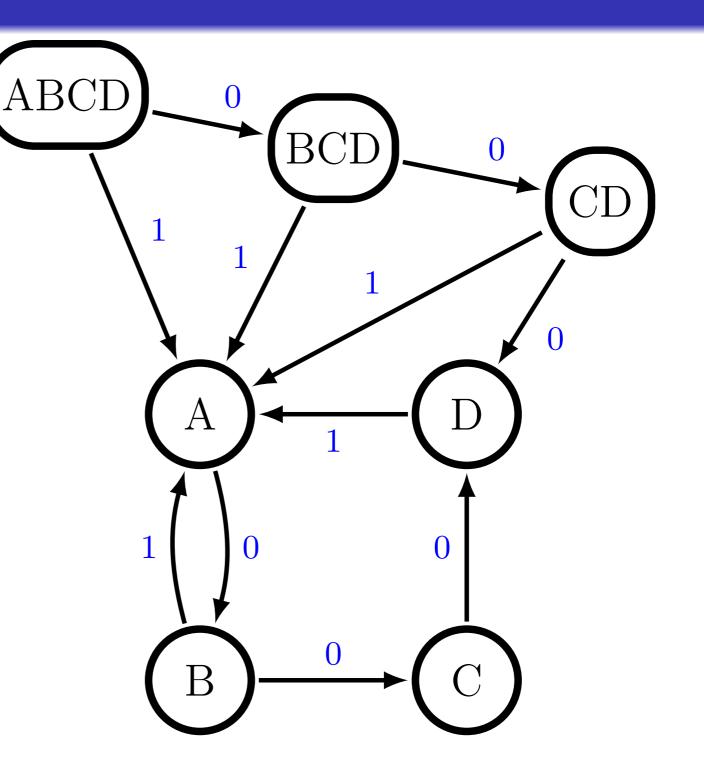
• R is the longest path from start to recurrent





Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

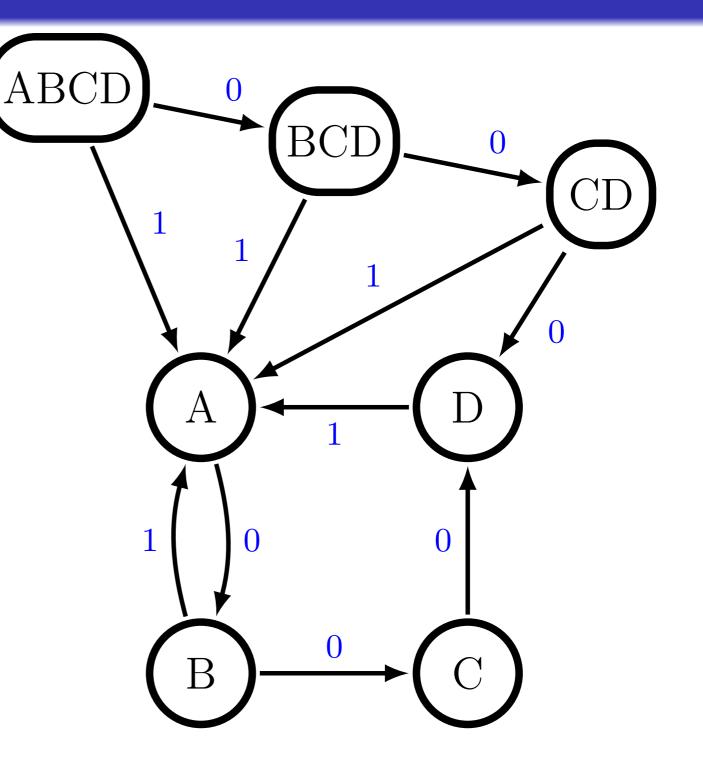
- R is the longest path from start to recurrent
- Min-weight path is easy: Bellman-Ford





Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

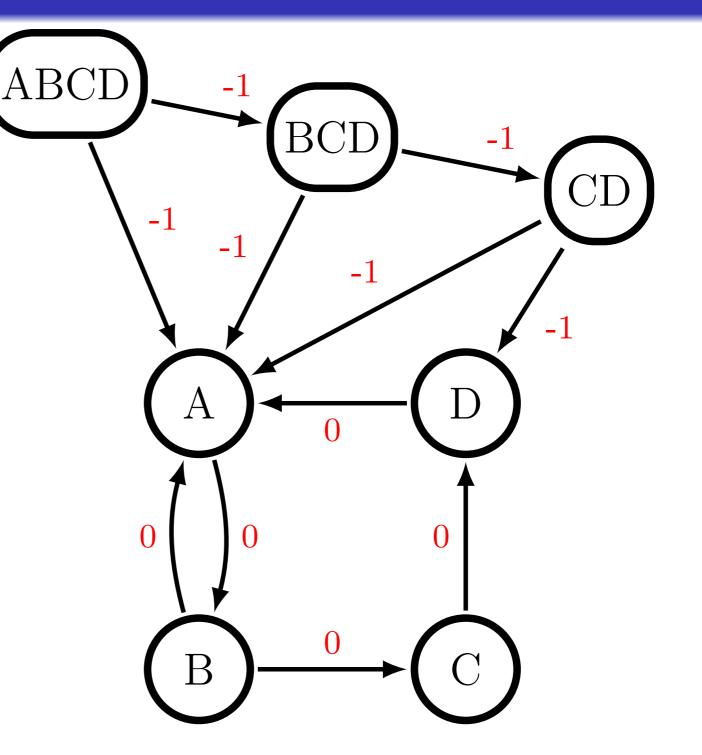
- R is the longest path from start to recurrent
- Min-weight path is easy: Bellman-Ford
- By weighting edges like so...





Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

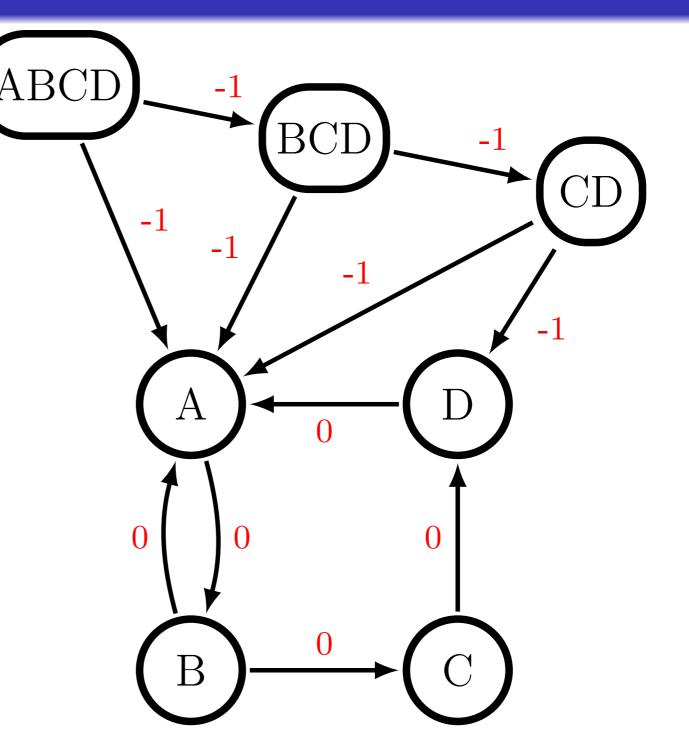
- R is the longest path from start to recurrent
- Min-weight path is easy: Bellman-Ford
- By weighting edges like so...





Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

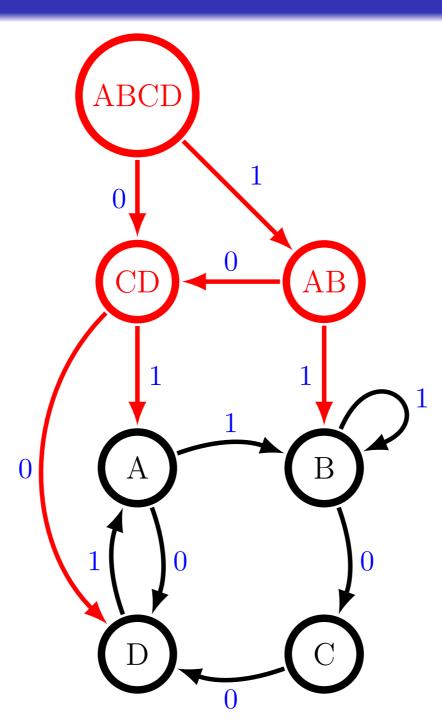
- R is the longest path from start to recurrent
- Min-weight path is easy: Bellman-Ford
- By weighting edges like so...
- Min-weight path from ABCD to A *is* longest path!





Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Markov Order			

Example the First

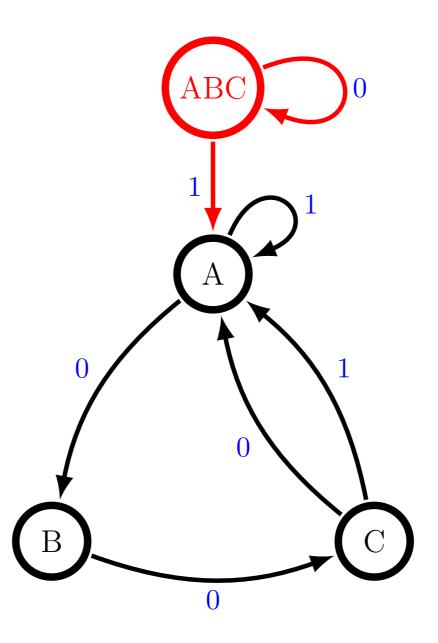


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Example the	e Second		
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Markov Order			
Example th	e Third		



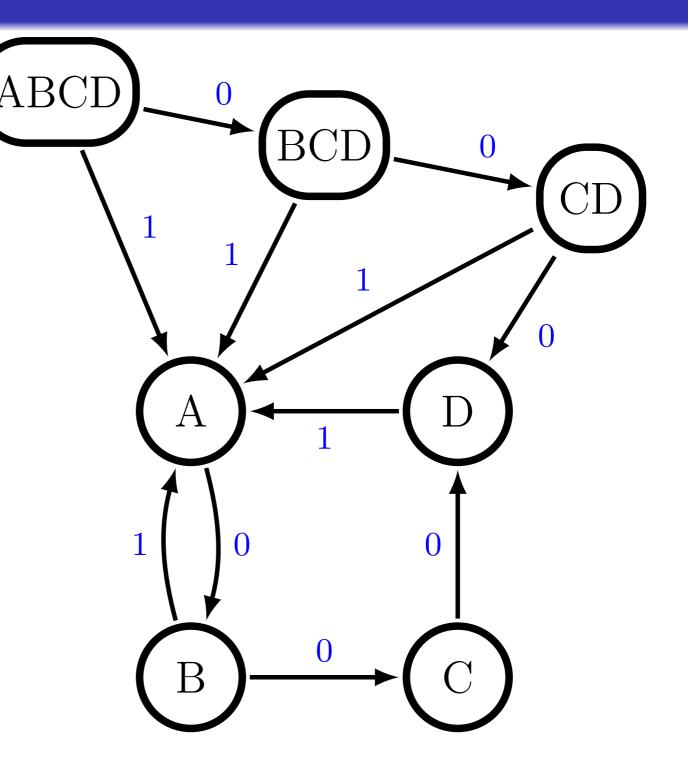


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			

Legend:

- checking
- removing
- adding
- good
- to check

Each edge: what states could have actually made that transition?



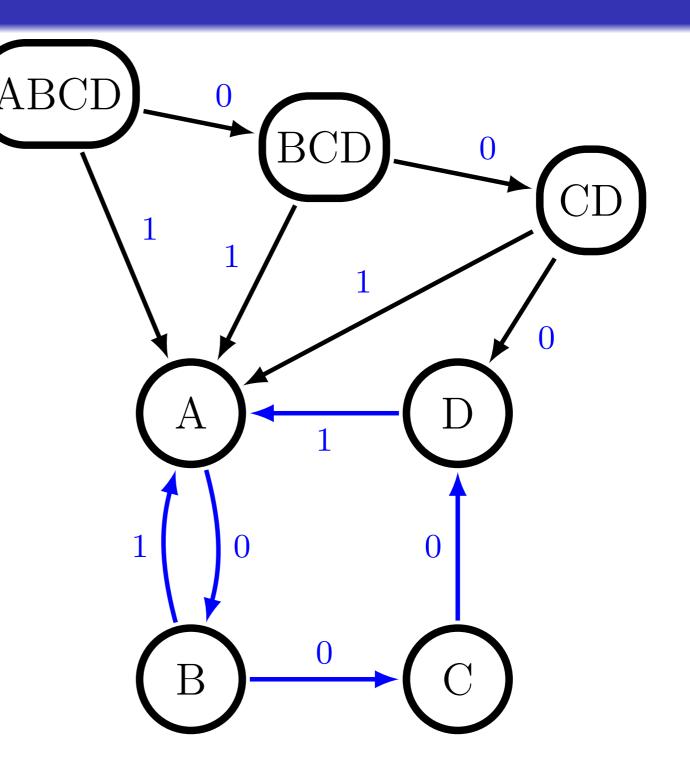


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			

Legend:

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- removing
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Each edge: what states could have actually made that transition?



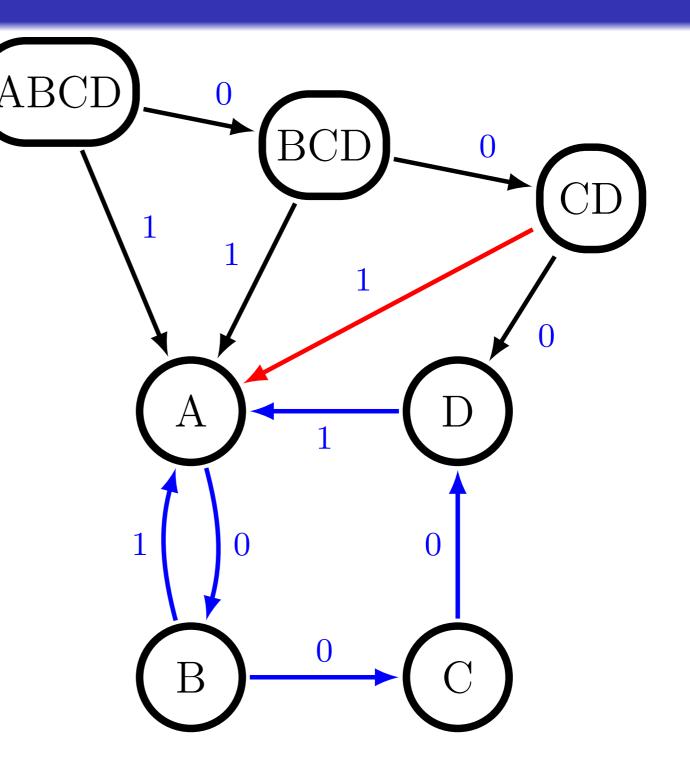


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			

Legend:

- checking
- removing
- adding
- good
- to check

Each edge: what states could have actually made that transition?



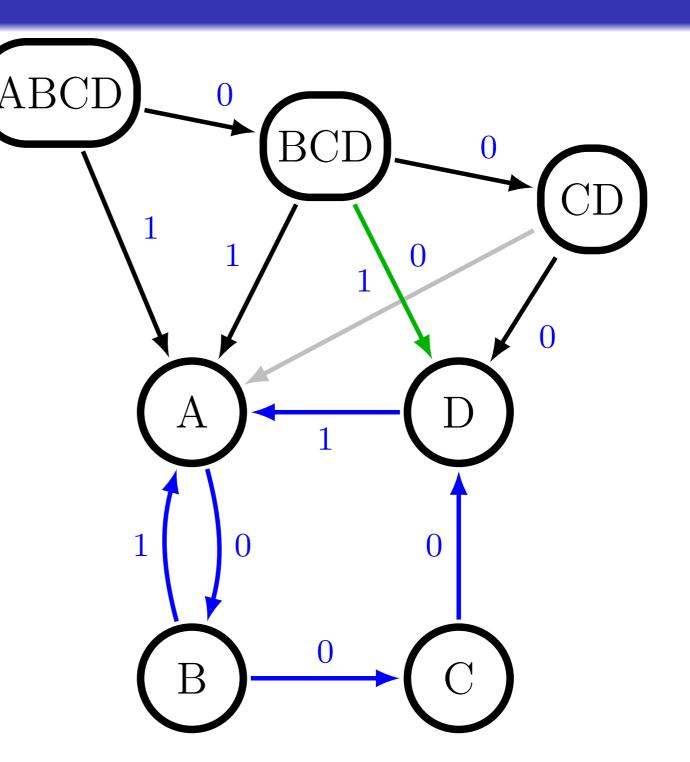


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
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Legend:

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Each edge: what states could have actually made that transition?



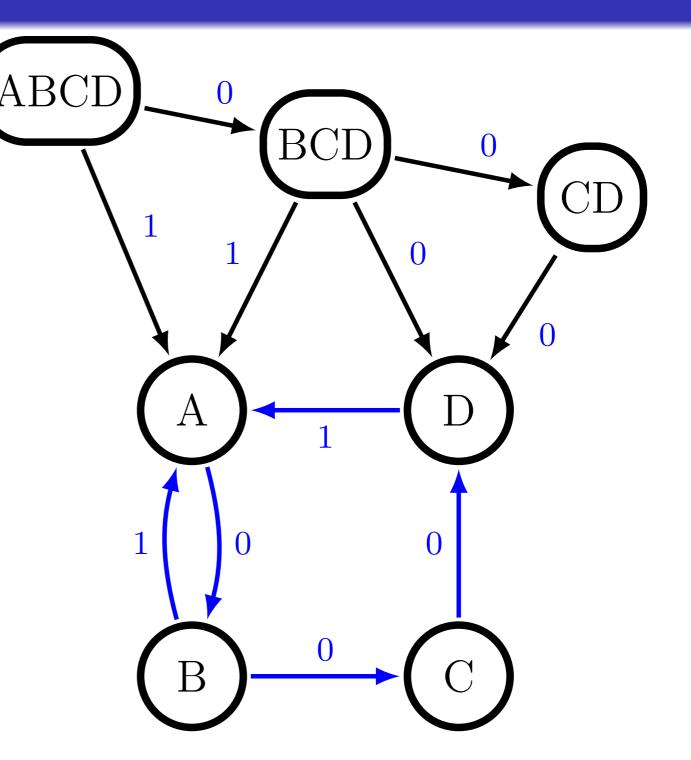


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			

Legend:

- checking
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- adding
- good
- to check

Each edge: what states could have actually made that transition?



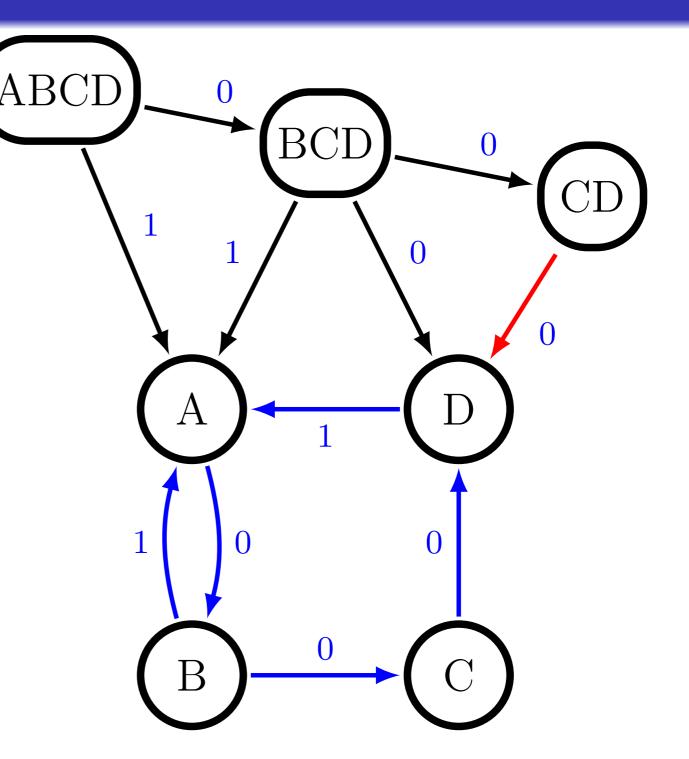


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			

Legend:

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Each edge: what states could have actually made that transition?



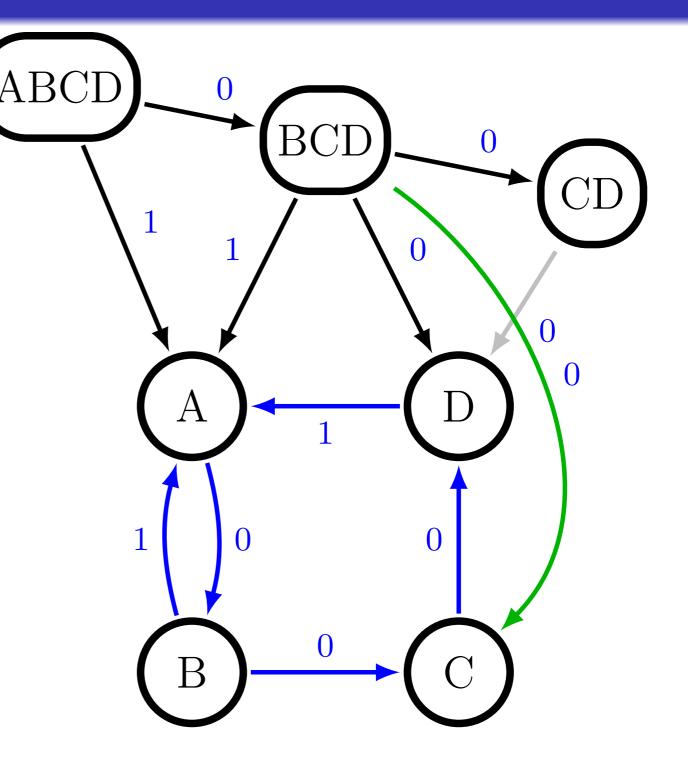


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Legend:

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Each edge: what states could have actually made that transition?



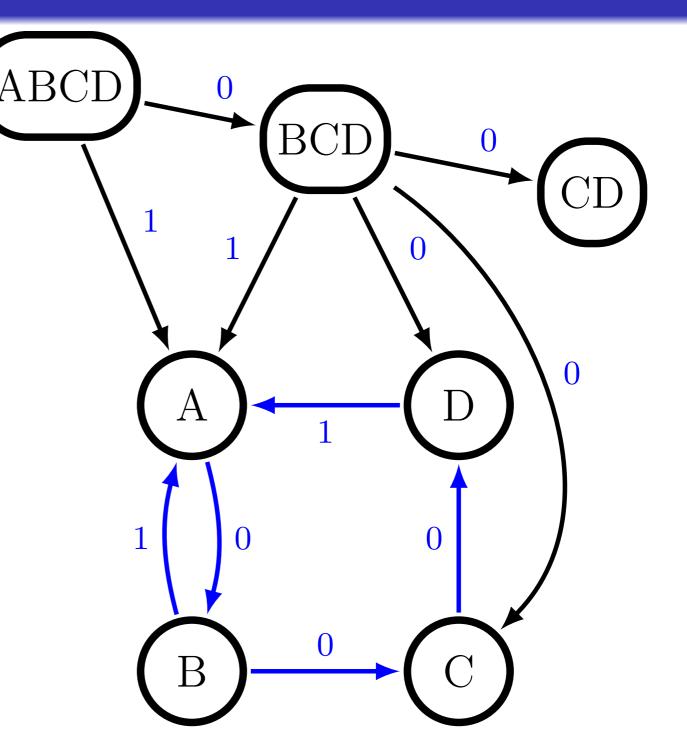


Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
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Each edge: what states could have actually made that transition?



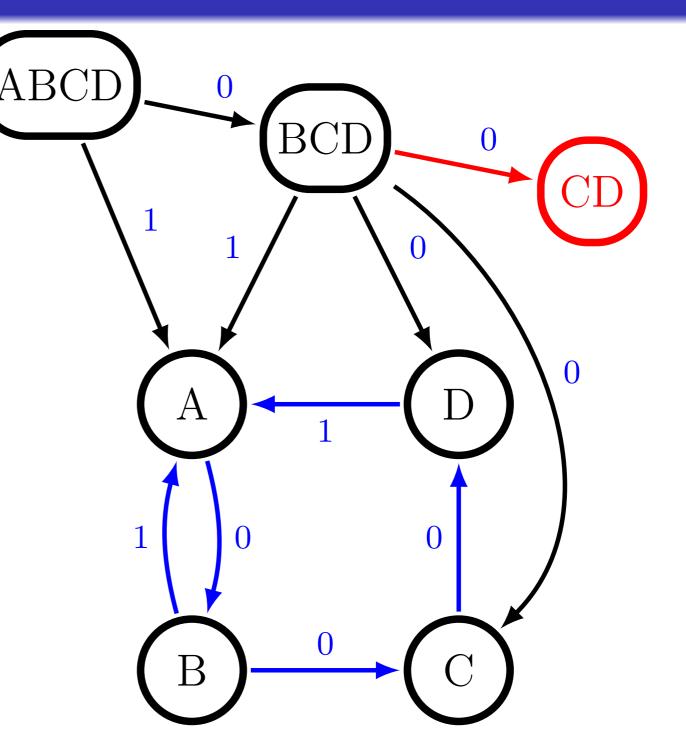


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Each edge: what states could have actually made that transition?



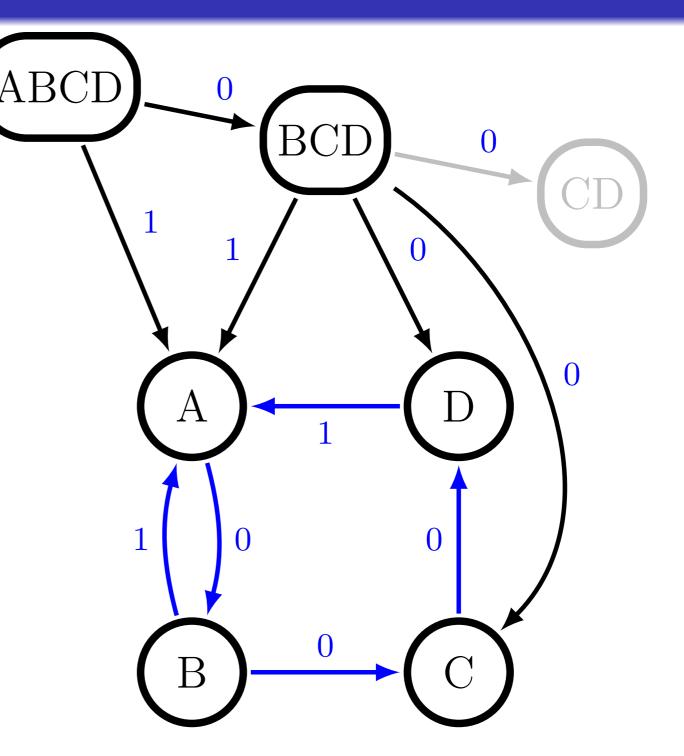


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Each edge: what states could have actually made that transition?



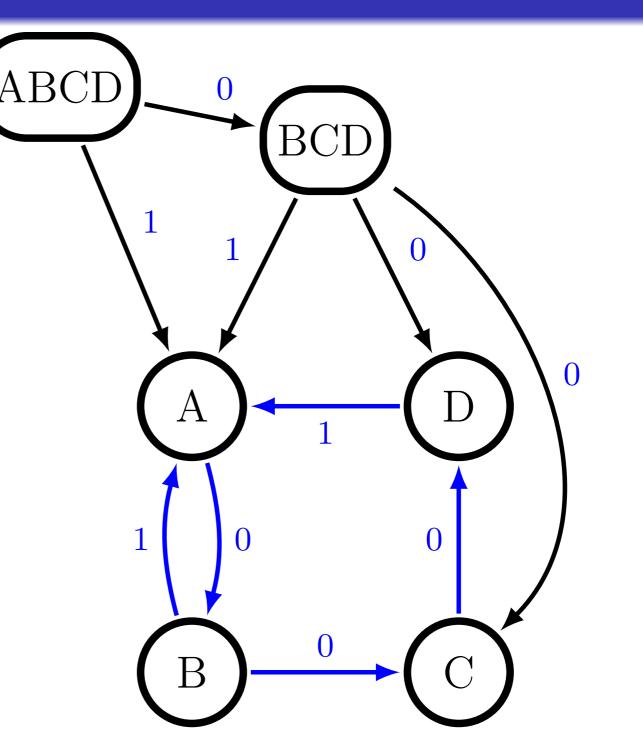


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Each edge: what states could have actually made that transition?



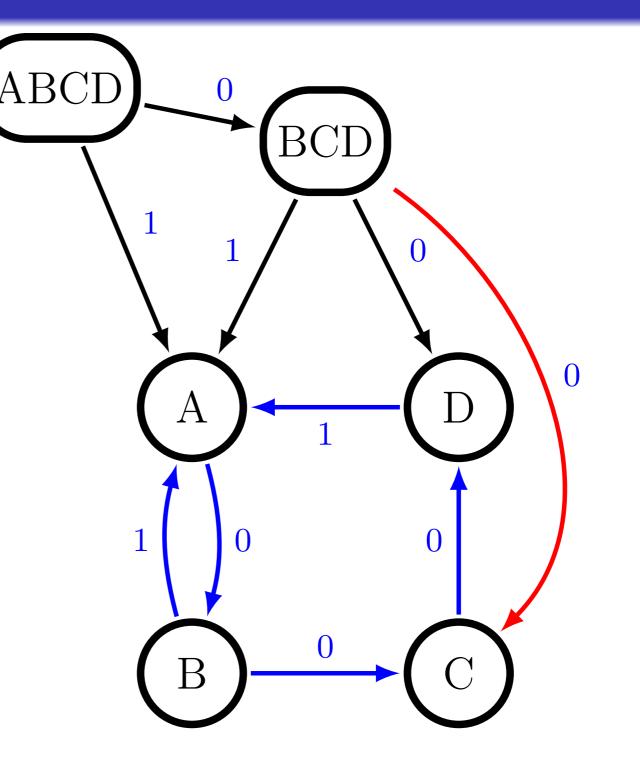


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Legend:

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Each edge: what states could have actually made that transition?



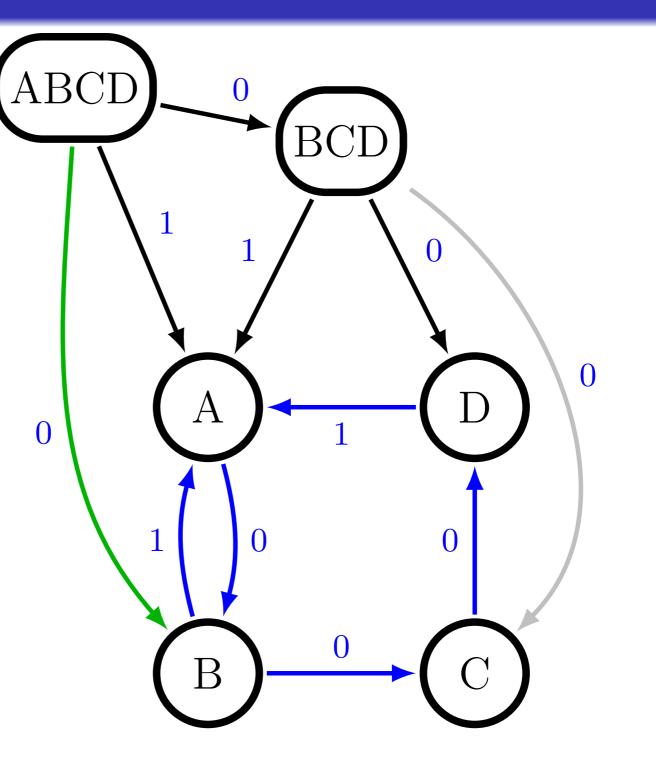


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Cryptic Order			

Legend:

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Each edge: what states could have actually made that transition?



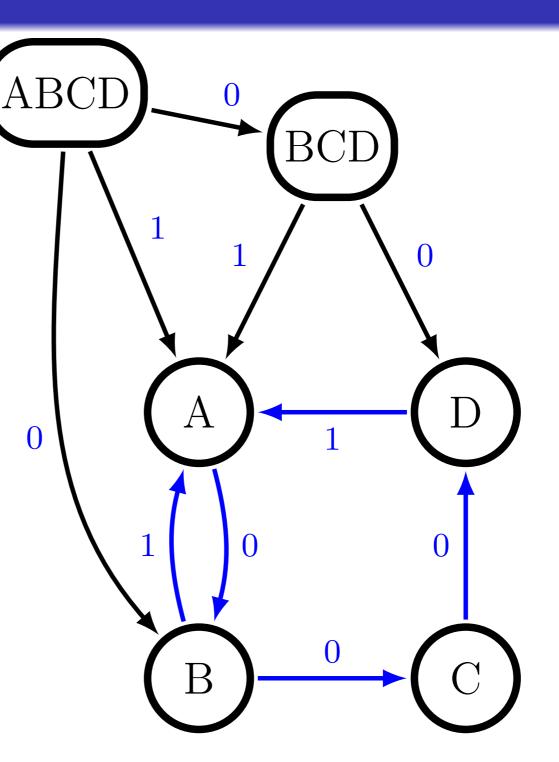


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Legend:

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Each edge: what states could have actually made that transition?



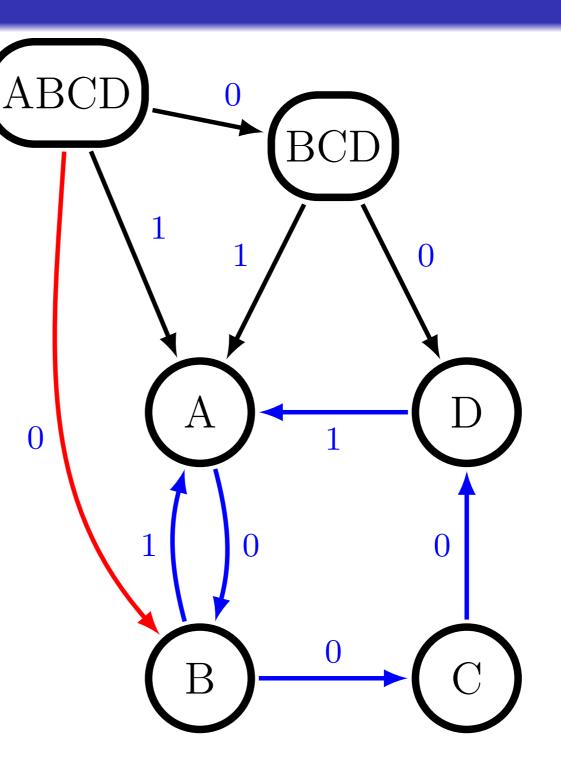


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Each edge: what states could have actually made that transition?



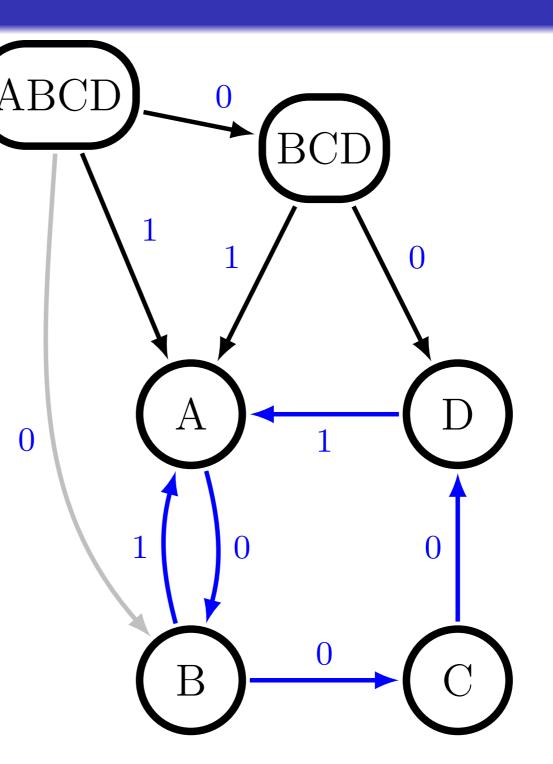


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Each edge: what states could have actually made that transition?



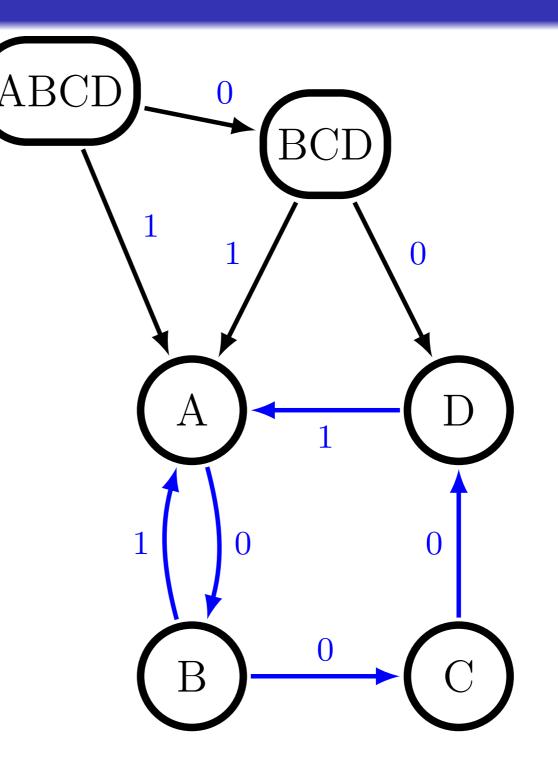


Introduction	Via Synchronizing Words	Via Topology	Results & Survey
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Legend:

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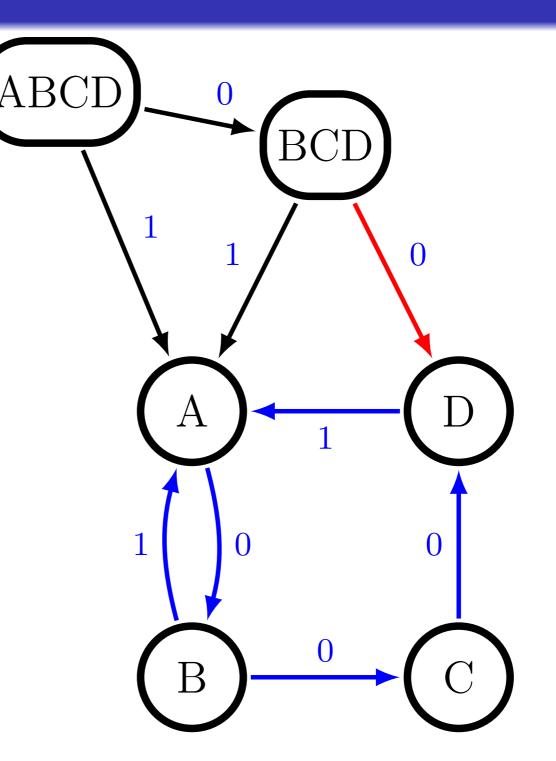


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Legend:

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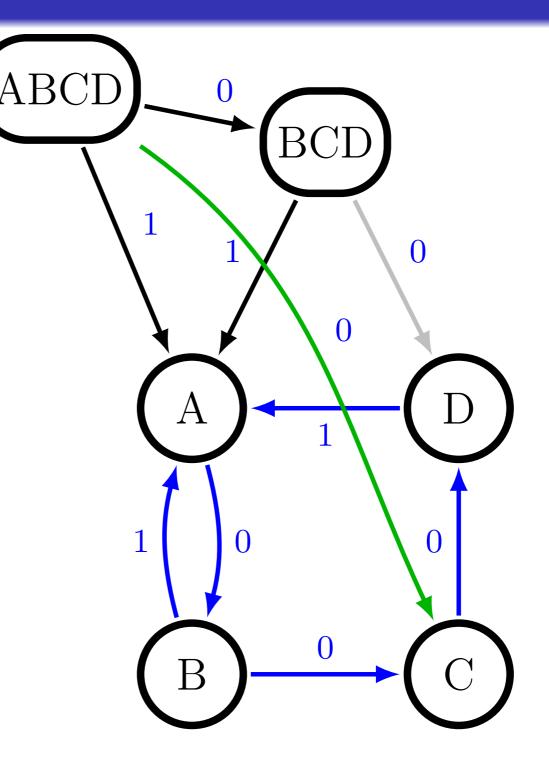


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Legend:

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Each edge: what states could have actually made that transition?



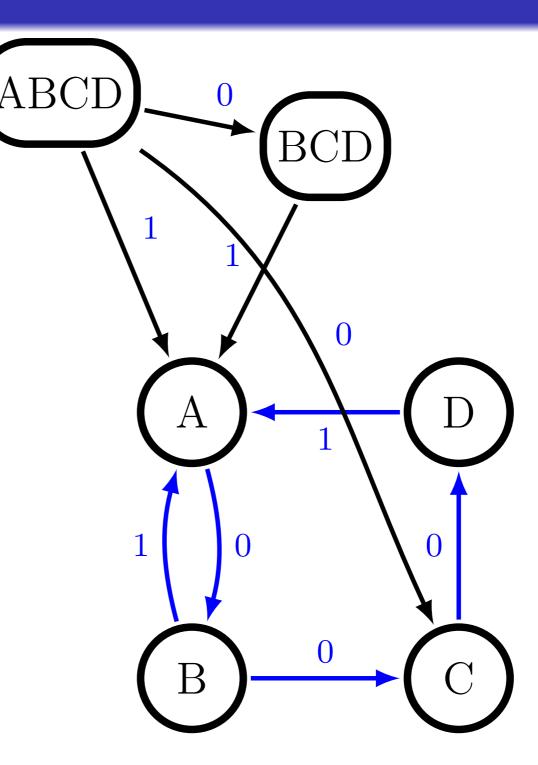


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Each edge: what states could have actually made that transition?

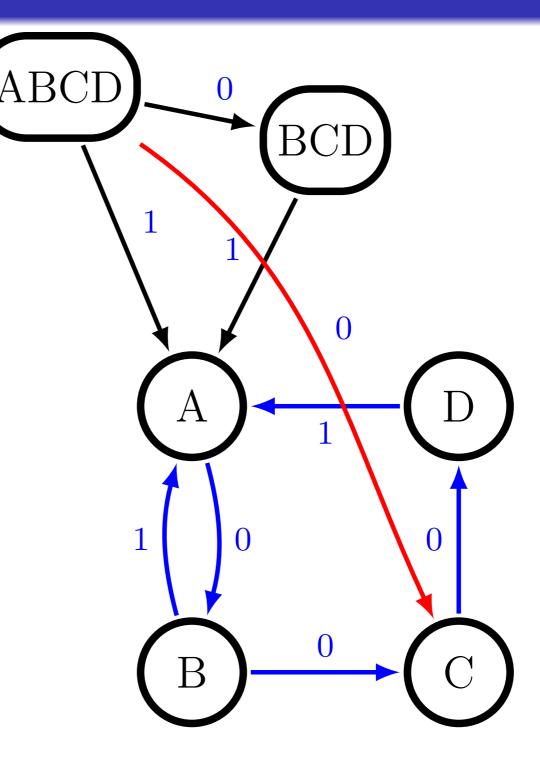




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- removing
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Each edge: what states could have actually made that transition?

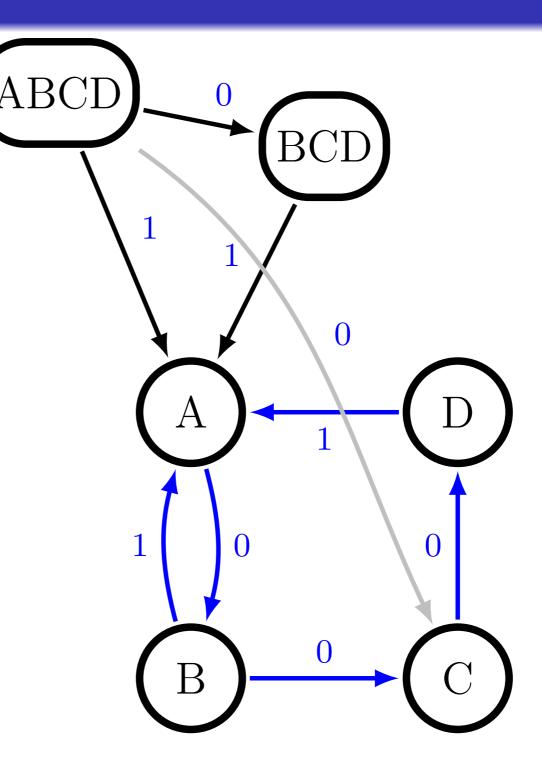




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Each edge: what states could have actually made that transition?

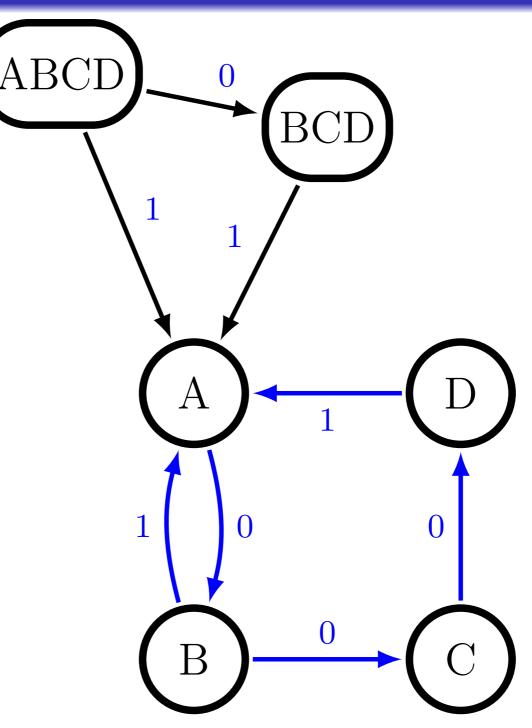




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Each edge: what states could have actually made that transition?

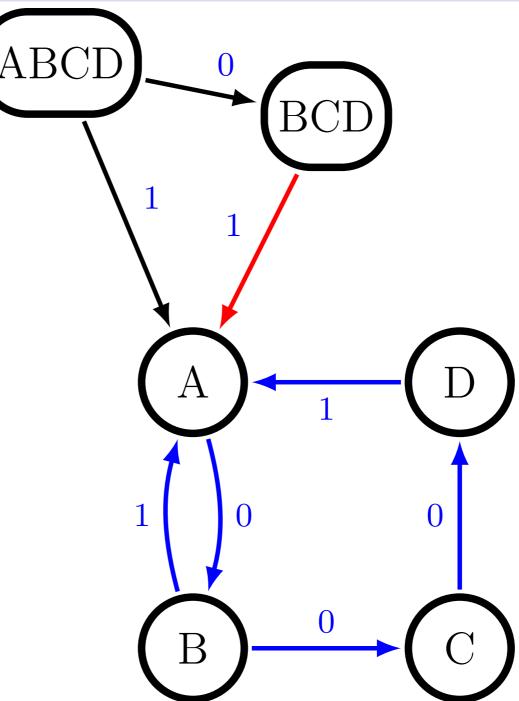


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- to check

Each edge: what states could have actually made that transition?



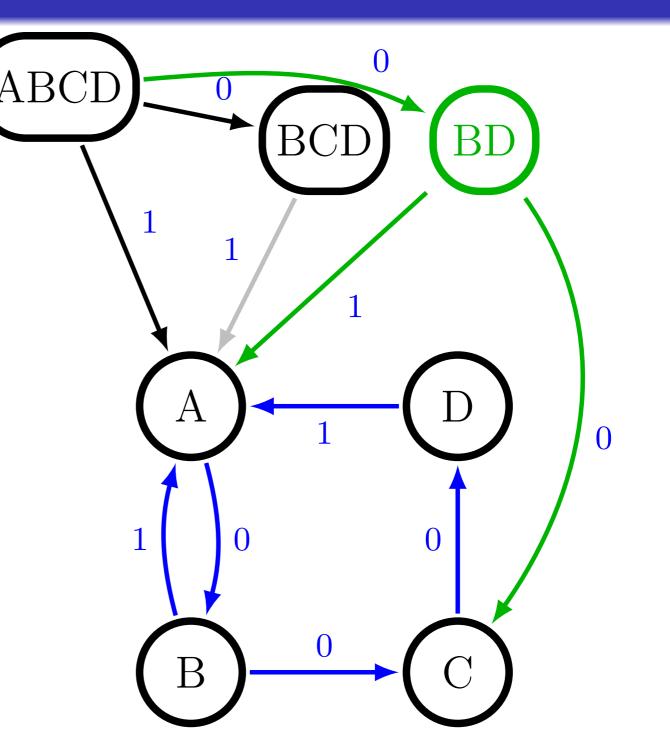


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- good
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Each edge: what states could have actually made that transition?



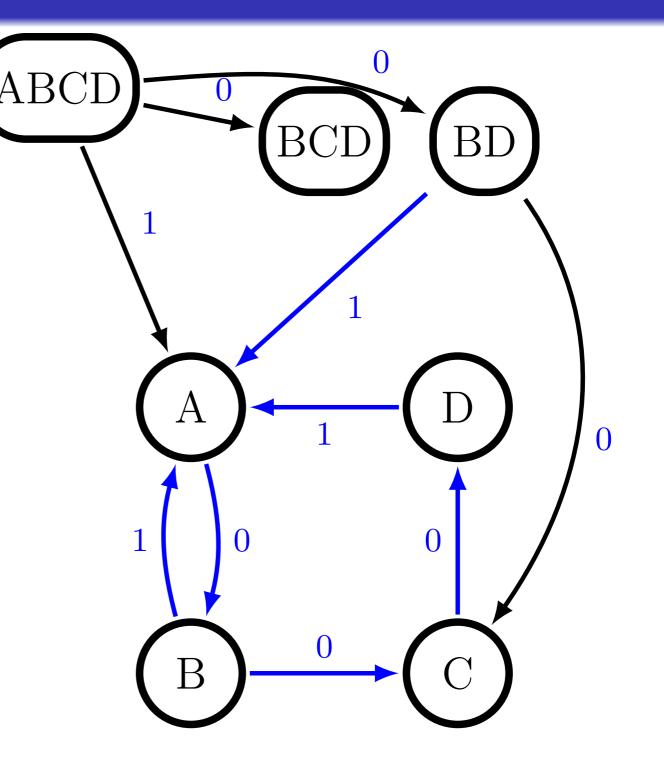


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Each edge: what states could have actually made that transition?



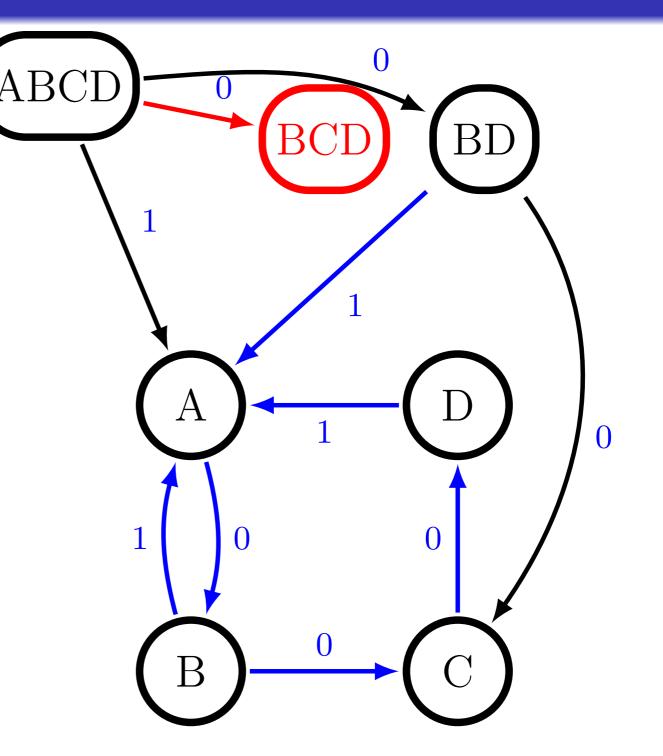


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Each edge: what states could have actually made that transition?



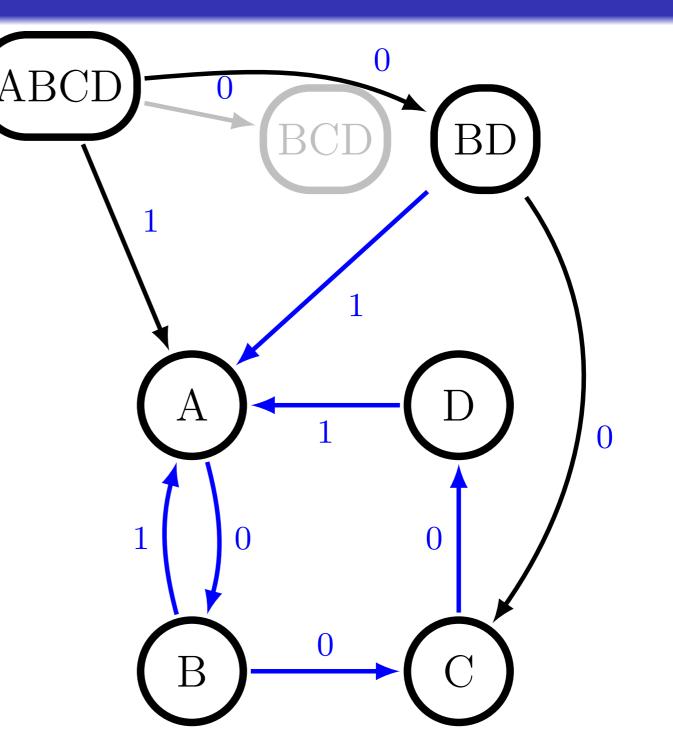


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Each edge: what states could have actually made that transition?

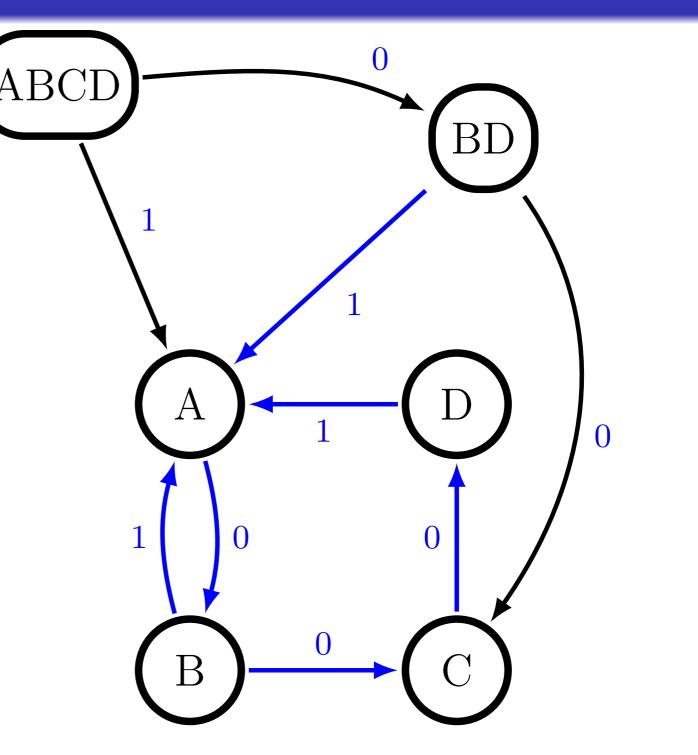




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Each edge: what states could have actually made that transition?

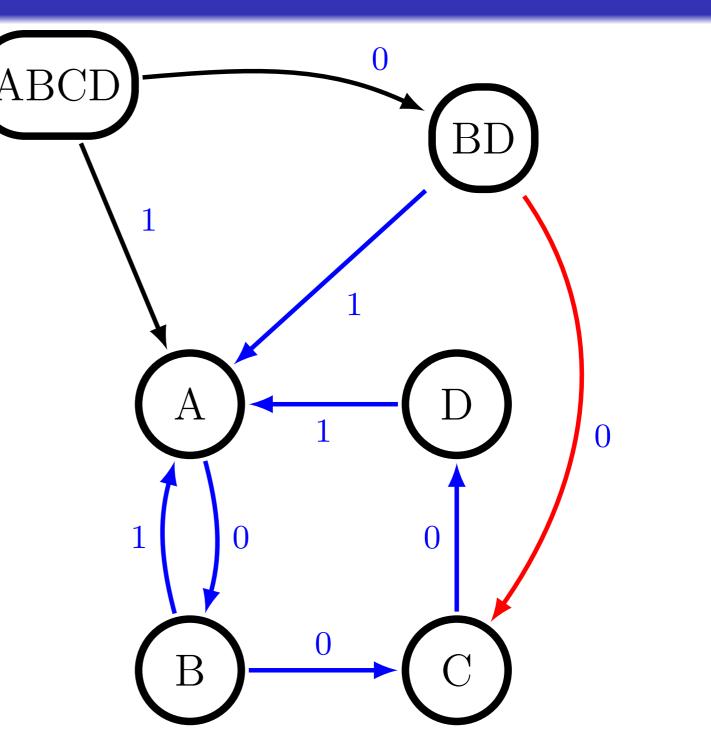




Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey 00
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Each edge: what states could have actually made that transition?

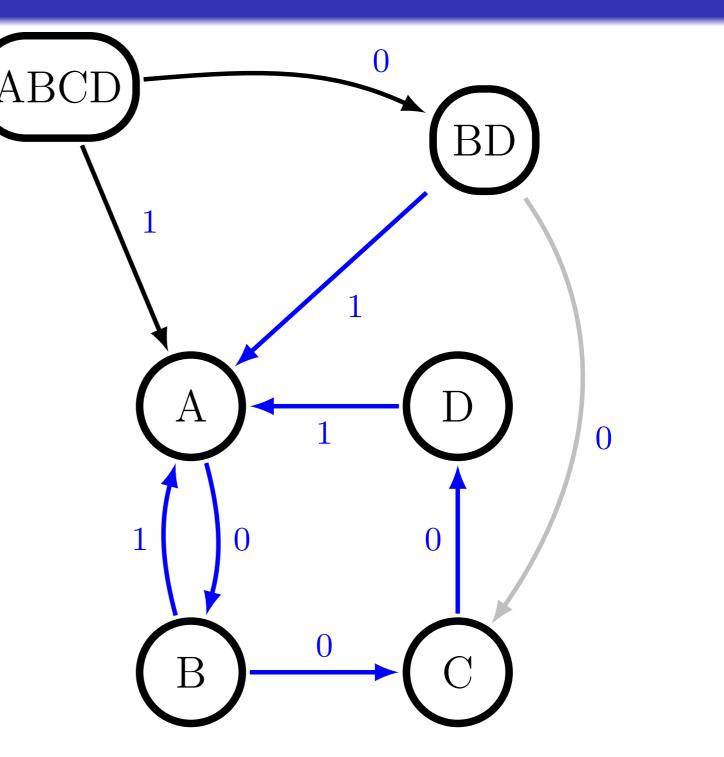




Introduction 000000	Via Synchronizing Words 0000	Via Topology 000000000	Results & Survey 00
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Each edge: what states could have actually made that transition?

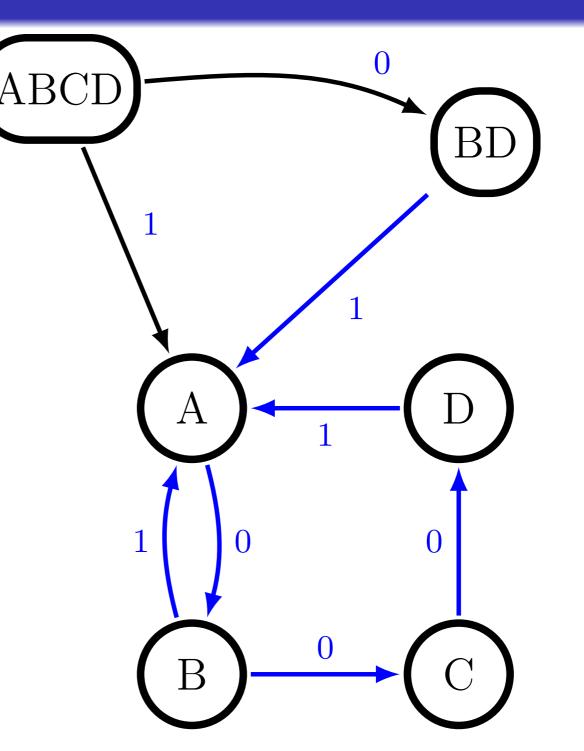




Introduction 000000	Via Synchronizing Words 0000	Via Topology 000000000	Results & Survey 00
Cryptic Order			
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Each edge: what states could have actually made that transition?

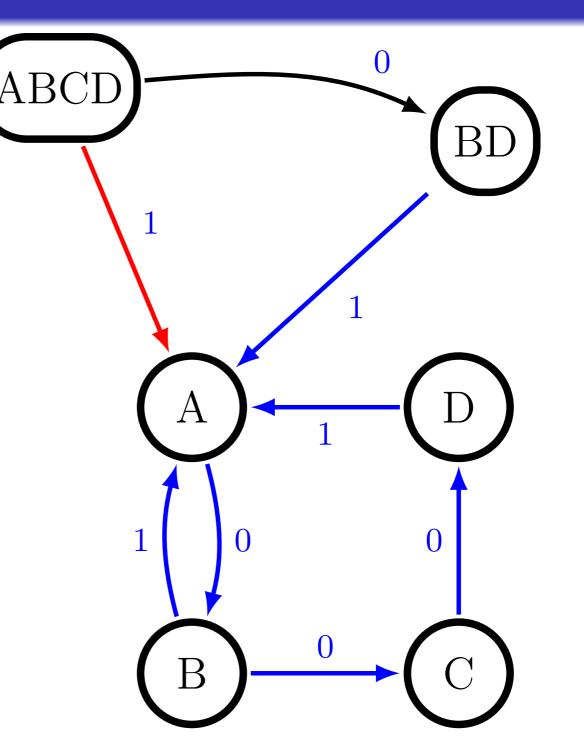




Introduction 000000	Via Synchronizing Words 0000	Via Topology 000000000	Results & Survey 00
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Each edge: what states could have actually made that transition?

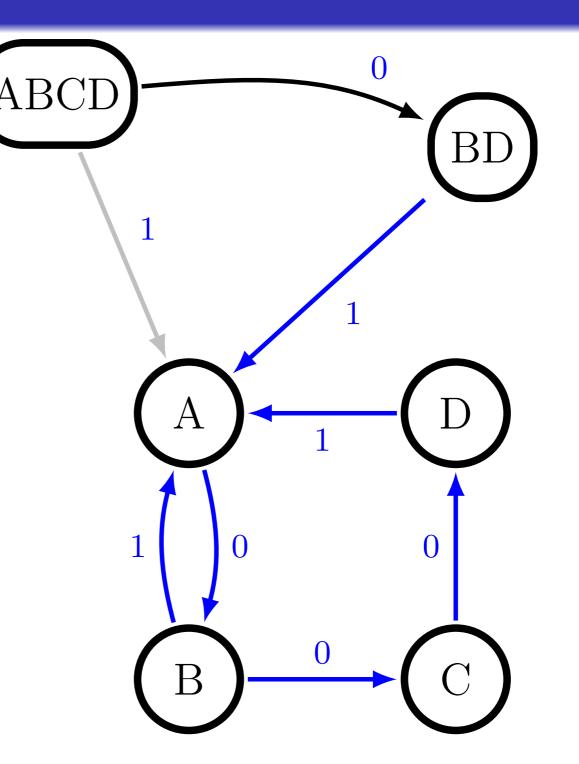




Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			
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- to check

Each edge: what states could have actually made that transition?





Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey 00
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Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey 00
Cryptic Order			
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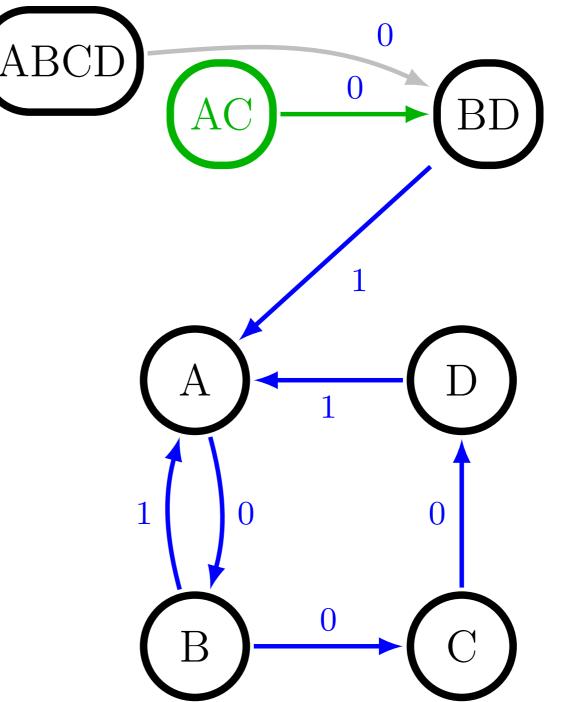
Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

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Cryptic Order			
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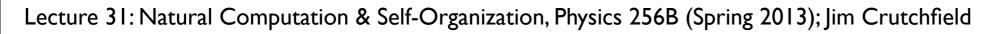
- checking
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- to check

Each edge: what states could have actually made that transition?





Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey 00
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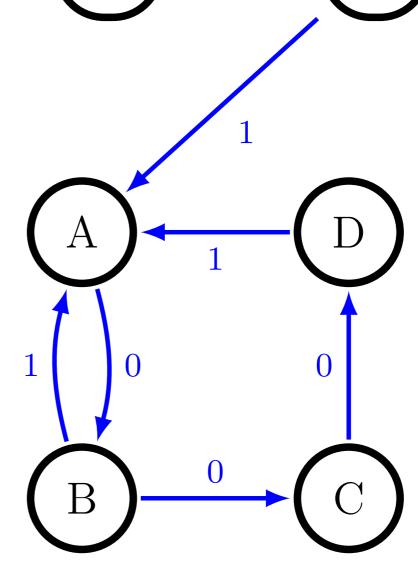
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Each edge: what could states have actually made that transition?





Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

Introduction 000000	Via Synchronizing 0000	Words	Via Topology 00000●000	Results & Survey 00
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Retrodiction				
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Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

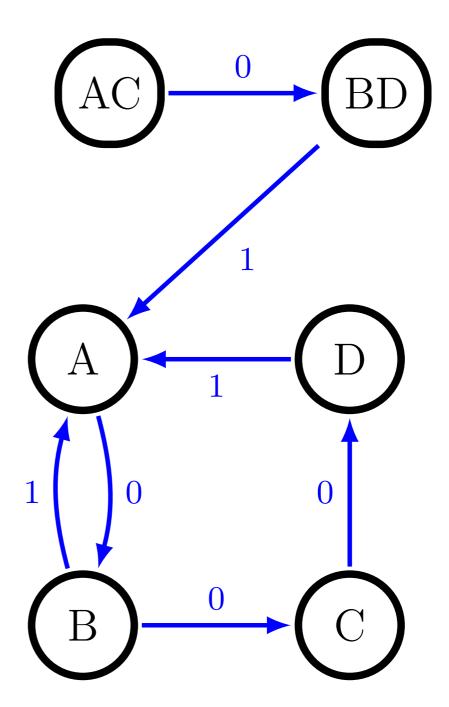
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Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
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- checking
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- good
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Each edge: what states could have actually made that transition?

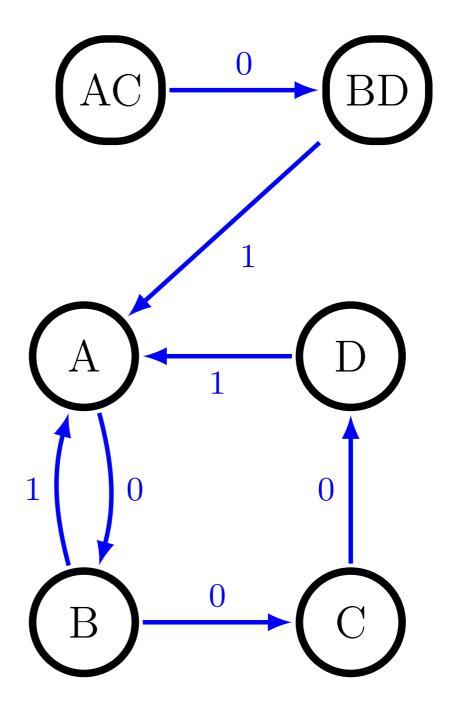




Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			
Retrodicti	on		

- checking
- removing
- adding
- good
- to check

Each edge: what states could have actually made that transition? Now do Bellman-Ford again





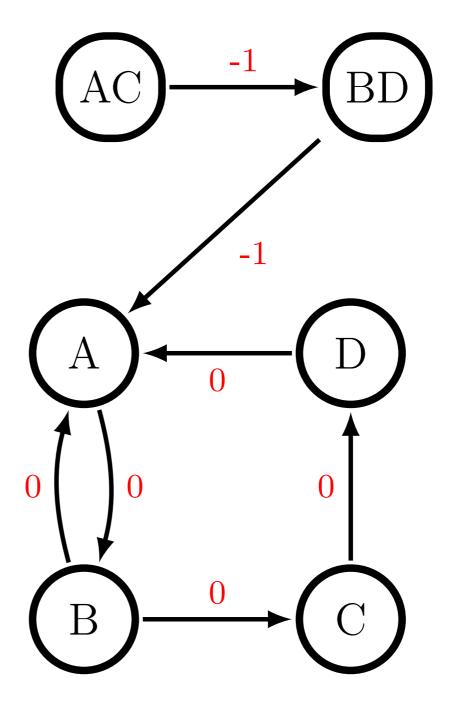
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Sunday, May 4, 14

Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000●000	Results & Survey 00
Cryptic Order			
Retrodicti	on		

- checking
- removing
- adding
- good
- to check

Each edge: what states could have actually made that transition? Now do Bellman-Ford again

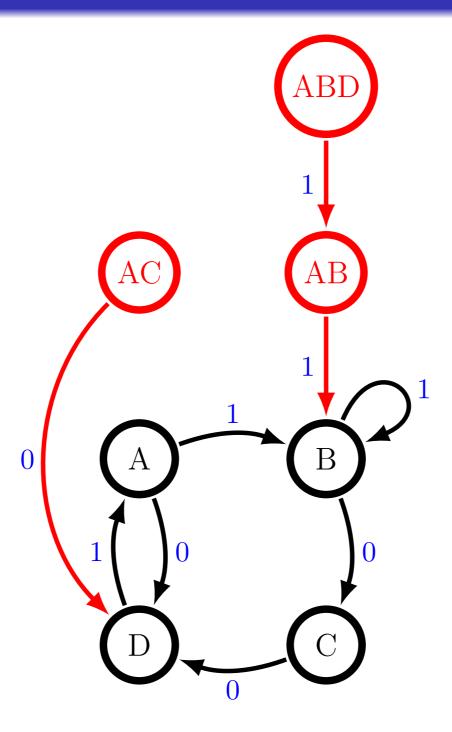




Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

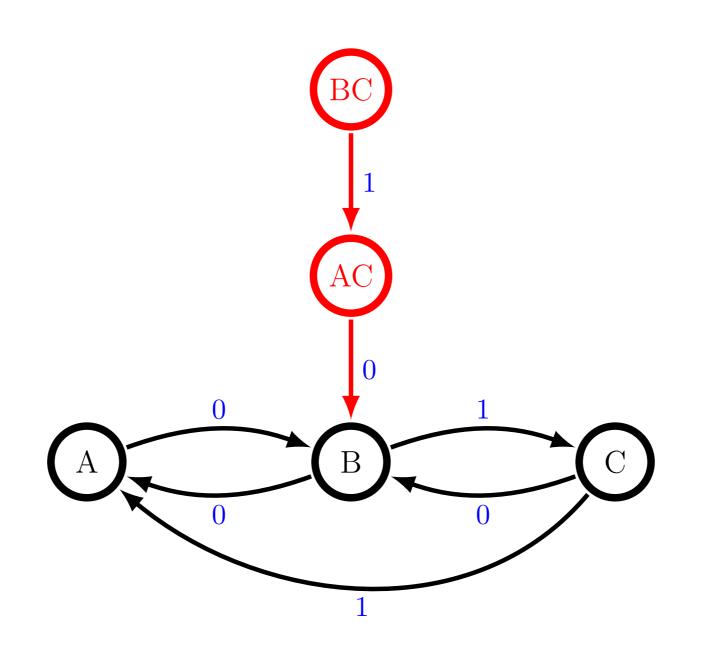
Sunday, May 4, 14

Introduction 000000	Via Synchronizing Words 0000	Via Topology 000000€00	Results & Survey 00
Cryptic Order			
Example t	he First		



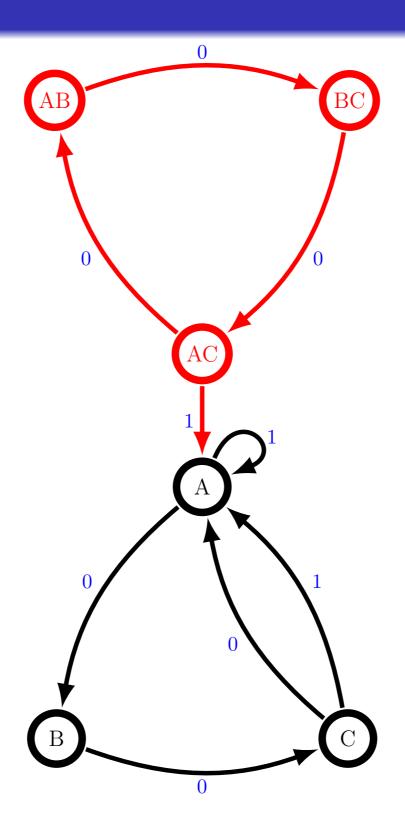
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Introduction 000000	Via Synchronizing Words 0000	Via Topology ○○○○○○●○	Results & Survey 00
Cryptic Order			
Example t	the Second		





Introduction 000000	Via Synchronizing Words 0000	Via Topology 0000000●	Results & Survey 00
Cryptic Order			
Example th	le Third		





Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey $\bullet \circ$
Results			
Further In	nprovements		



Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey •0
Results			
Further In	nprovements		
Now whe	ere are we?		

• Need to know the structure of the $\epsilon\text{-machine,}$ but not transition probabilities



Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey ●0
Results			
Further Imp	rovements		

- Need to know the structure of the ϵ -machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed



Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey ●0
Results			
Further Imp	rovements		

- Need to know the structure of the ϵ -machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed
- Integer based, avoiding comparisons of nearby floating point numbers



Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey $\bullet \circ$
Results			
Further Im	provements		

- Need to know the structure of the ϵ -machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed
- Integer based, avoiding comparisons of nearby floating point numbers
- Encapsulates all synchronizing words in a finite structure



Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey ●○
Results			
Further Imp	orovements		

- Need to know the structure of the ϵ -machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed
- Integer based, avoiding comparisons of nearby floating point numbers
- Encapsulates all synchronizing words in a finite structure
- Operates on that finite structure resulting in finite computation



Introduction	Via Synchronizing Words	Via Topology	Results & Survey	
000000	0000	00000000	$\bigcirc \bigcirc$	
Results				
Further Improvements				

- Need to know the structure of the ϵ -machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed
- Integer based, avoiding comparisons of nearby floating point numbers
- Encapsulates all synchronizing words in a finite structure
- Operates on that finite structure resulting in finite computation
- Utilizes Bellman-Ford (Floyd-Warshall works too) to compute the longest path or detect cycles efficiently

Introduction 000000	Via Synchronizing Words 0000	Via Topology 00000000	Results & Survey $\circ \bullet$
Survey			

$R \text{ vs } k_{\chi} \text{ for 6-state } \epsilon \text{-machines}$

