Length Scales in Complex Time Series: Markov and Cryptic Orders: A Rope of Sand

Ryan G. James

Complexity Sciences Center
Department of Physics
University of California, Davis
One Shields Avenue, Davis, CA 95616

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Lecture 31: Natural Computation & Self-Organization, Physics 256B (Spring 2013); Jim Crutchfield

Natural Computation: Processes

$$\mathcal{P} = (\mathbf{X}, \mu): \quad \mathbf{X} \subseteq \mathcal{A}^{\mathbb{Z}}, \sigma(\mathbf{X}) = \mathbf{X}$$

$$\cdots \quad X_{-3} \; X_{-2} \; X_{-1} \quad X_0 \quad X_1 \quad X_2 \quad X_3 \quad \cdots$$

Additional properties:

- Stationary
- Ergodic
- Discrete

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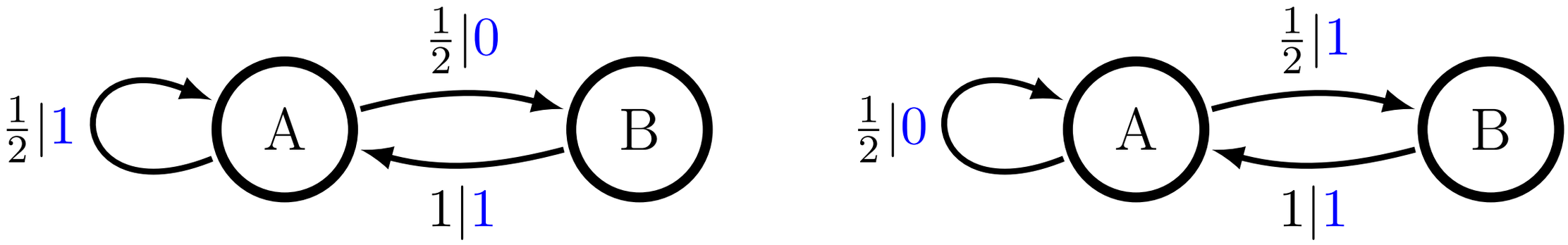
Natural Computation: ϵ-Machines



Properties:

- Minimal unifilar
- States are causal
- Causal states are sufficient statistics

Markov and Cryptic Orders

Definition

The *Markov order* of a process is:

$$R = \underset{\ell}{\operatorname{argmin}} \{\Pr(X_0|X_{-\ell:0}) = \Pr(X_0|X_{:0})\}$$

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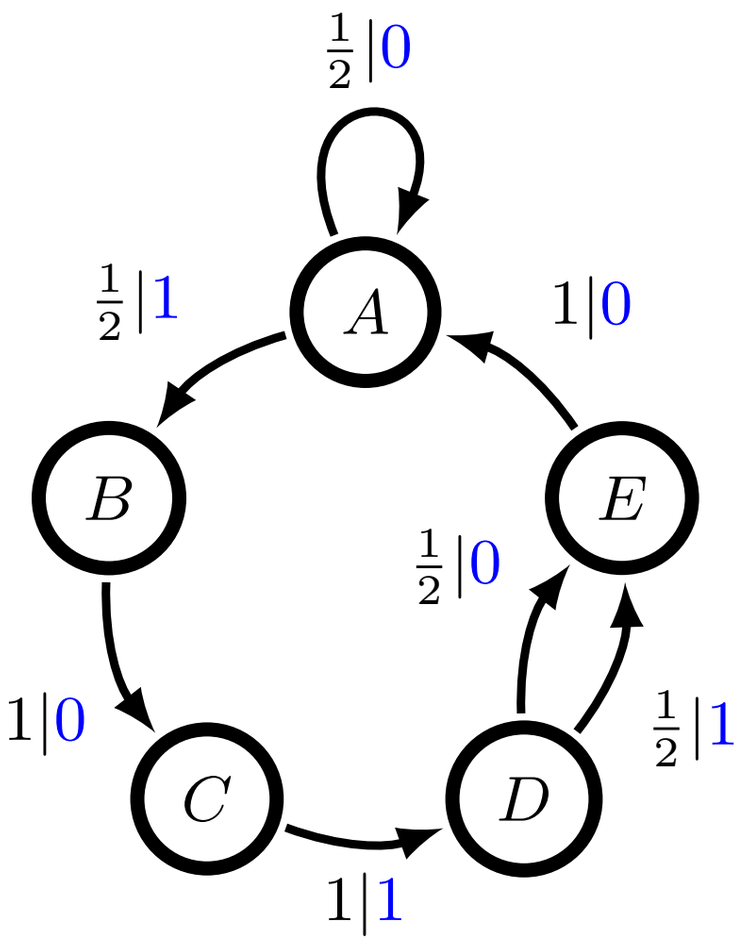
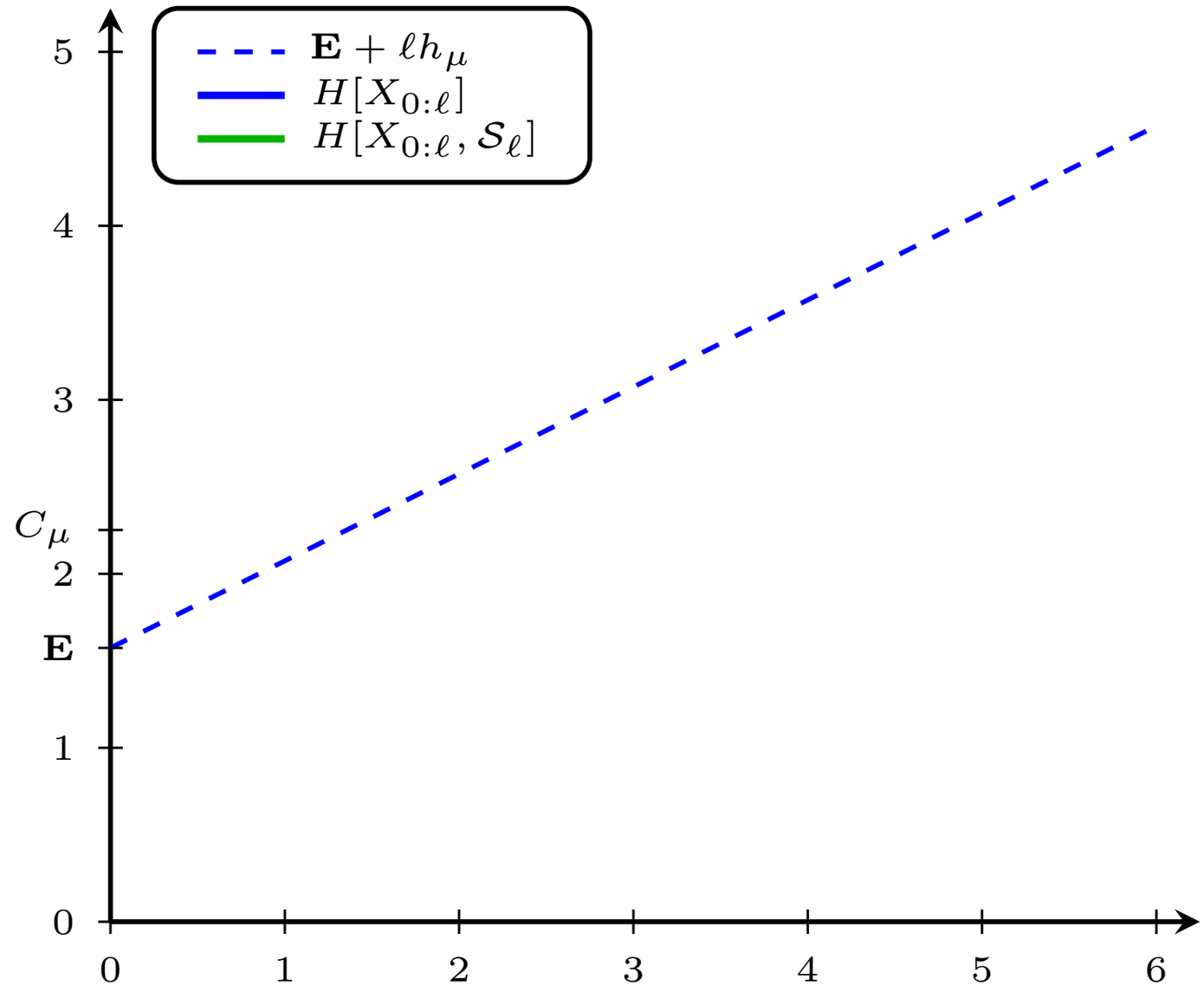
Block Entropy Curves





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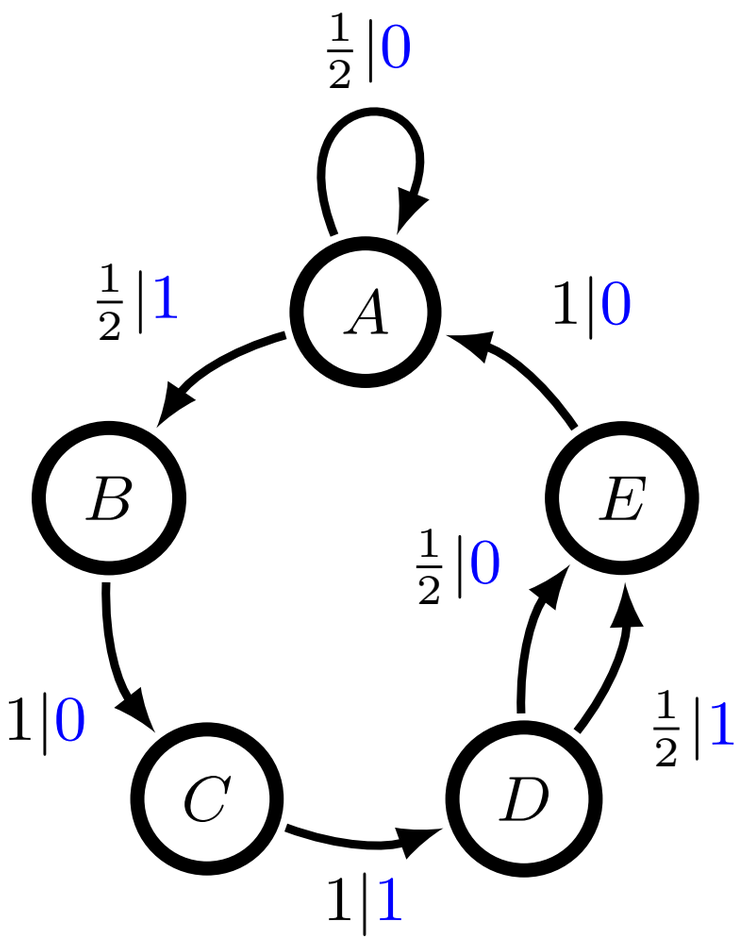

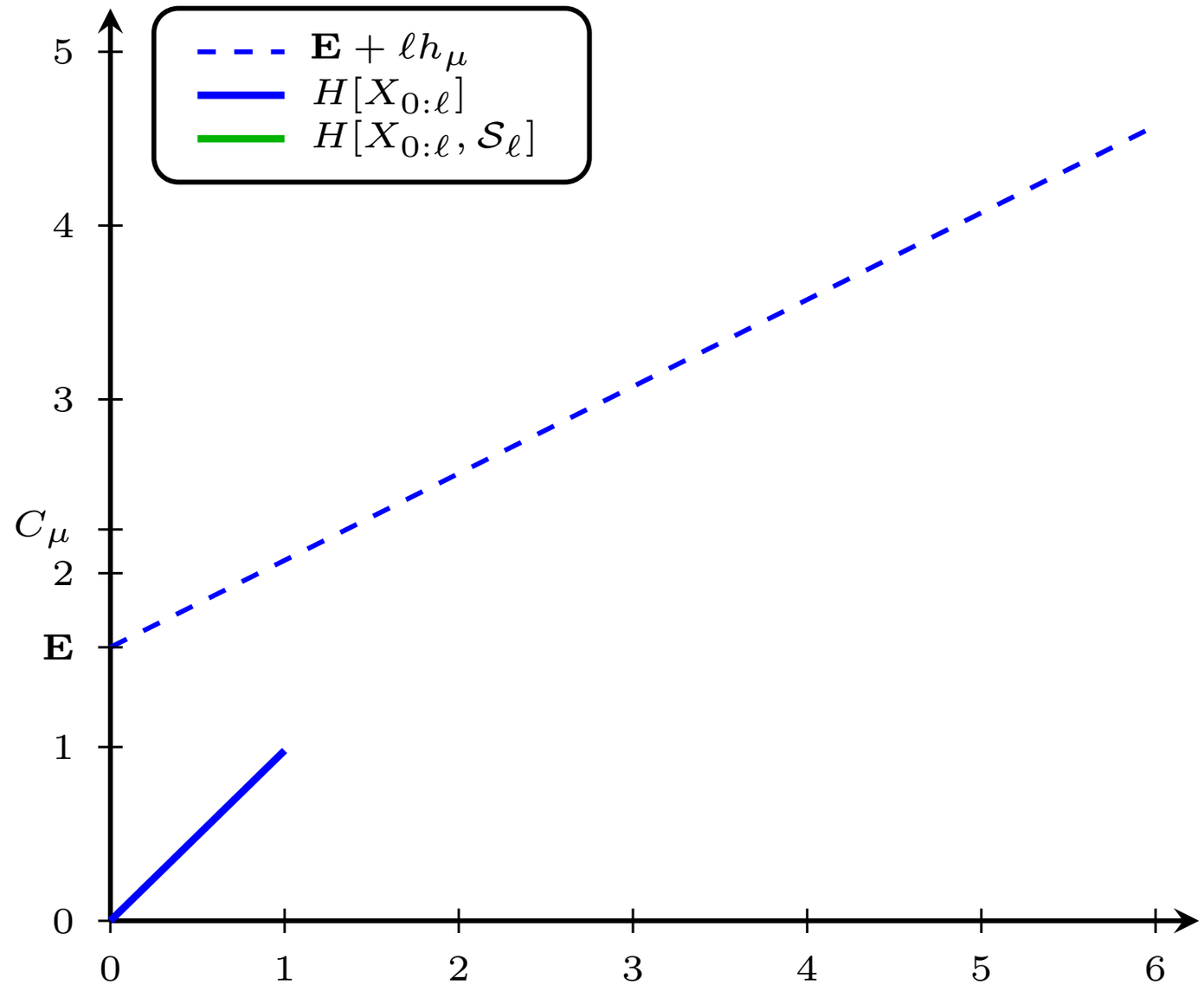

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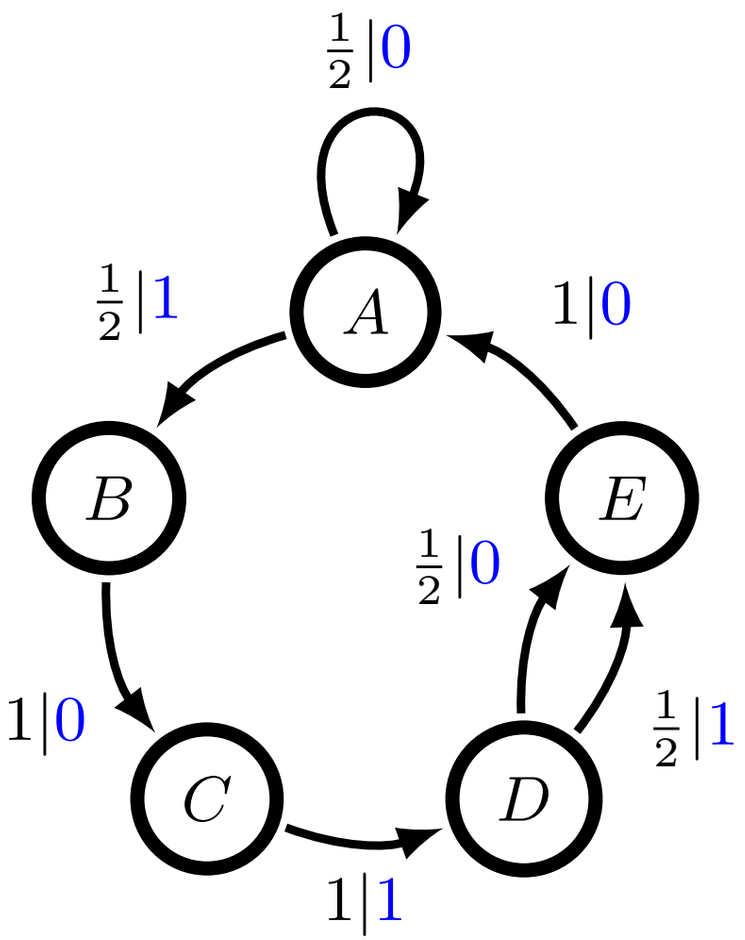

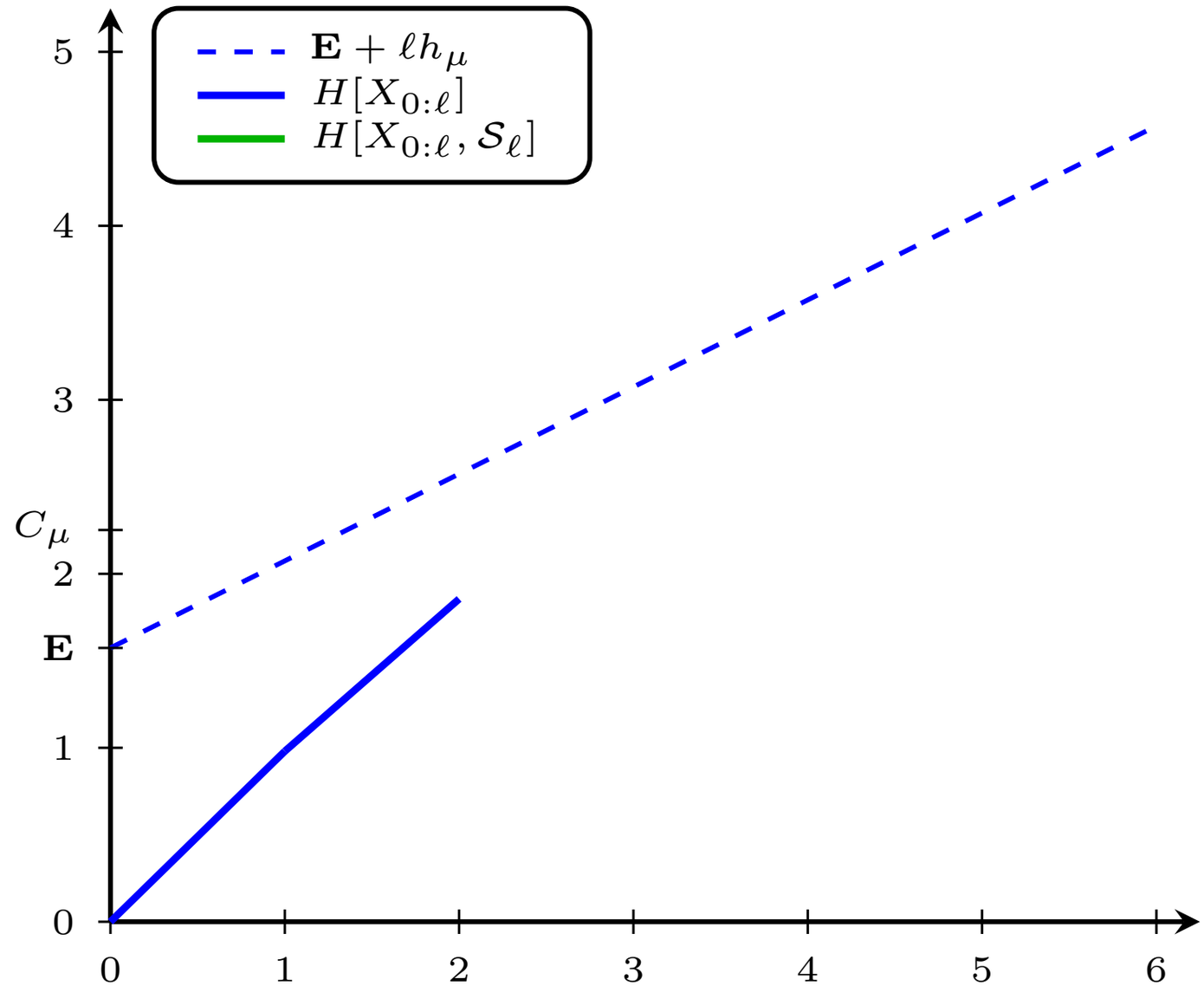

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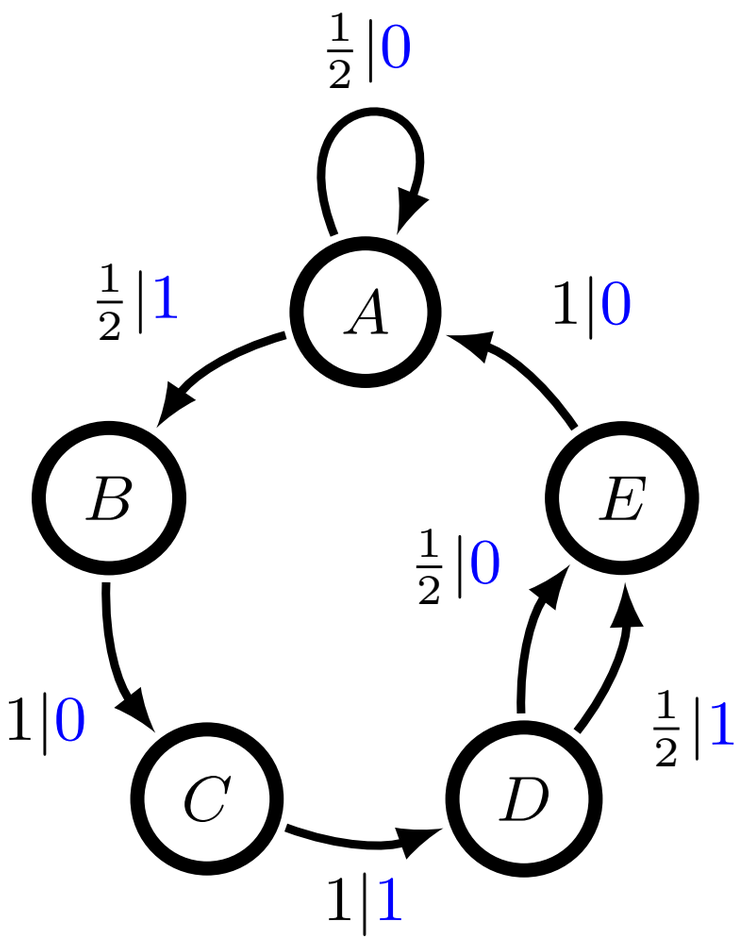

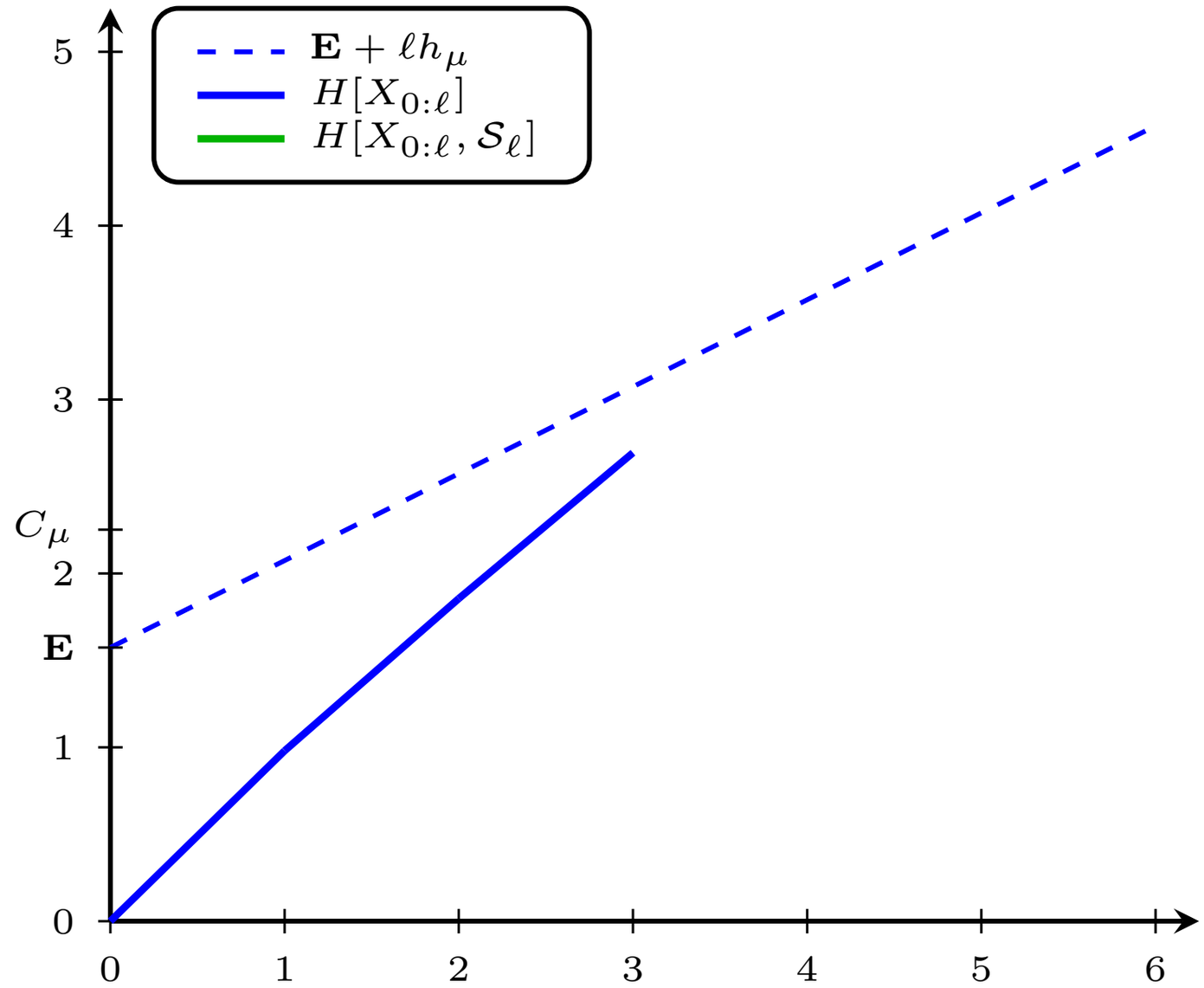

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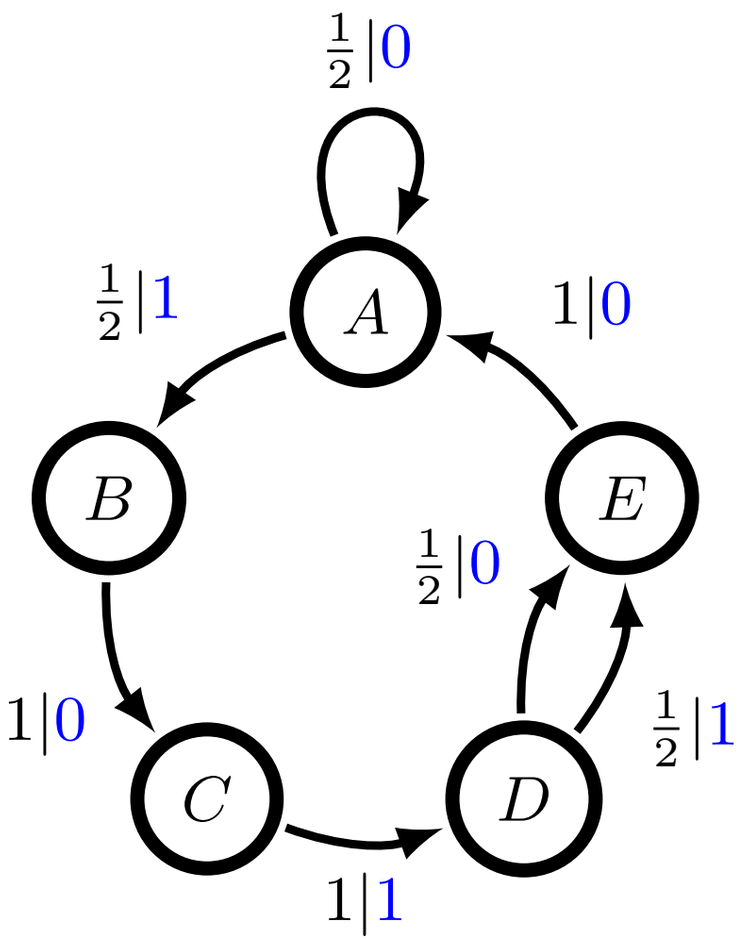

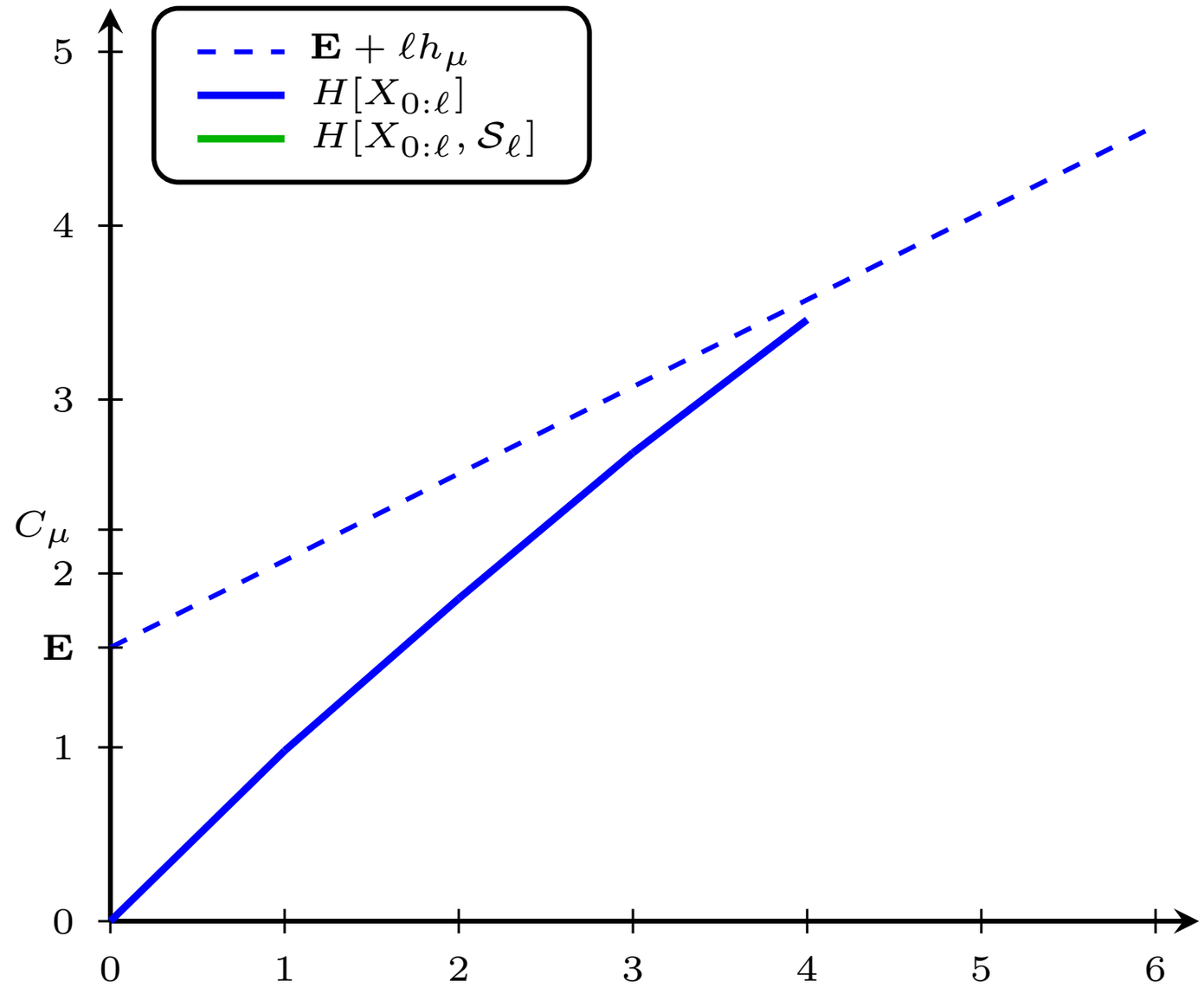

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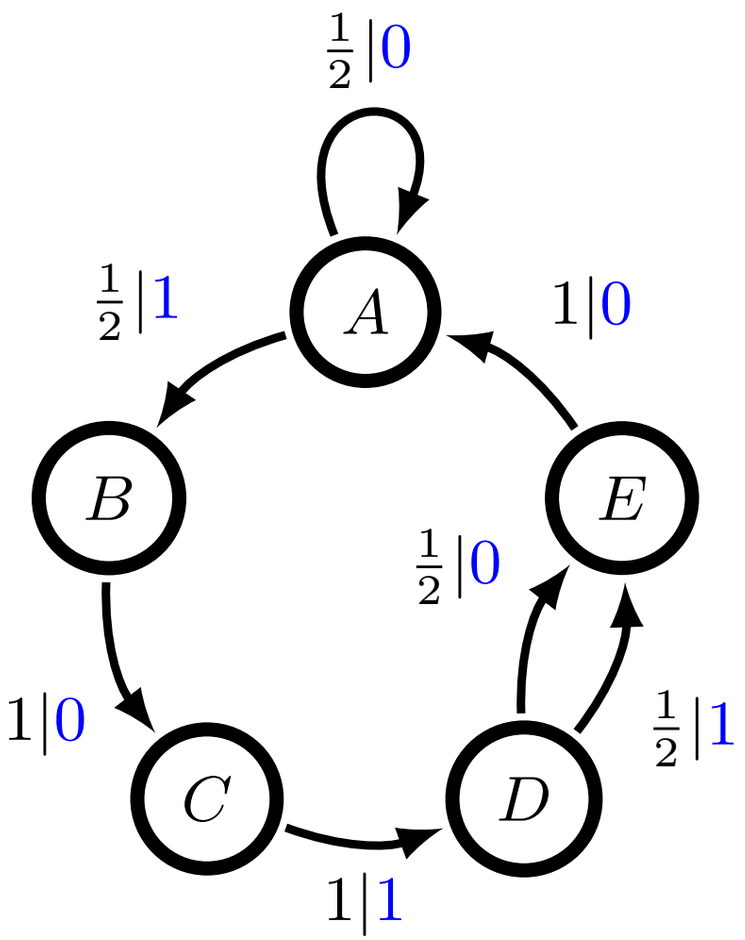

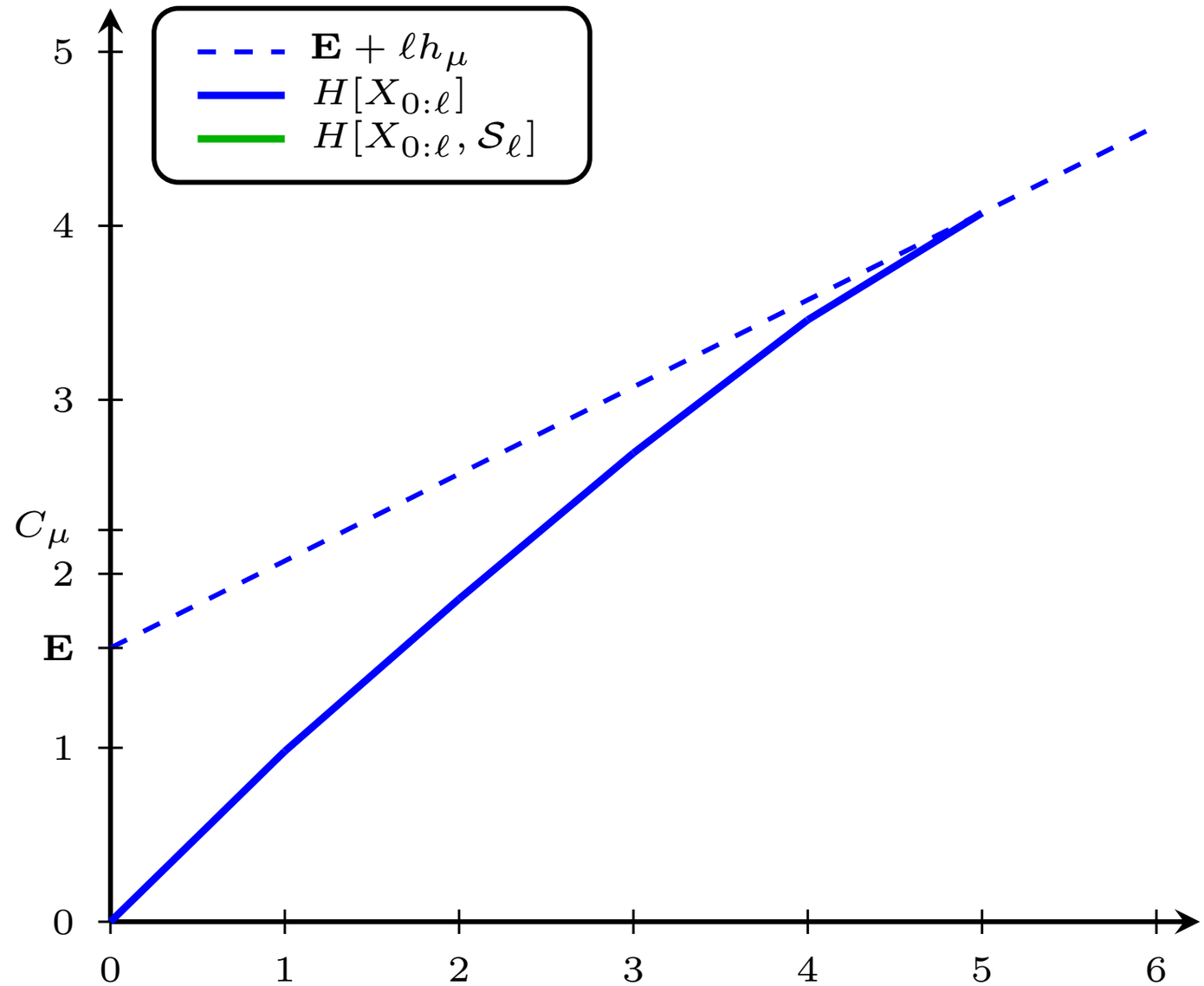

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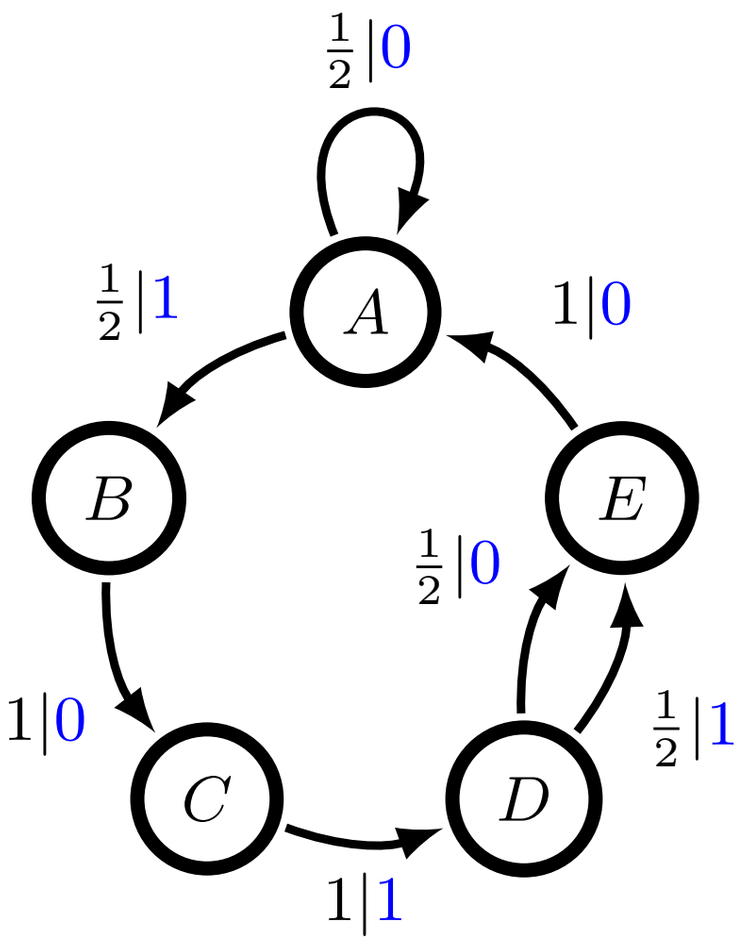

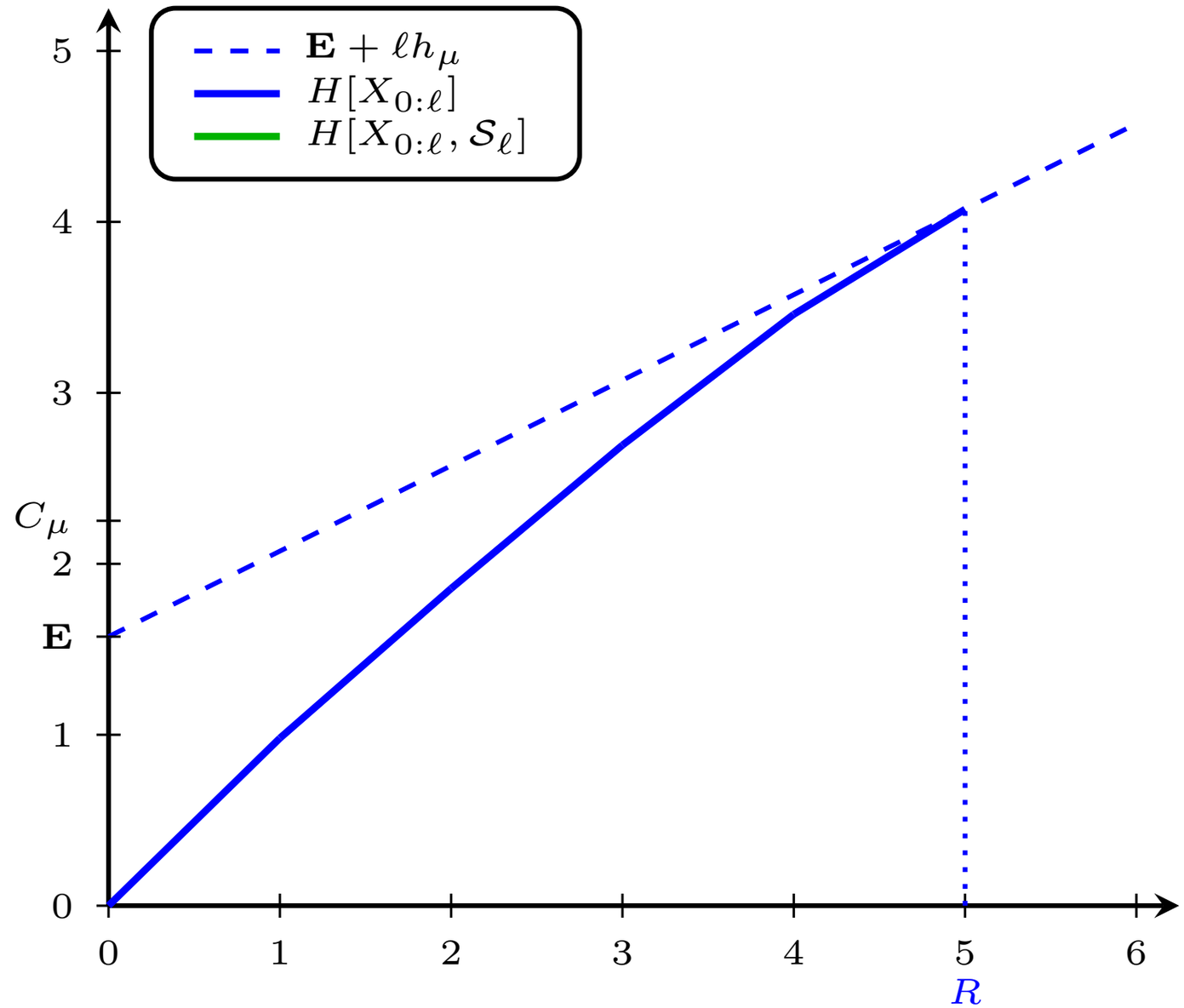

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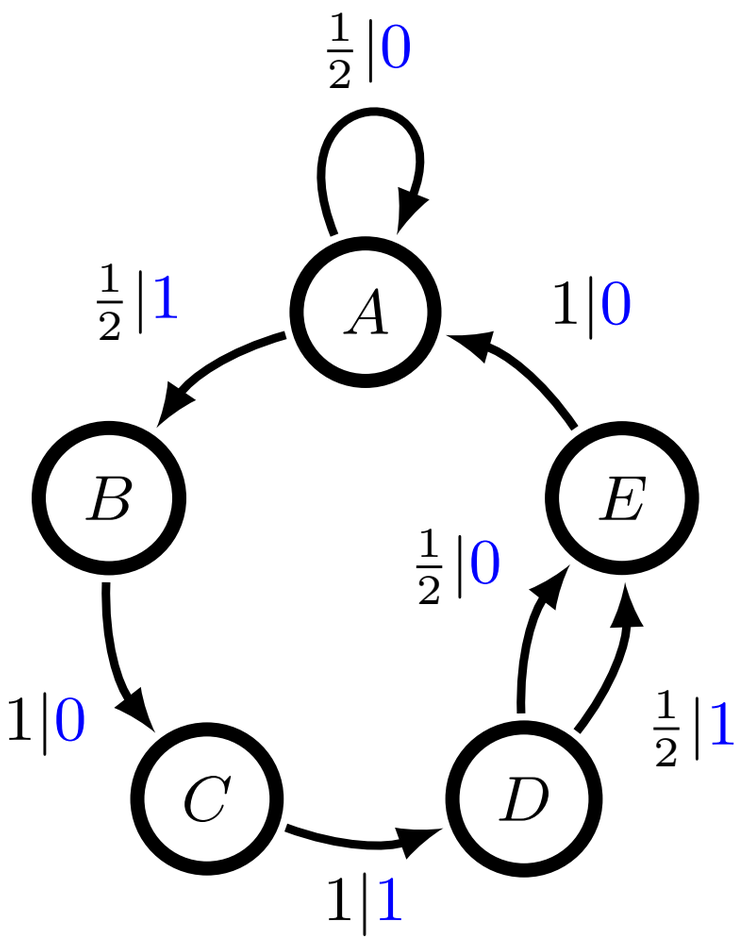

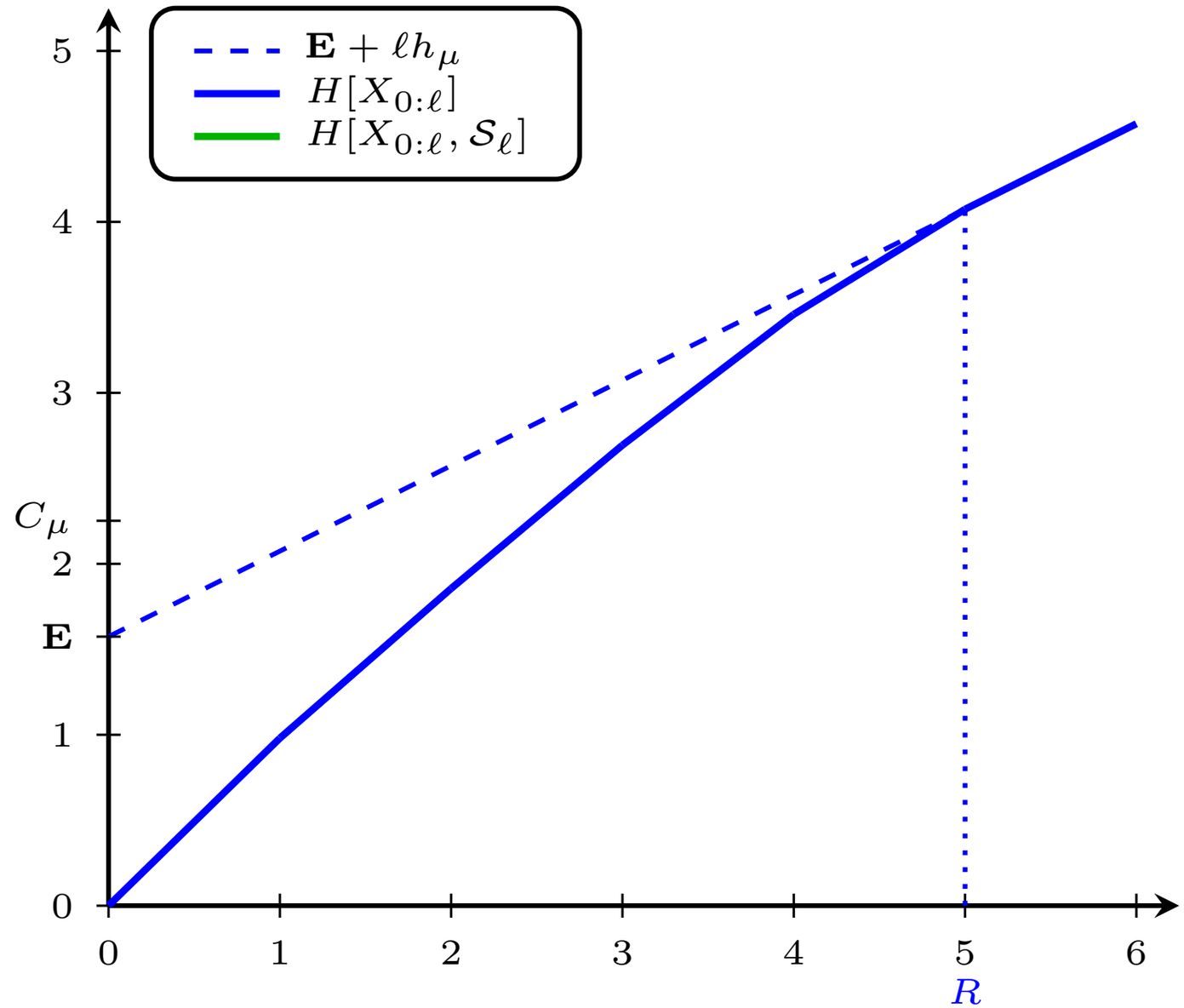

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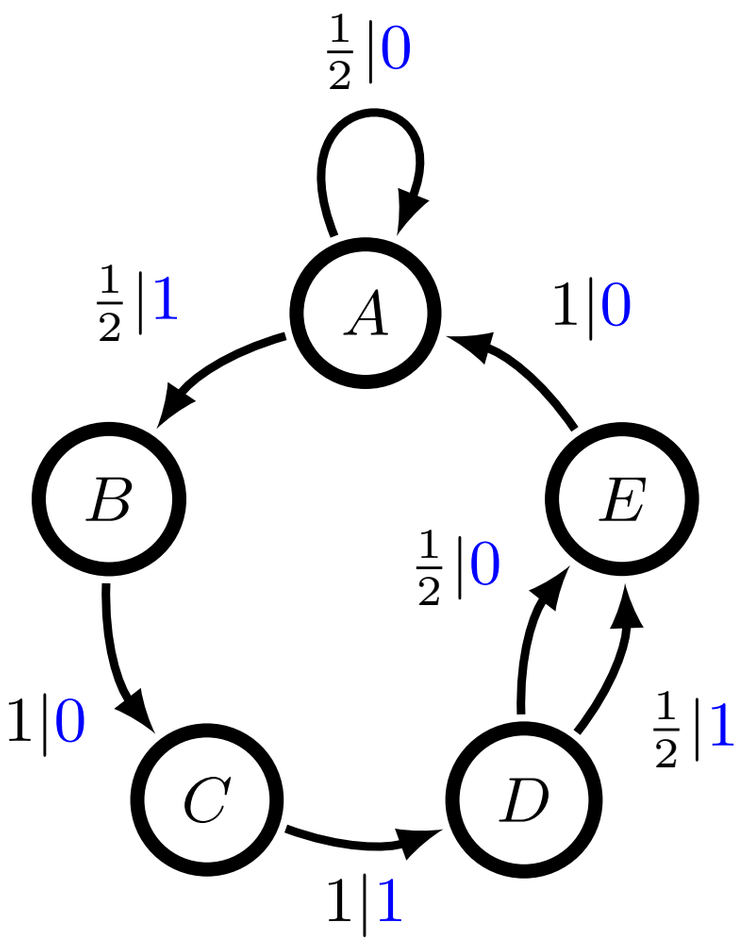

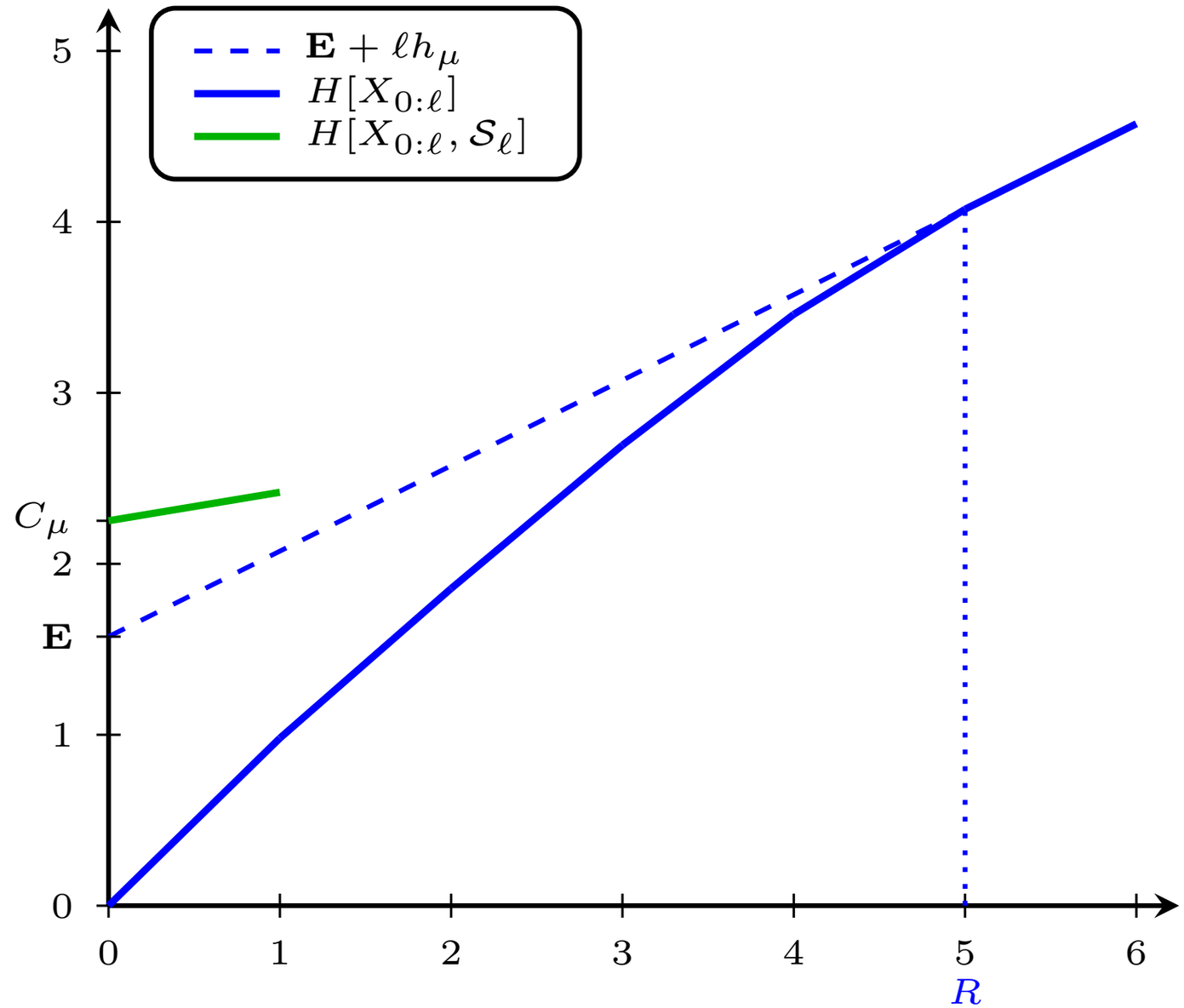

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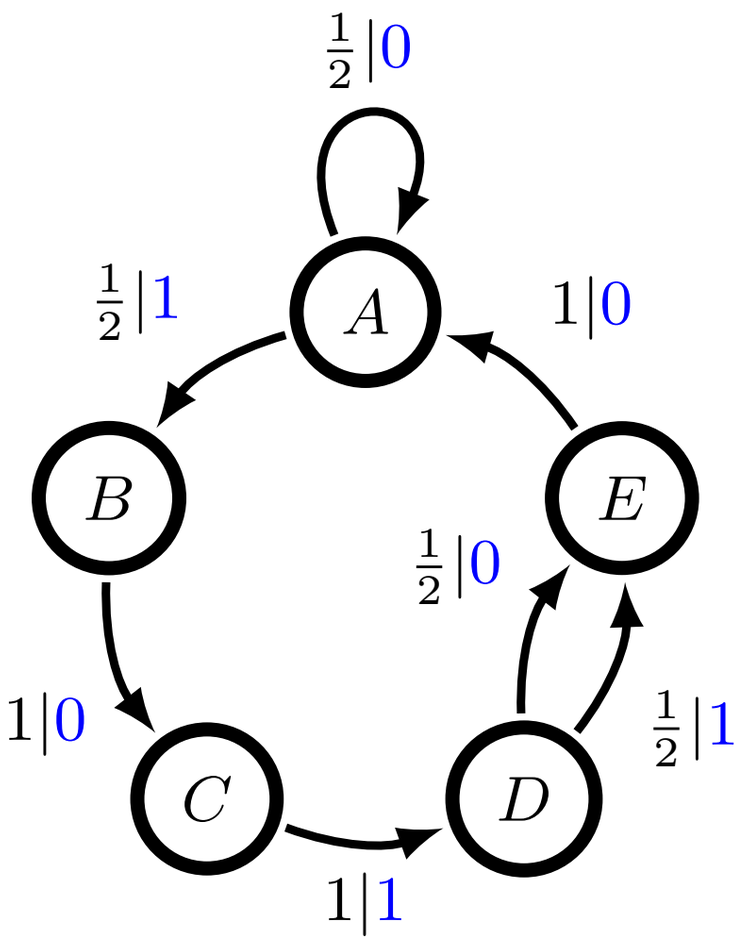

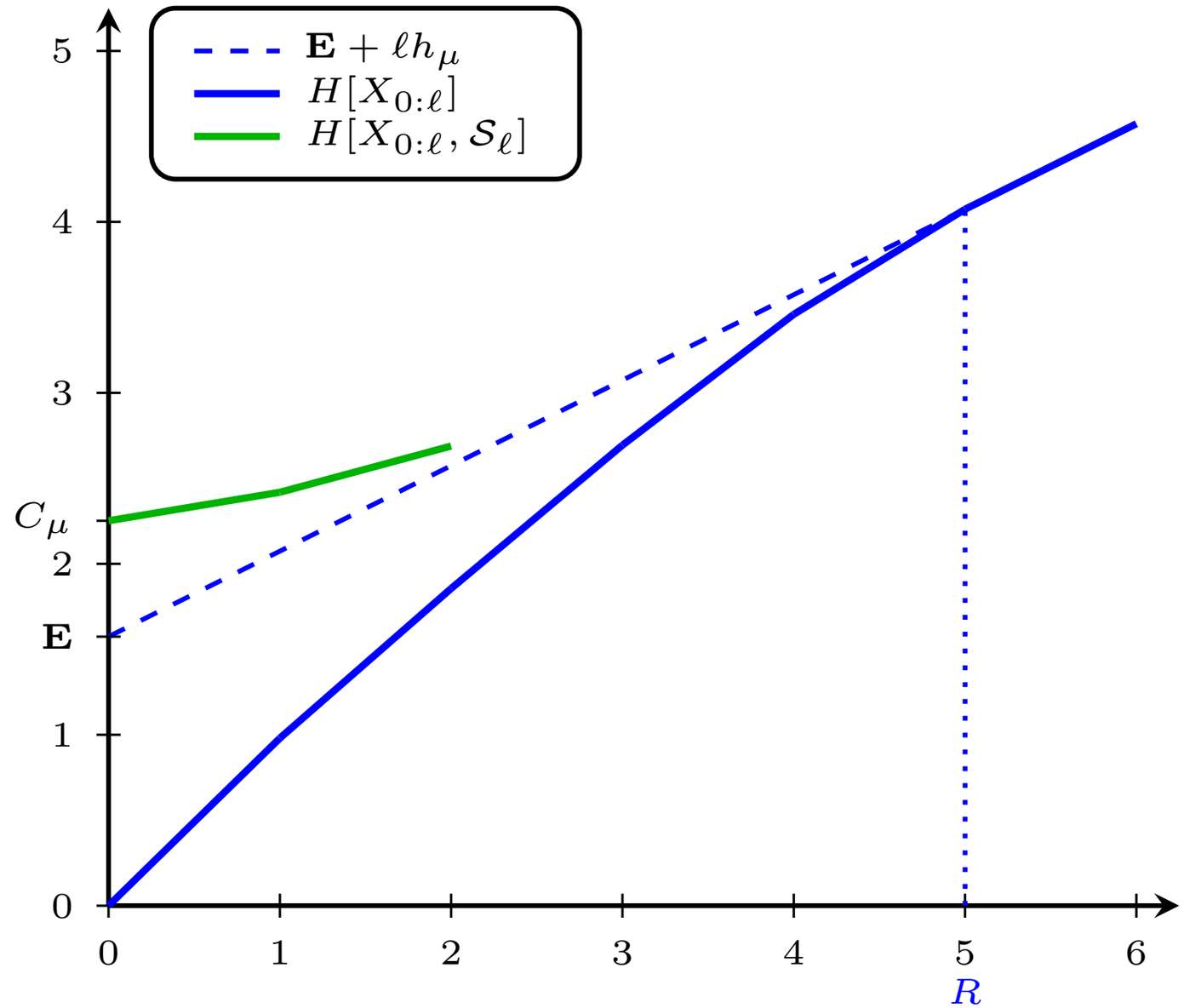

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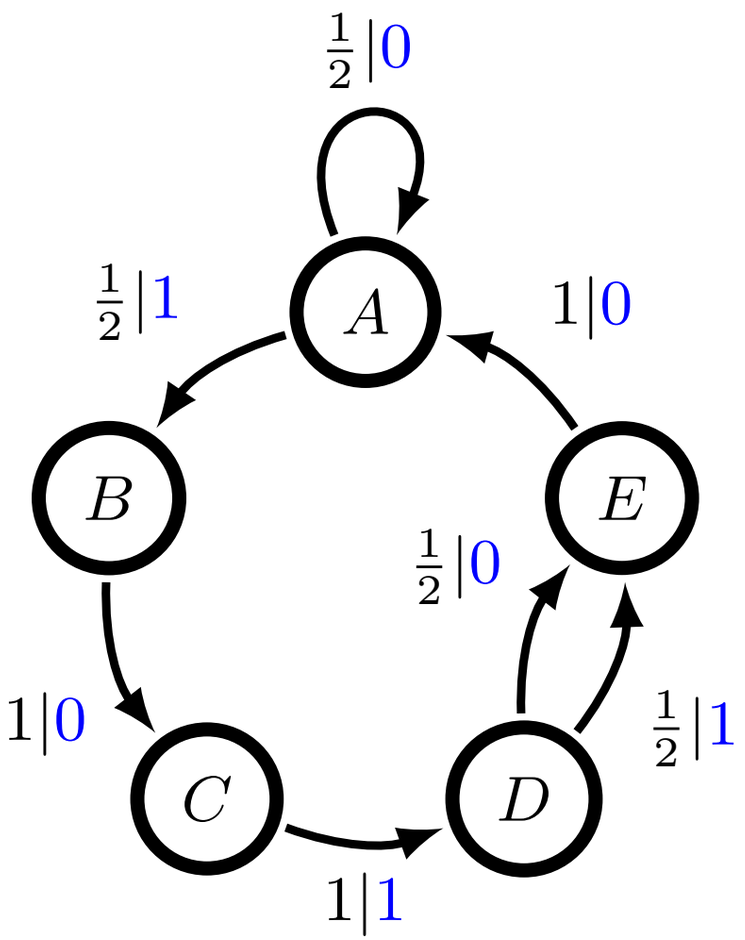

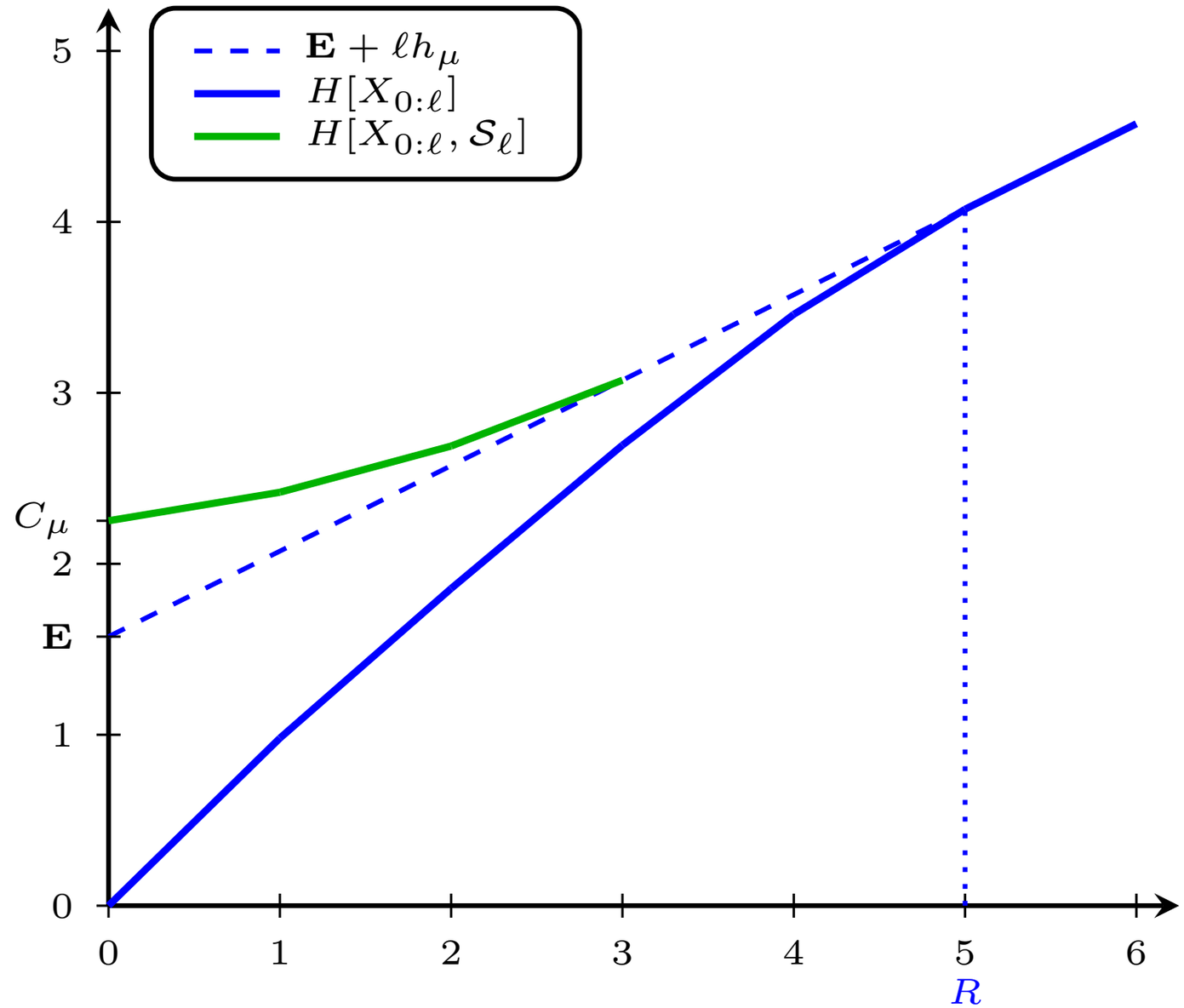

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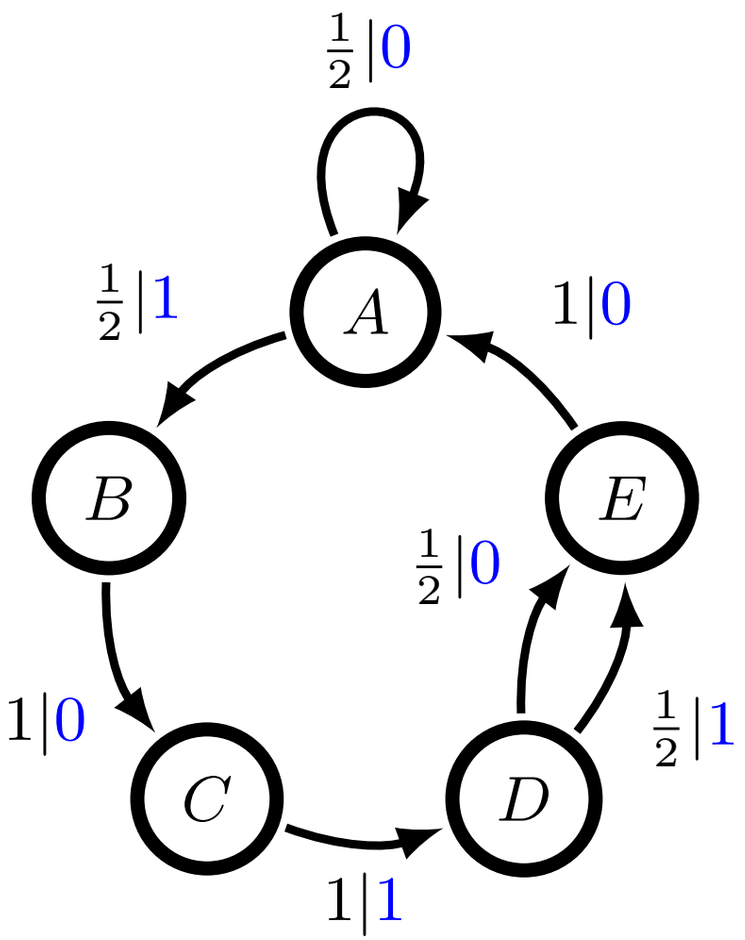

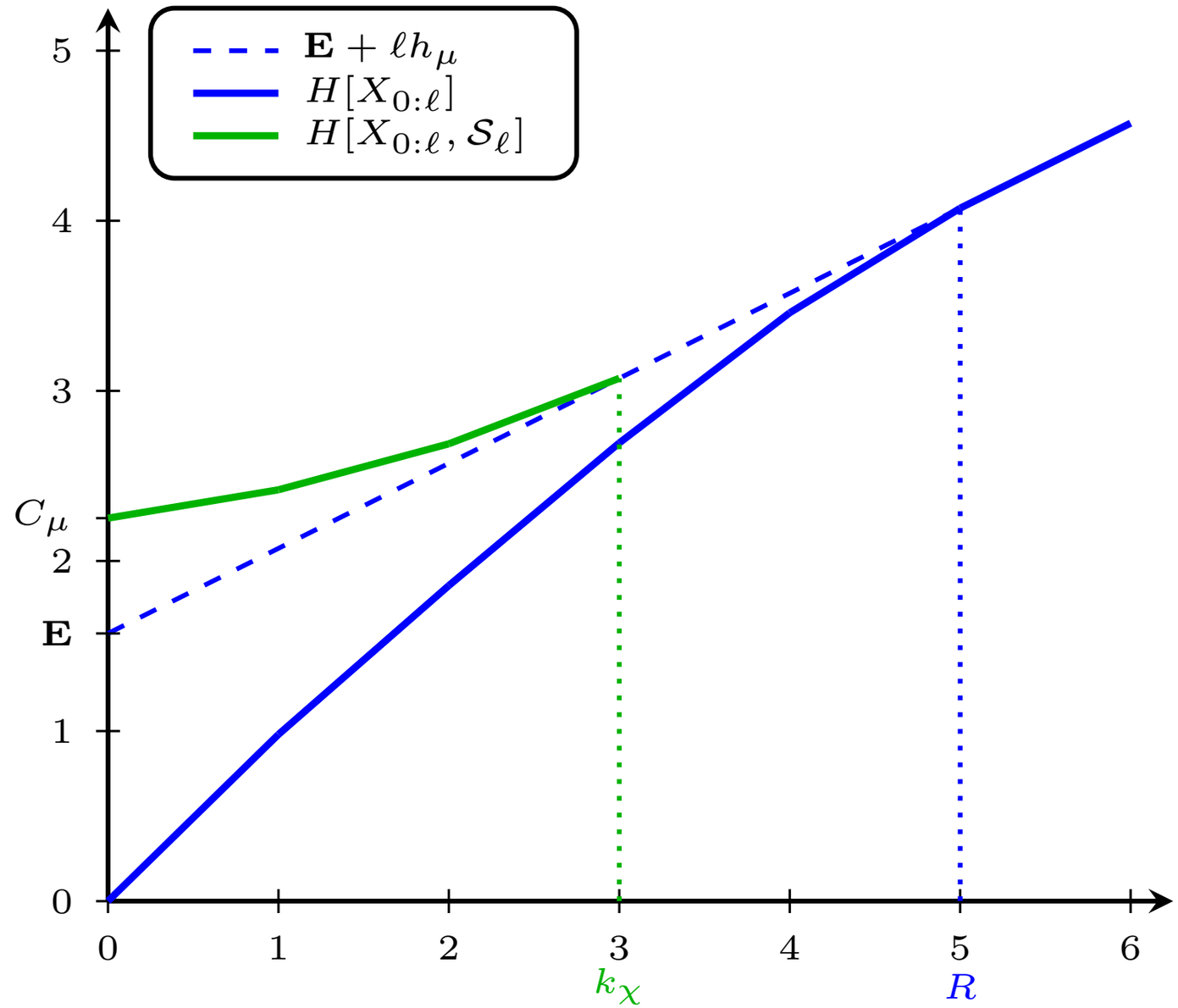

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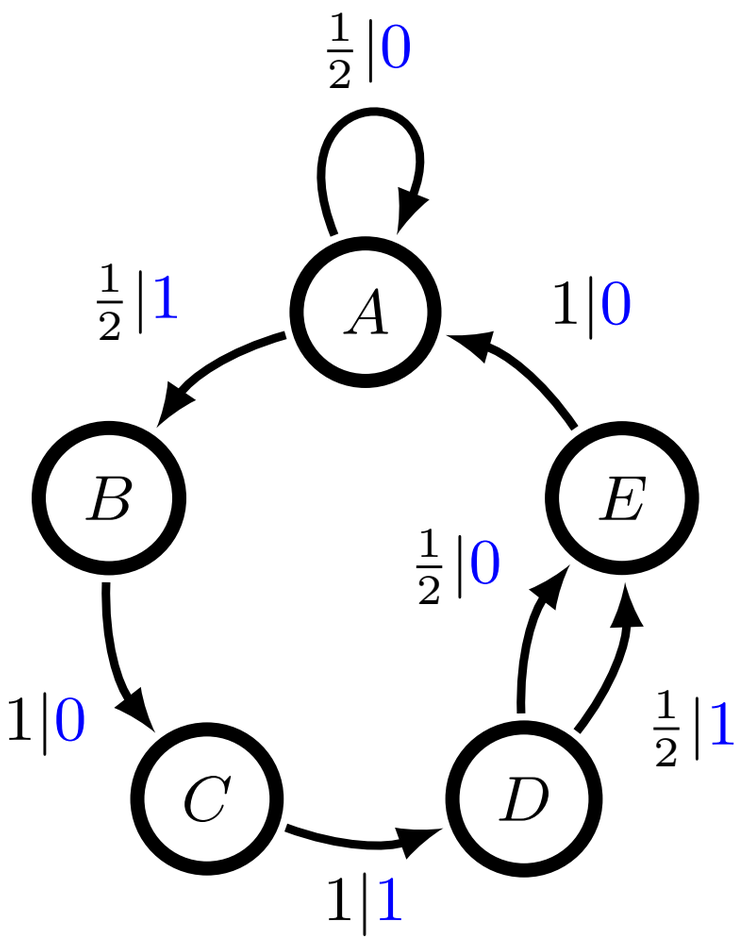

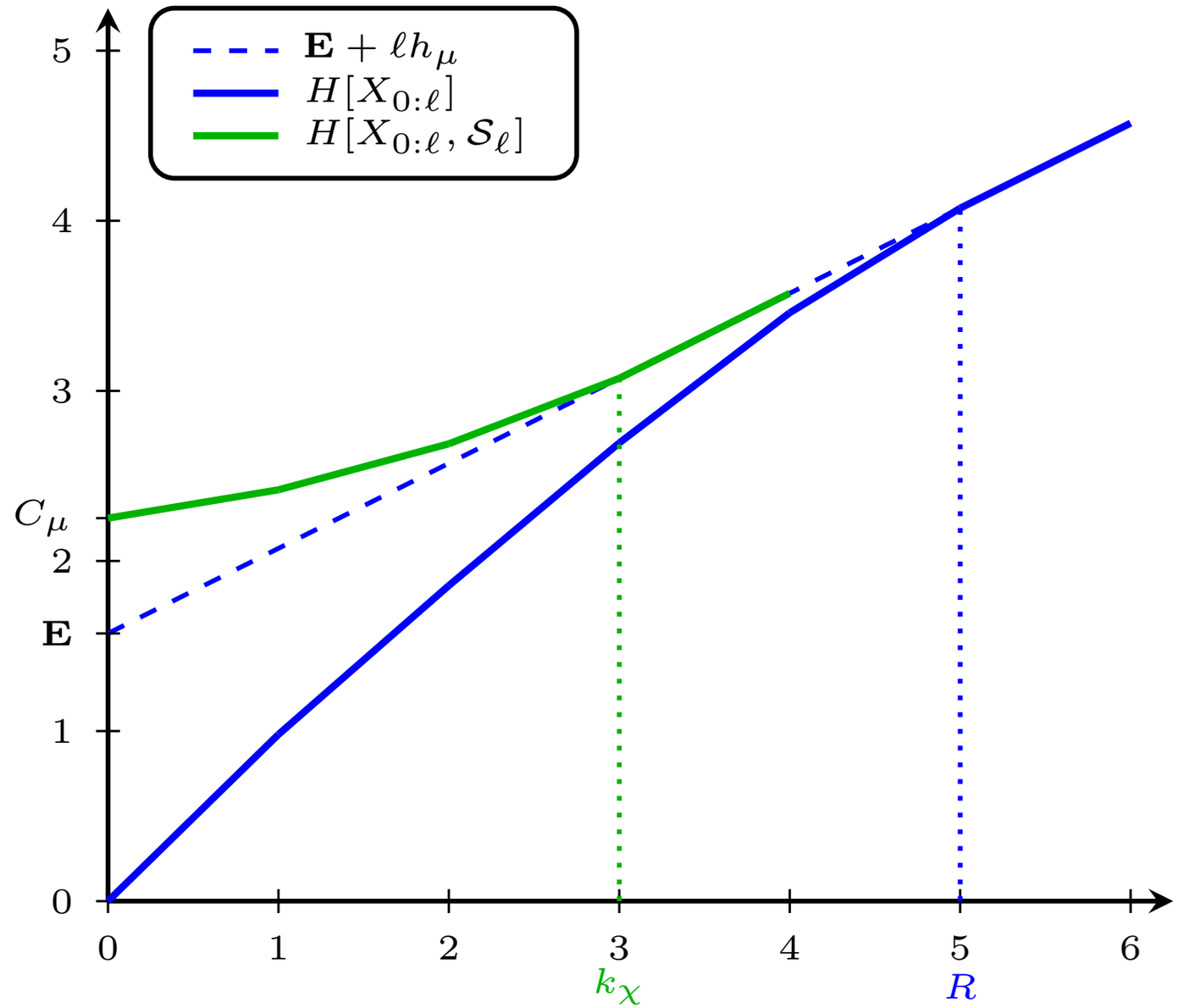

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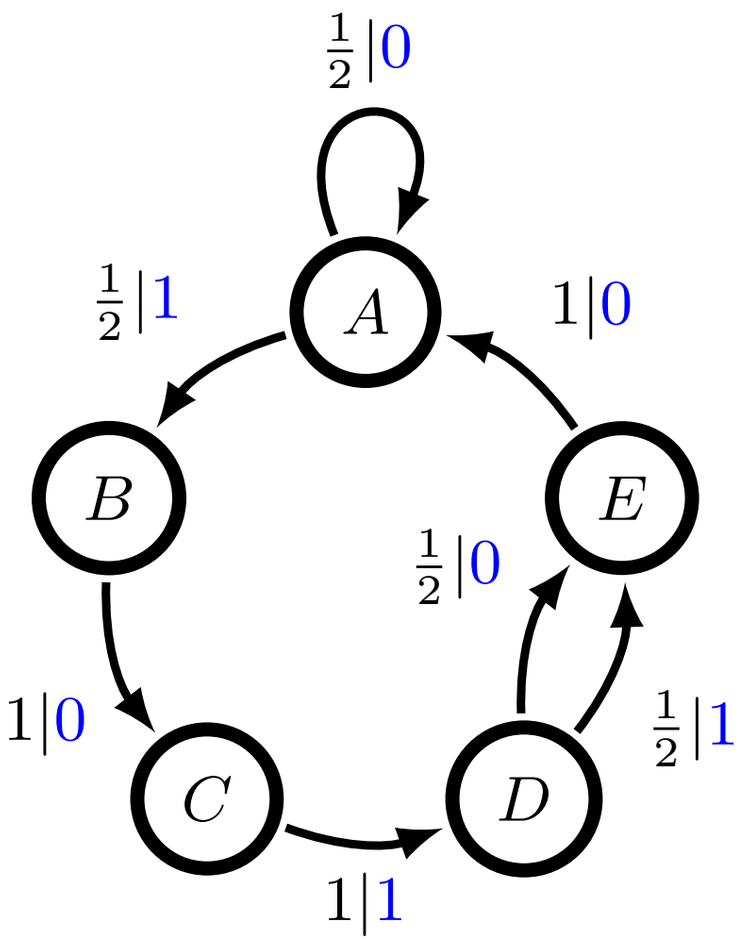

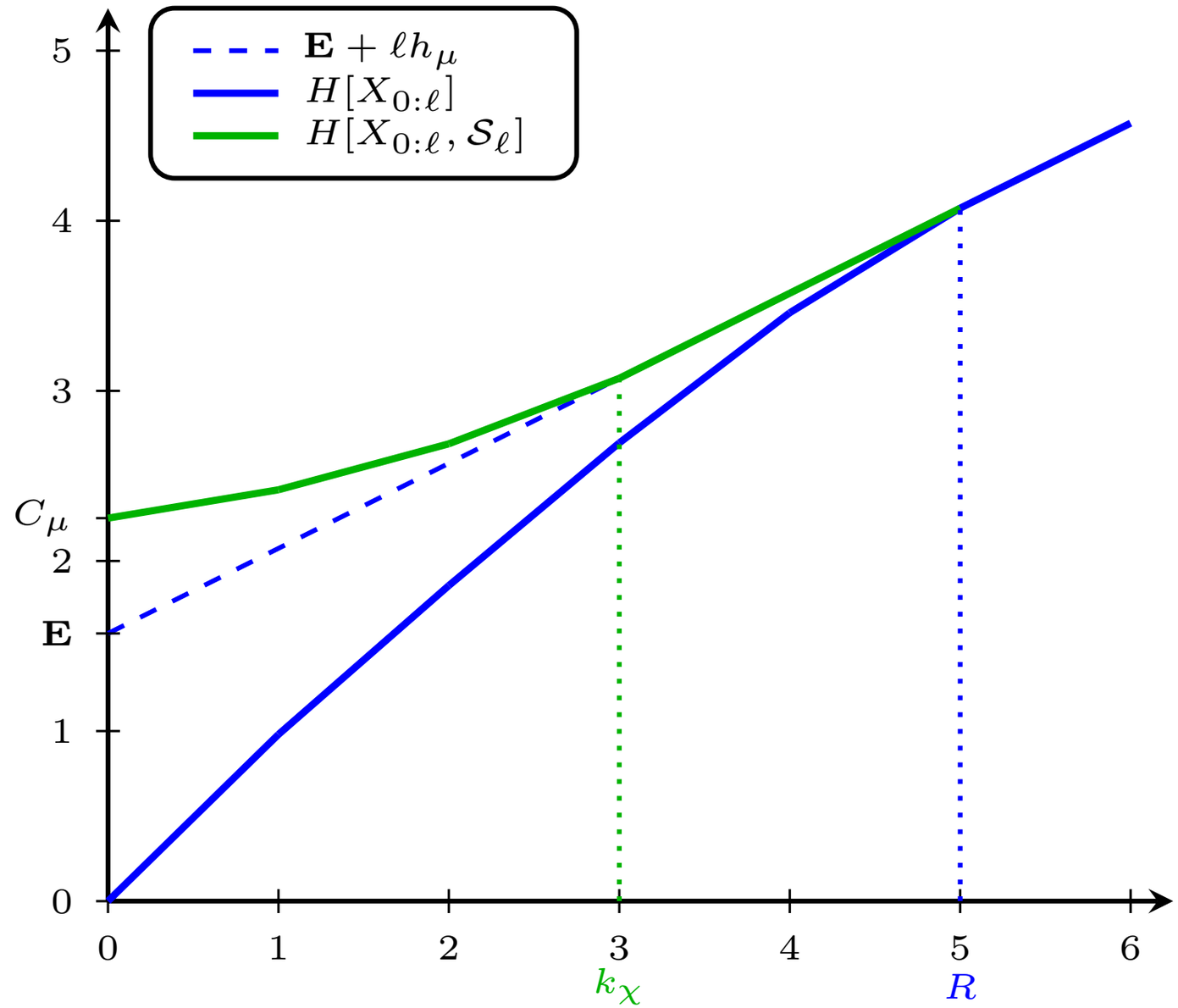

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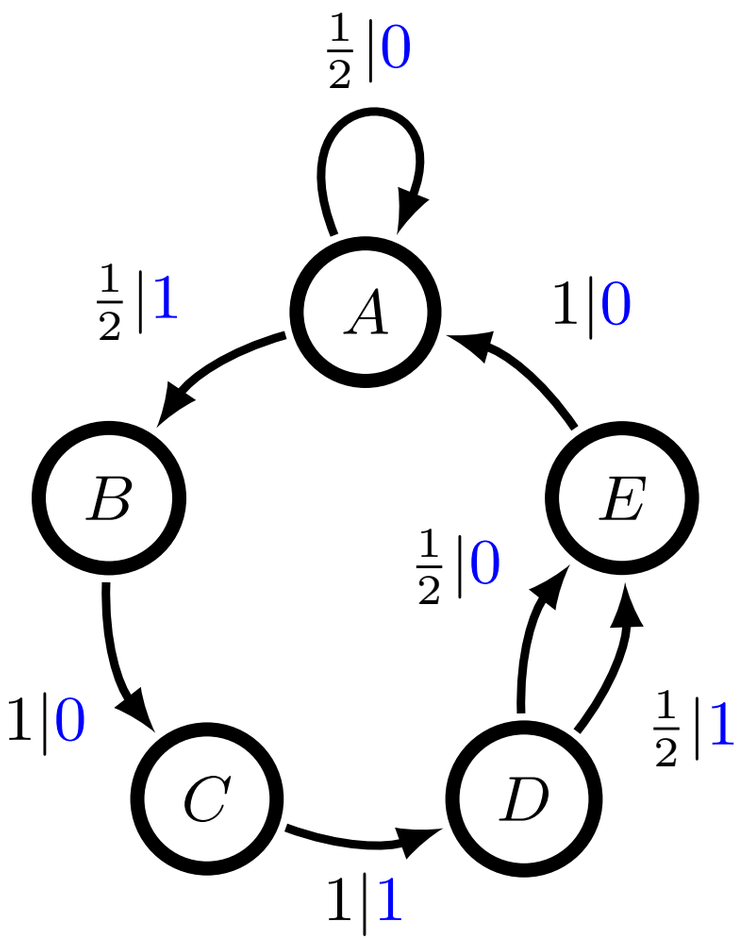

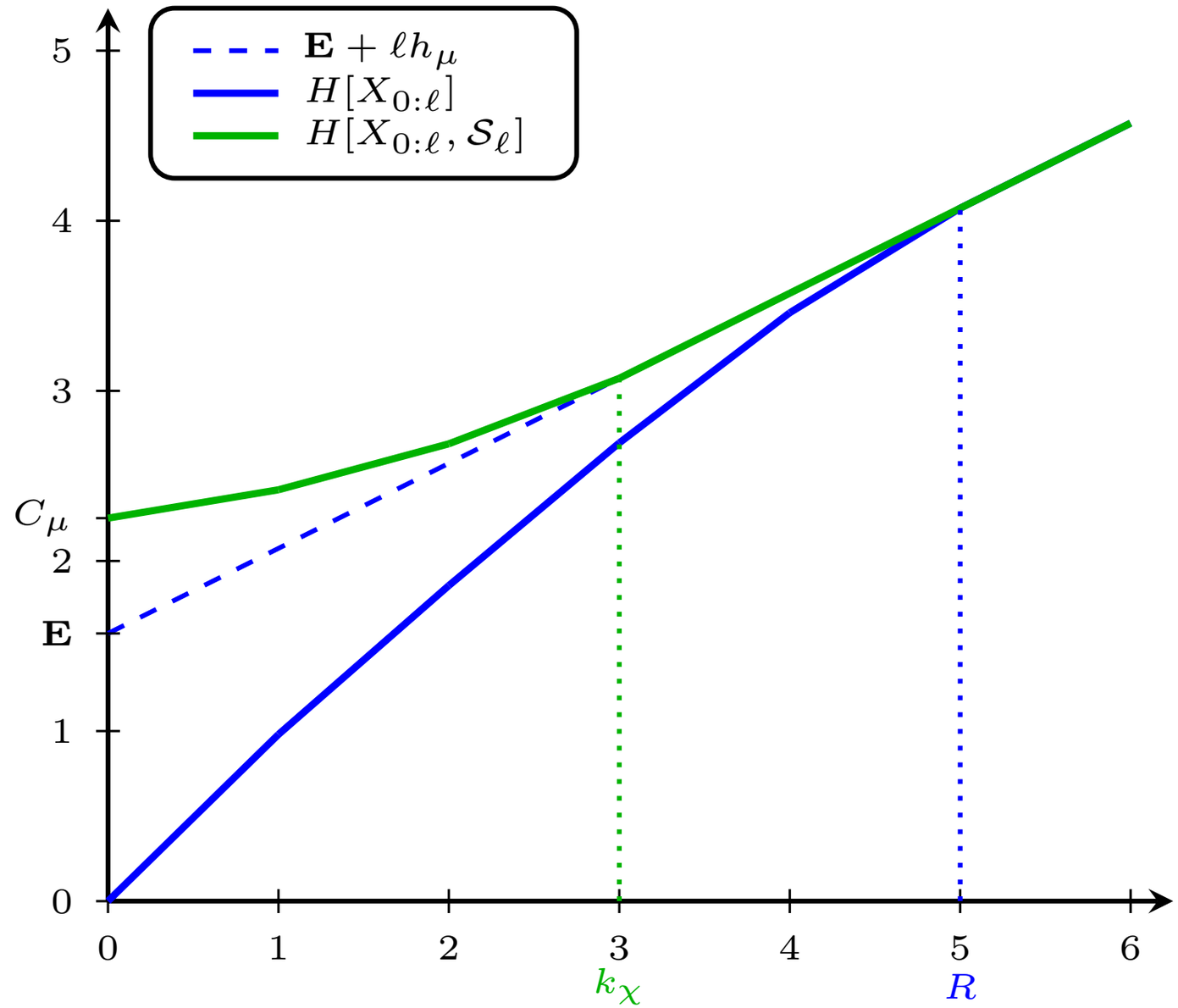

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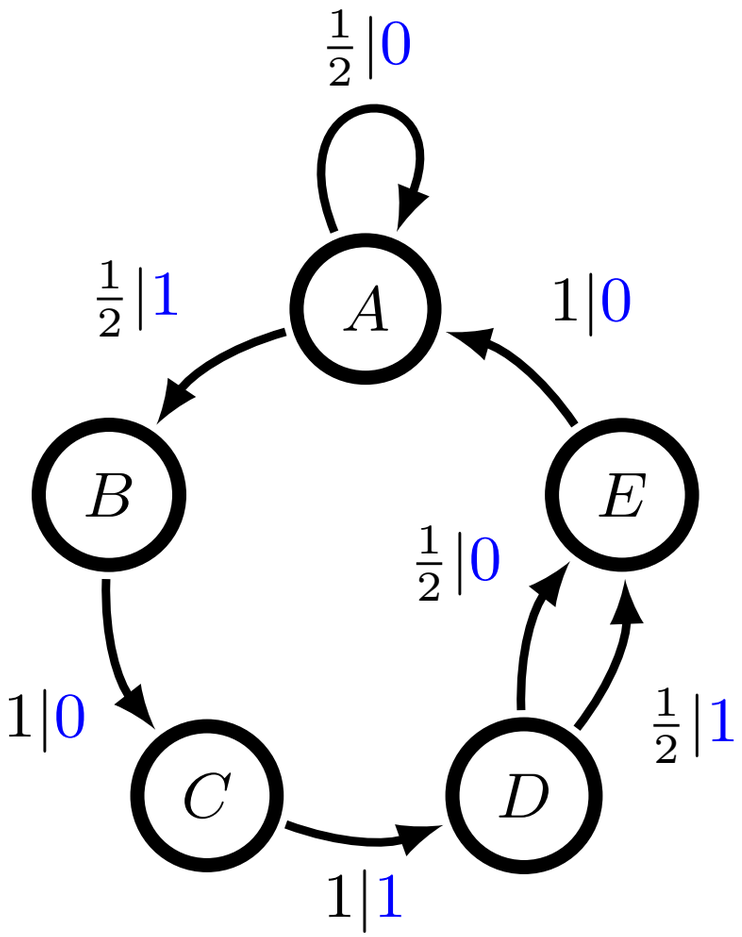
Problems With This Approach

What did we assume when we computed the Markov and cryptic orders?

- Knew $\mathbf{E}$ exactly
- Knew h_μ exactly
- Could differentiate *exactly on* the asymptote from *less than machine precision away from* the asymptote
- Could "guess" when R or k_χ were infinite, else we'd be computing block entropies indefinitely

A Hint of a Solution



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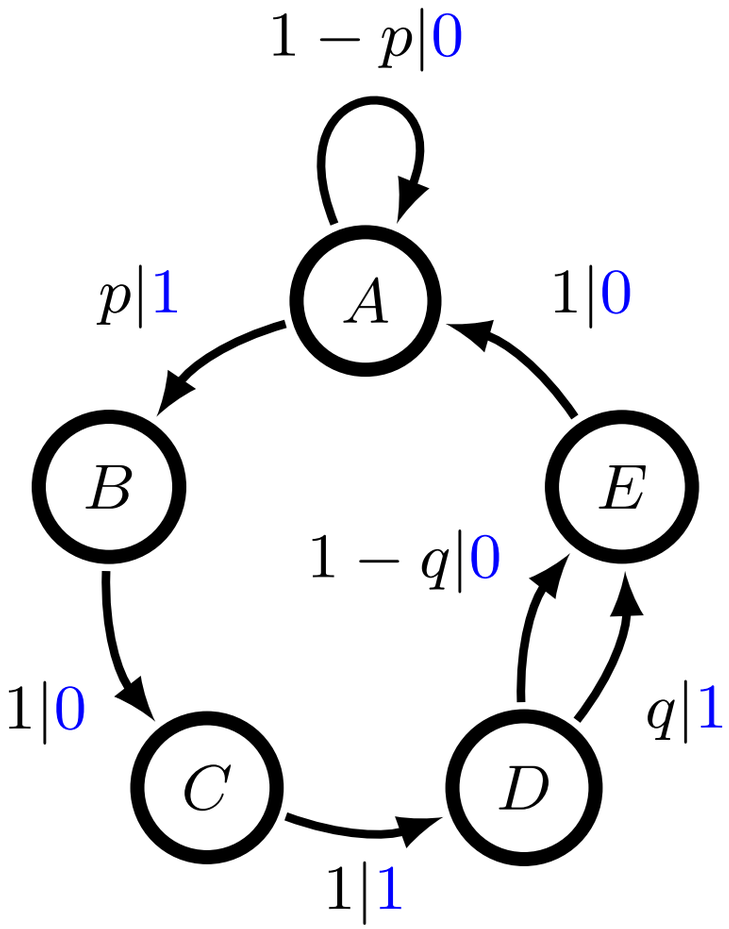

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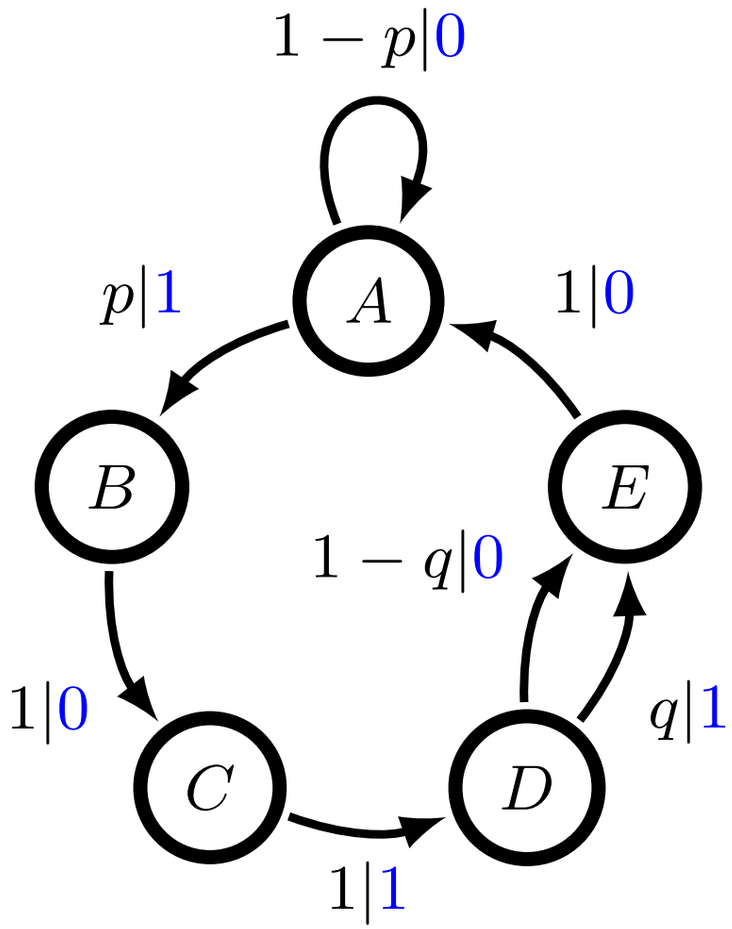

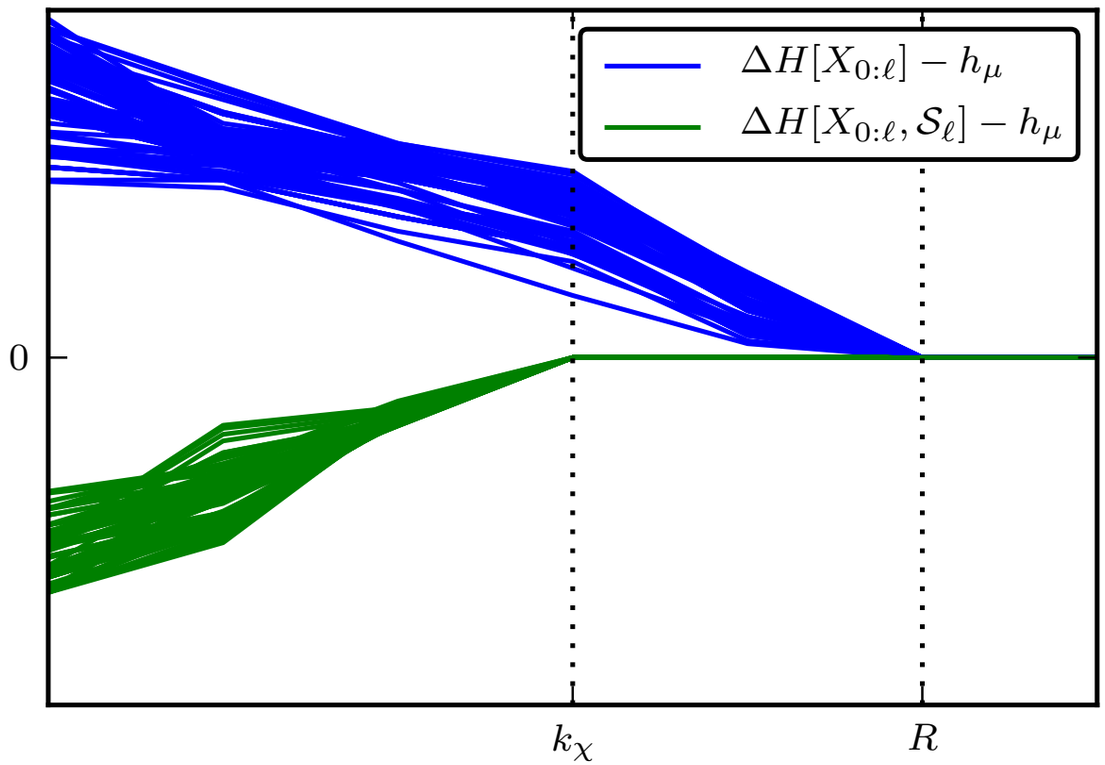

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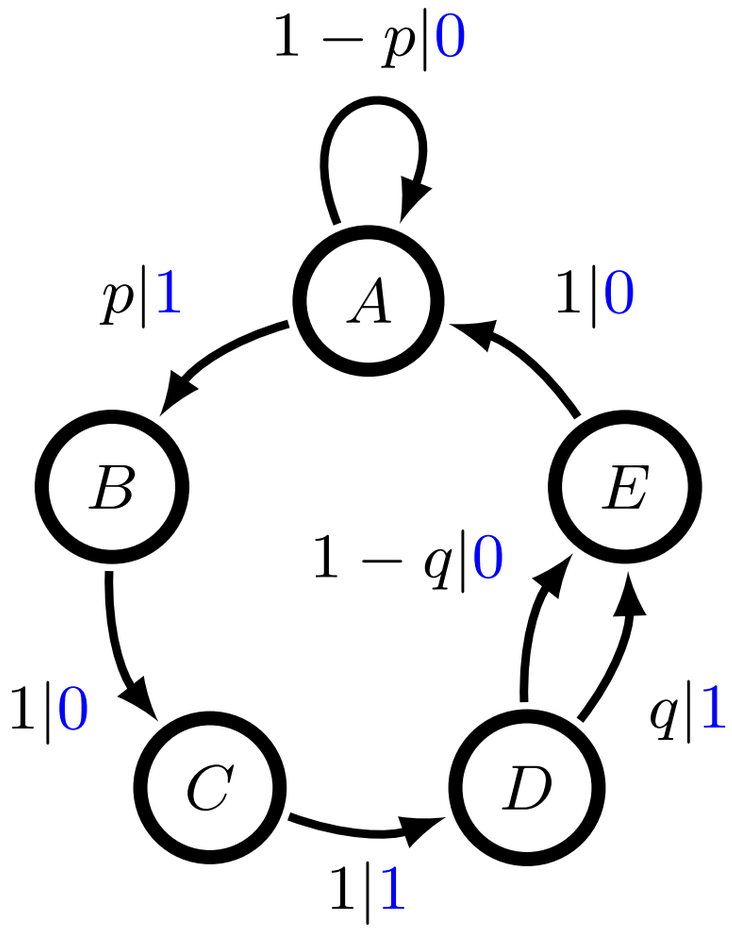

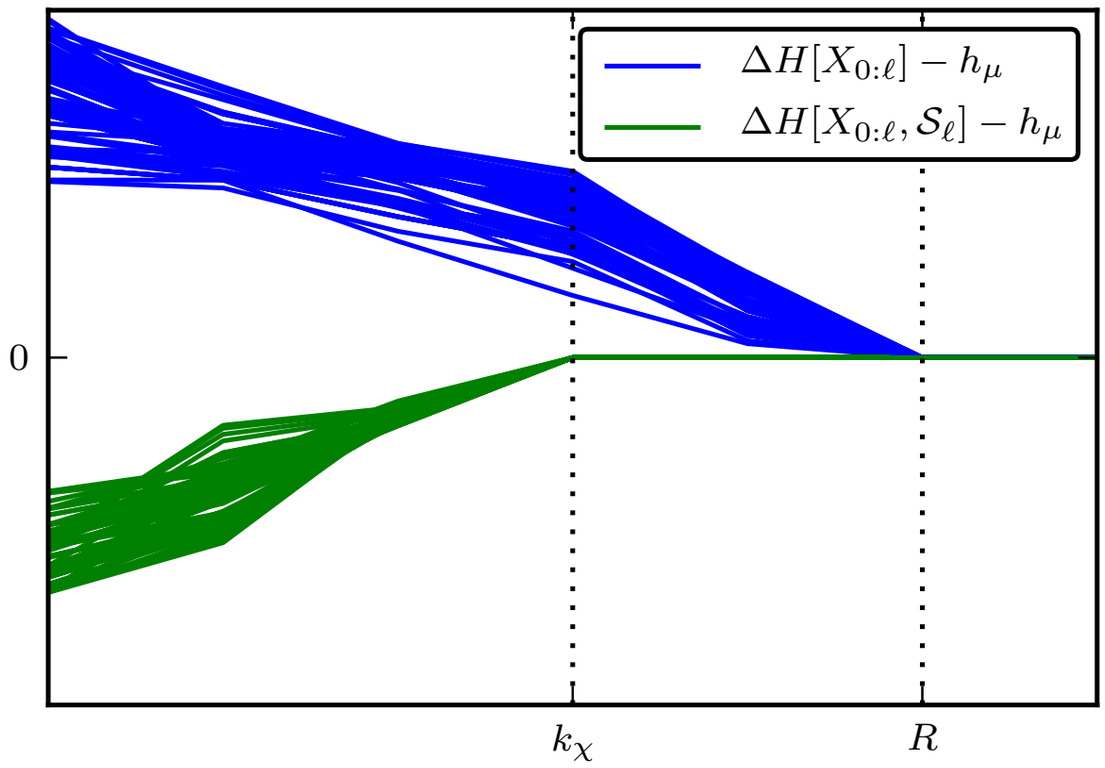

Markov and cryptic orders are *independent* of the probabilities!

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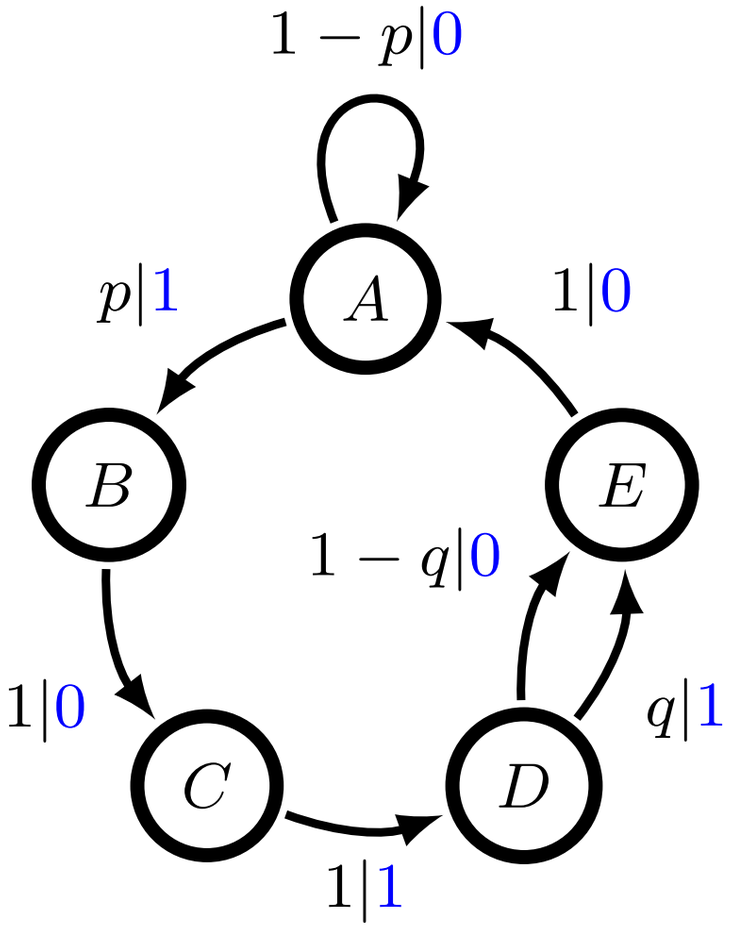

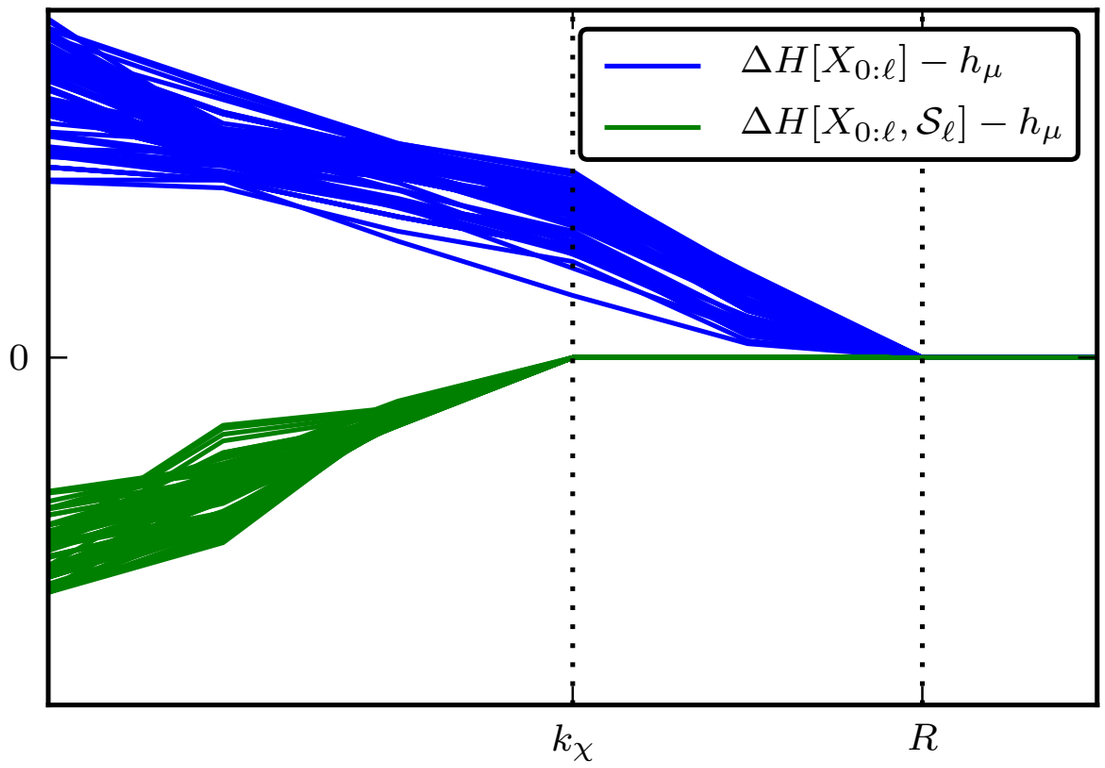

Markov and cryptic orders are *independent* of the probabilities!
They depend only on the *topology* of the ϵ-machine!

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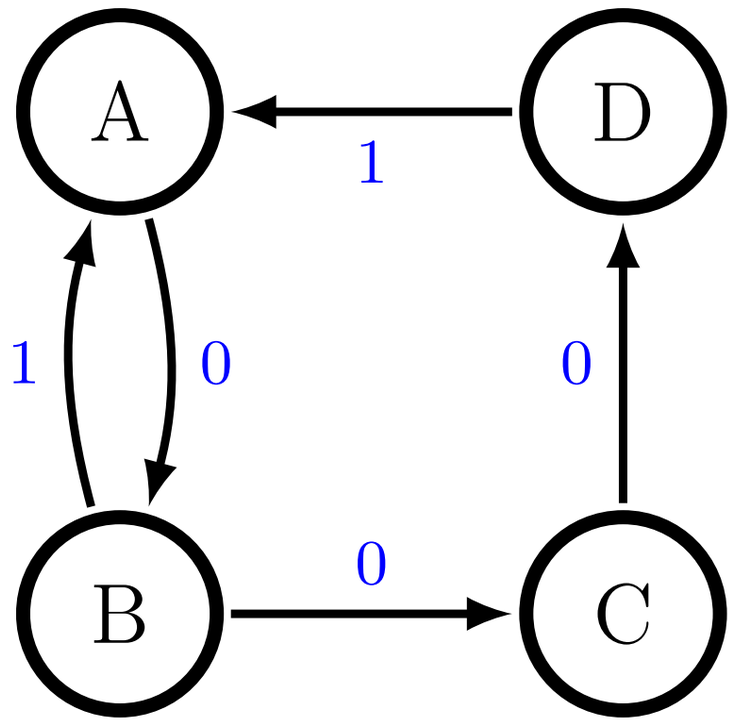
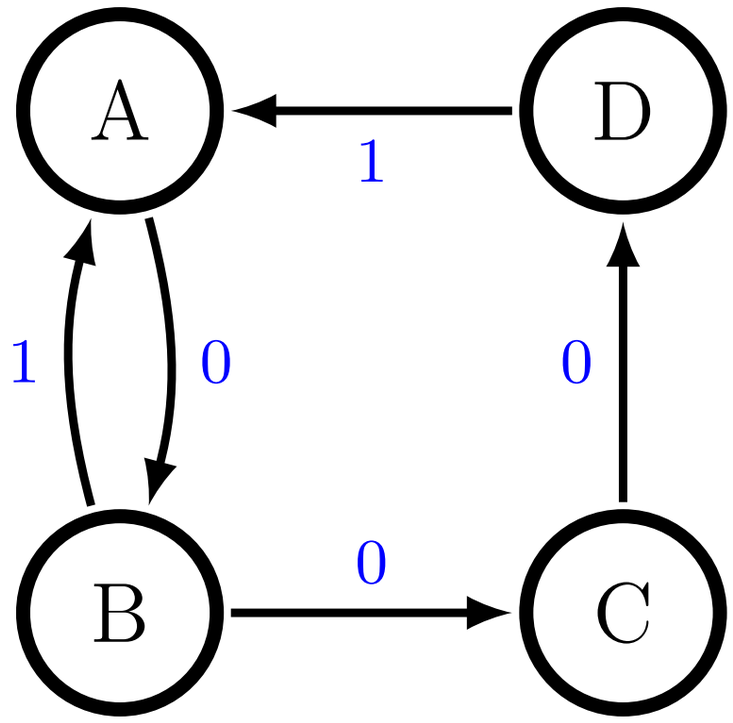
Walking Paths



Walking Paths

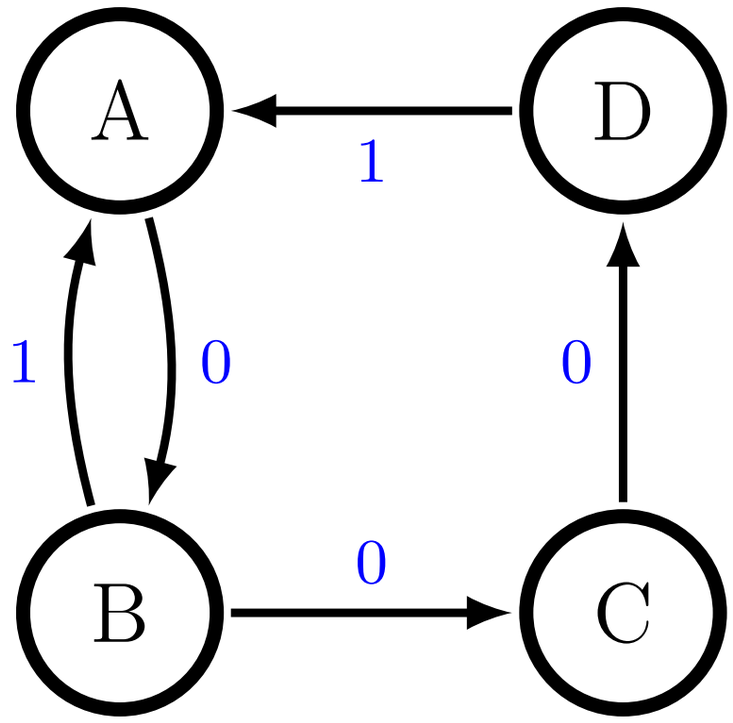
A
B
C
D



Walking Paths

A
B
C
D

0



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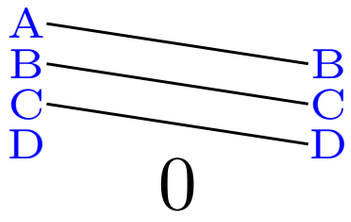

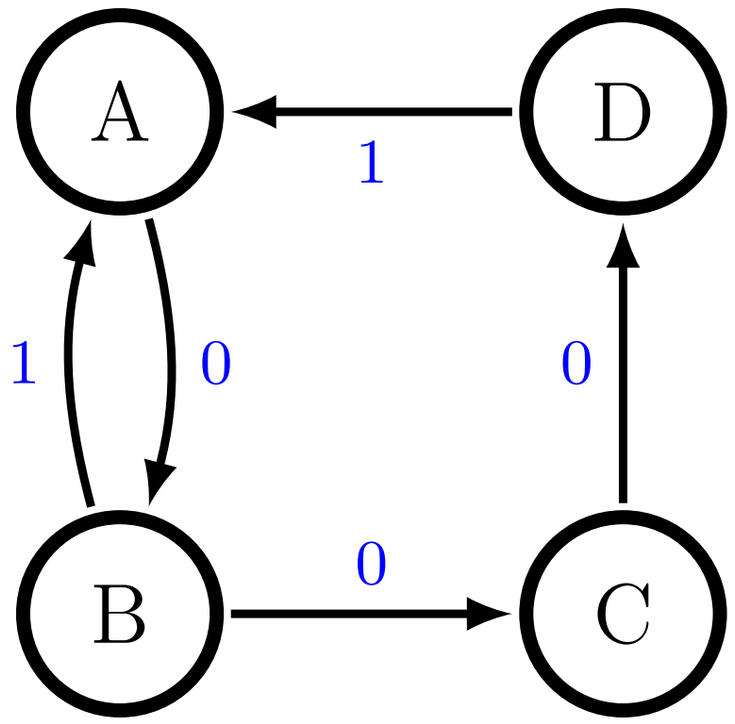

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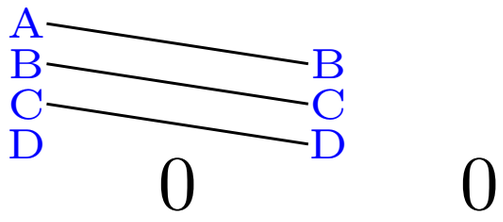

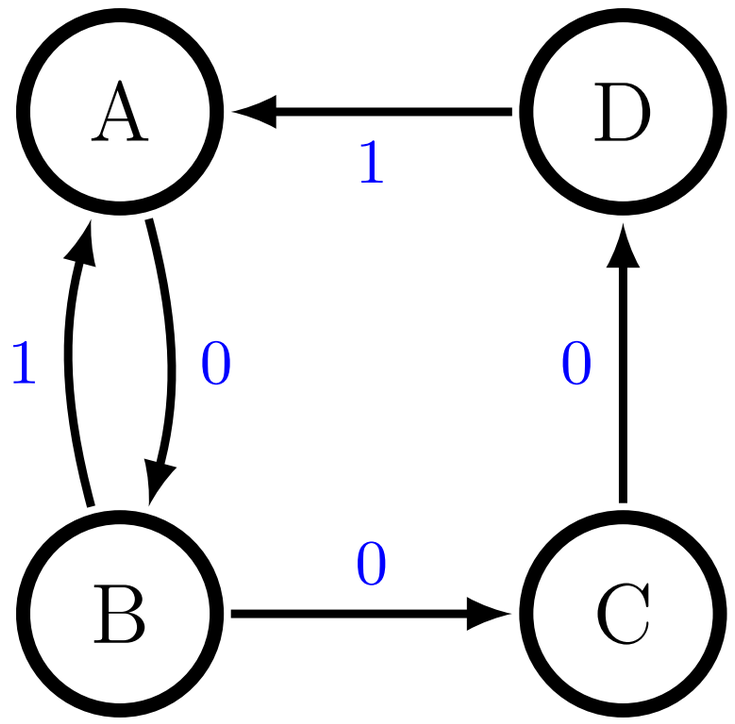

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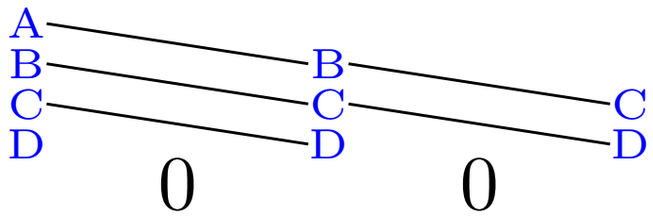

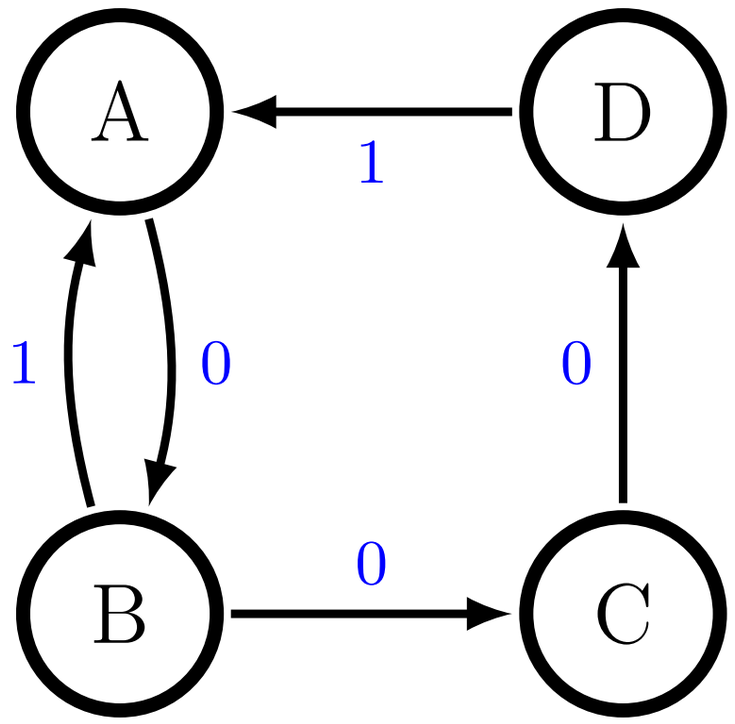

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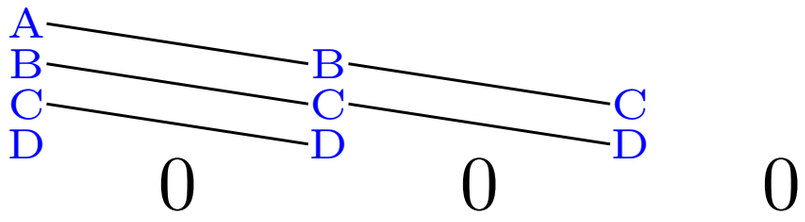

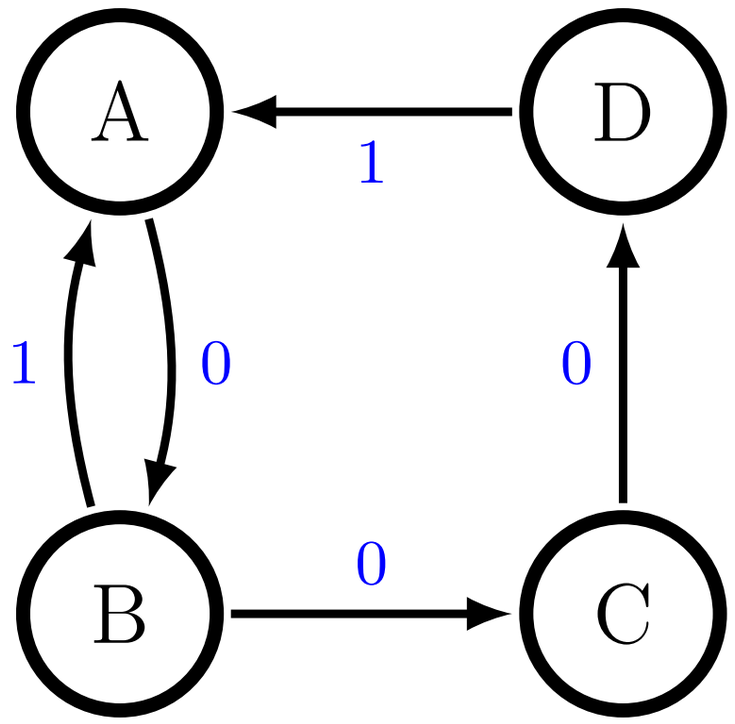

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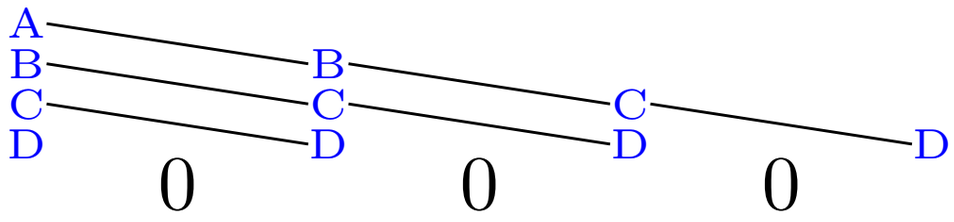

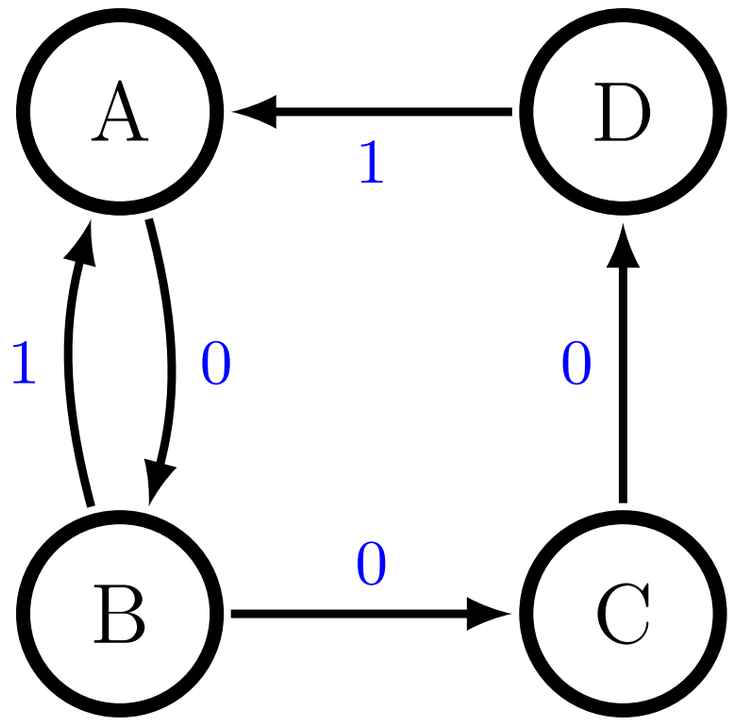

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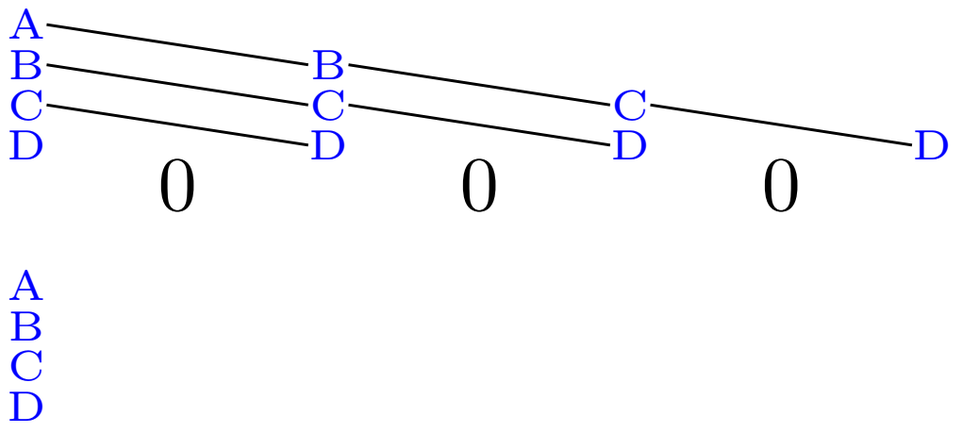

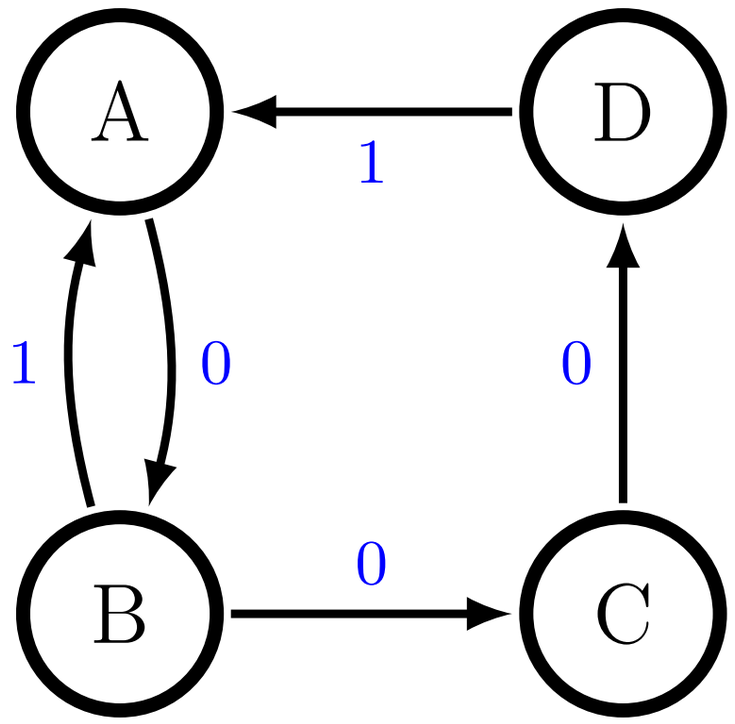

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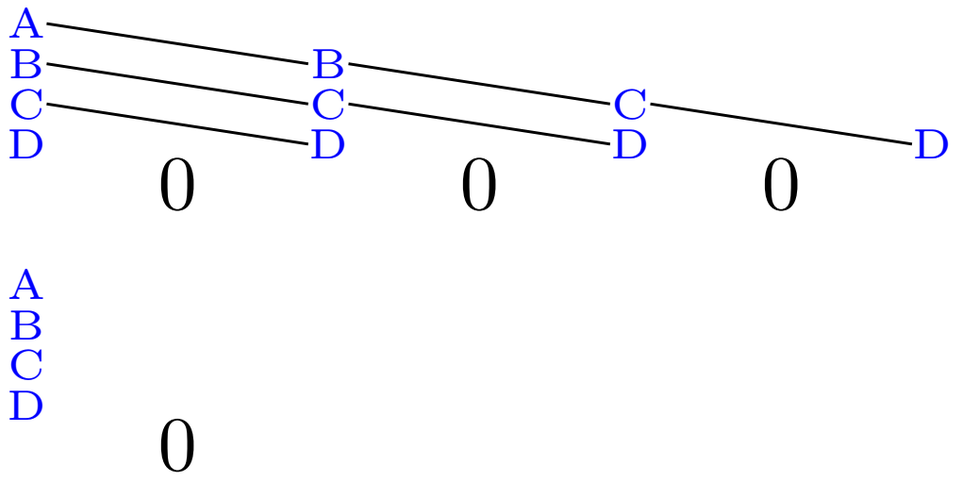

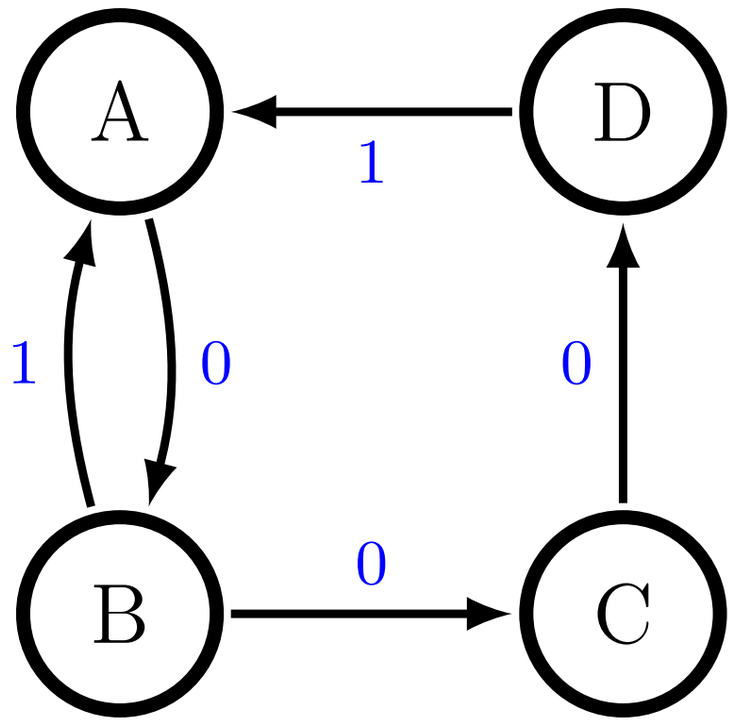

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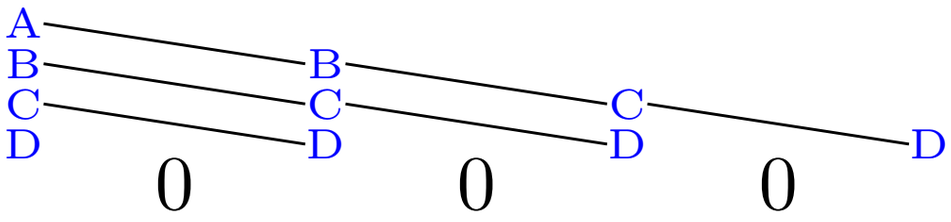

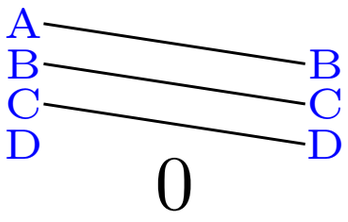

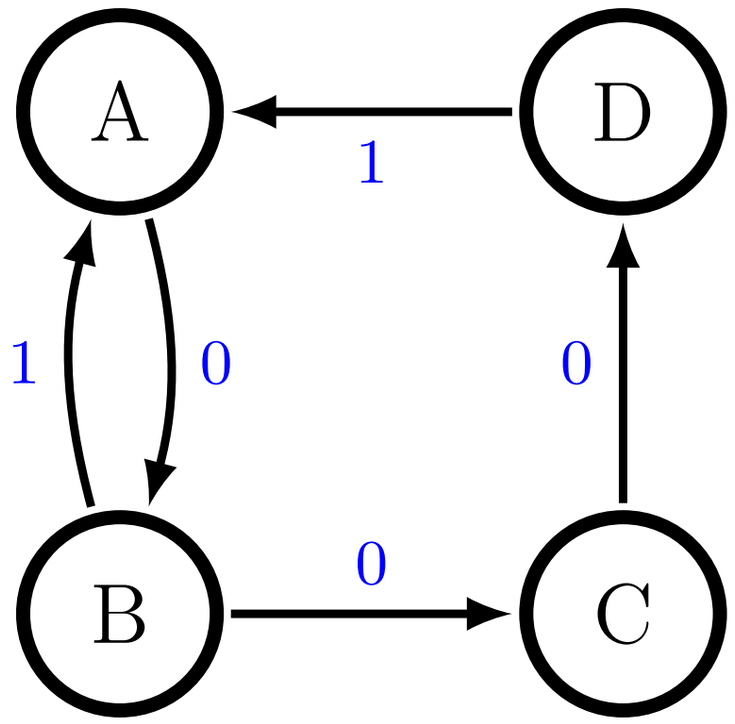

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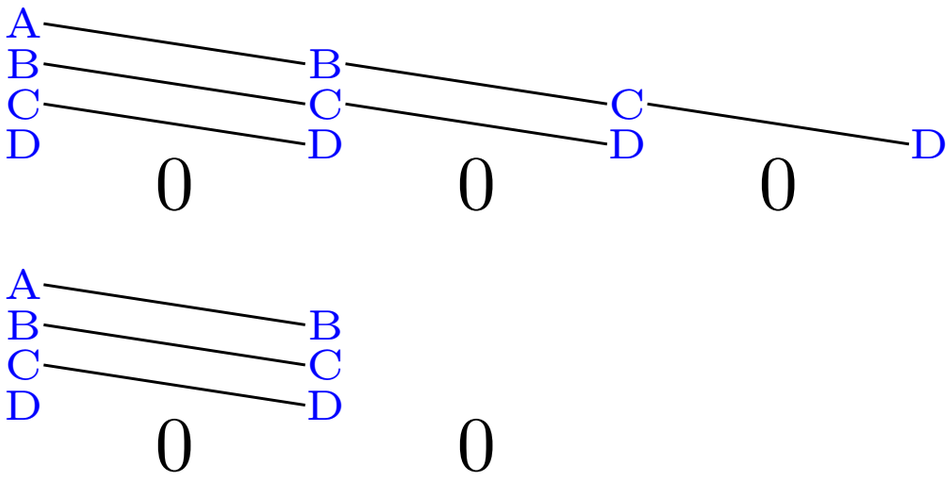

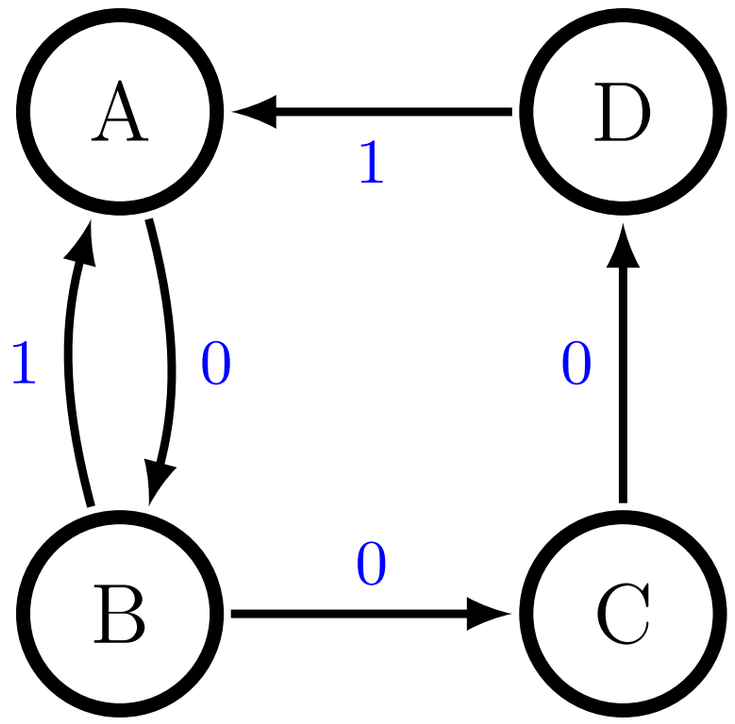

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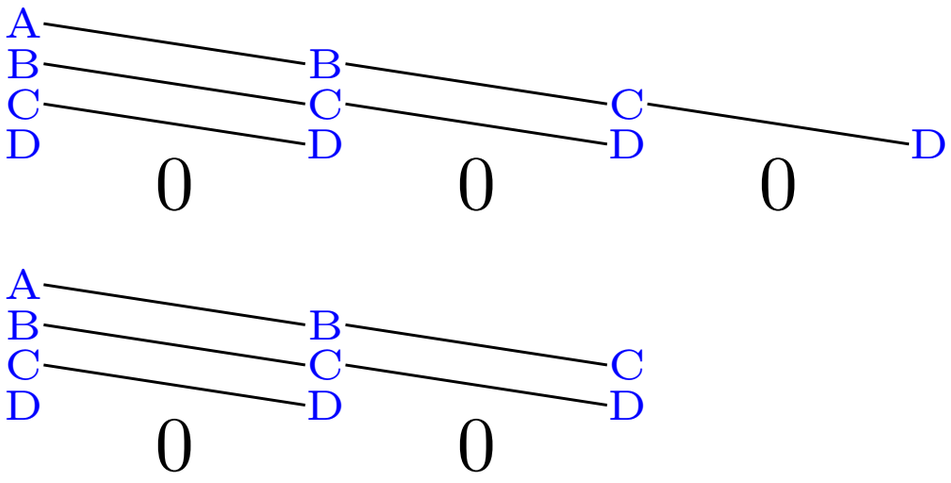

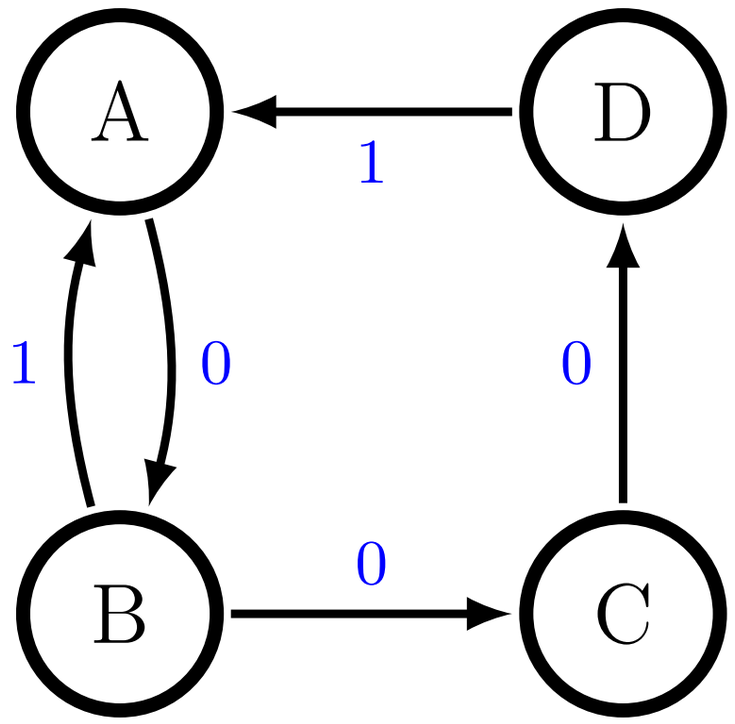

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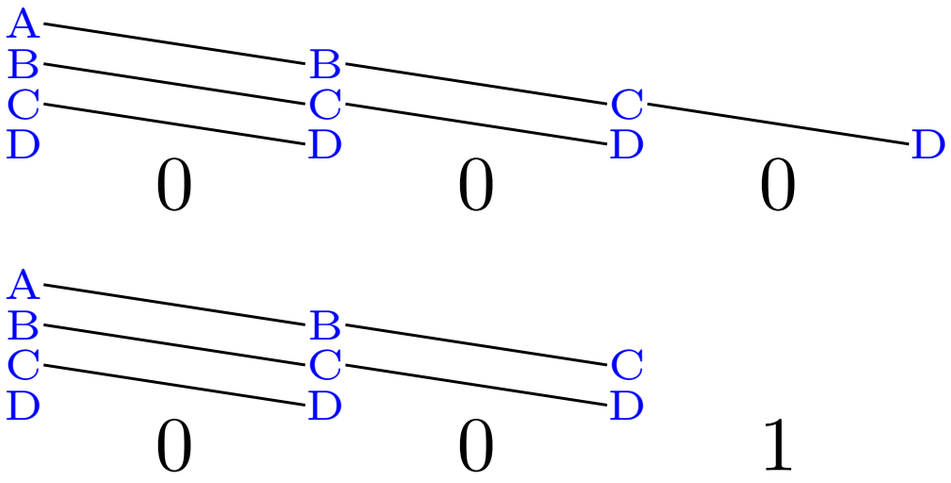

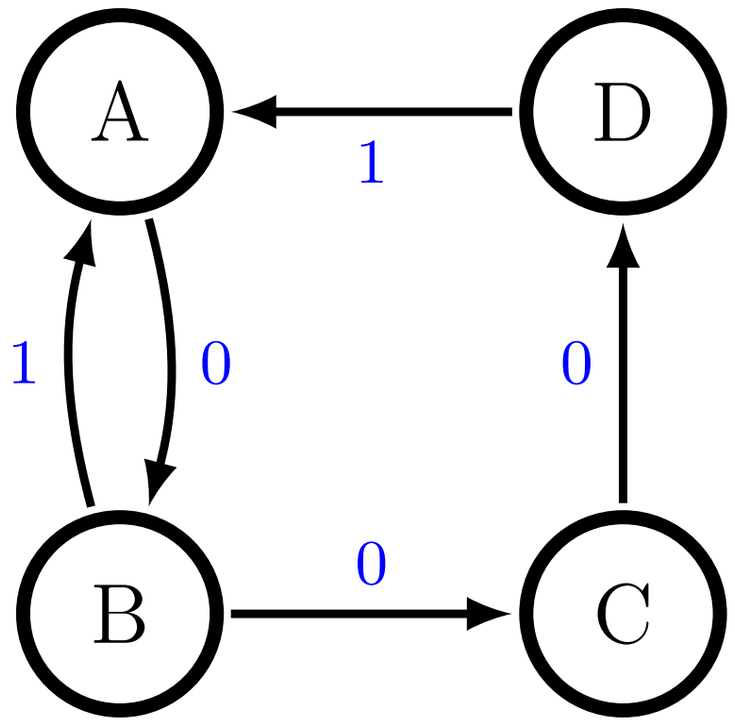

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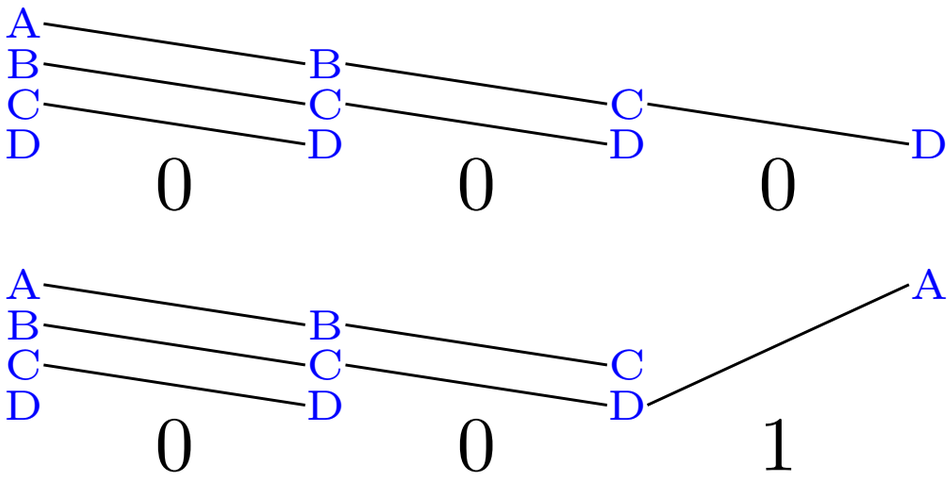

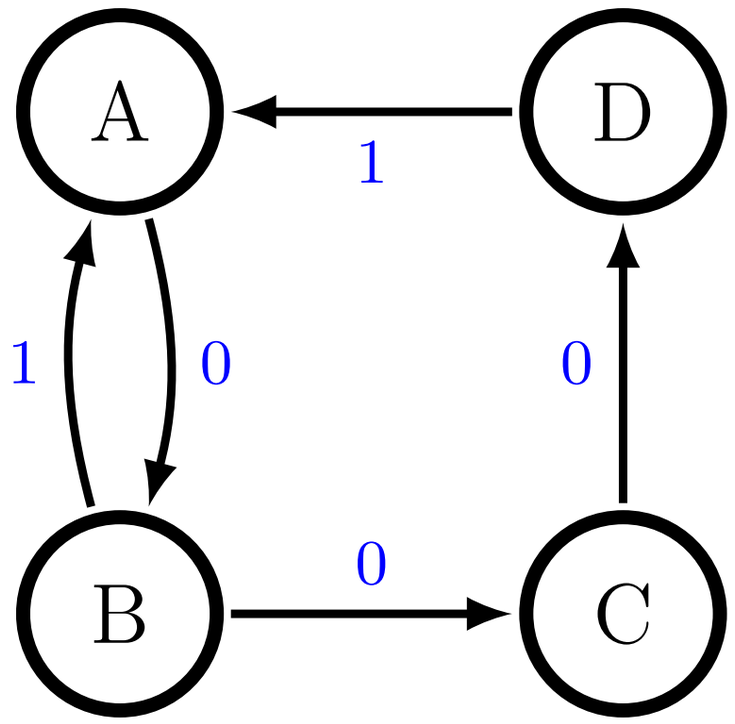

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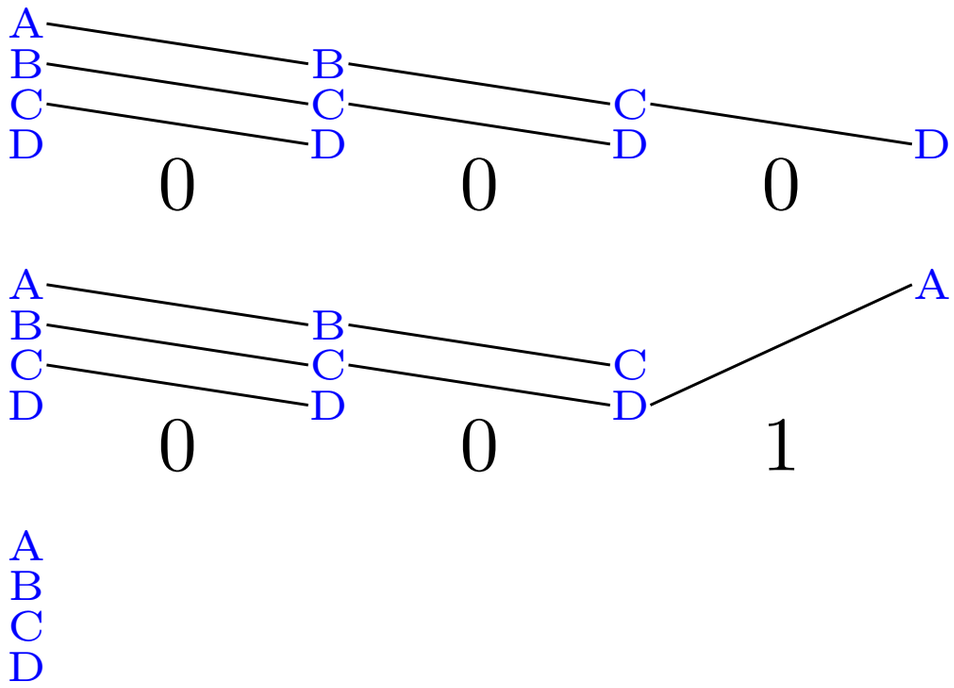

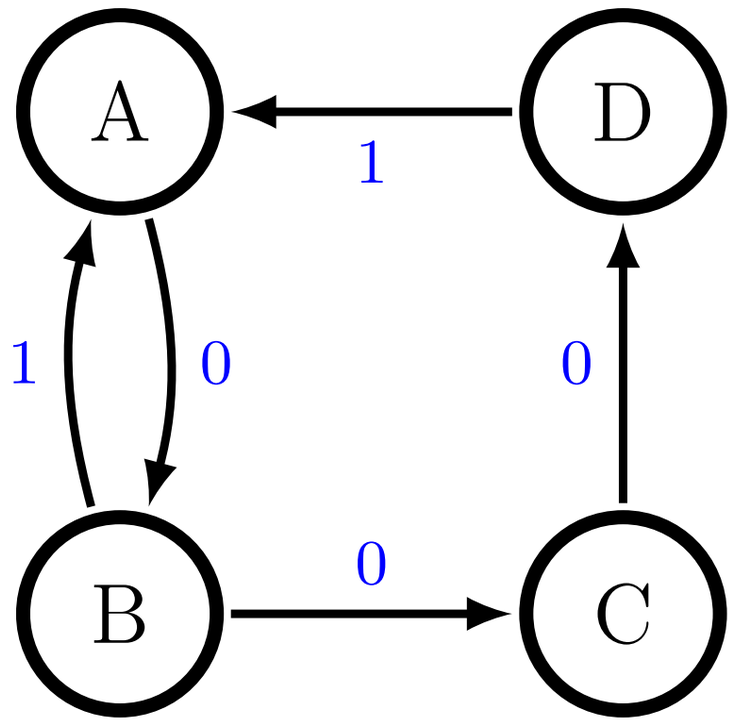

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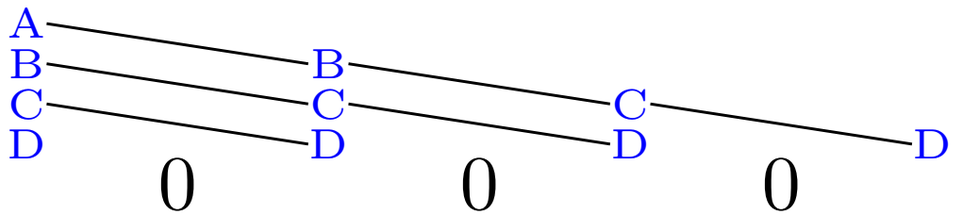

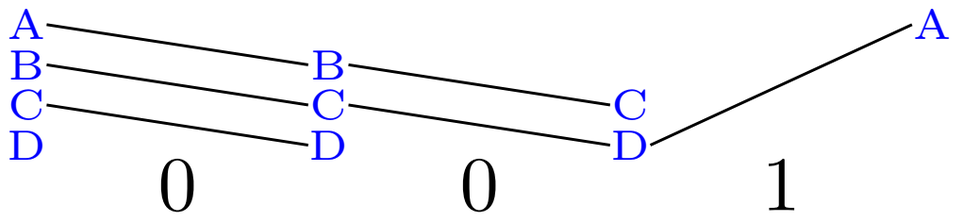

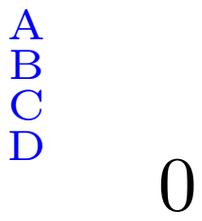

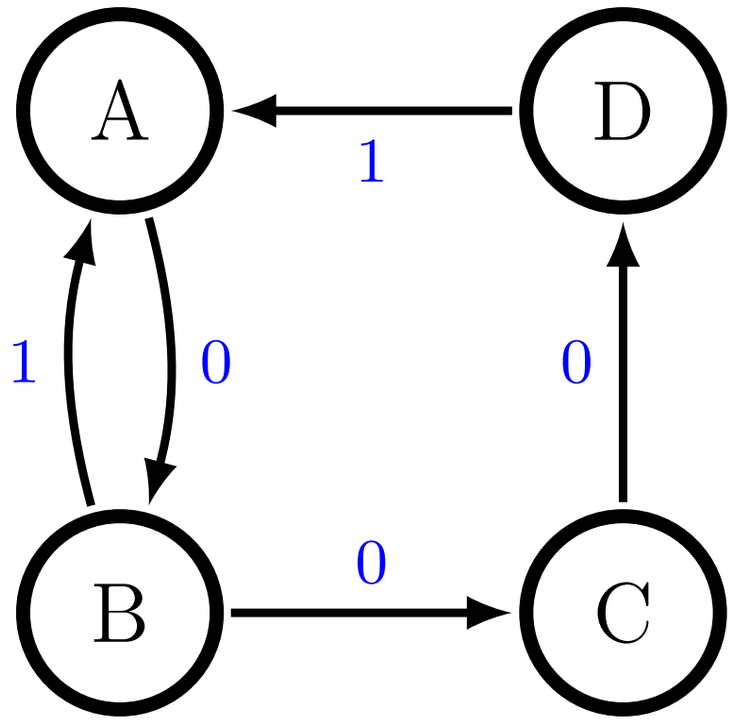

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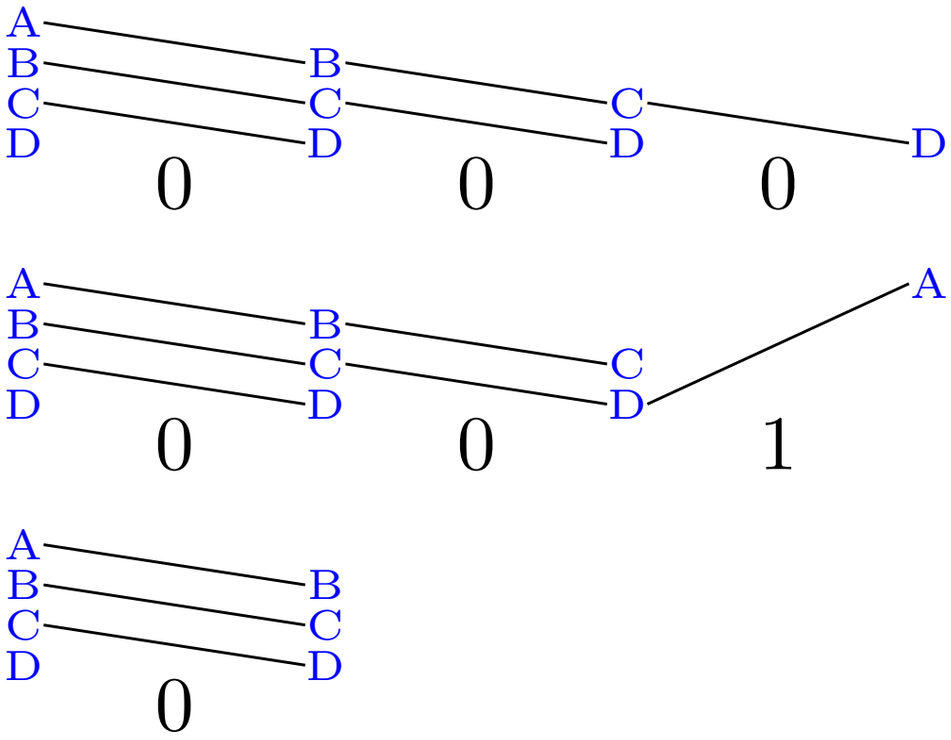

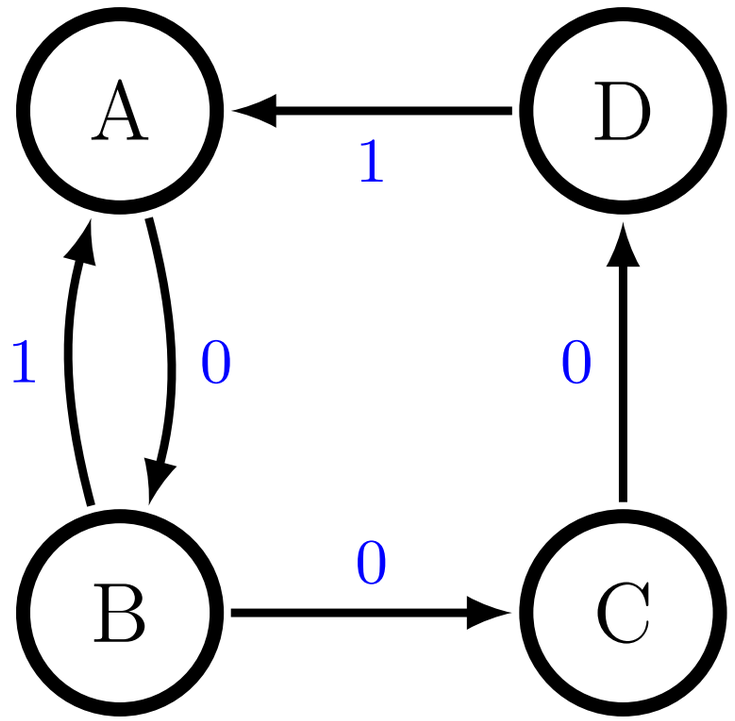

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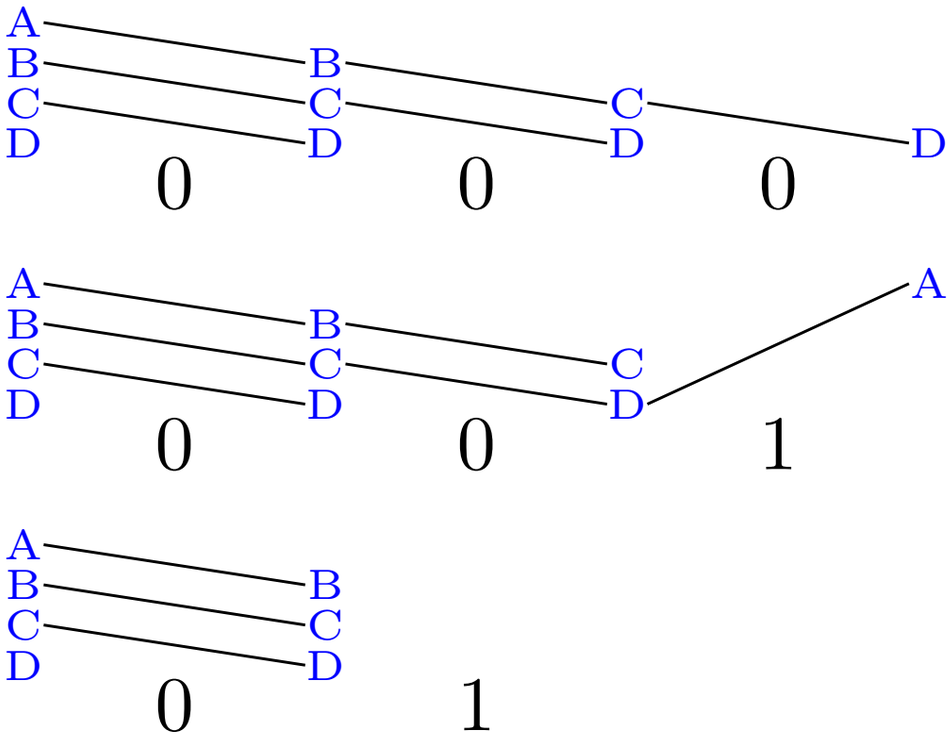

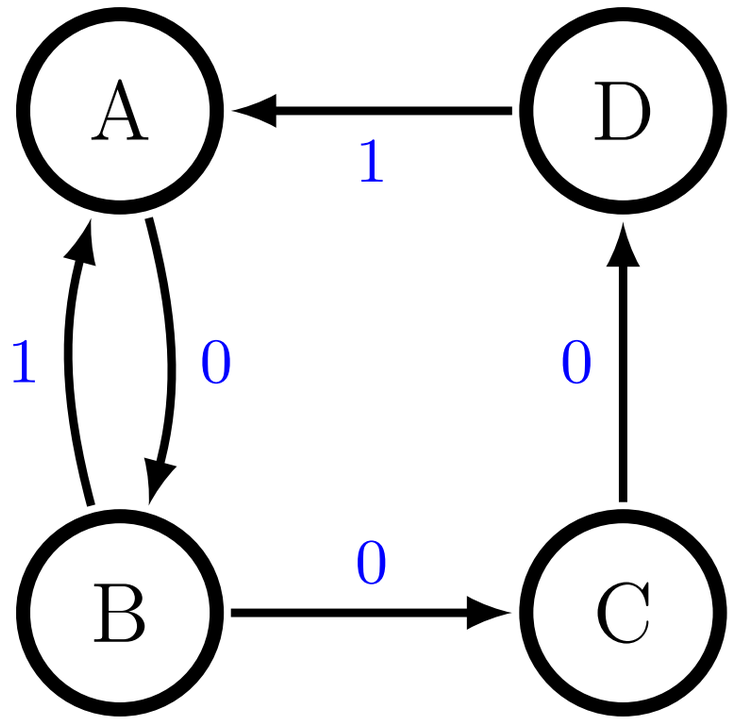

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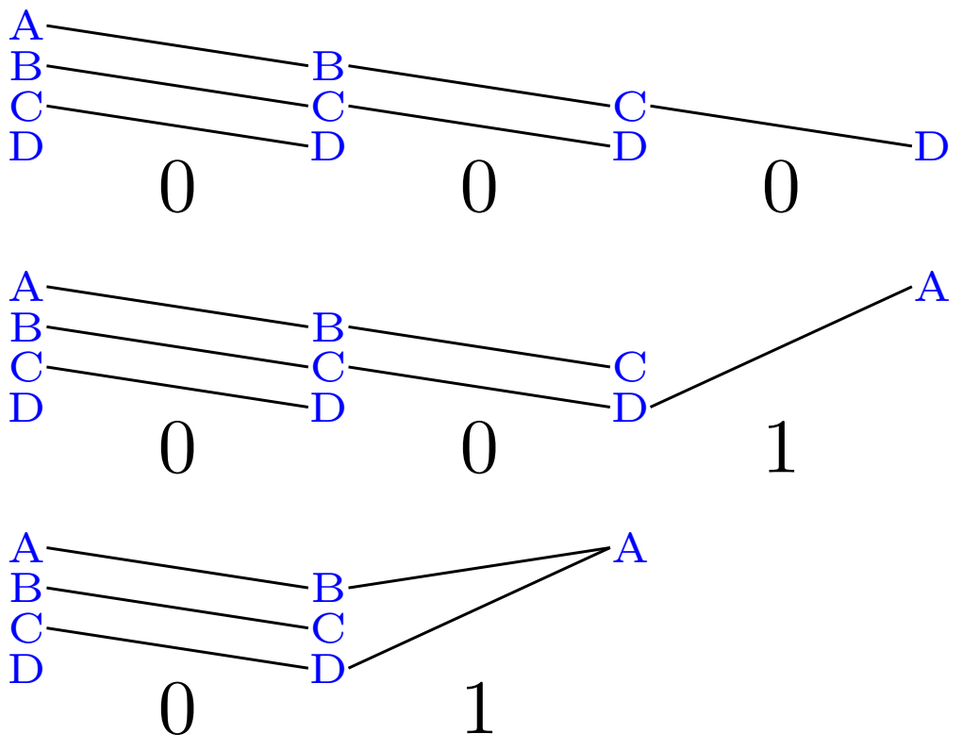

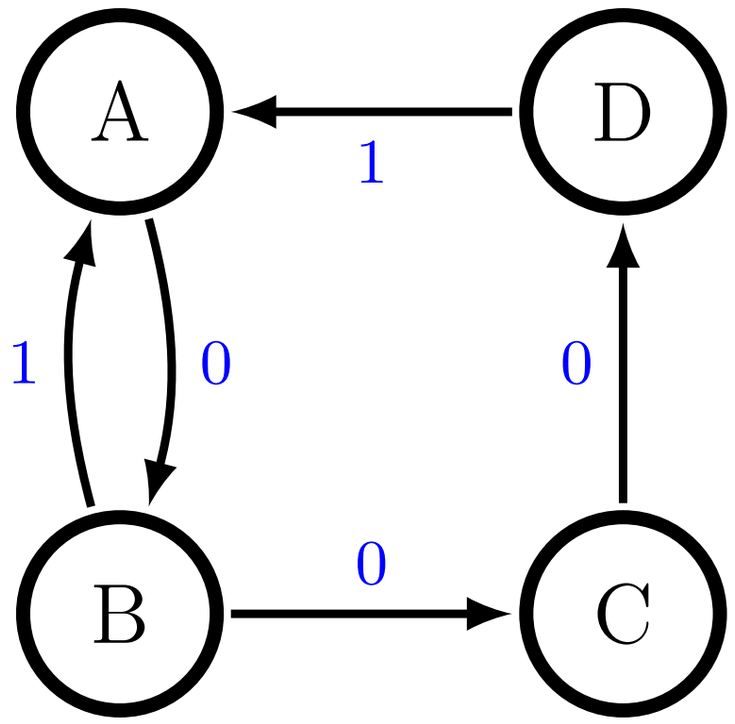

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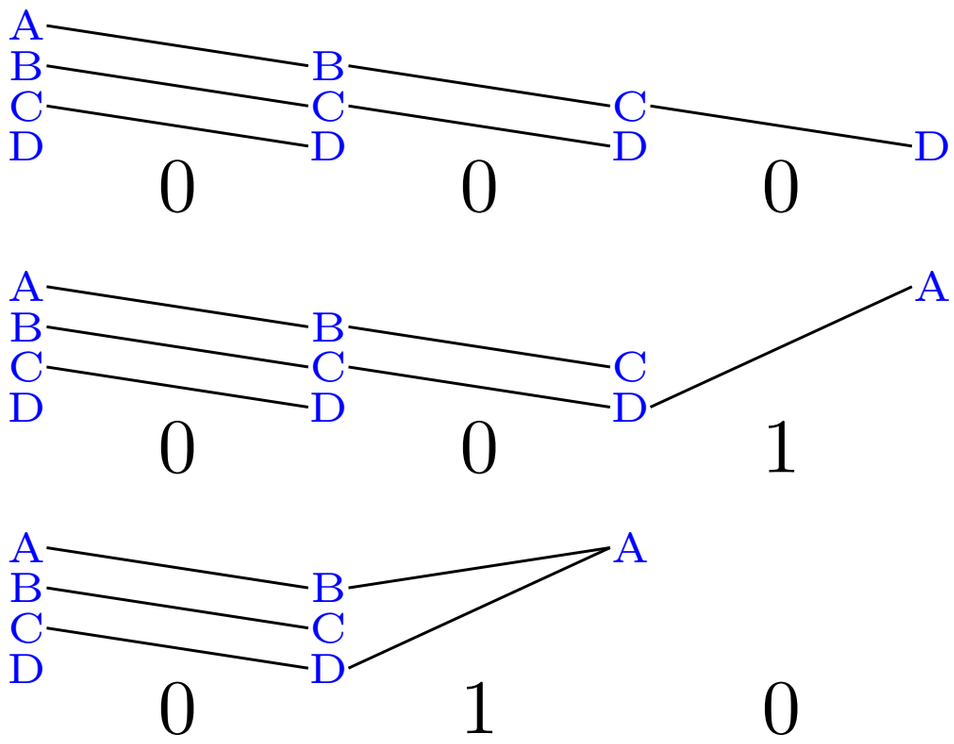

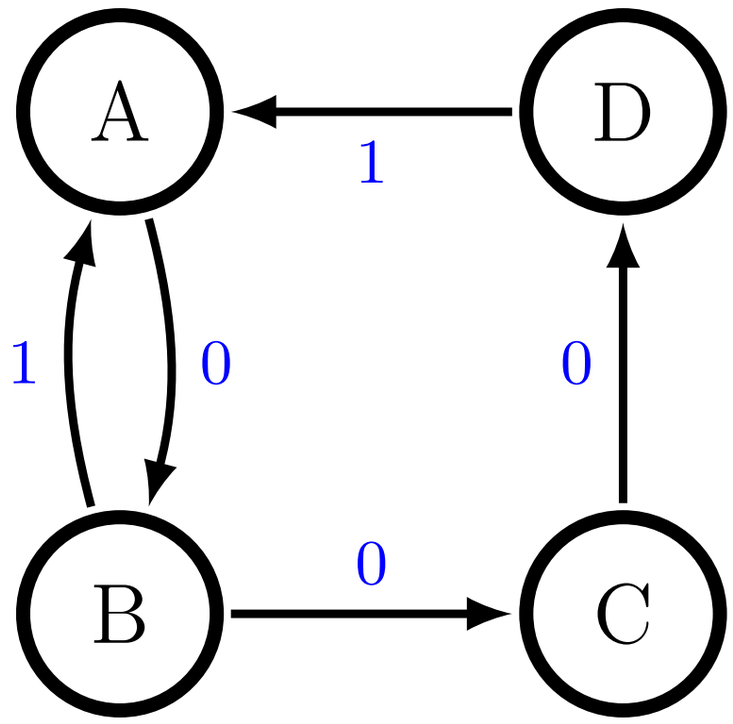

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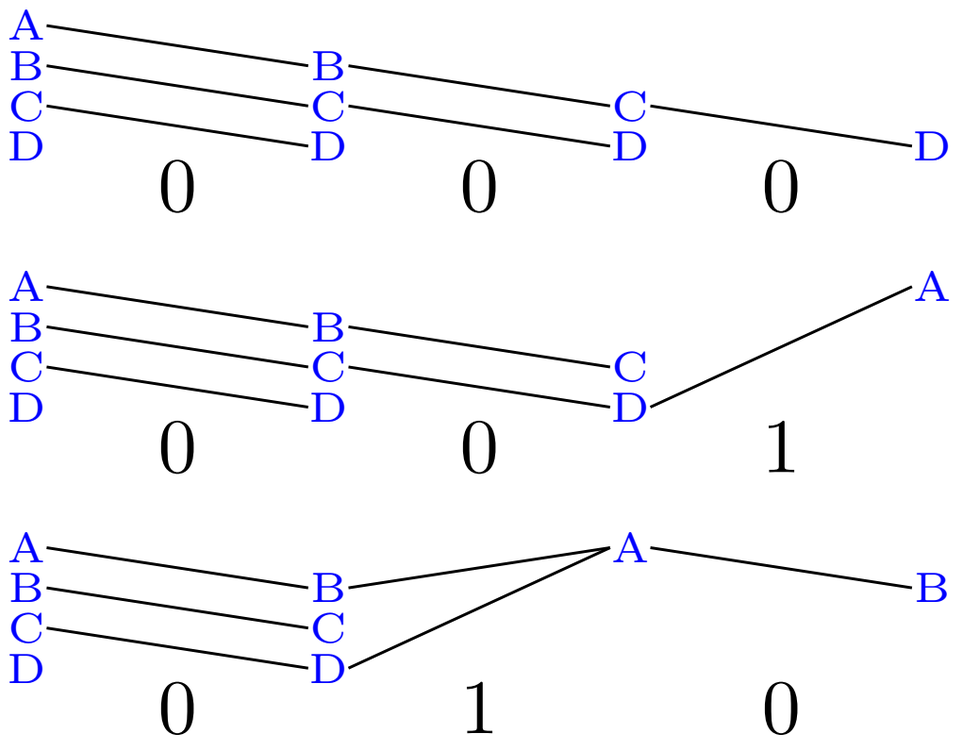

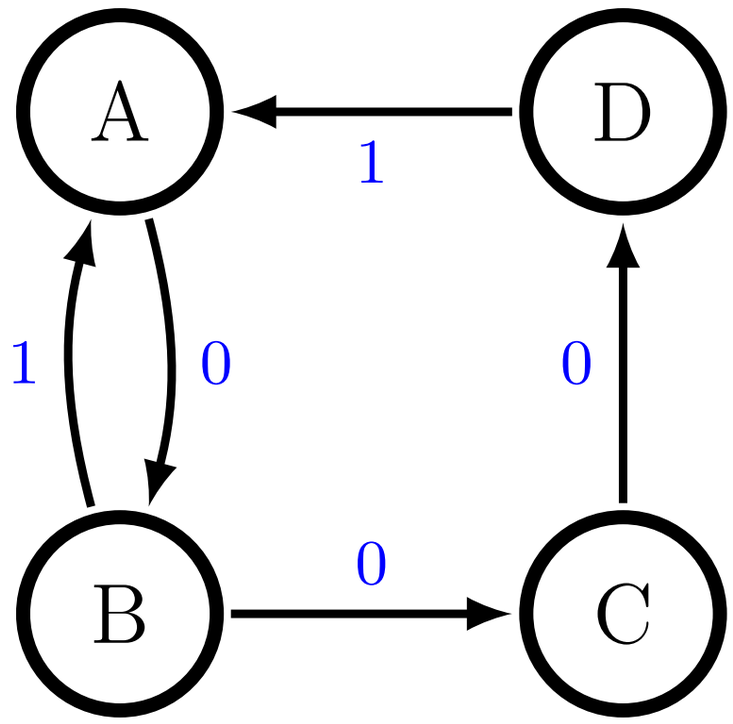

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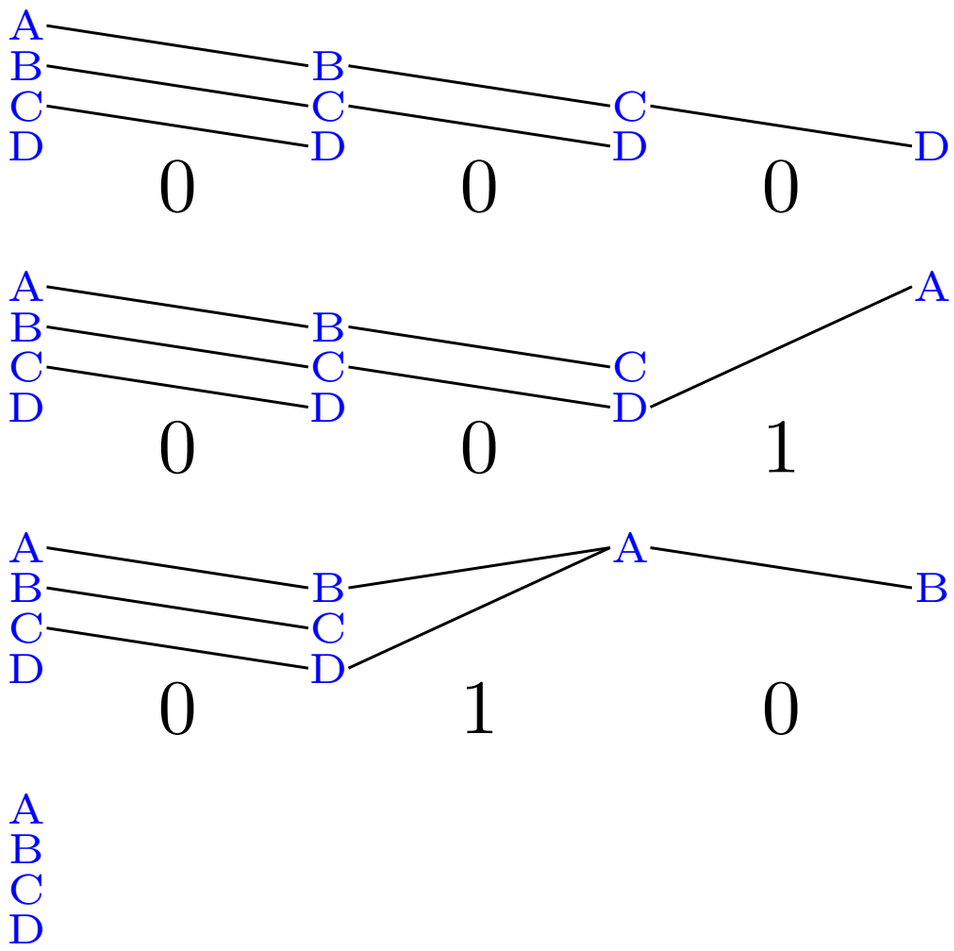

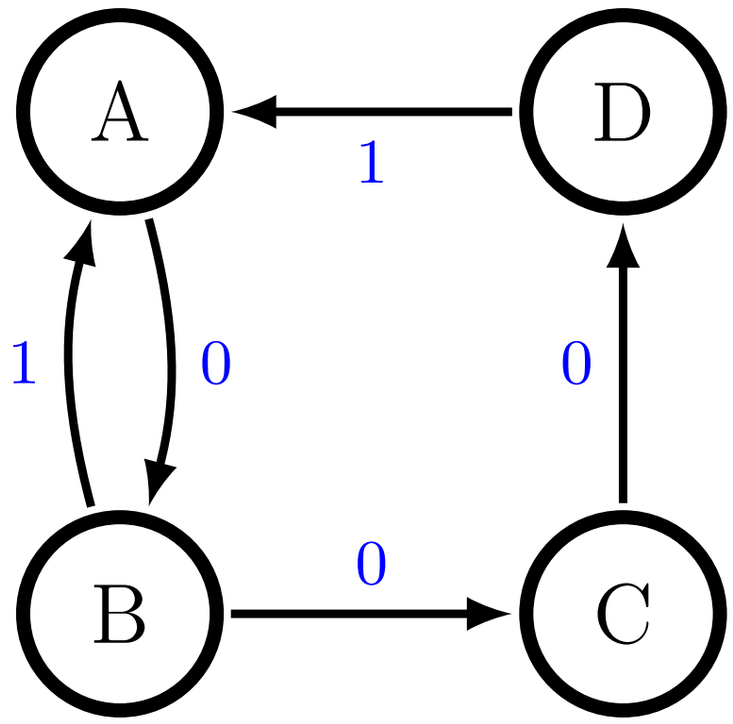

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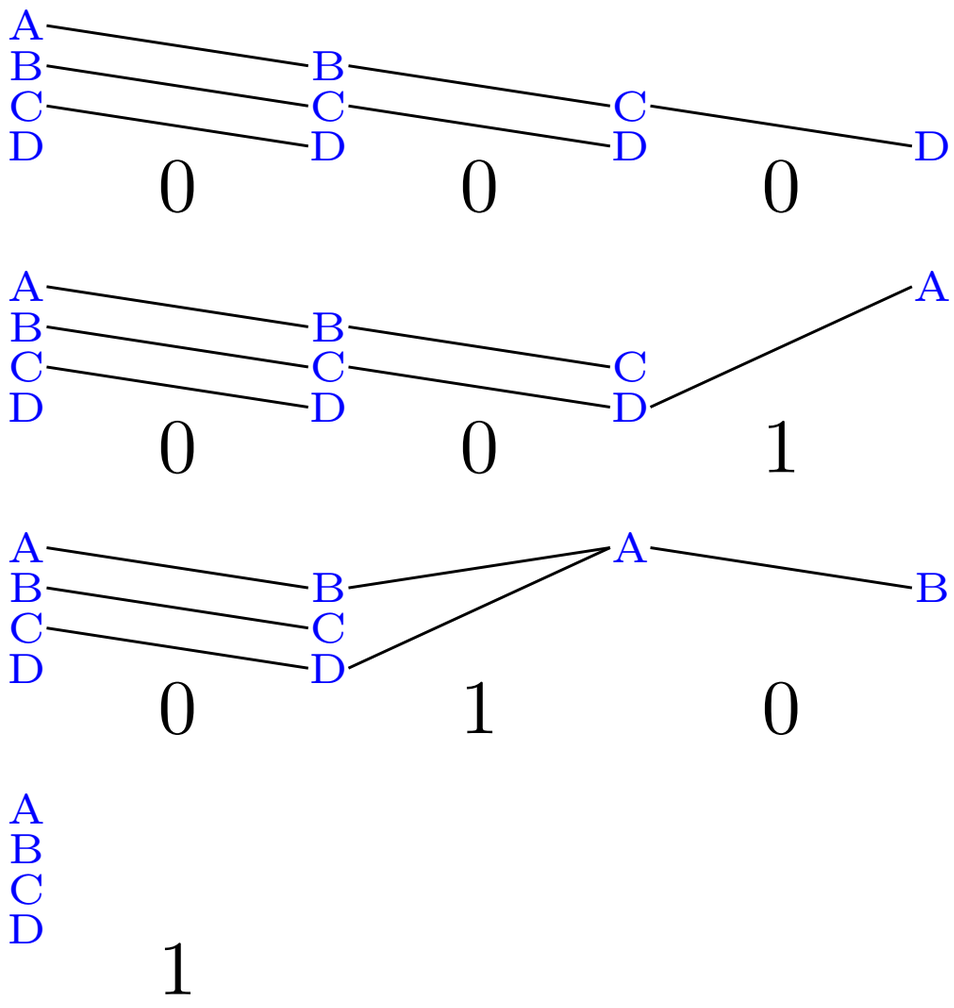

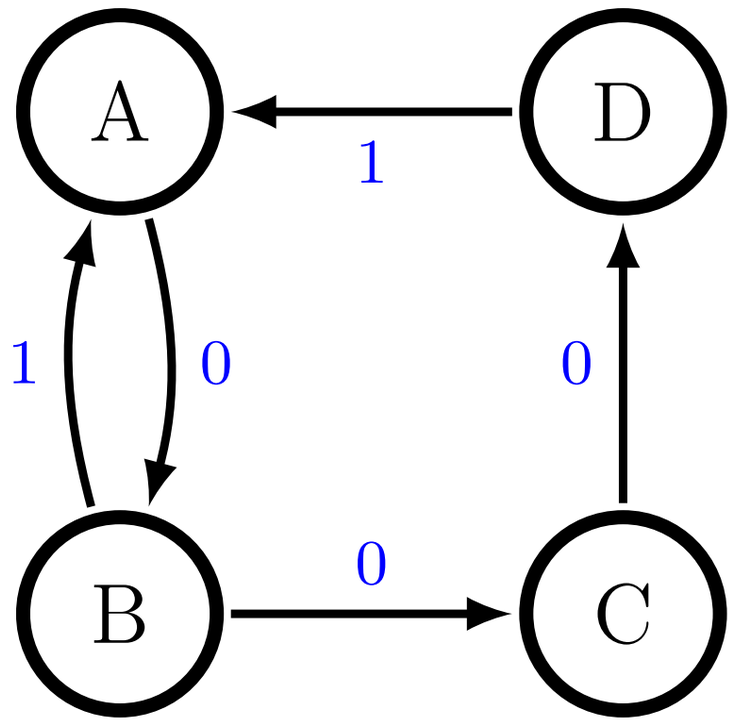

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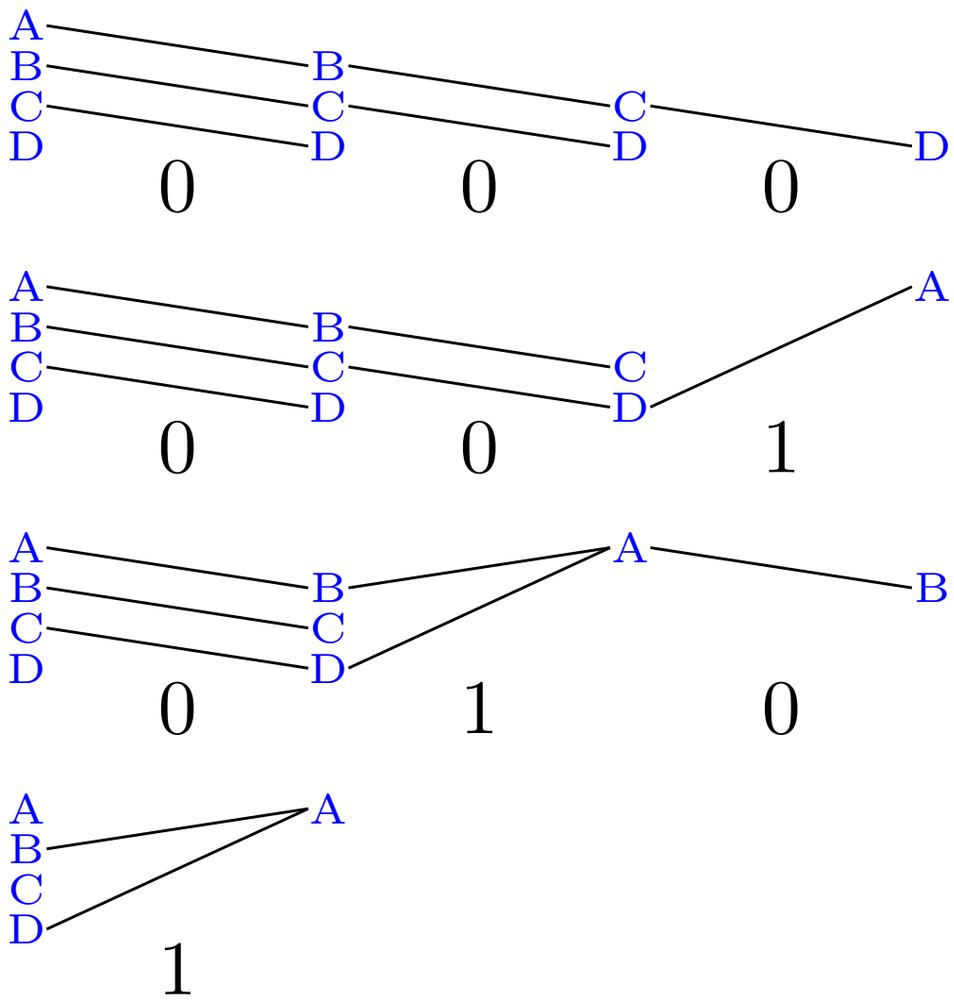

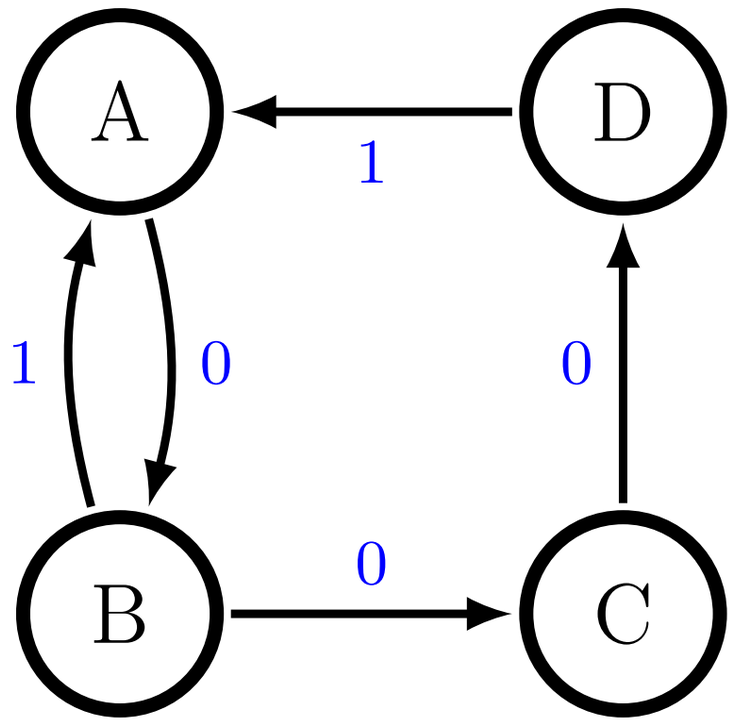

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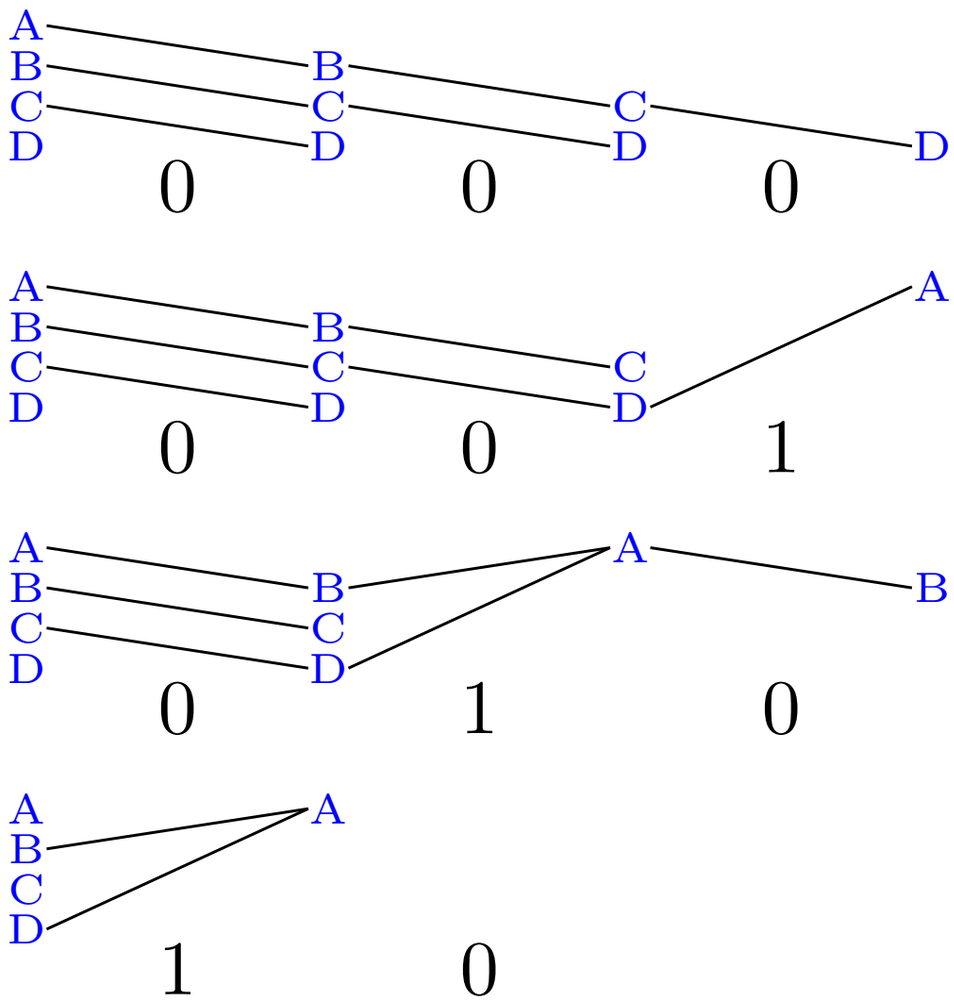

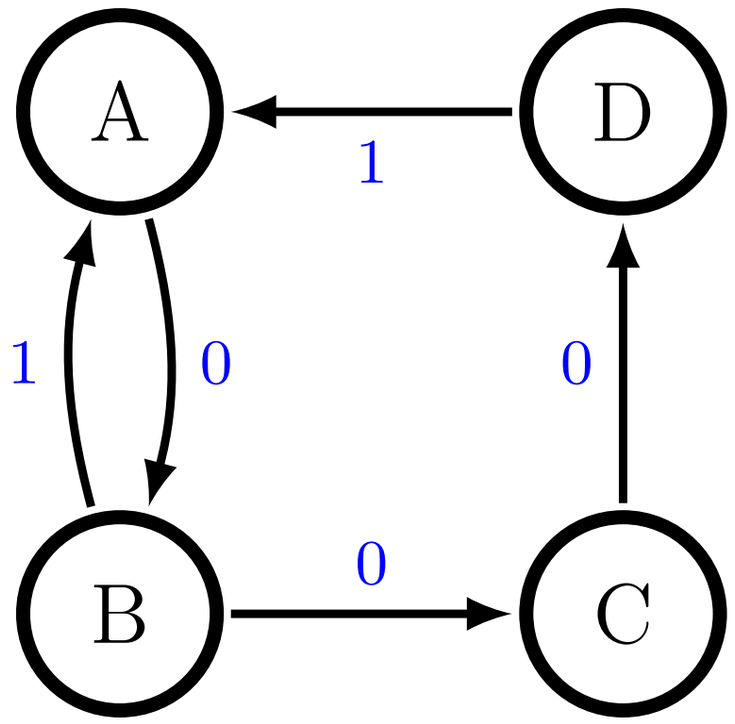

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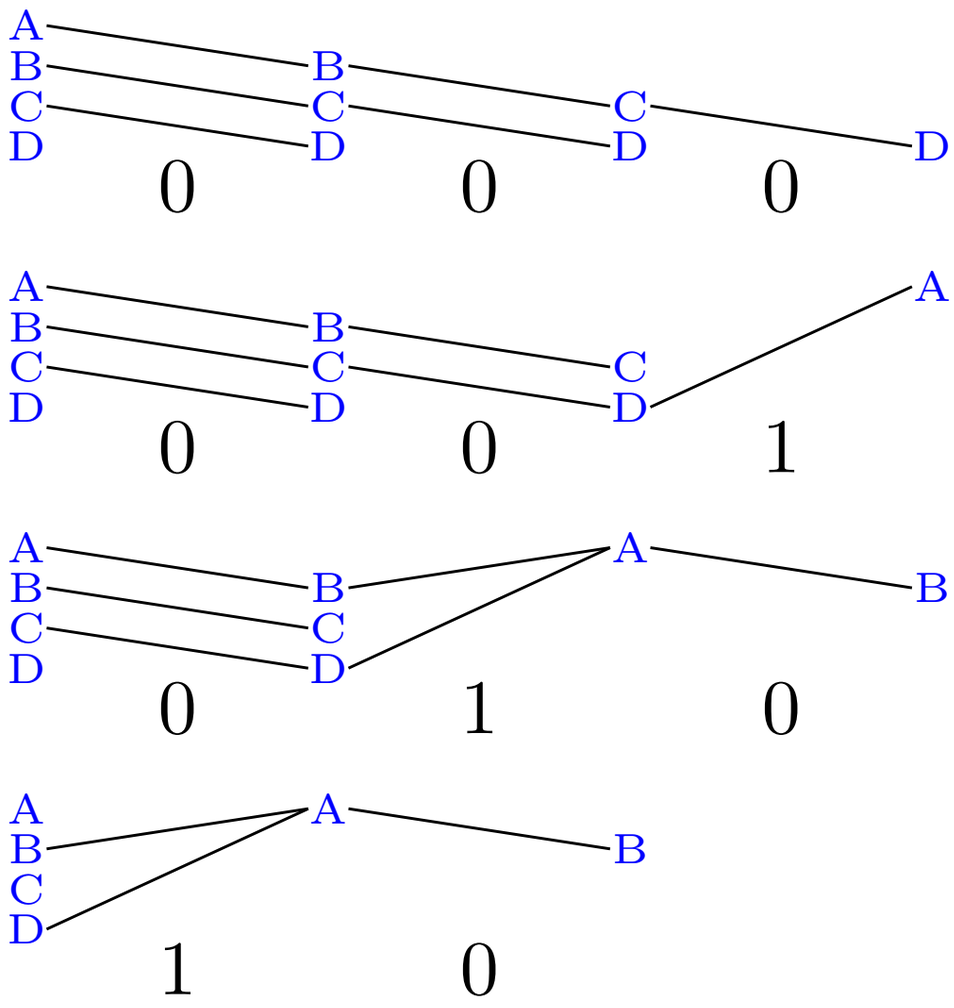

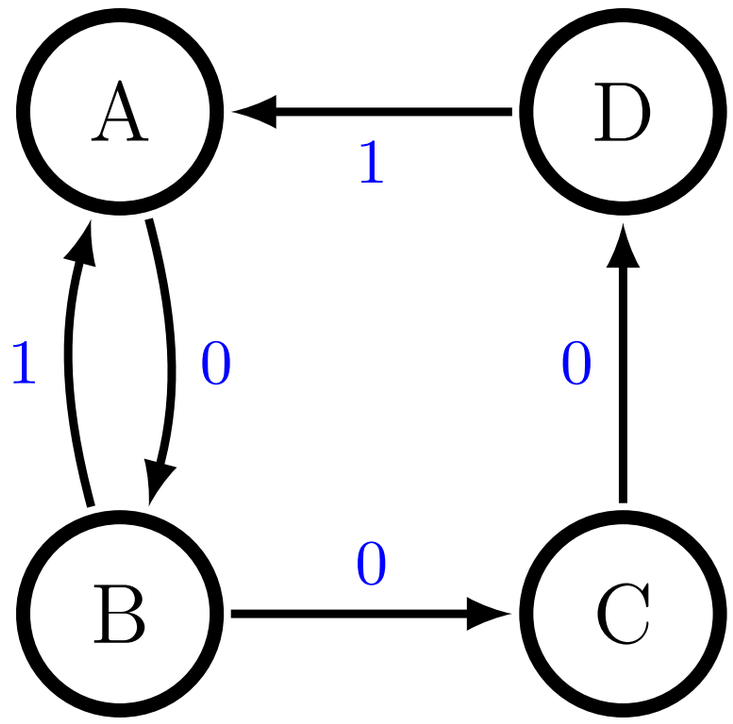

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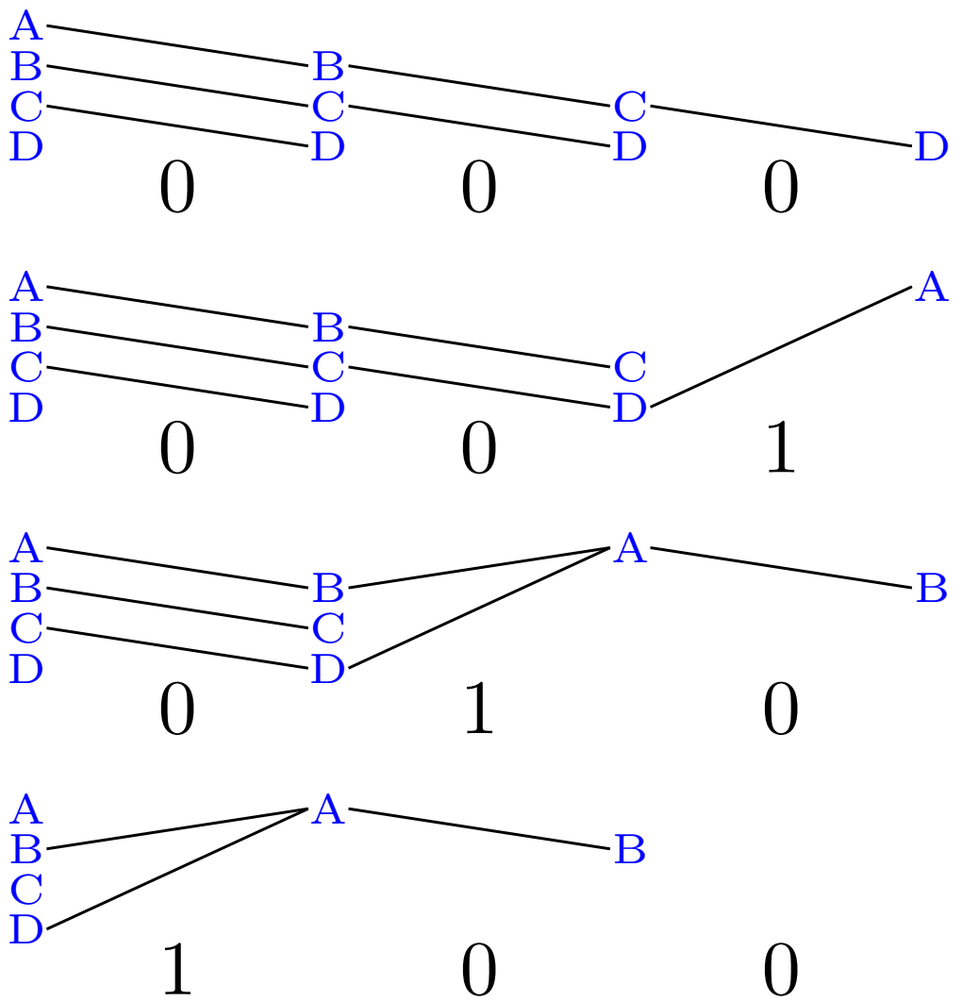

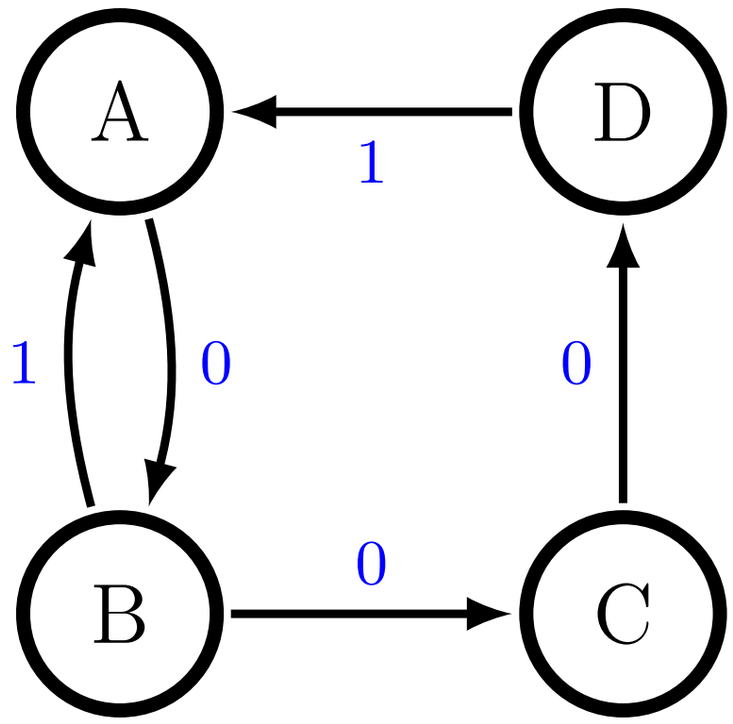

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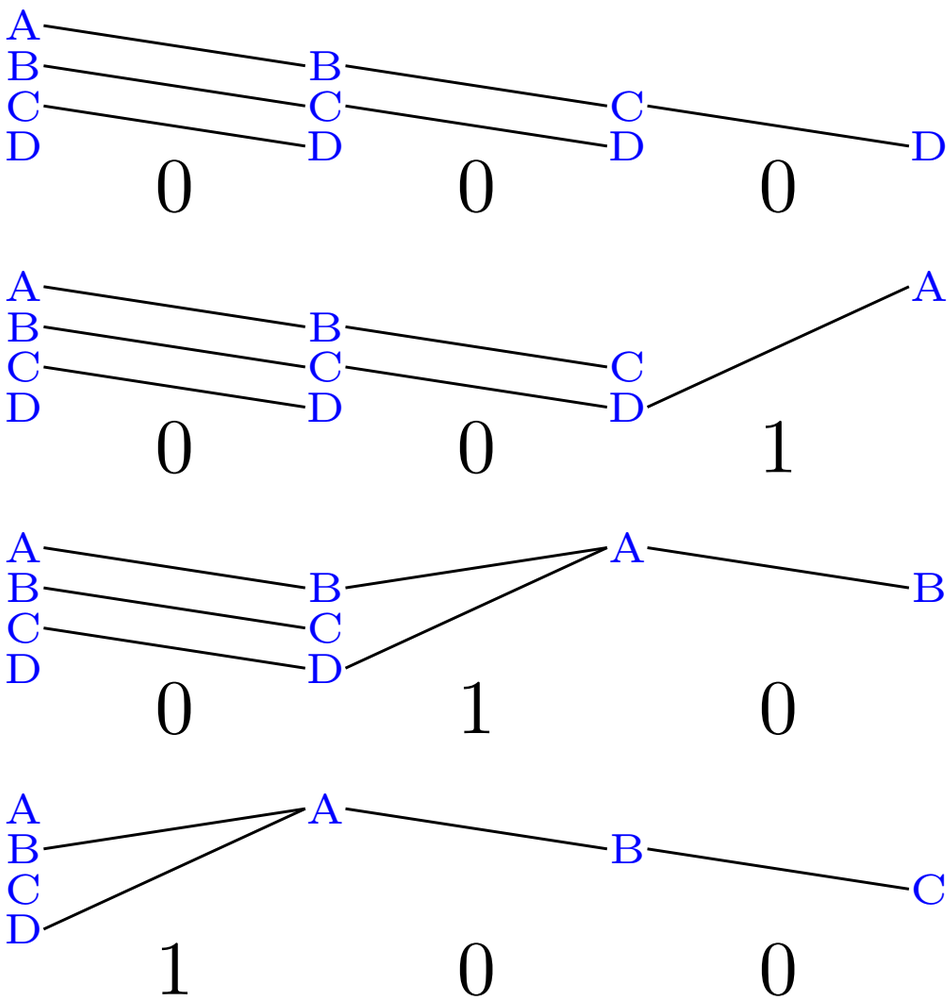

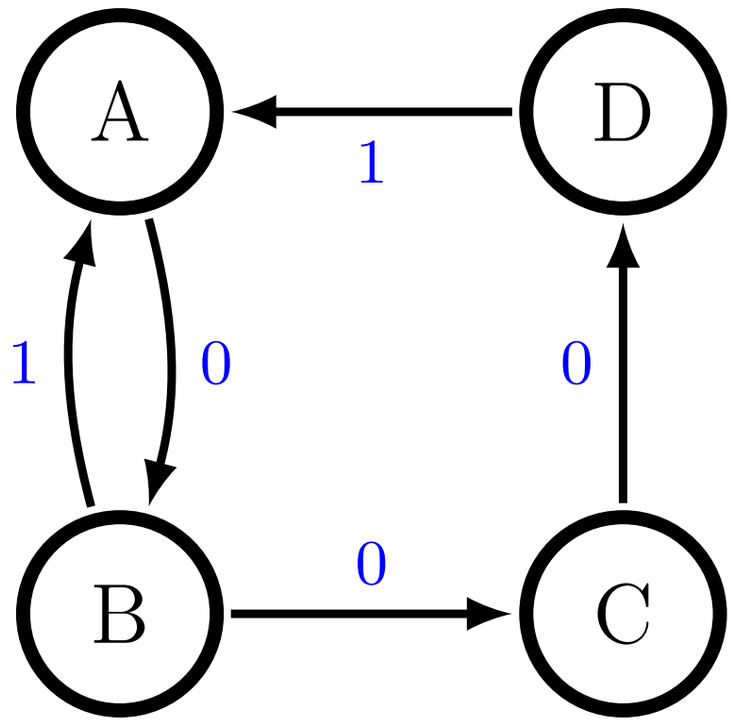

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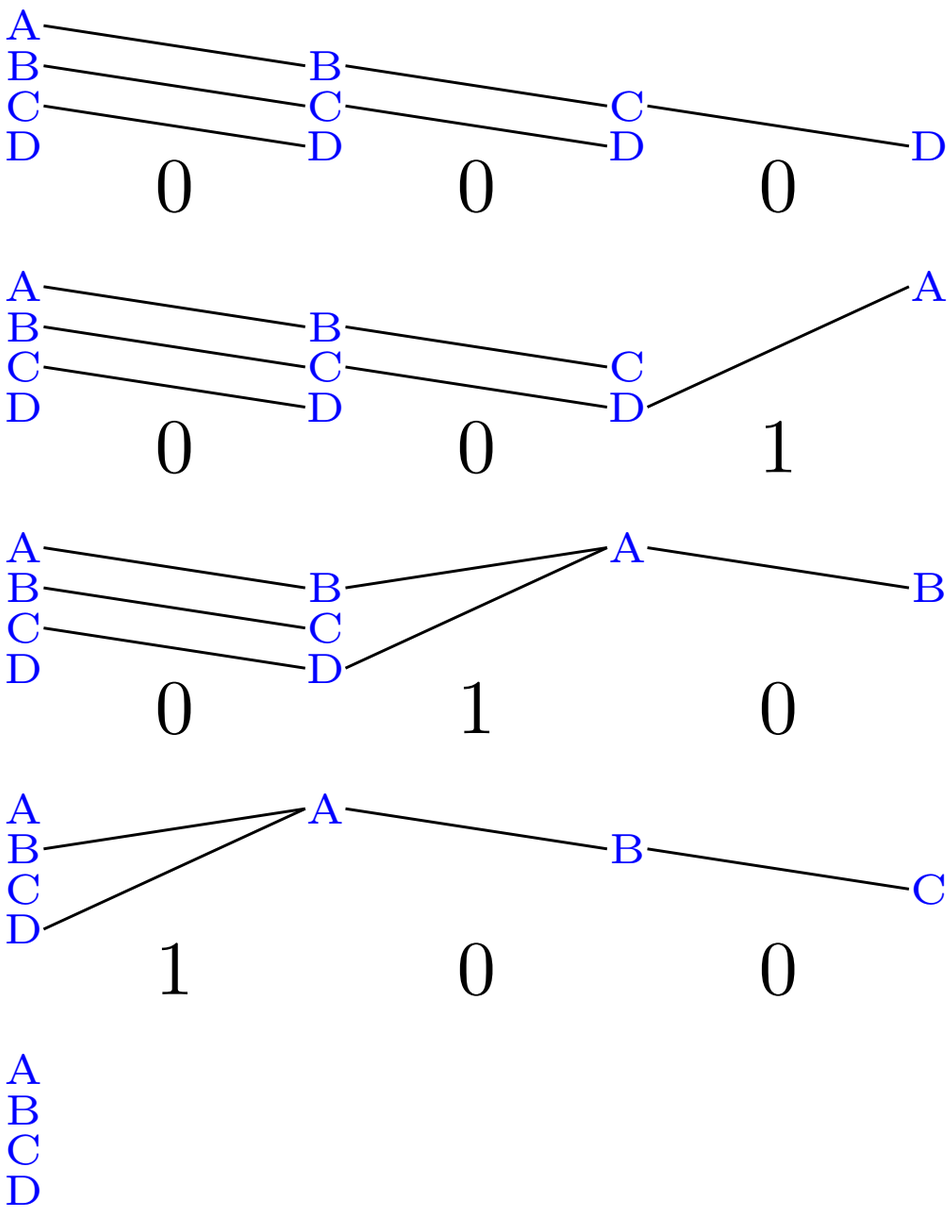

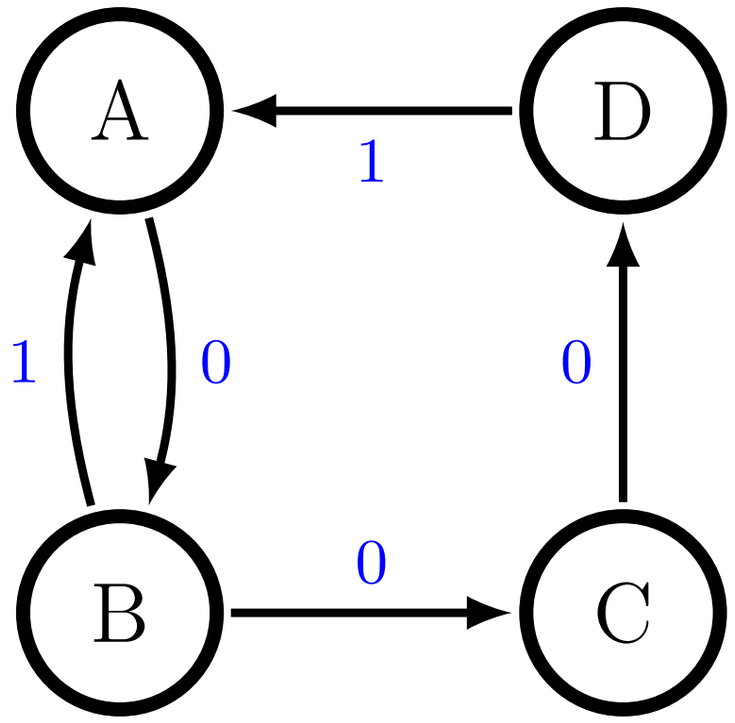

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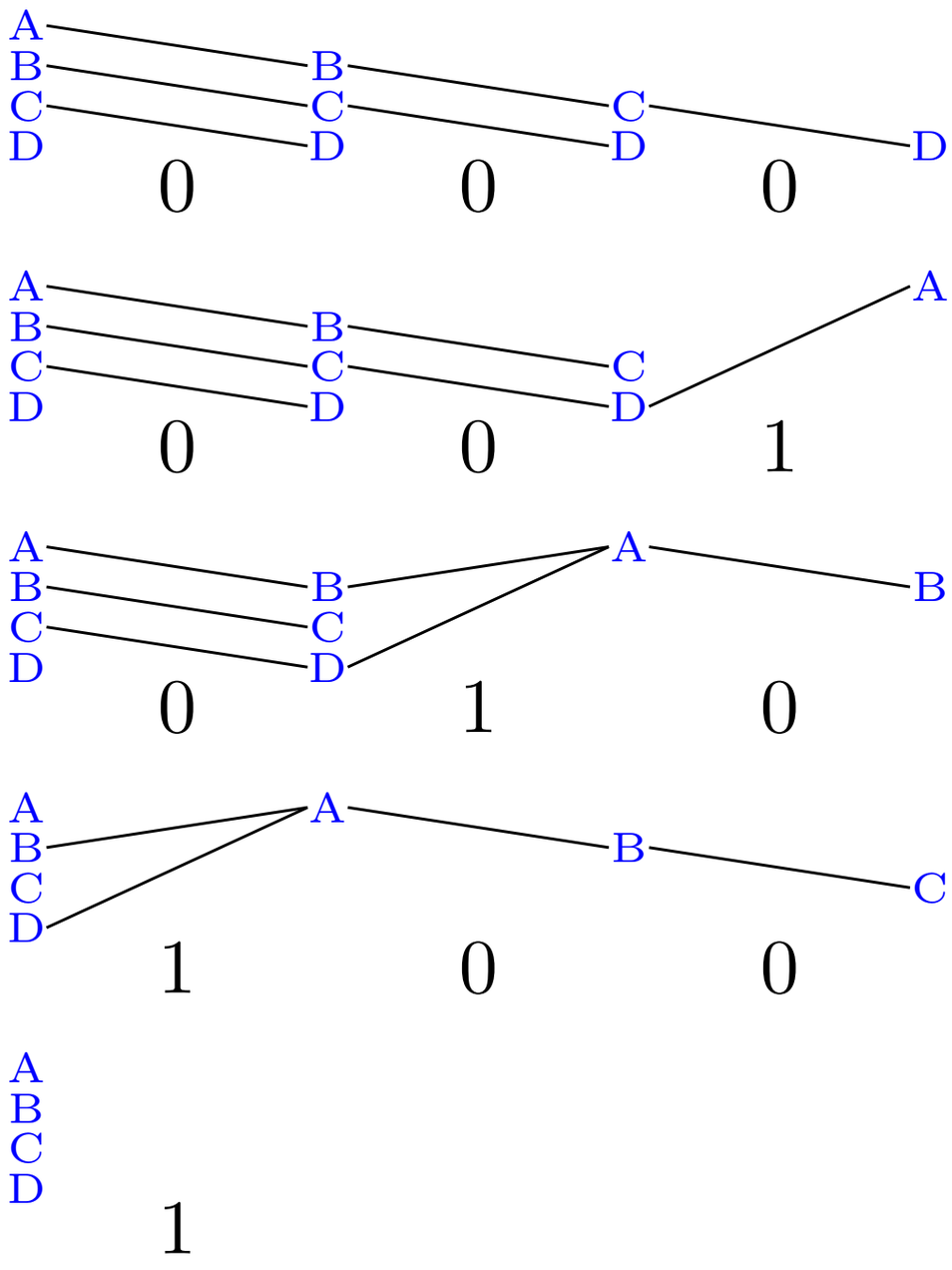

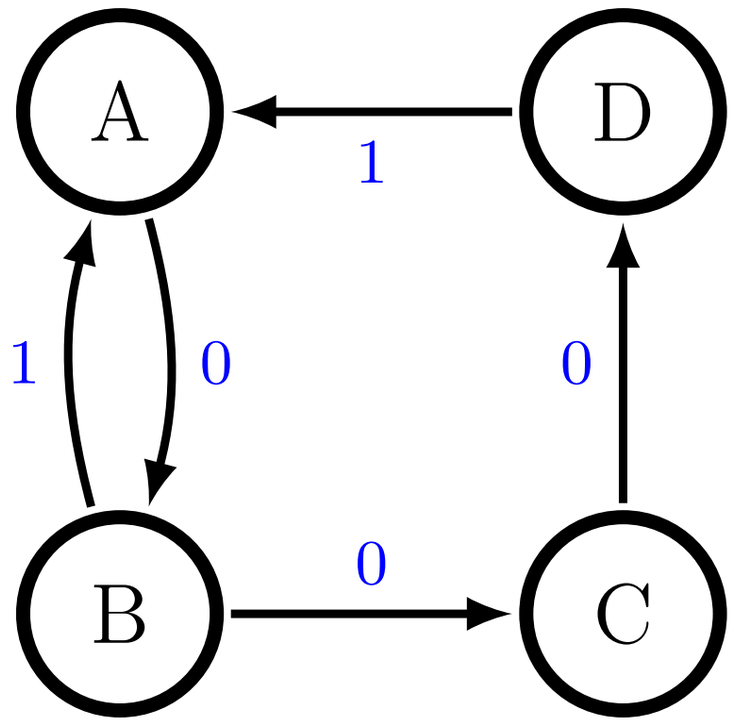

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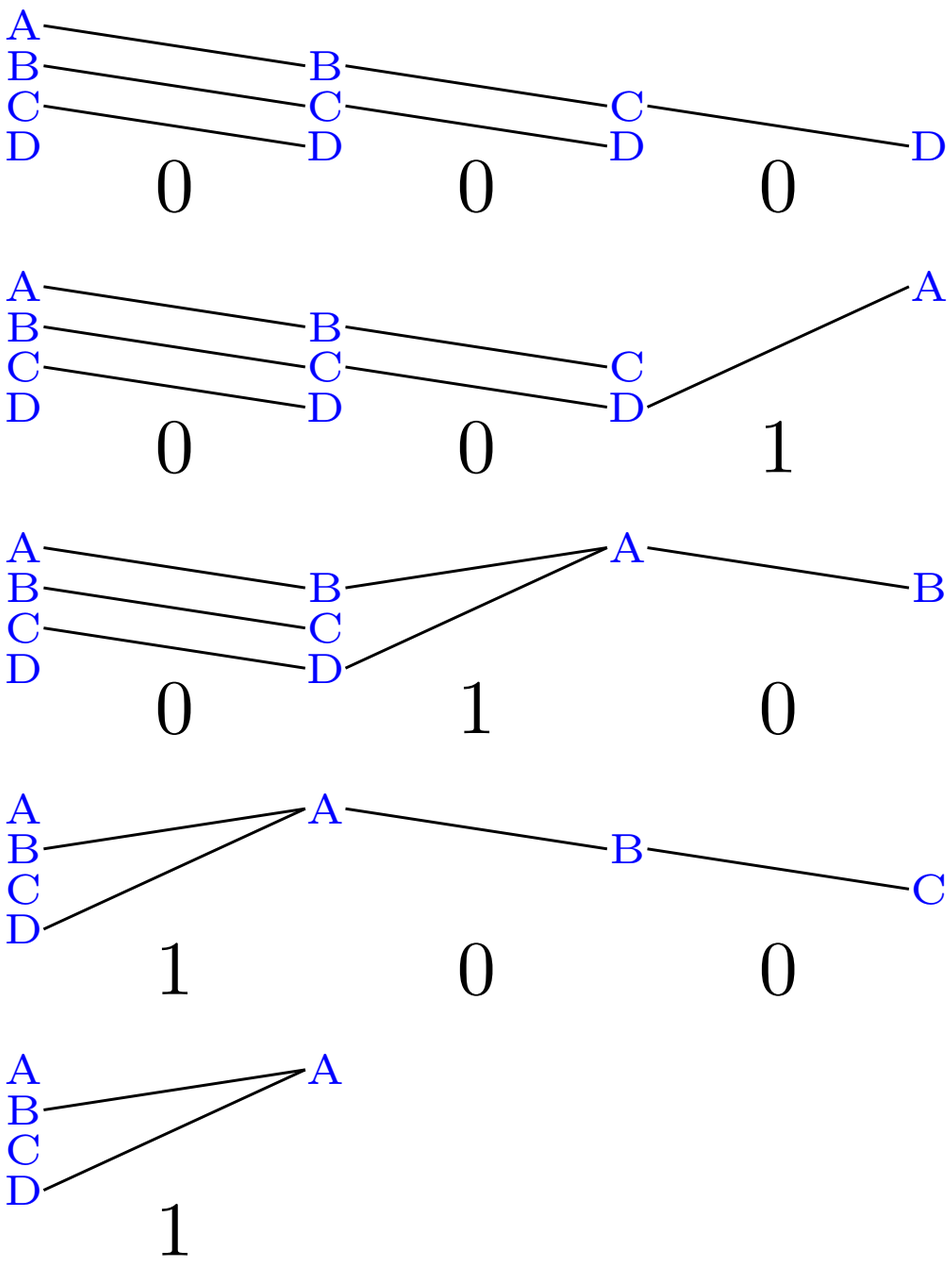

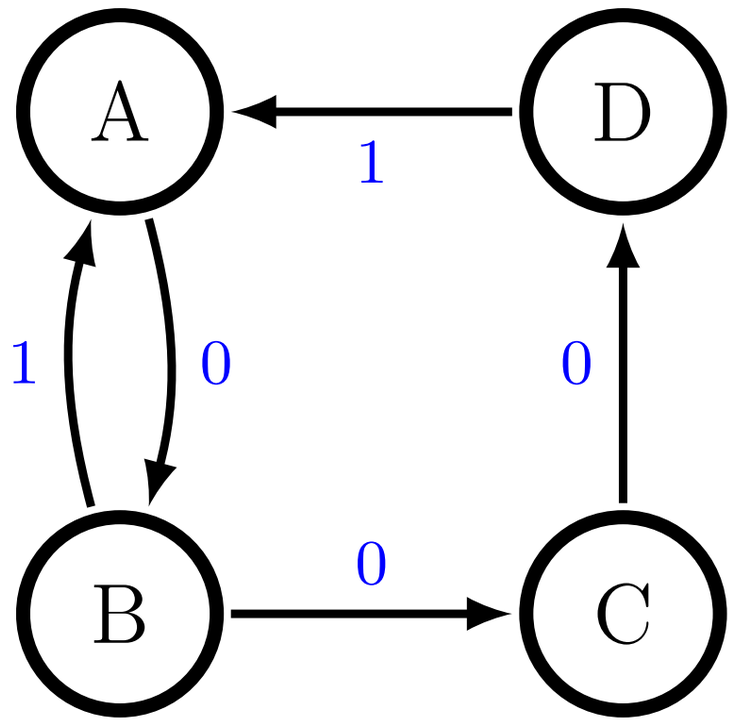

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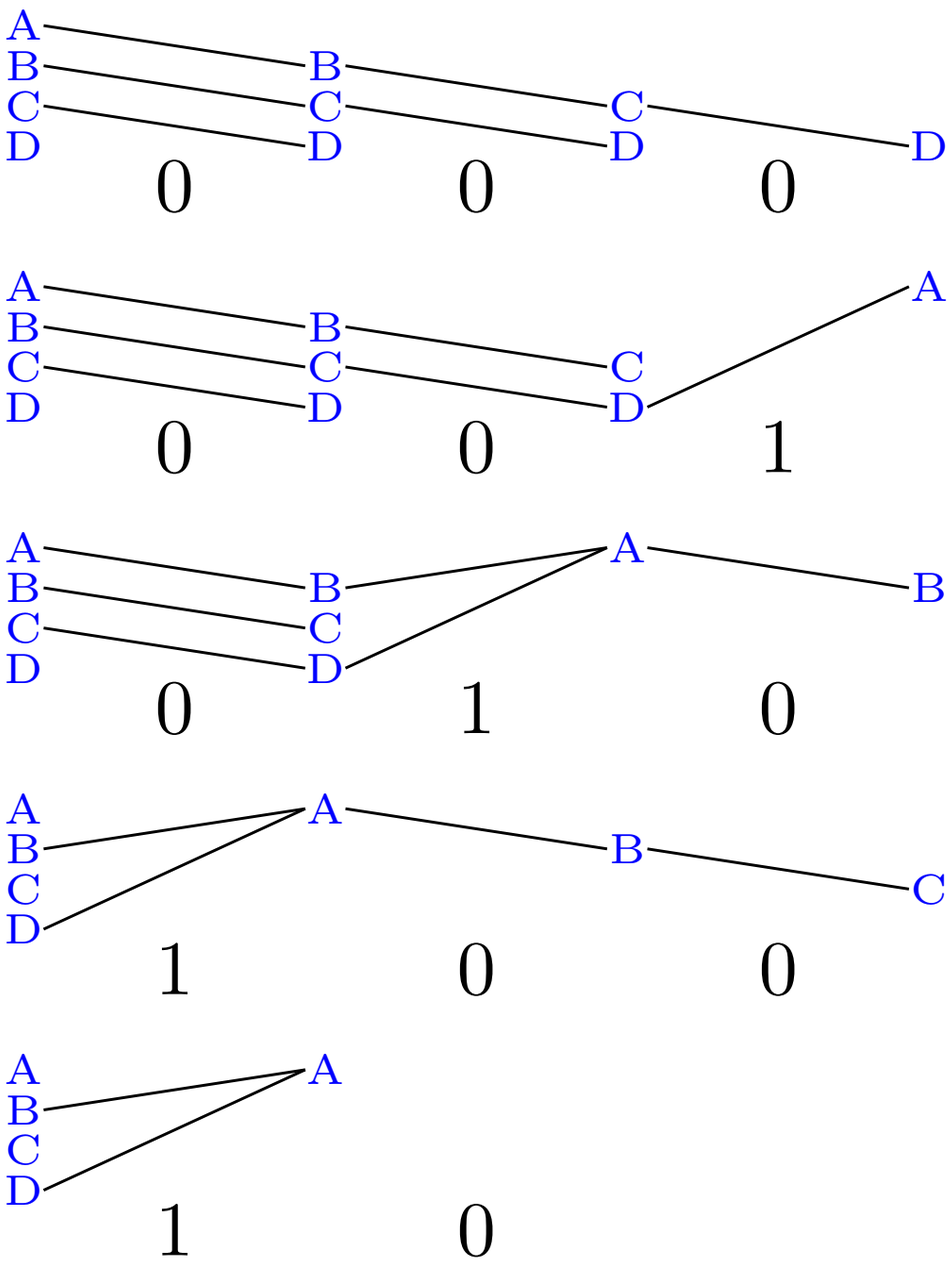

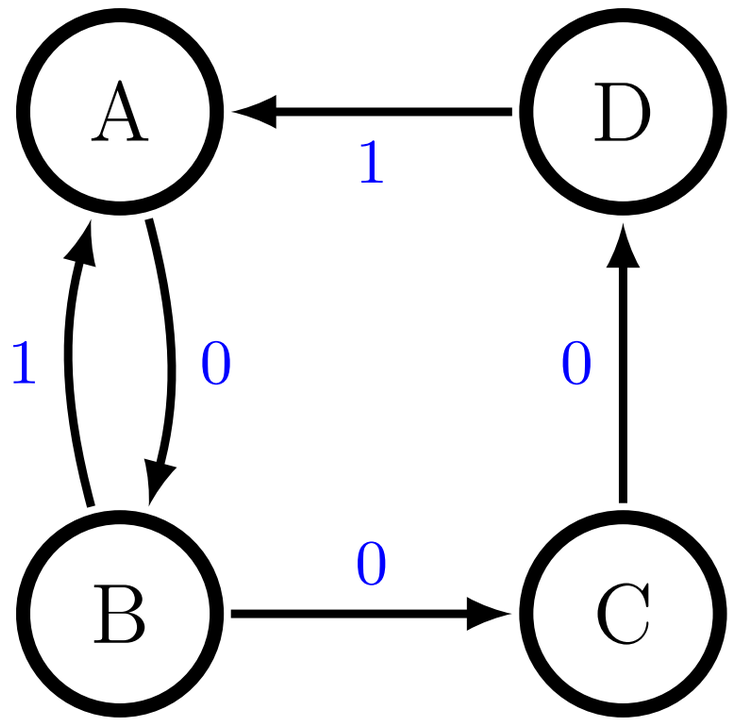

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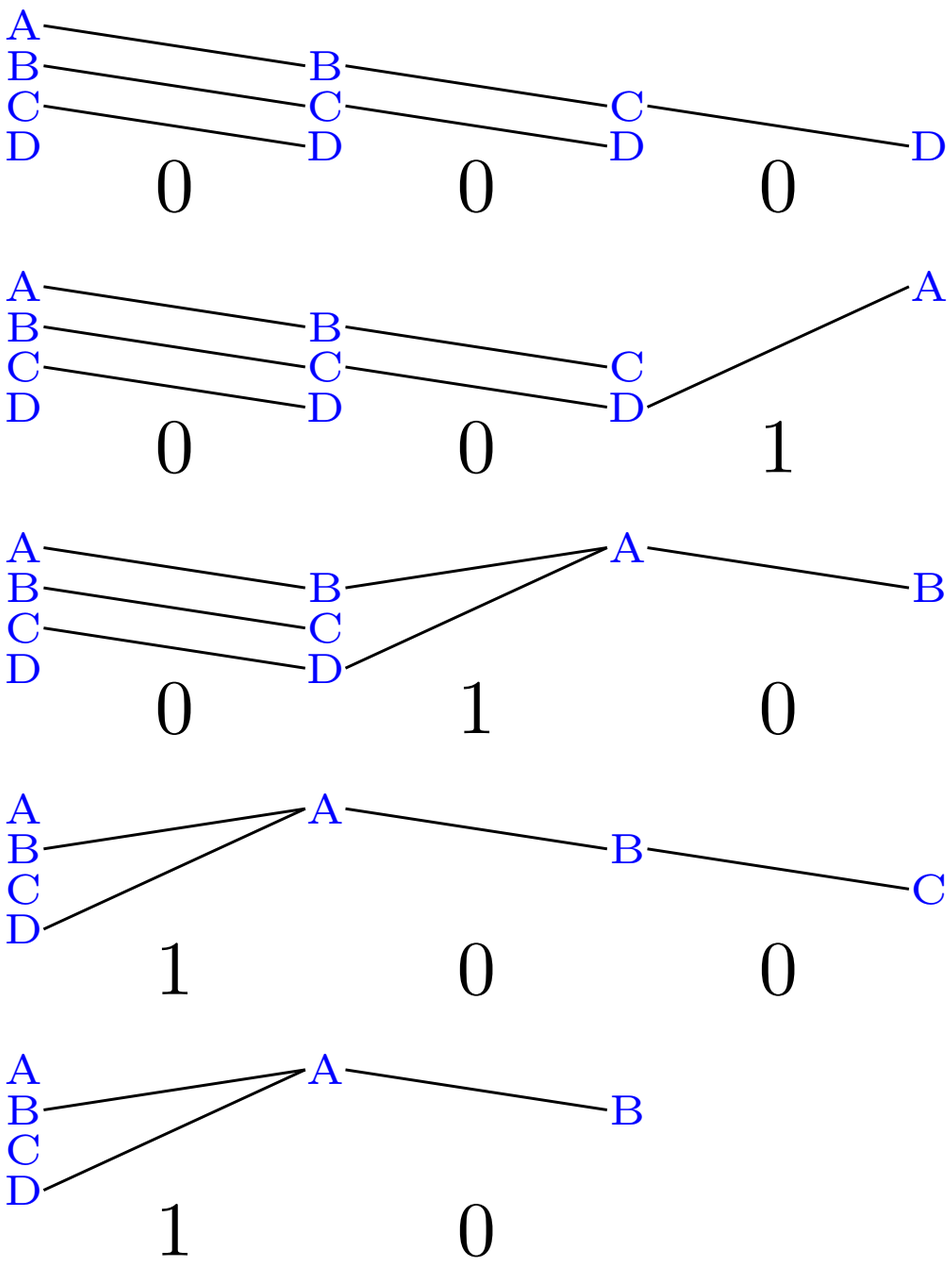

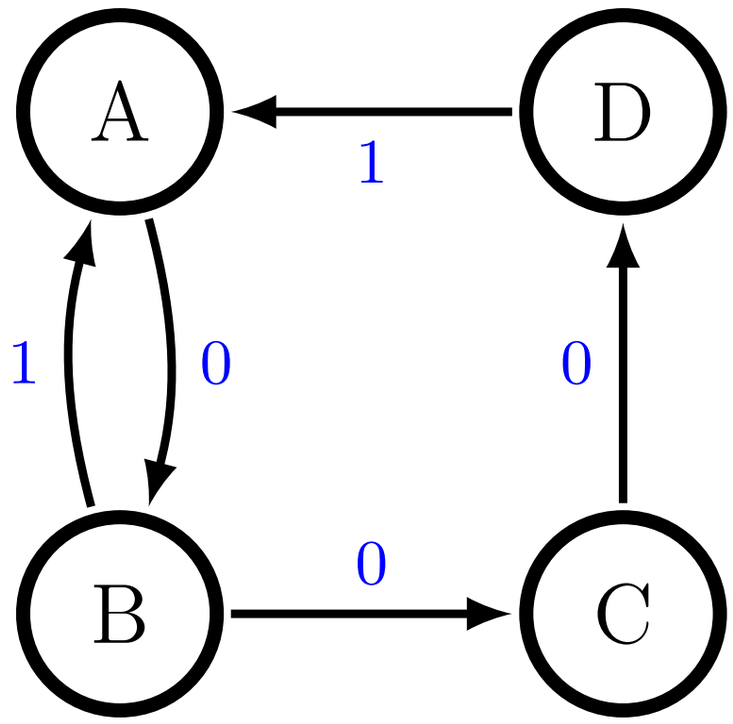

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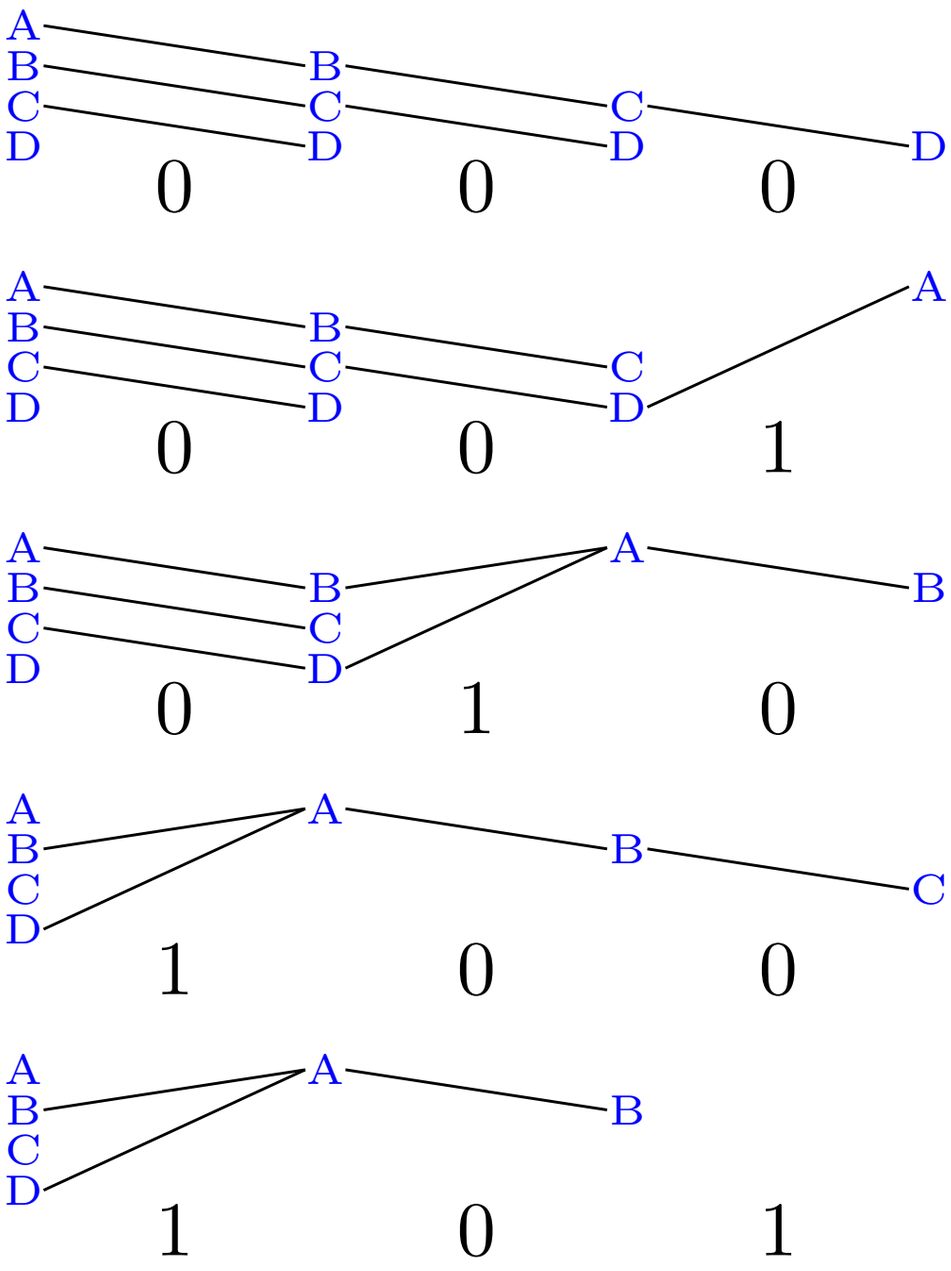

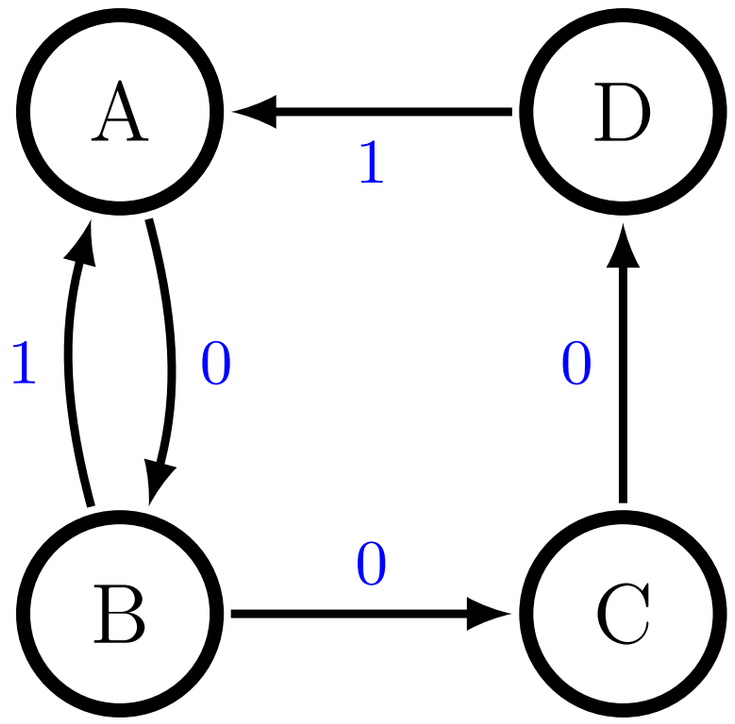

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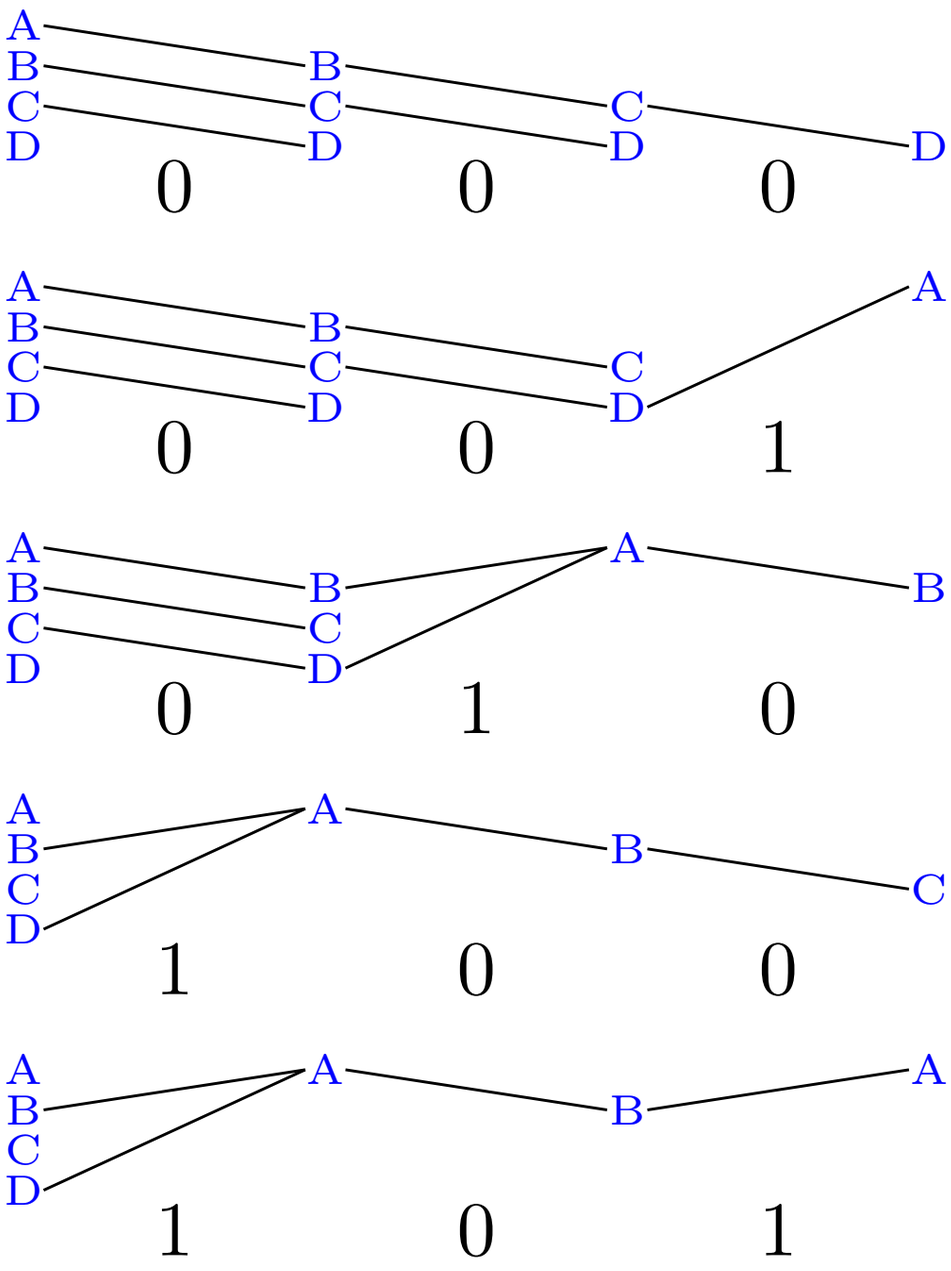
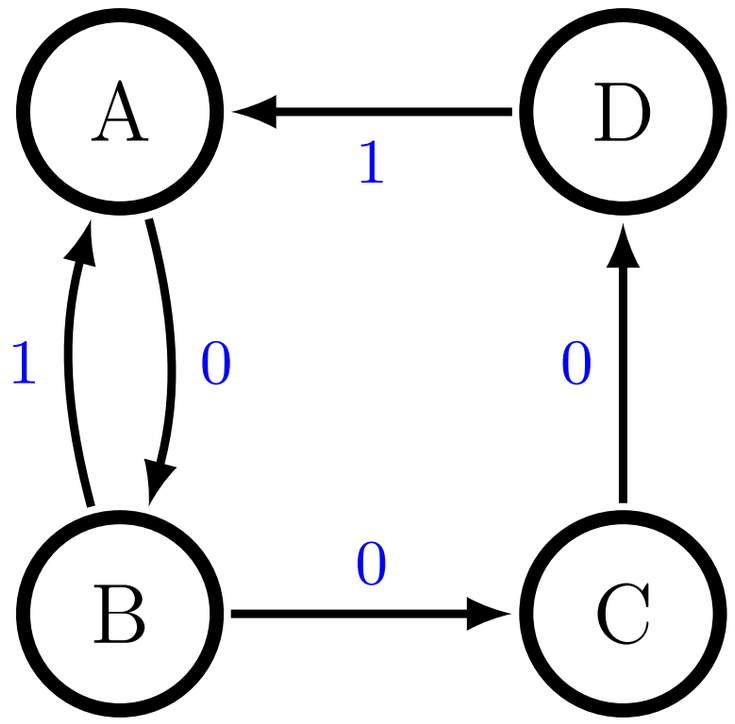
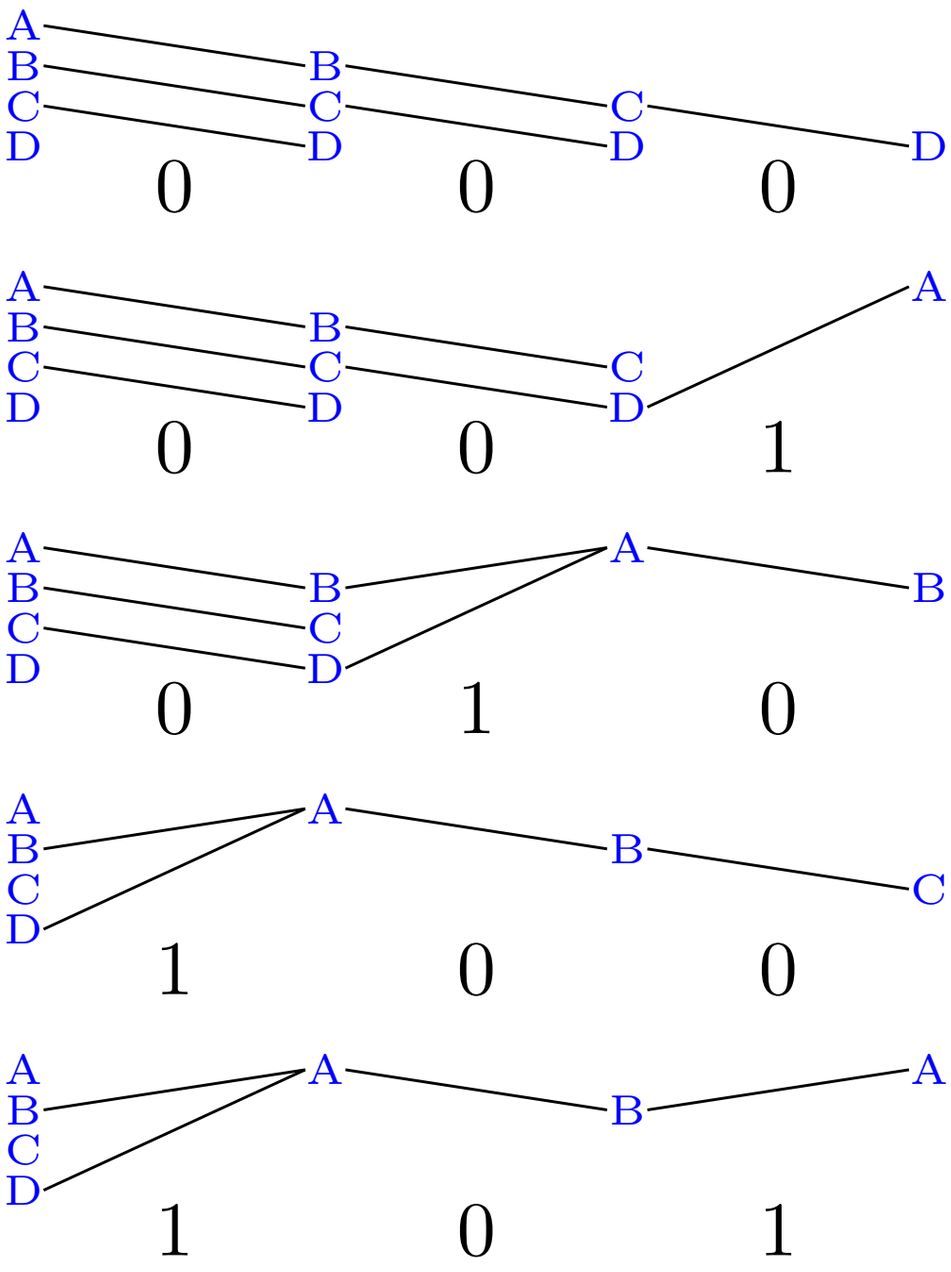
Walking Paths

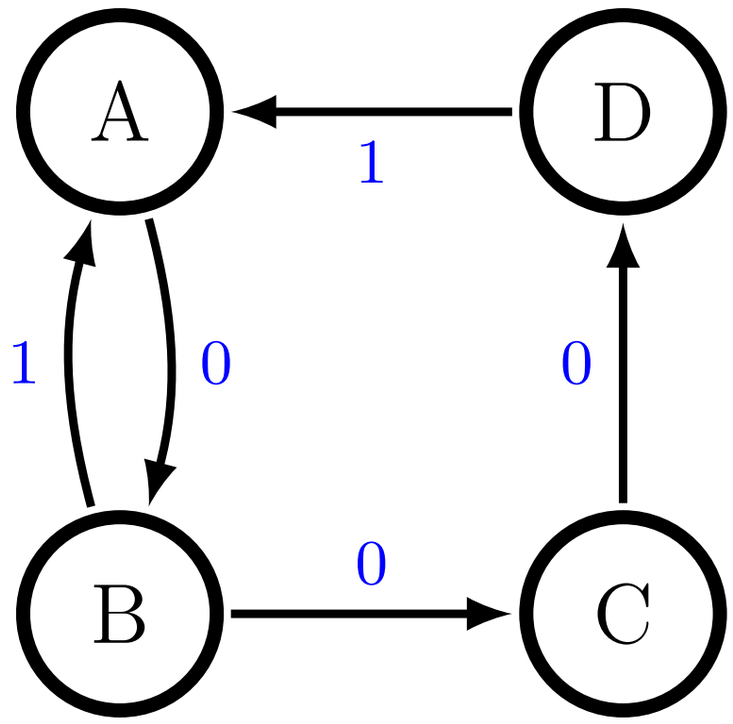




Are We There Yet?



$$H[\mathcal{S}_\ell | X_{0:\ell}] = 0$$



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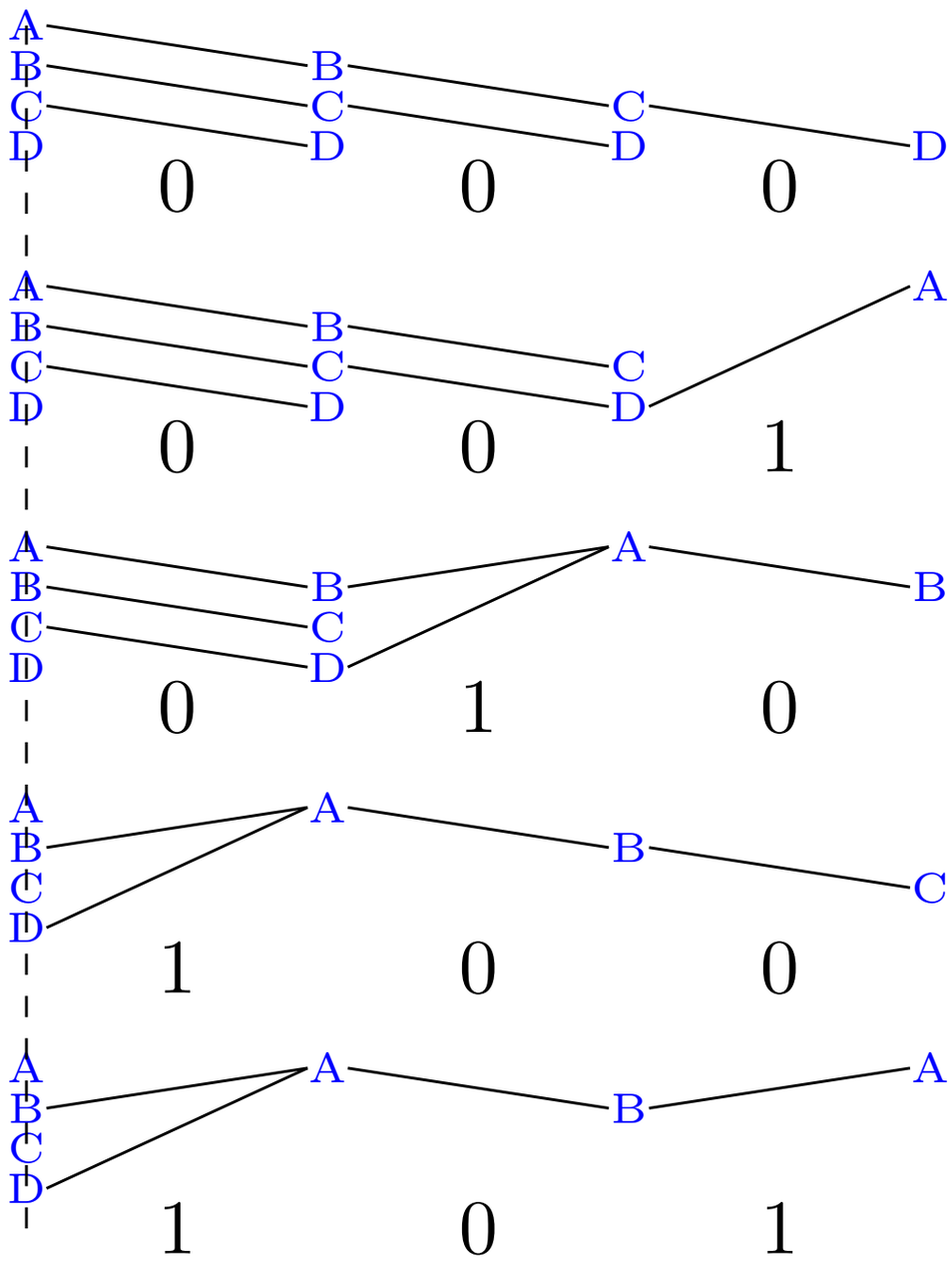

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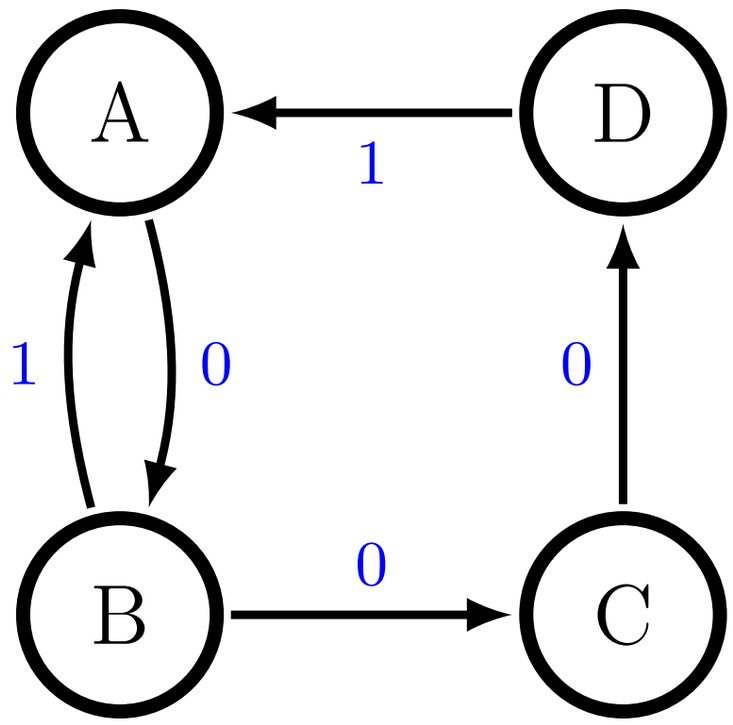

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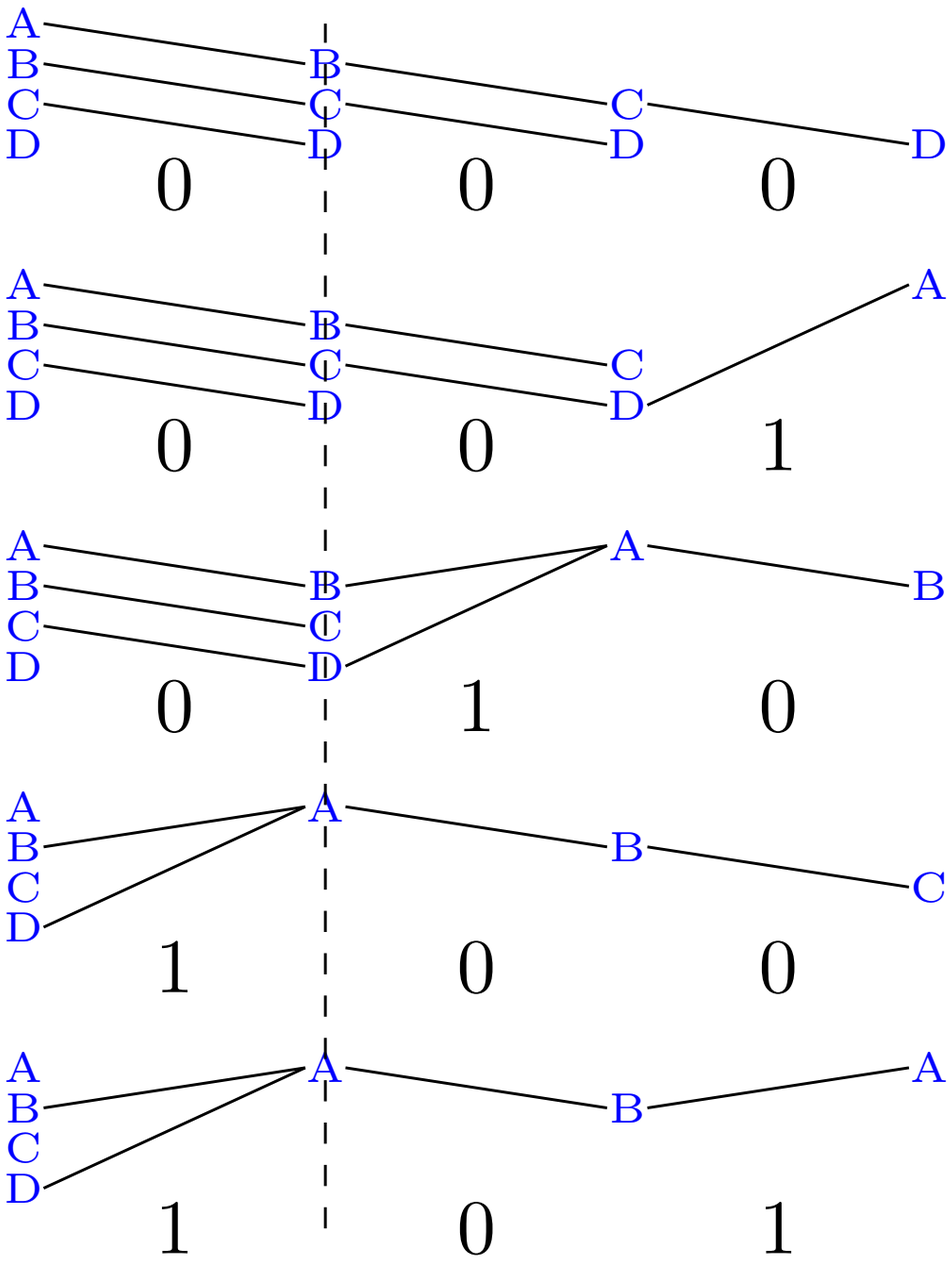

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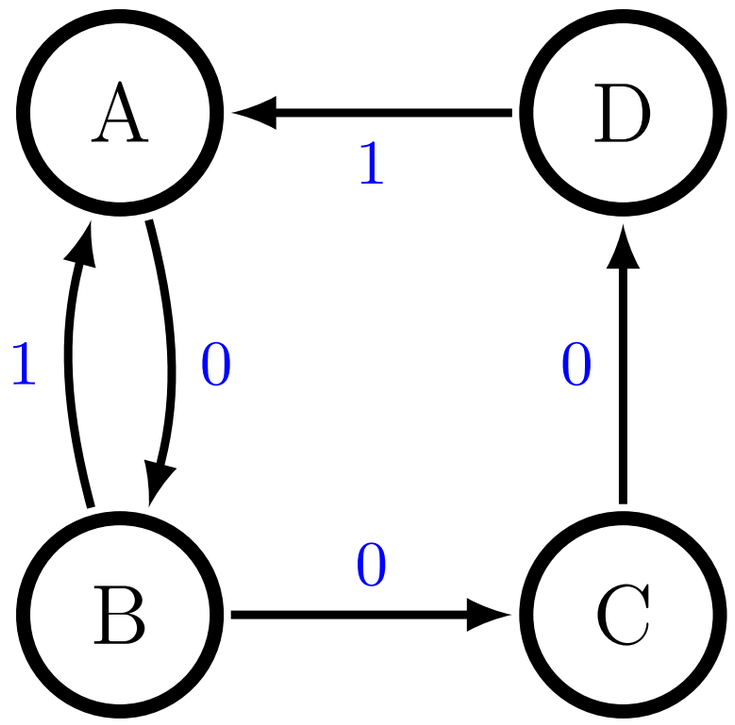

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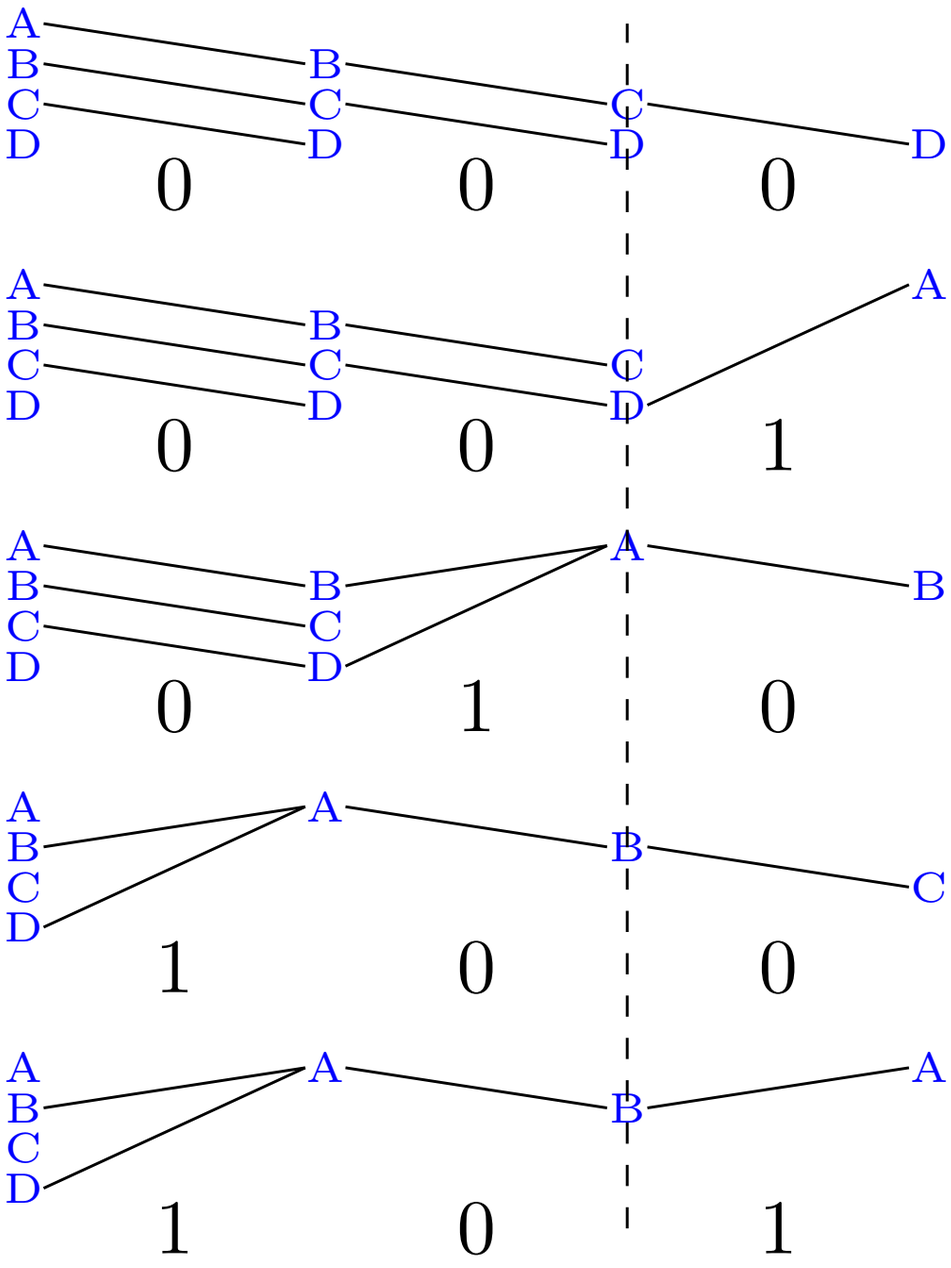

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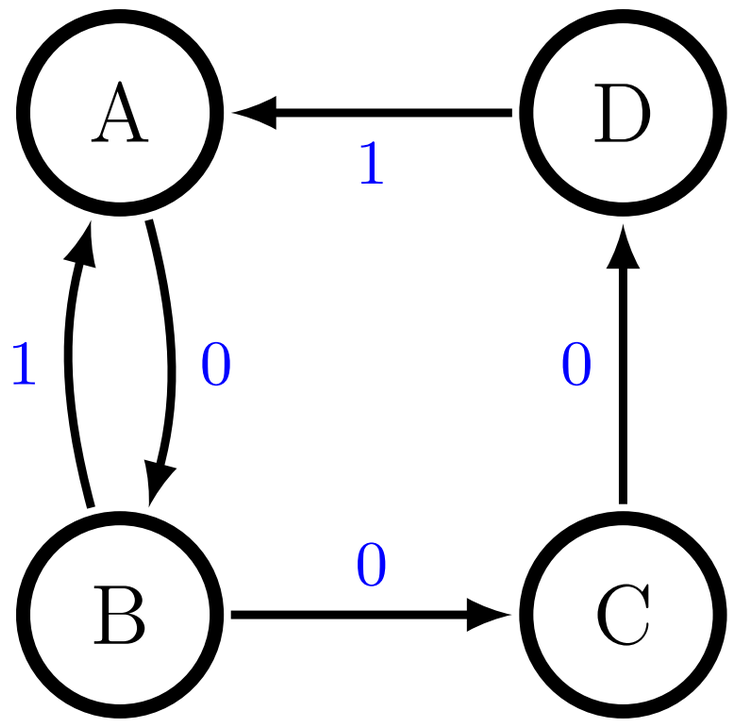

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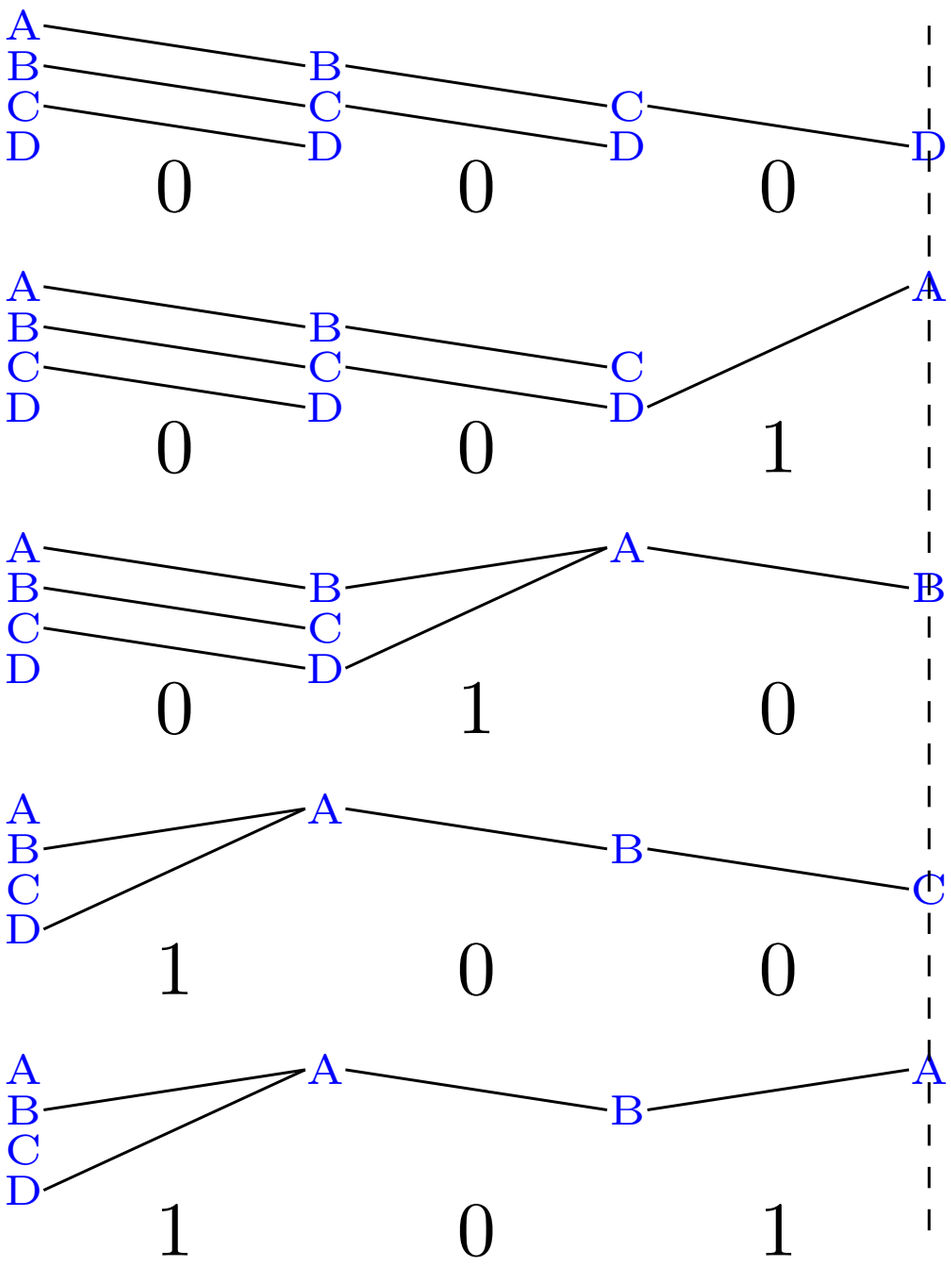

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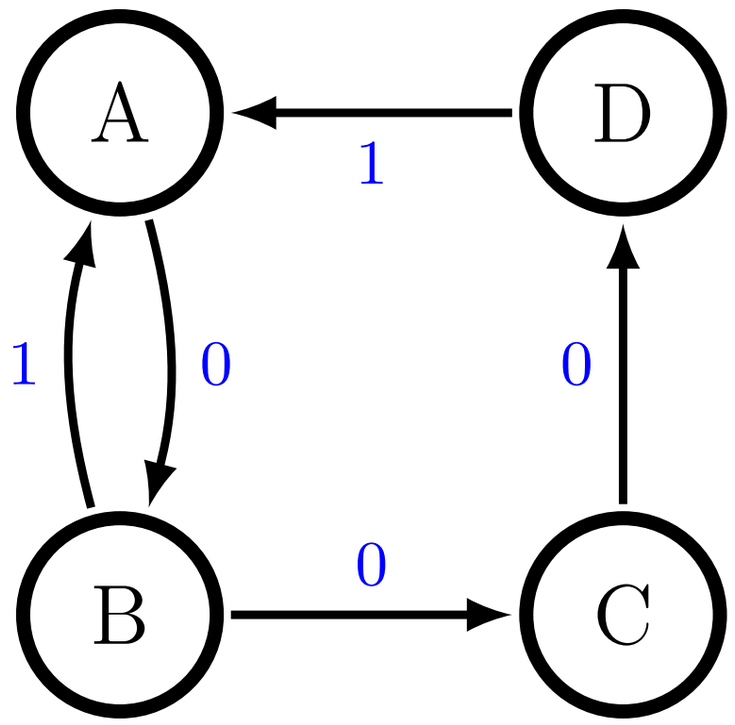

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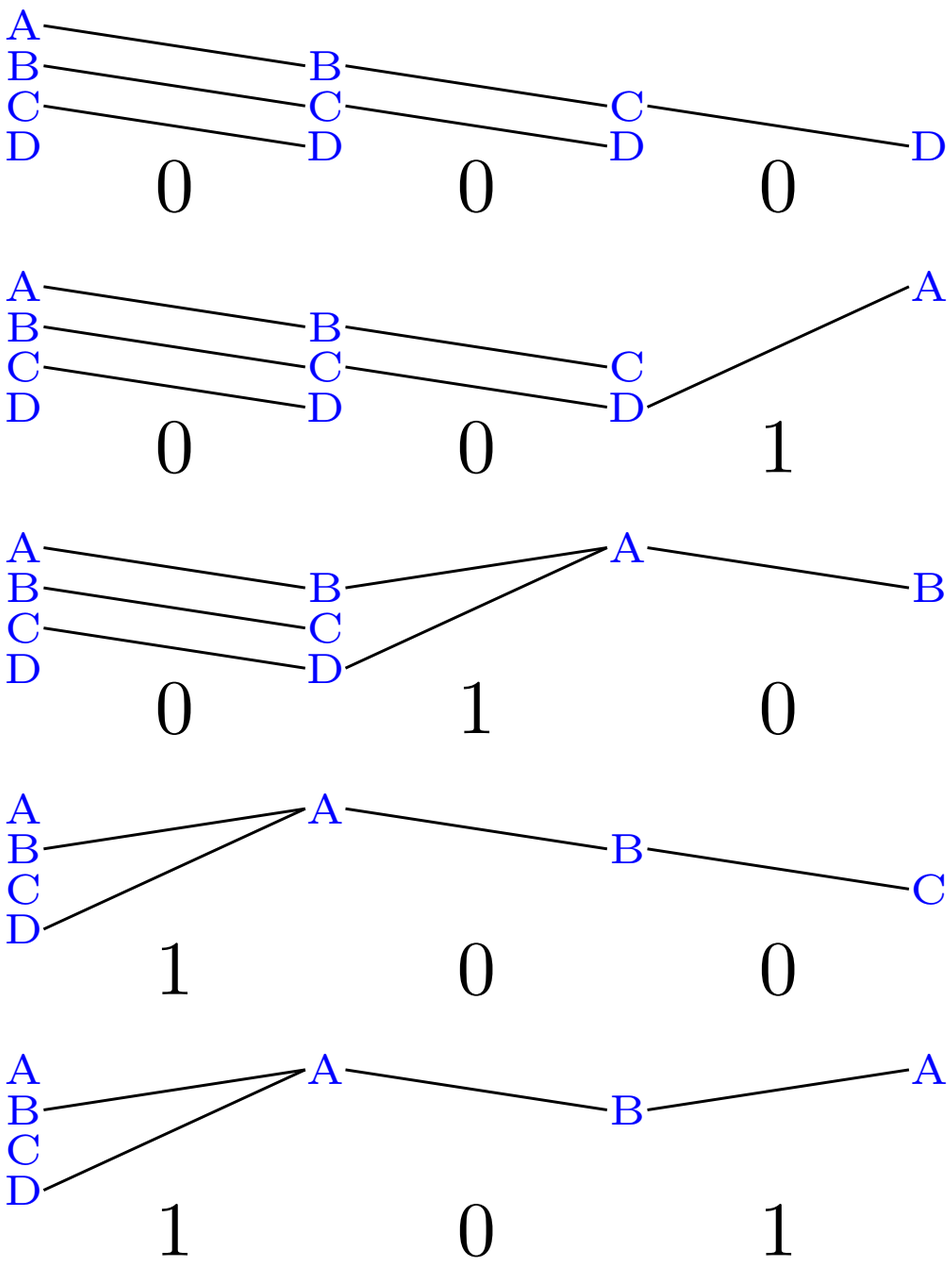

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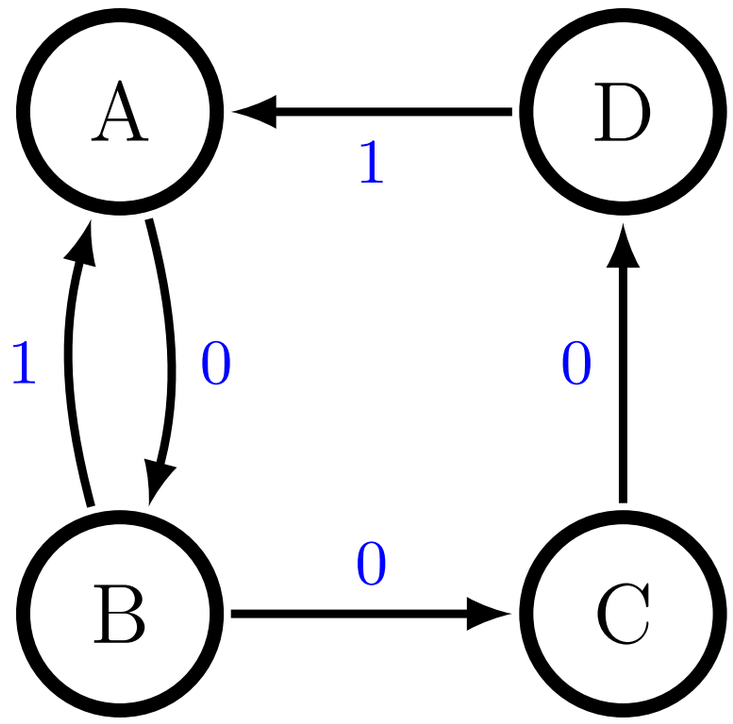
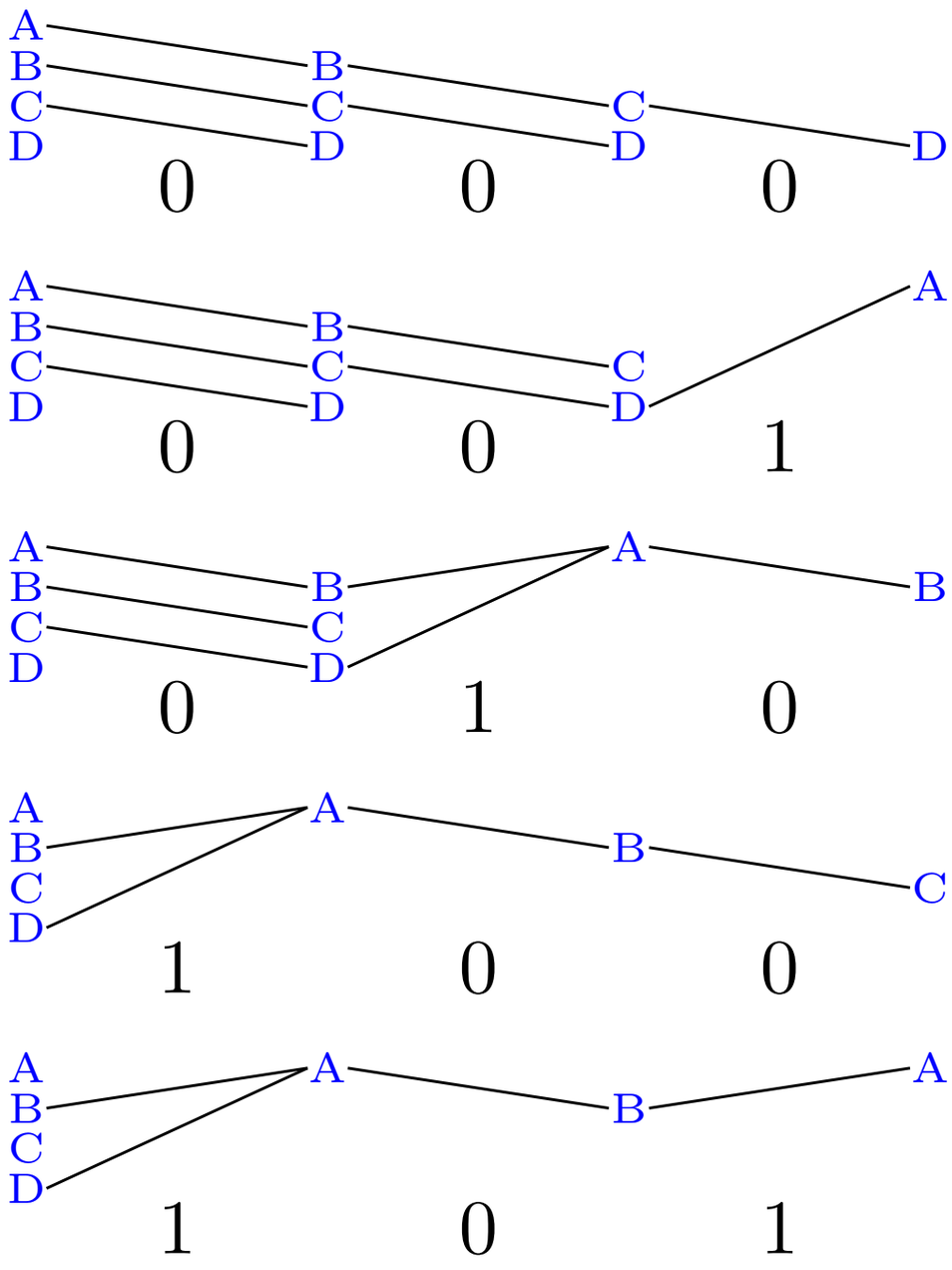


$$R = 3$$

This Time Only The Successful



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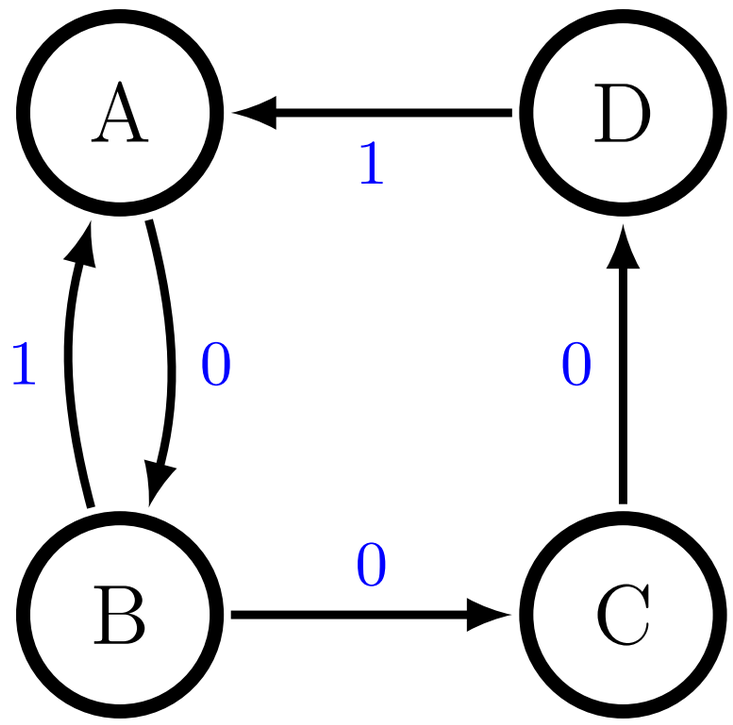

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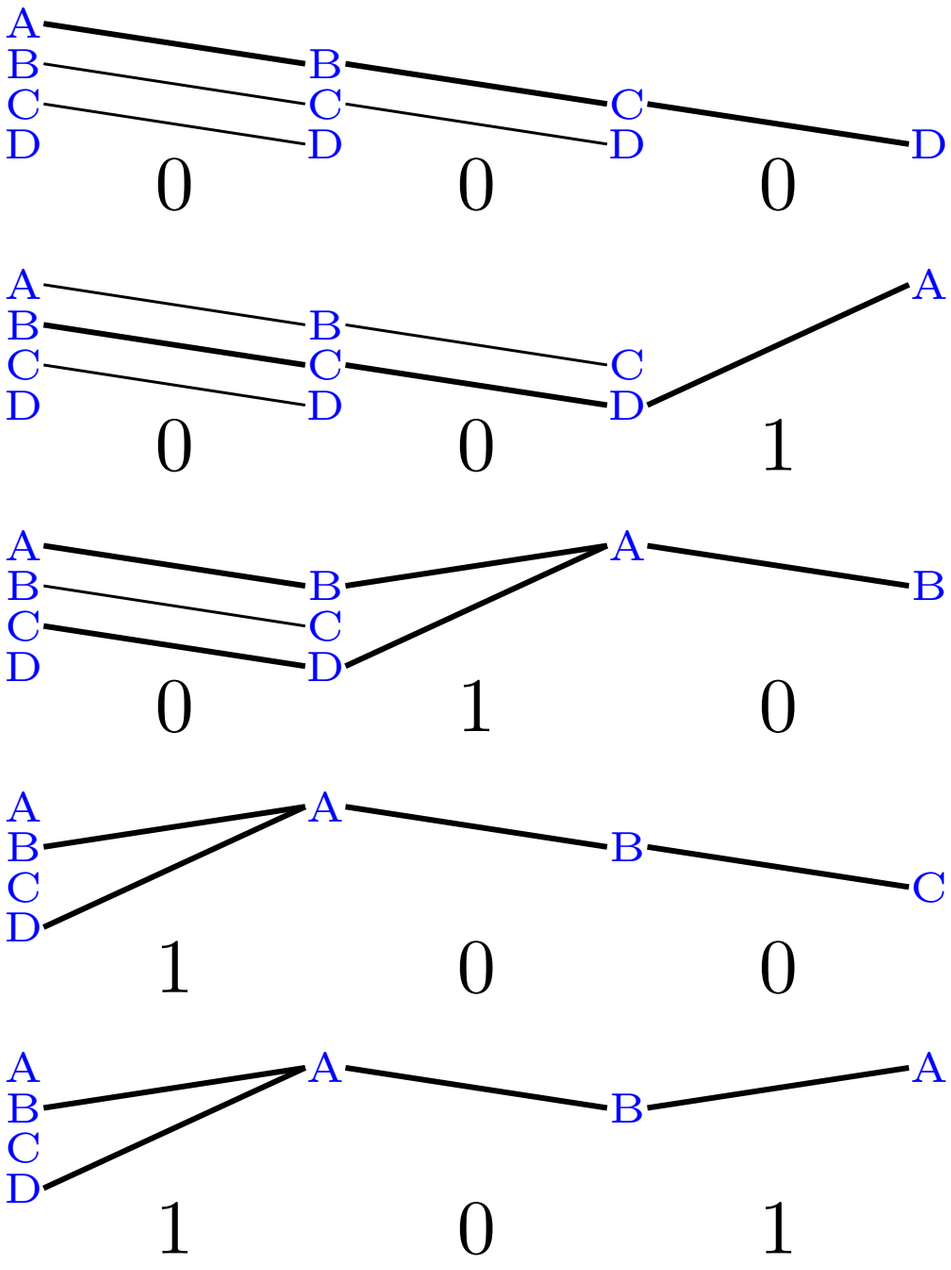

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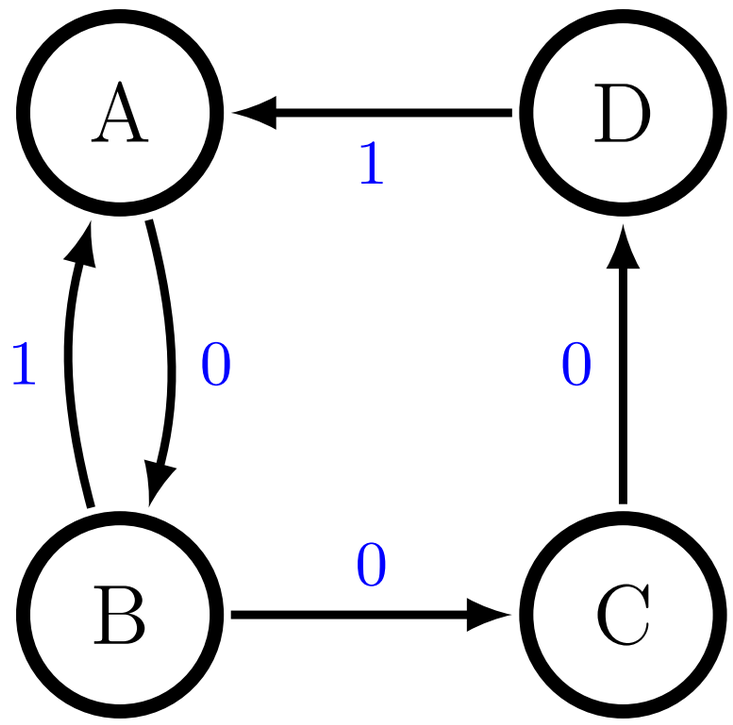

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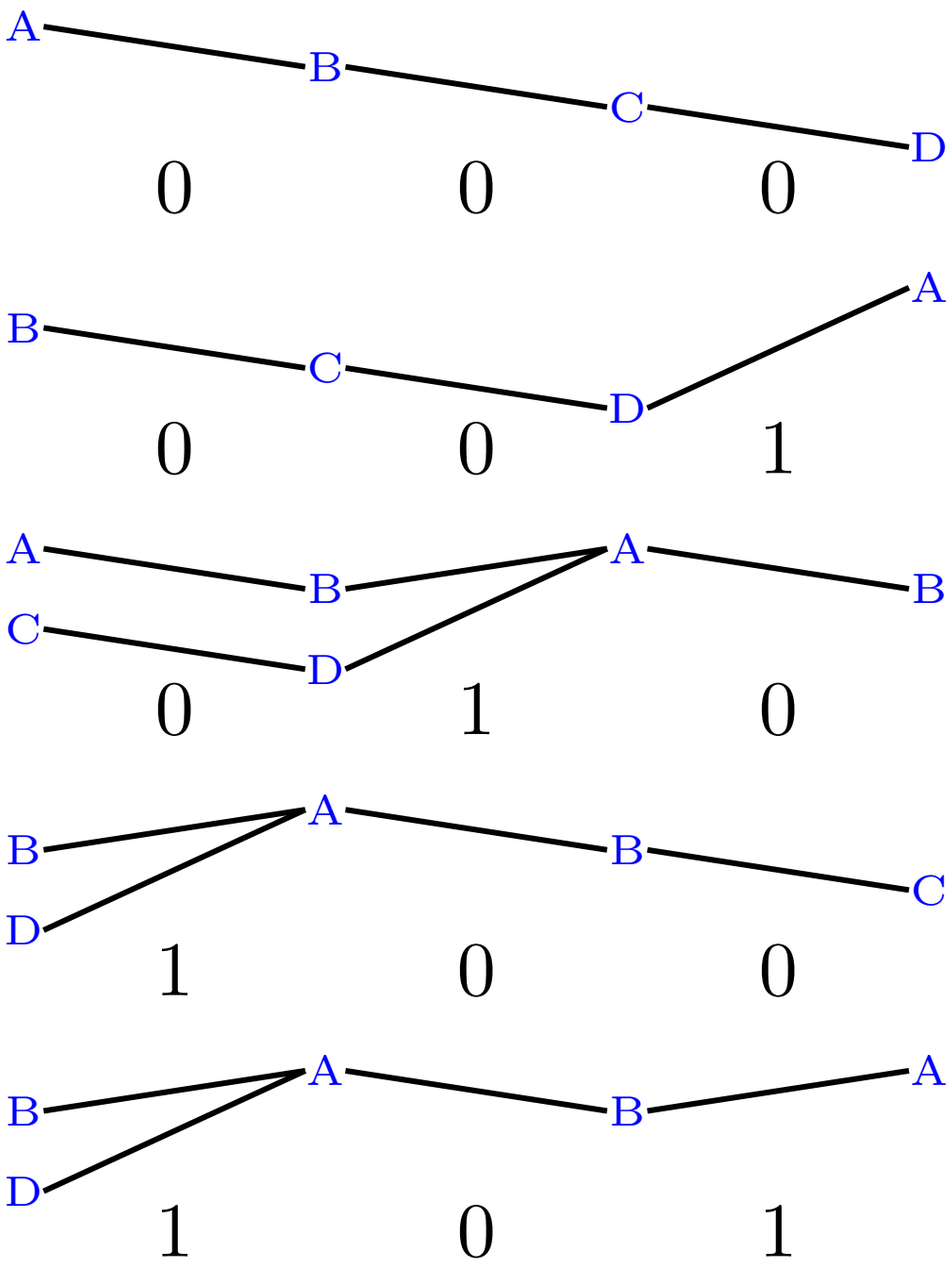

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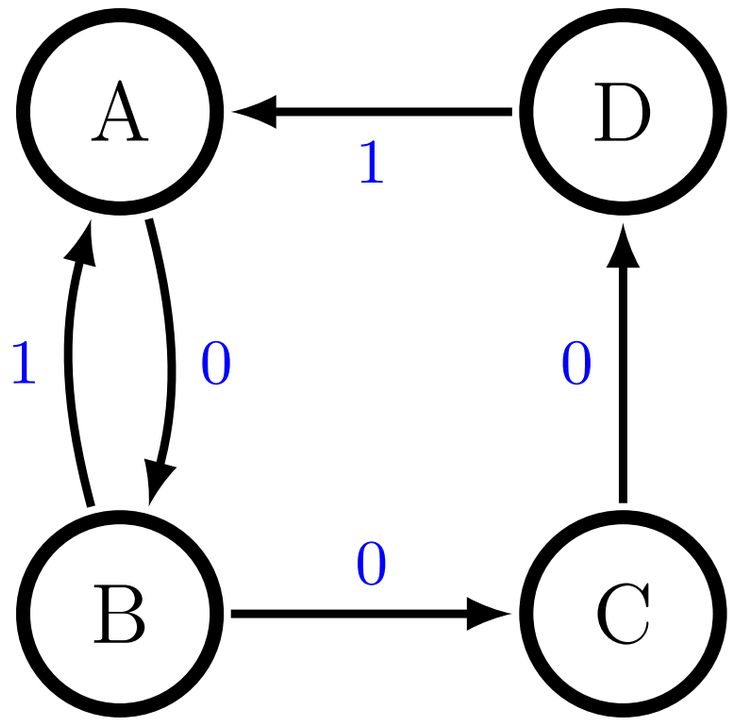

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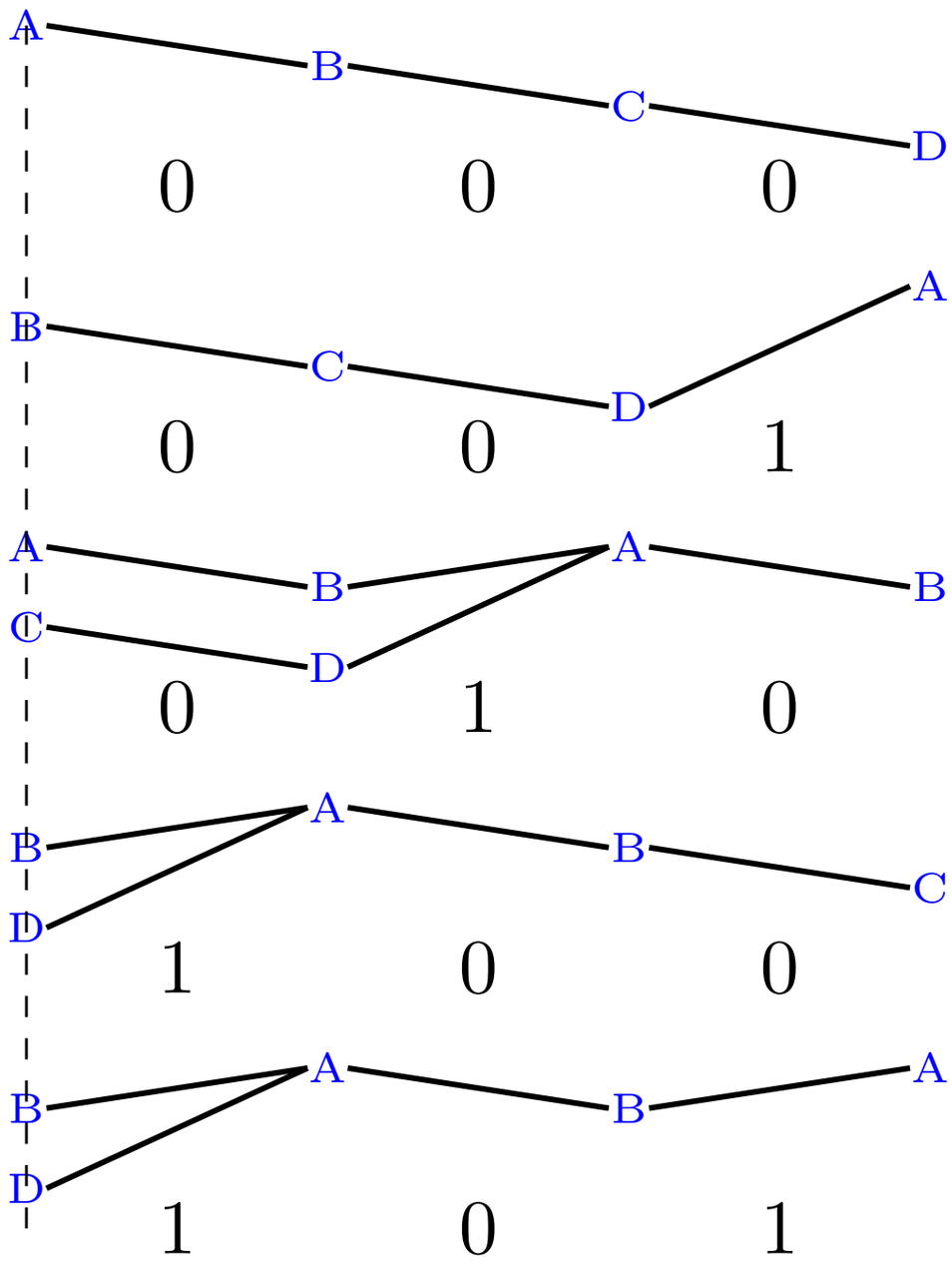

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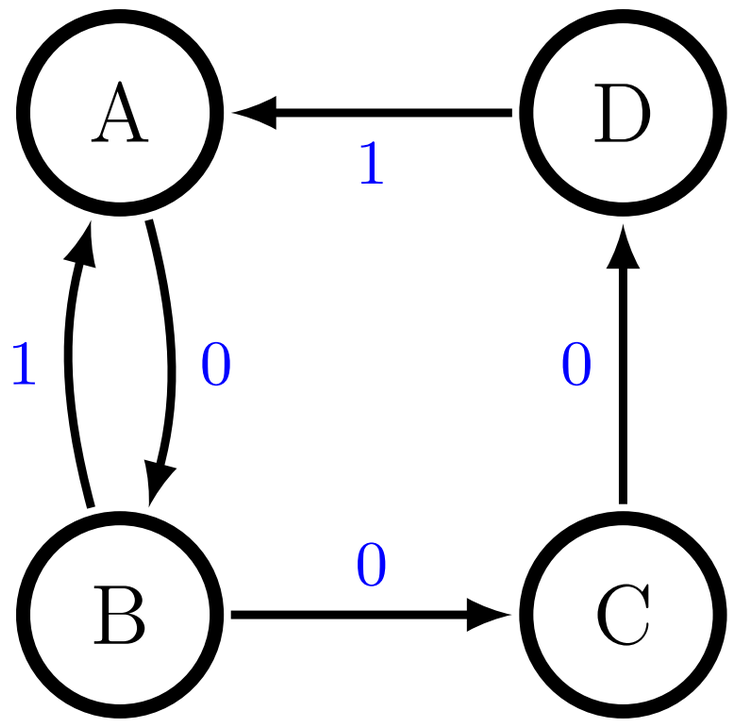

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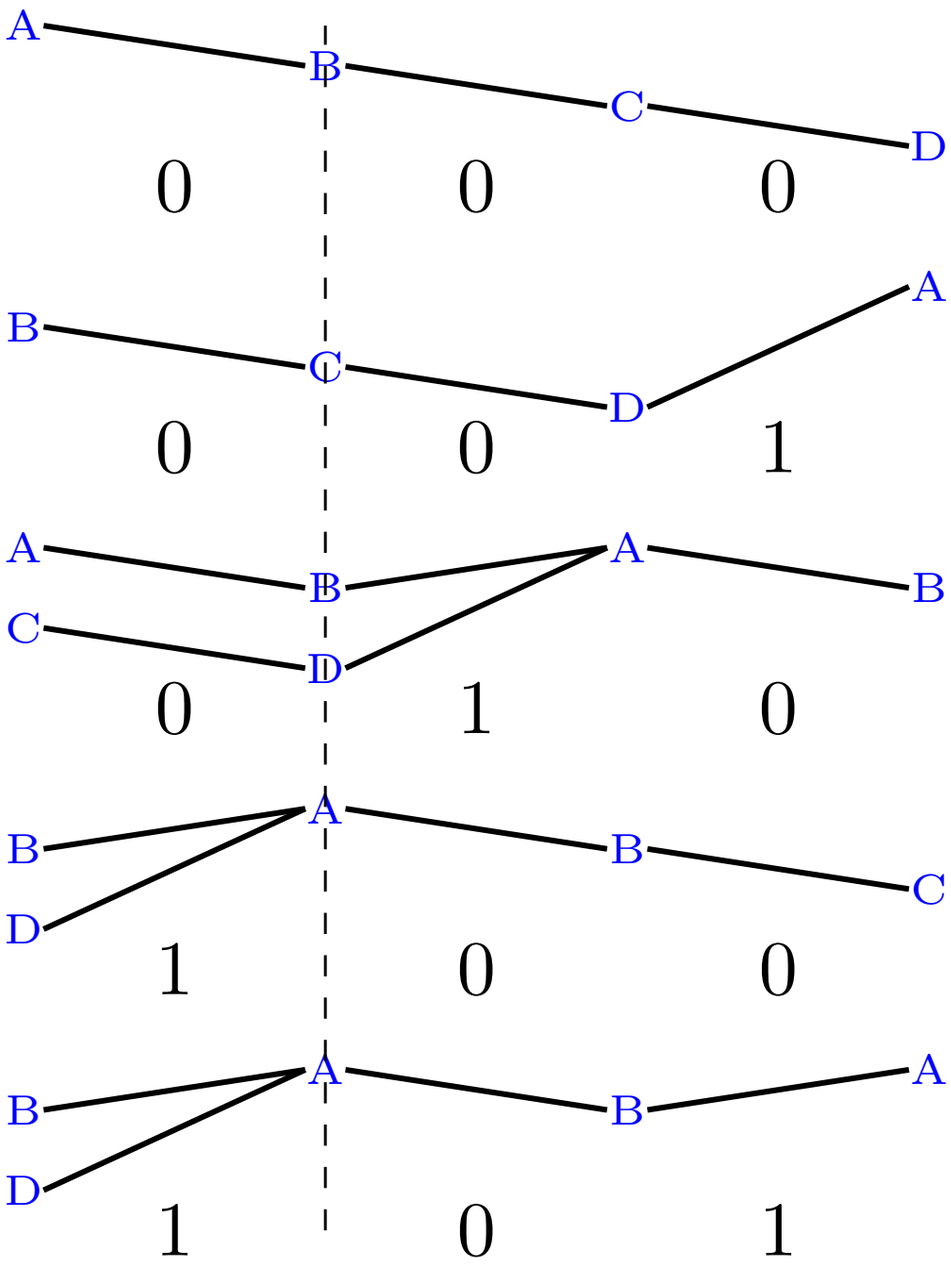

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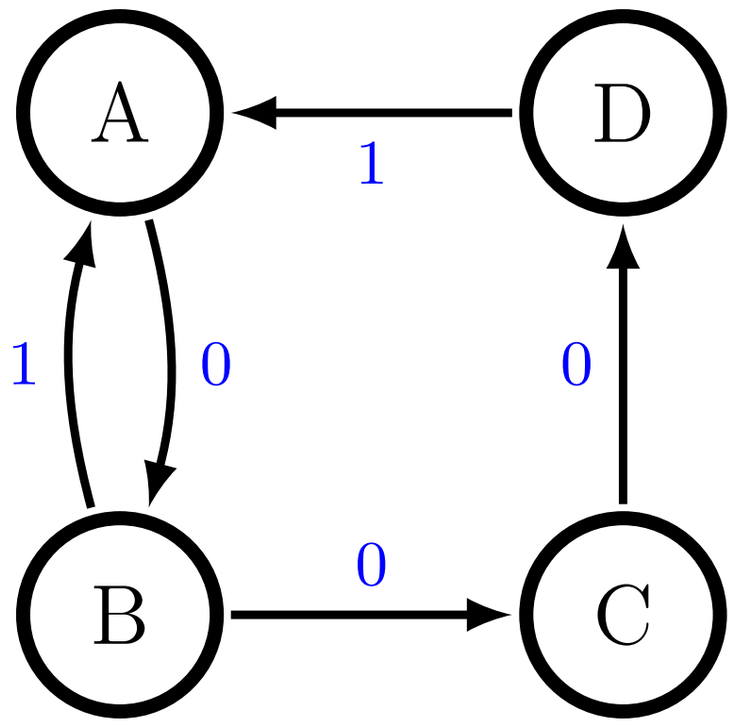

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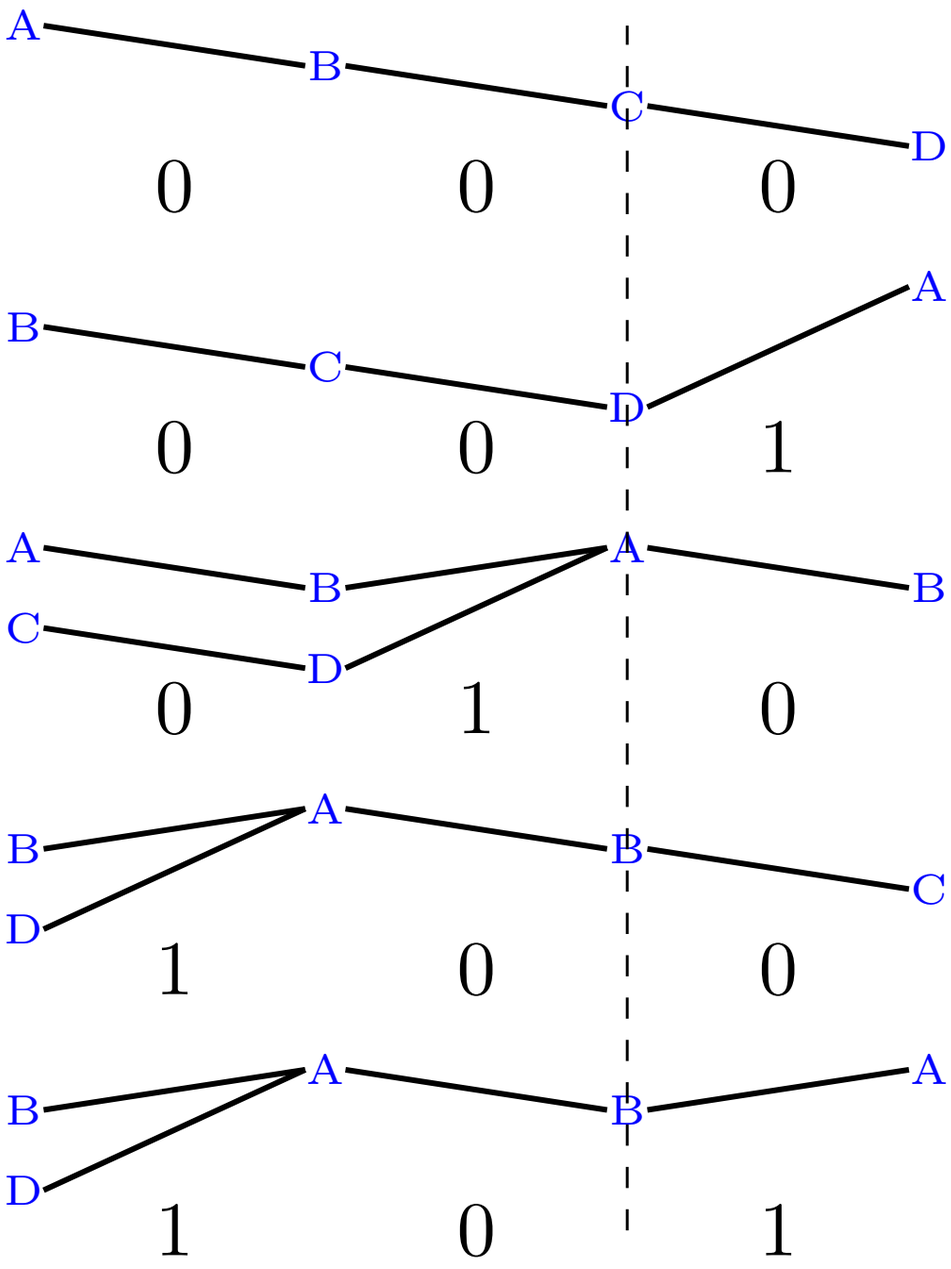
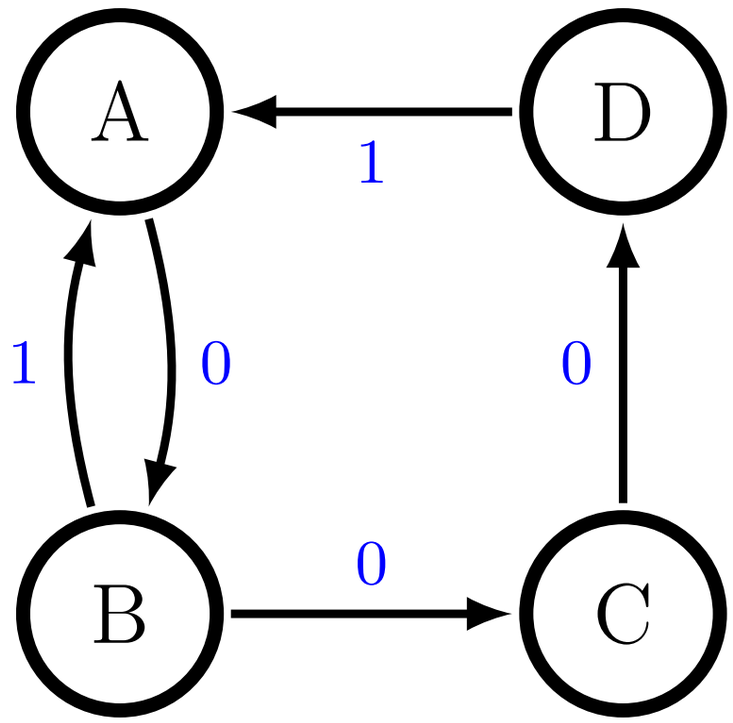
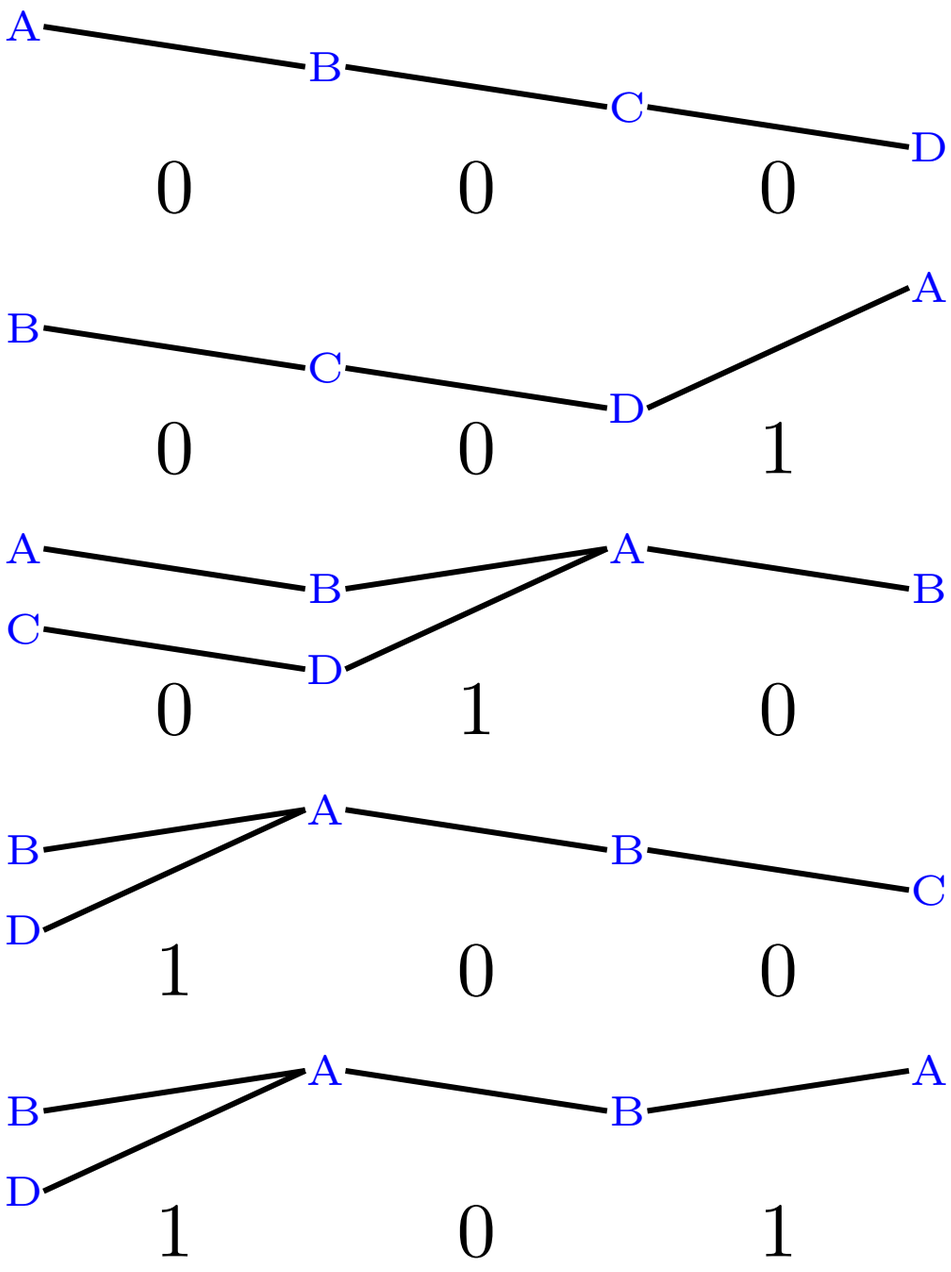


$$H[\mathcal{S}_\ell | X_{0:}] = 0$$

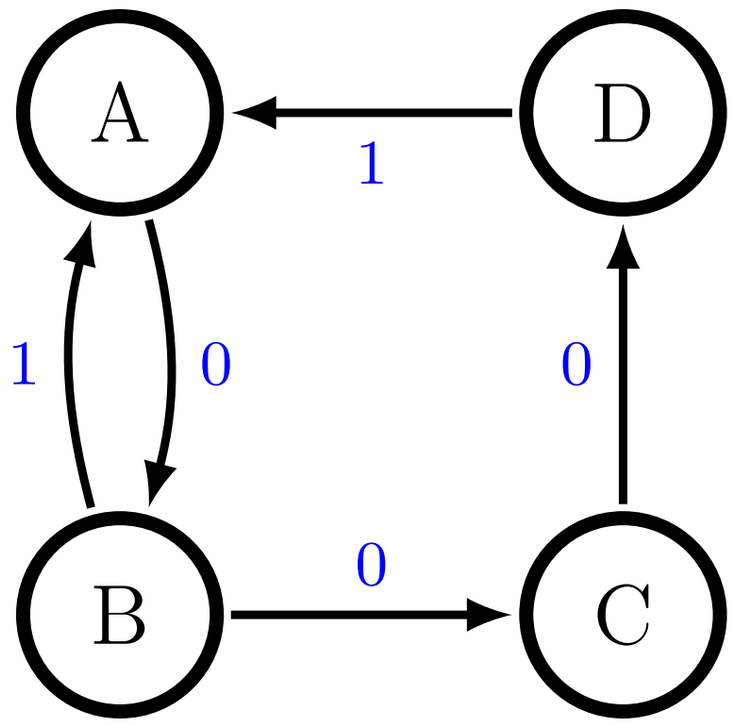


$$R = 3$$

This Time Only The Successful



$$H[\mathcal{S}_\ell | X_{0:}] = 0$$



$$R = 3$$

$$k_\chi = 2$$

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Improvements

In what ways have we improved over the original method of computing R and k_χ?

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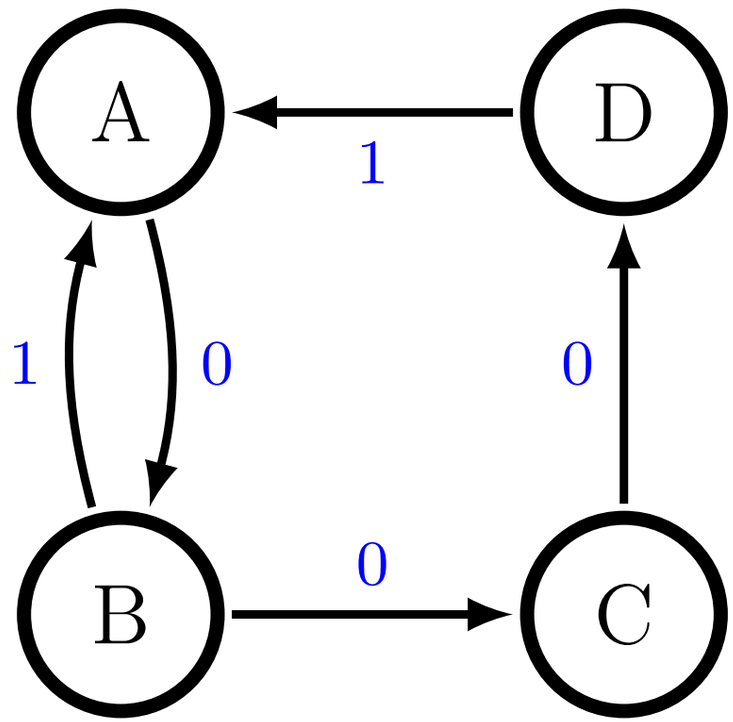
- Don't need $\mathbf{E}$ or h_μ
- Integer based, so don't need to worry about machine precision
- But . . . There could be an arbitrary number of synchronizing words, each of arbitrary length
- Can we do better?

Powersets



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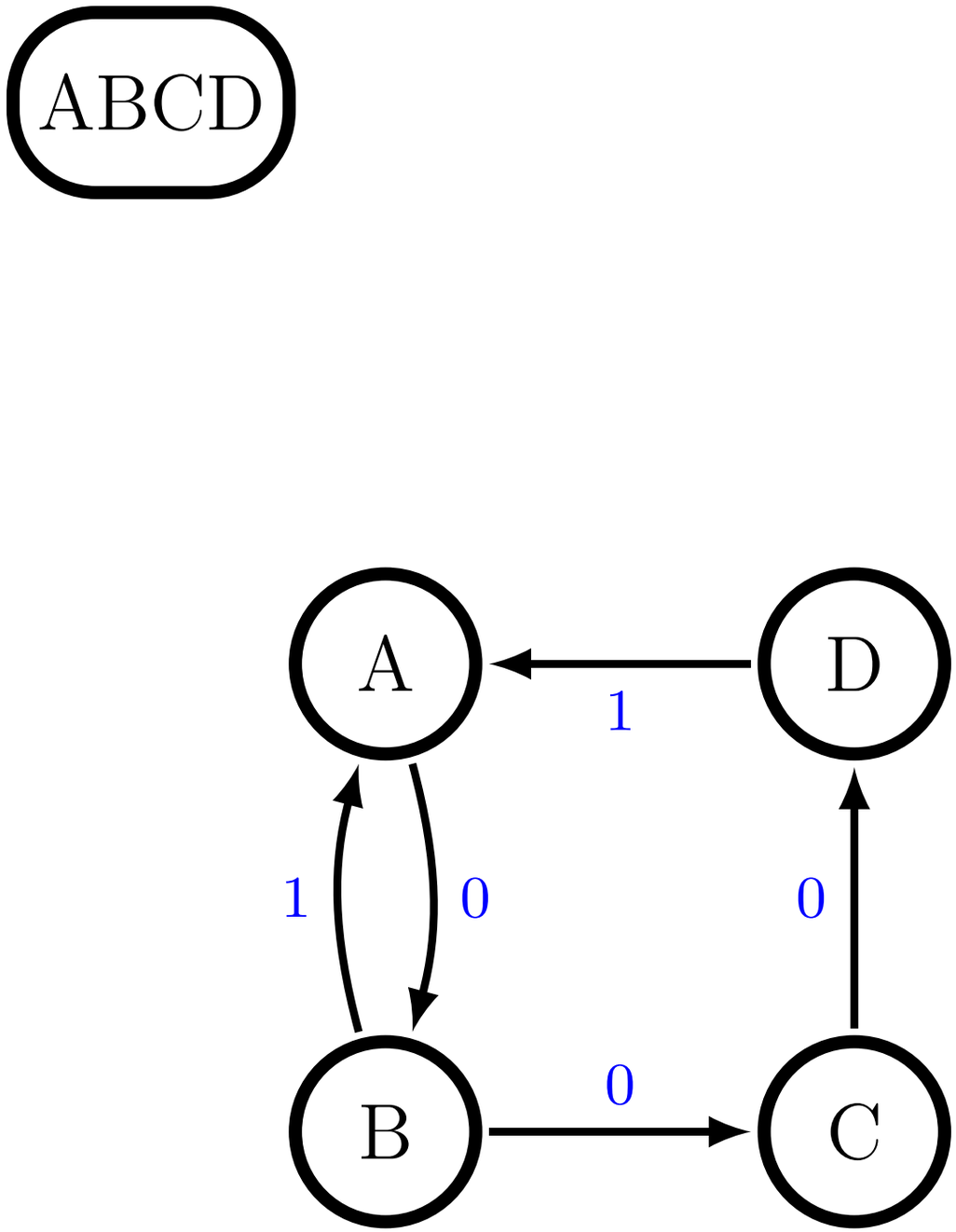
Powersets



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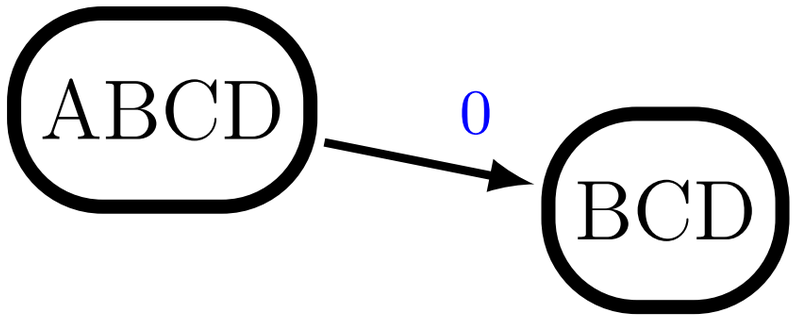

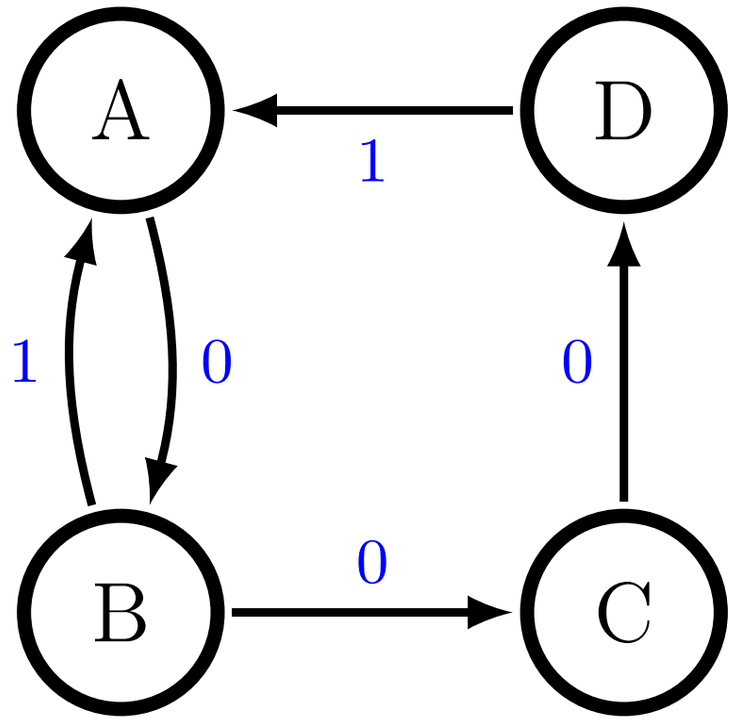

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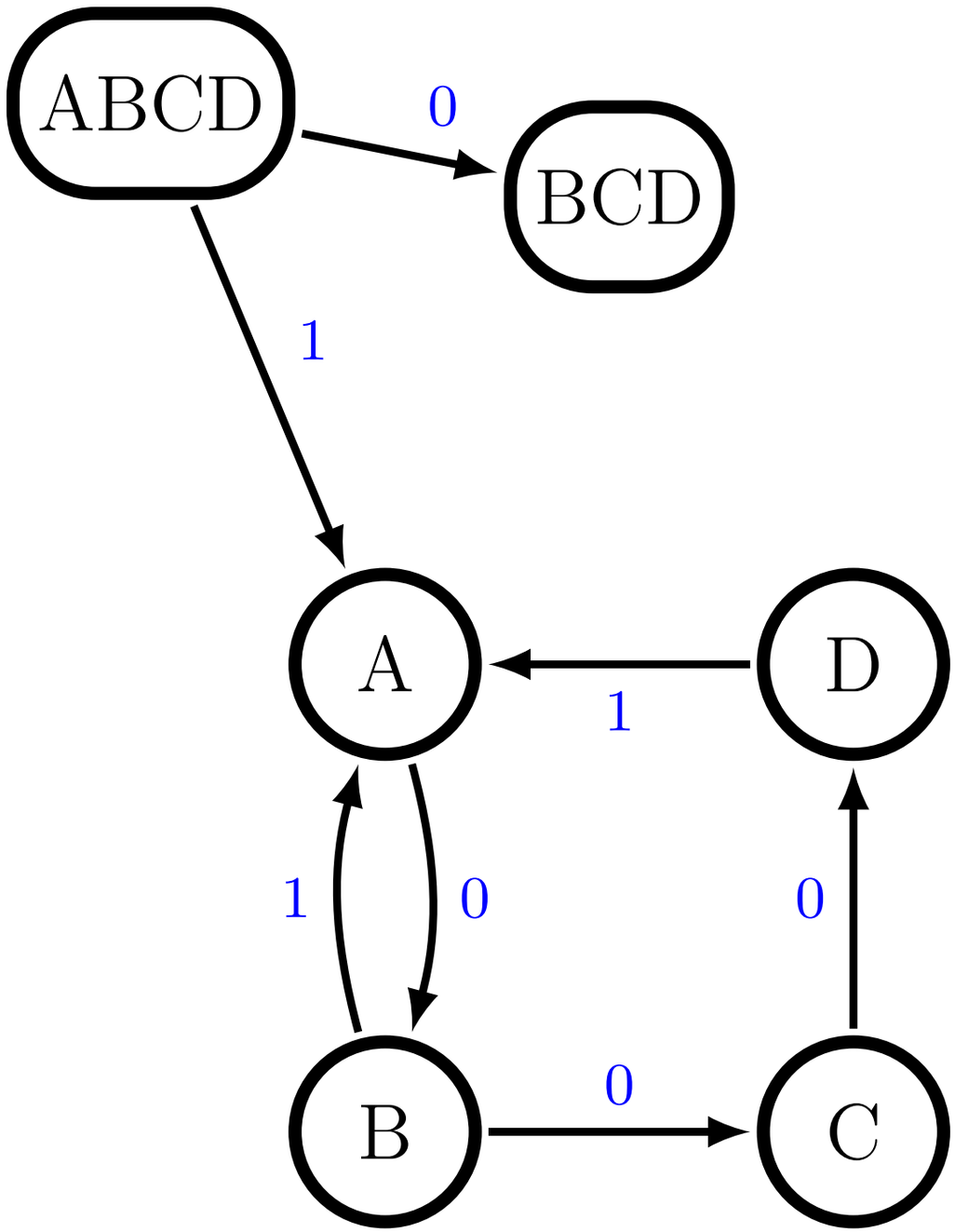

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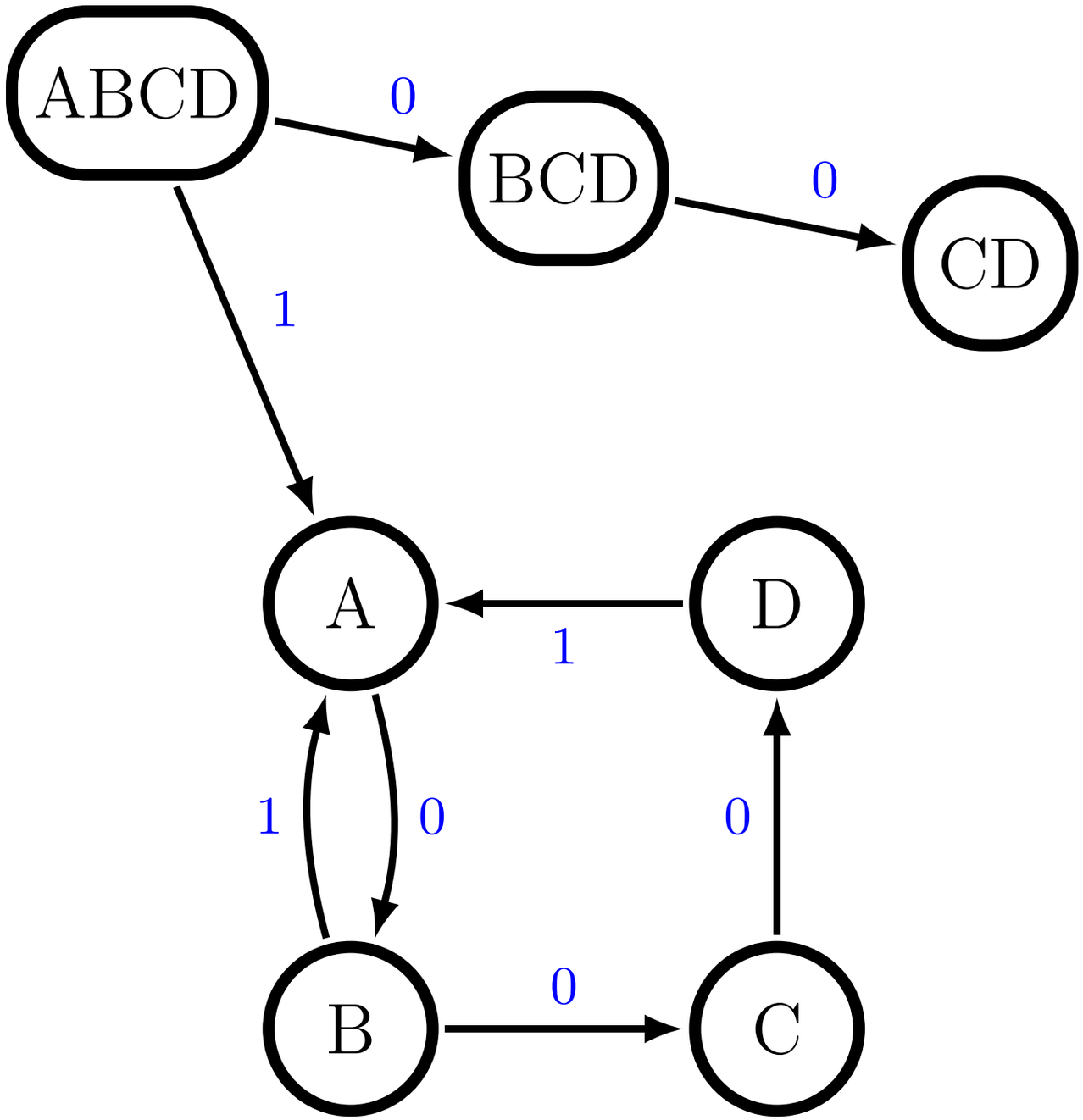

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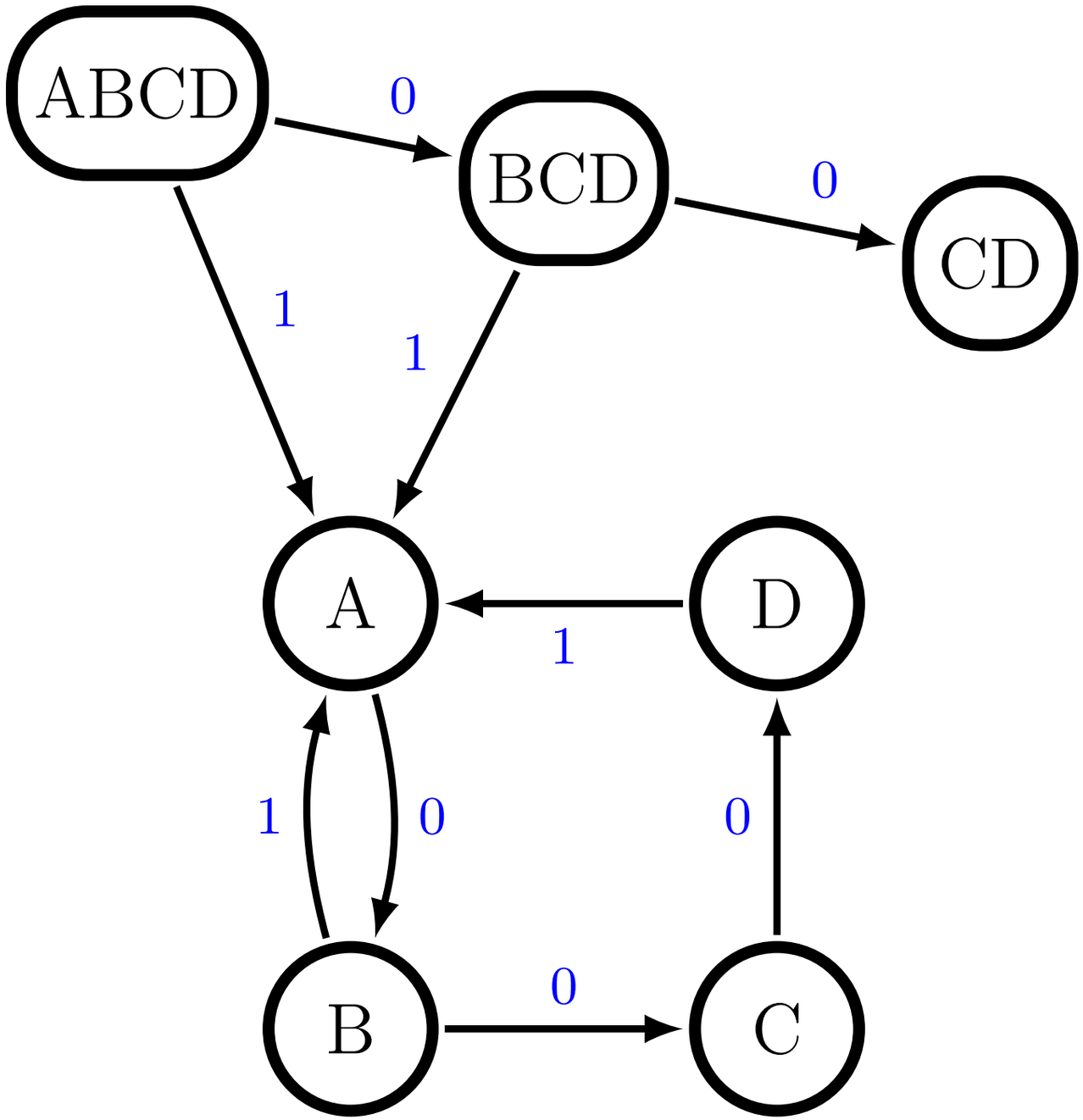

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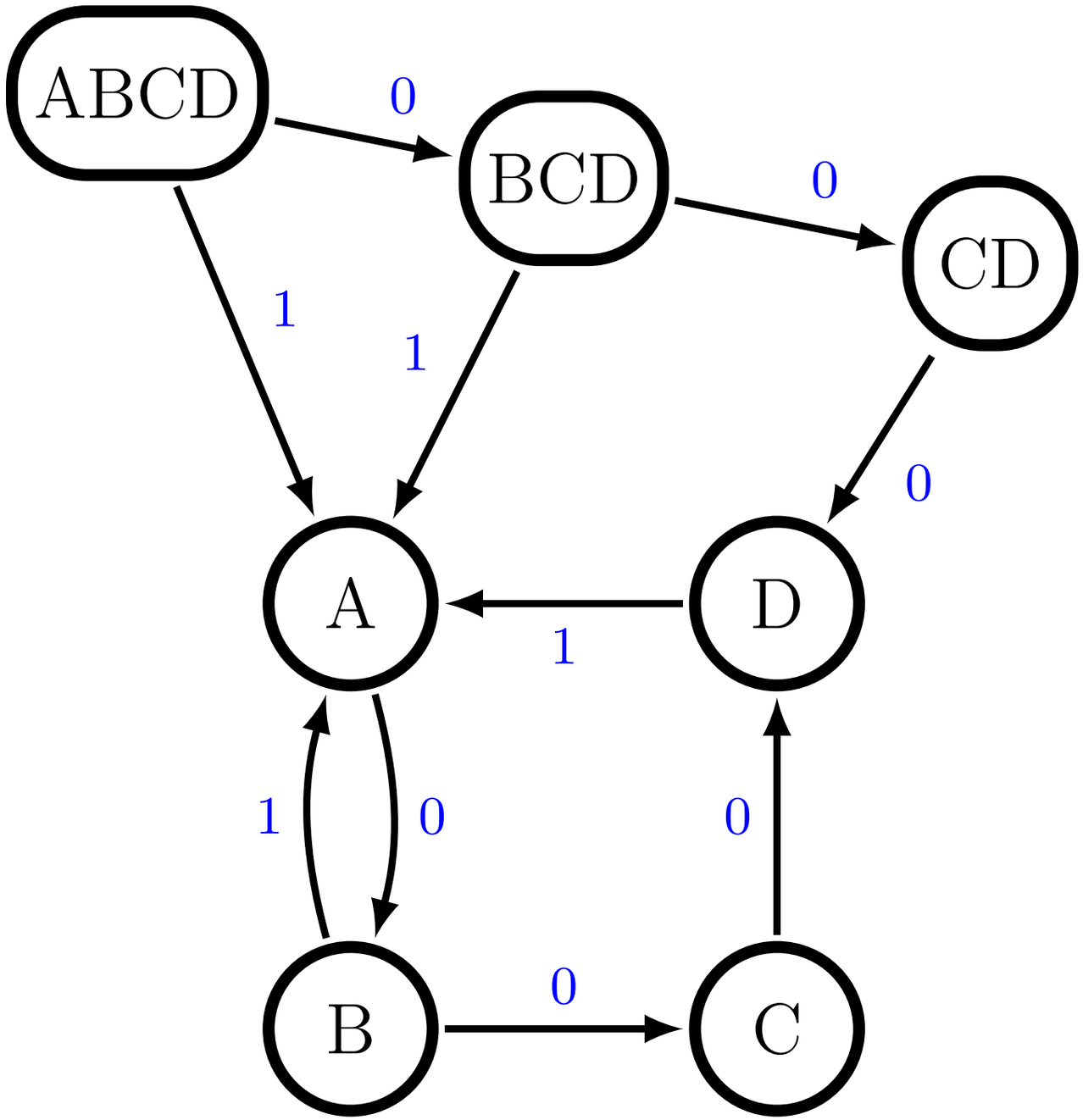

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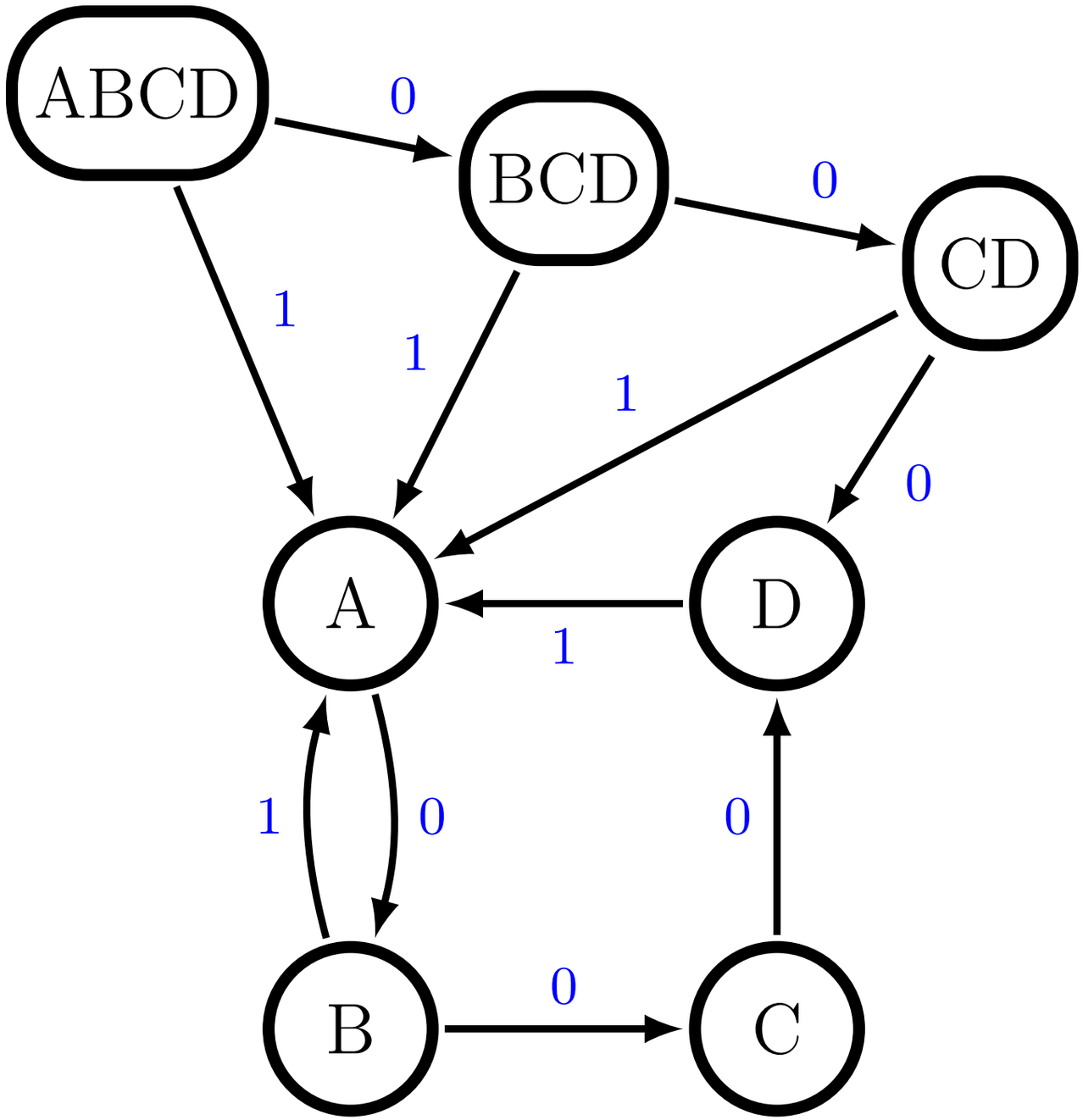
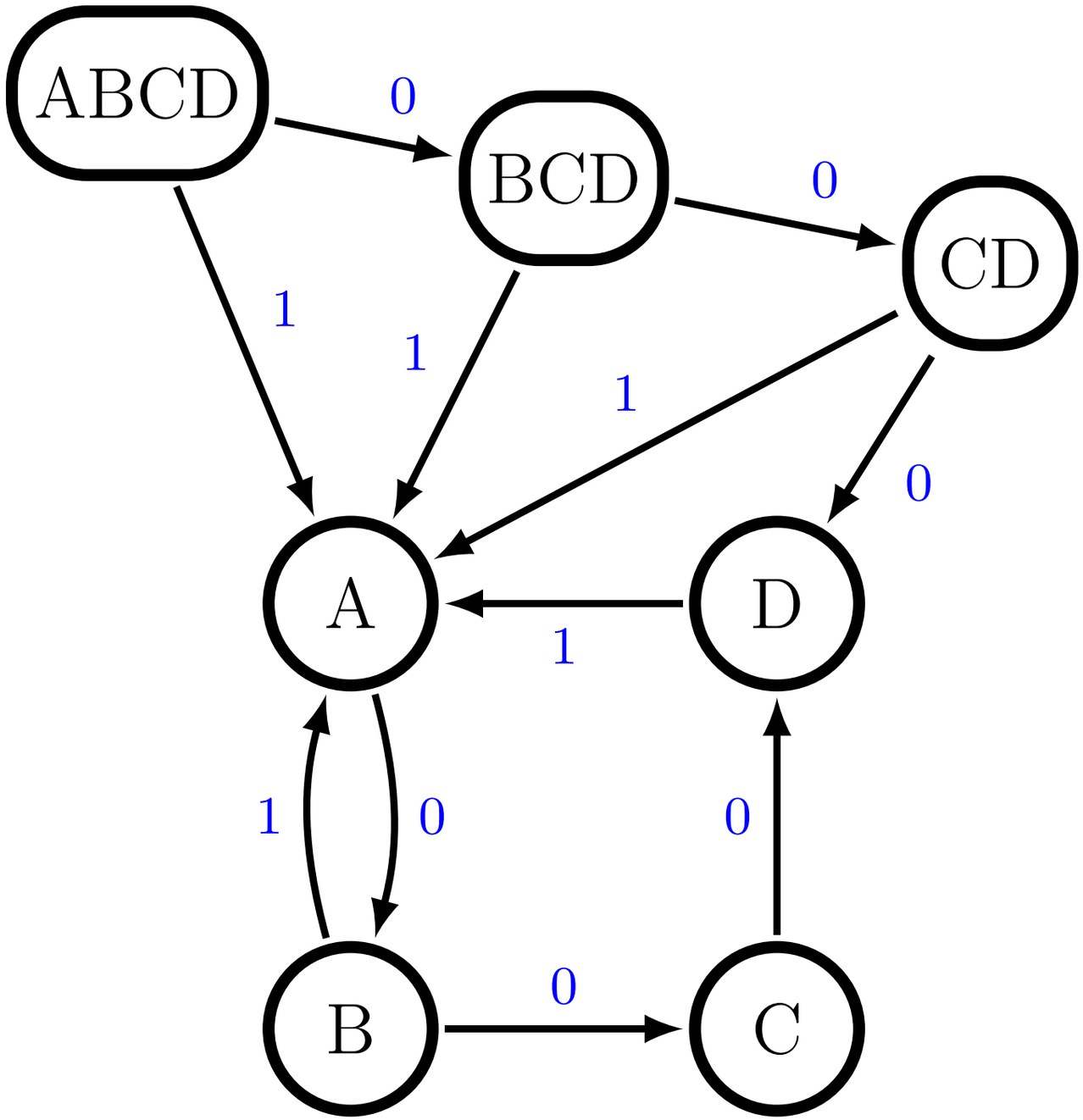


Powersets

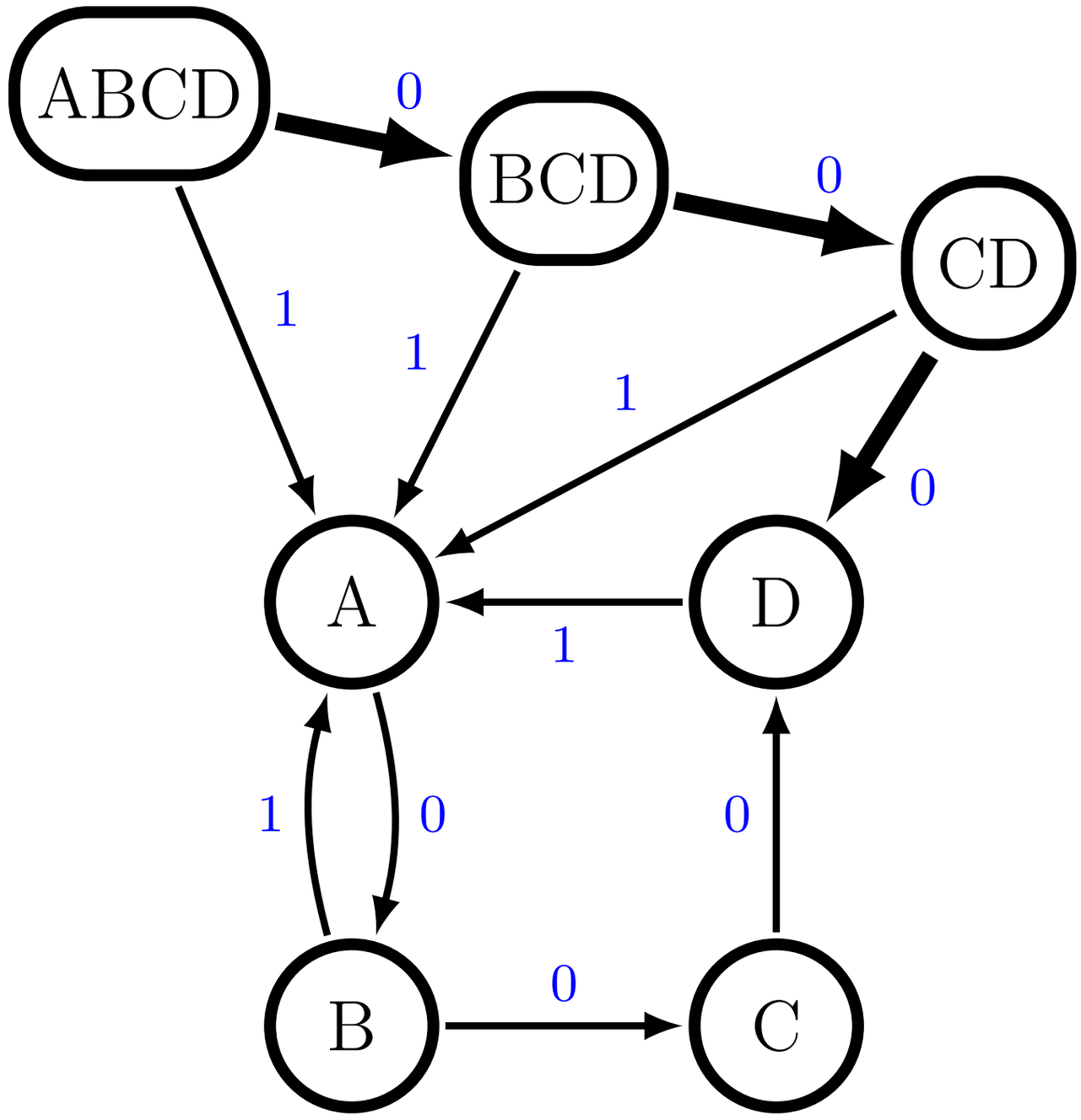
Synchronizing Words:



Powersets

Synchronizing Words:

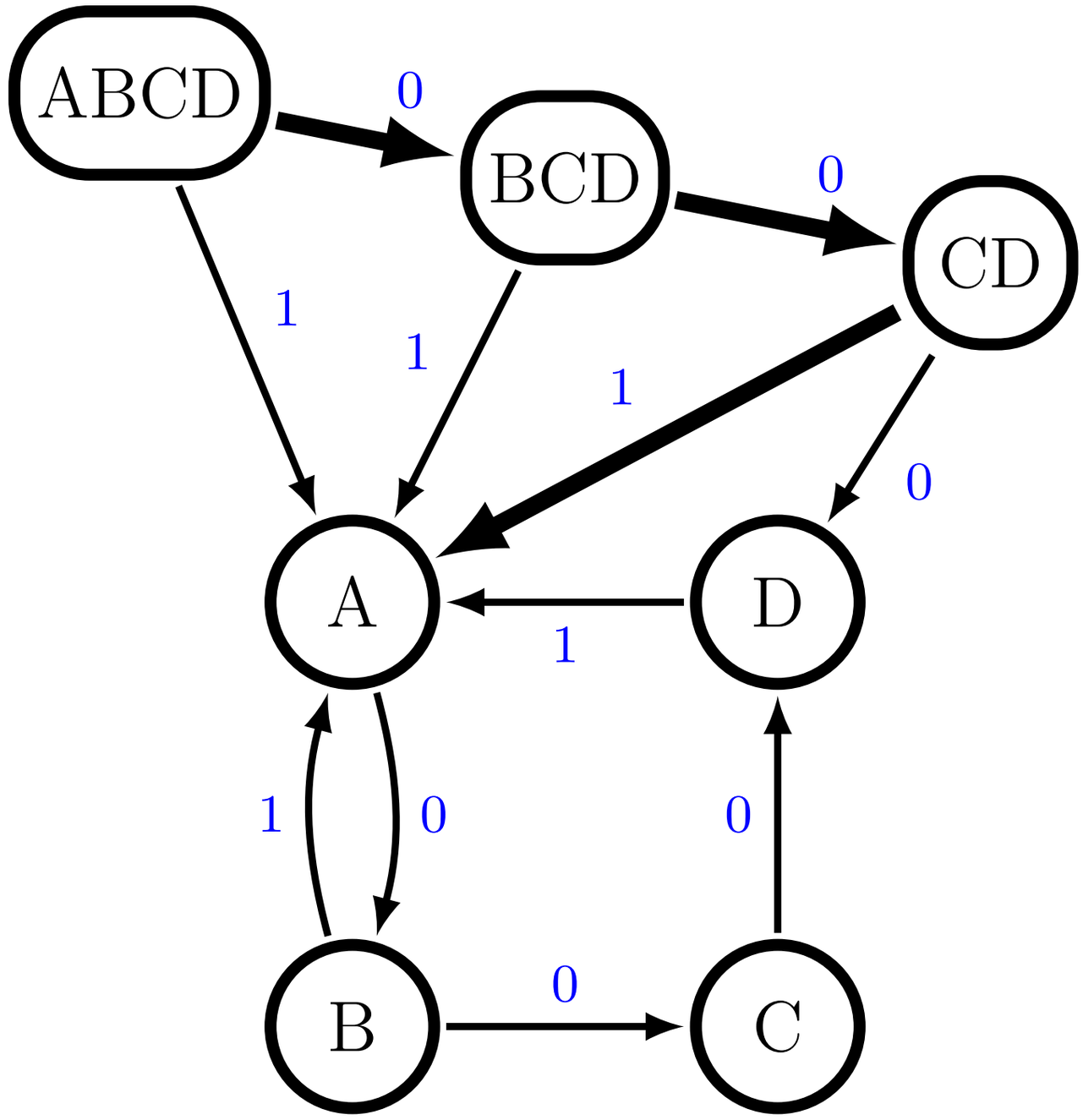
- 000



Powersets

Synchronizing Words:

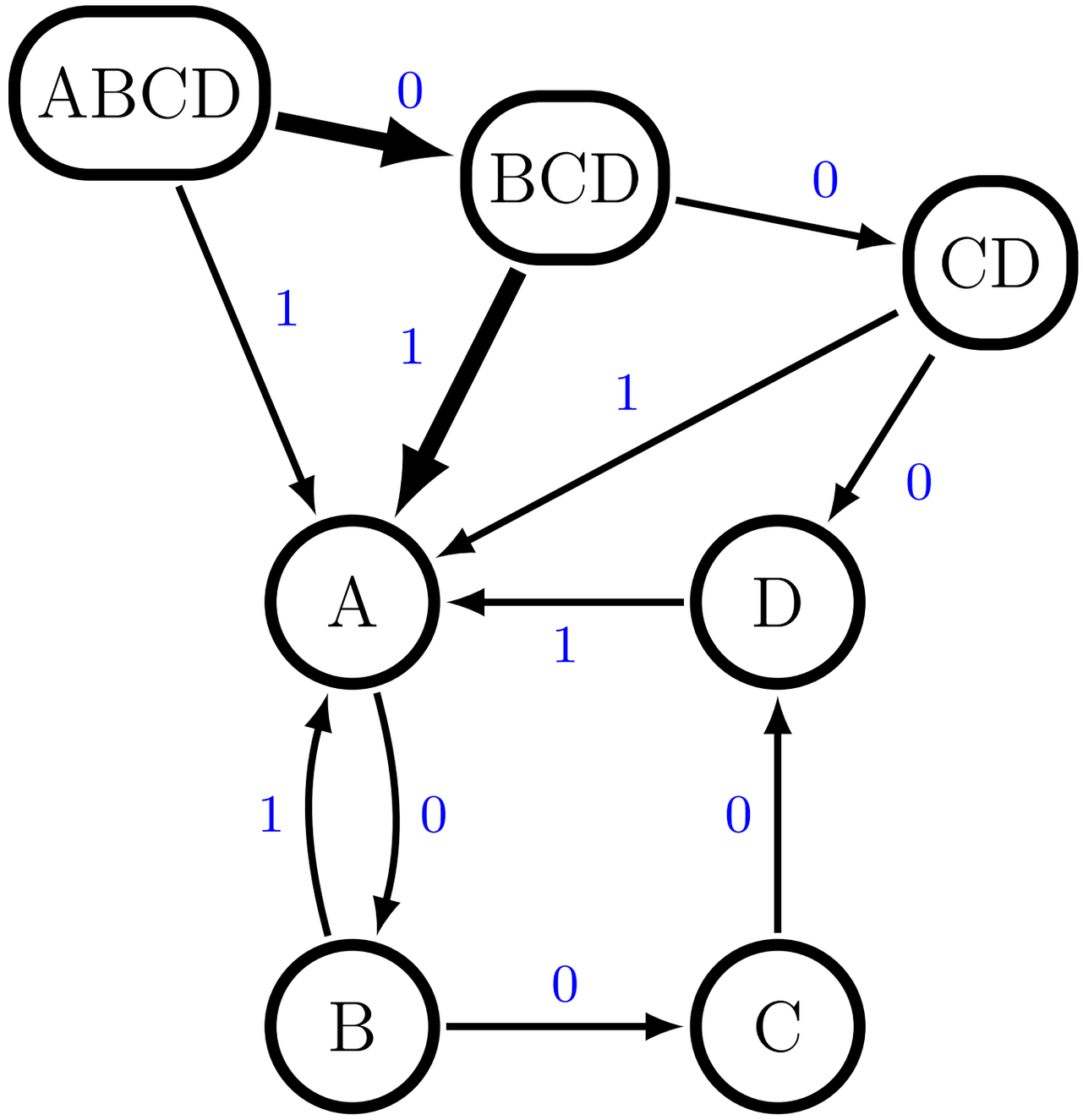
- 000
- 001



Powersets

Synchronizing Words:

- 000
- 001
- 01



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Powersets

Synchronizing Words:

- 000
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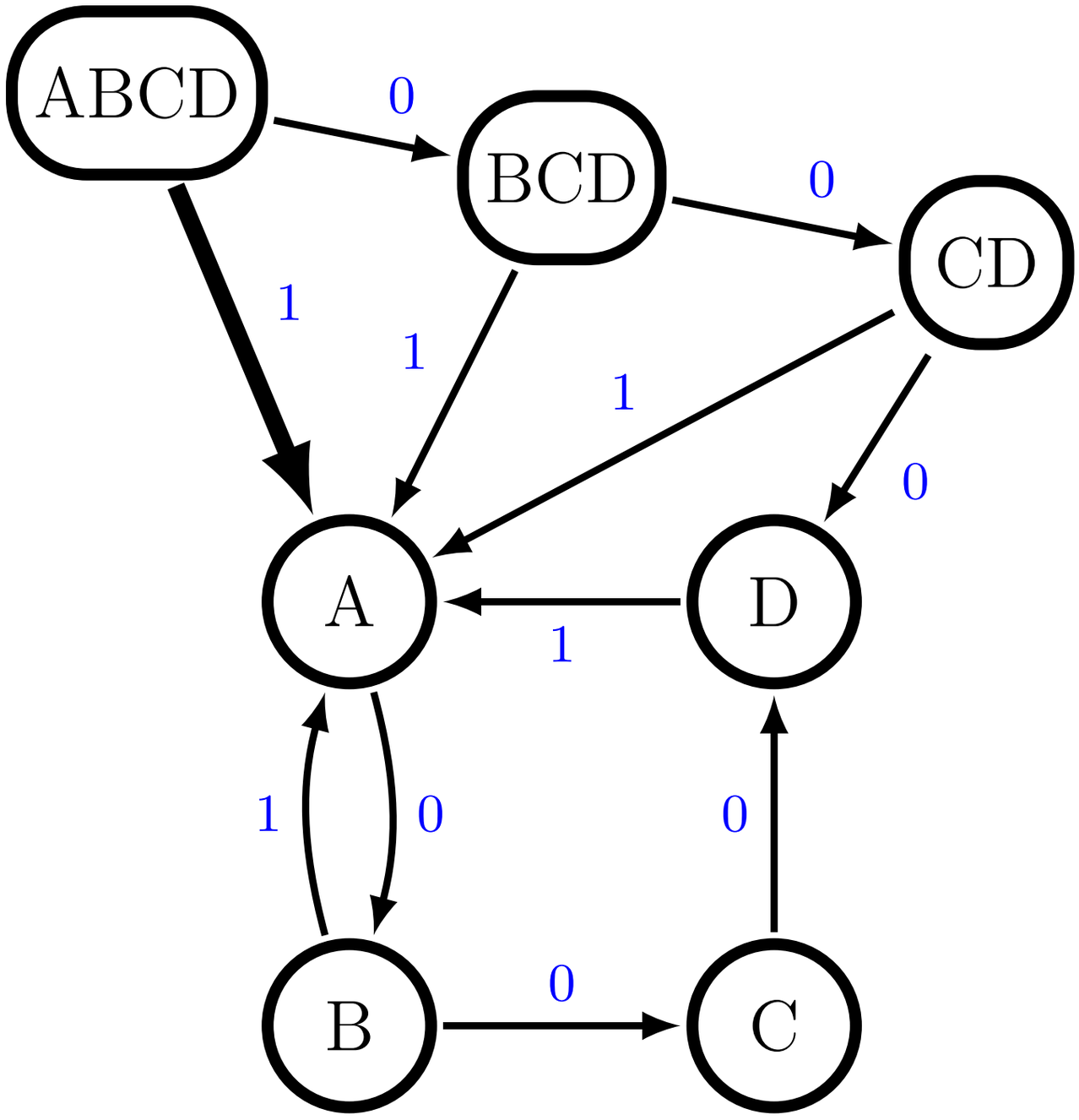
- 01
- 1



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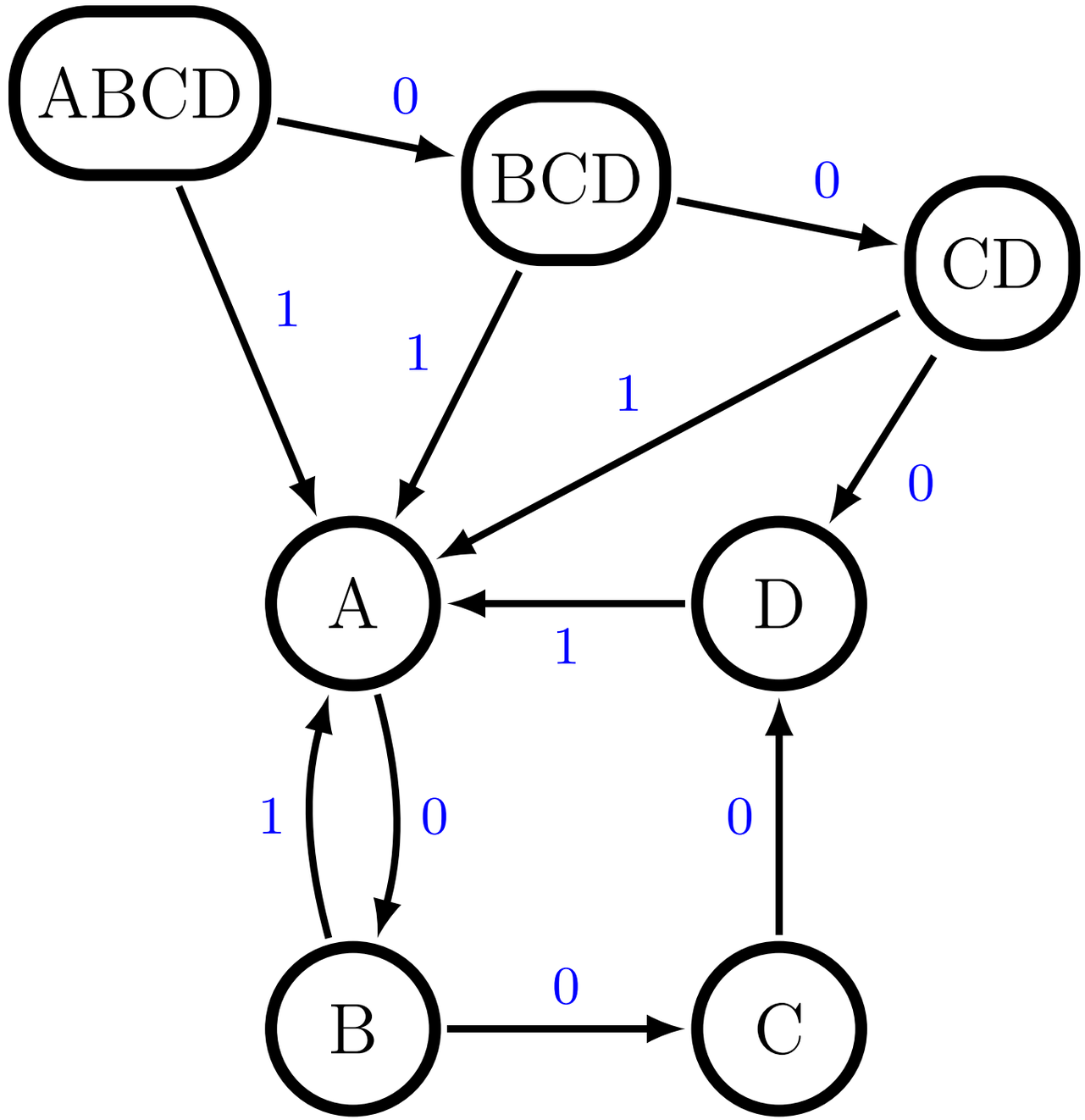
Longest Path

- R is the longest path from start to recurrent



Longest Path

- R is the longest path from start to recurrent
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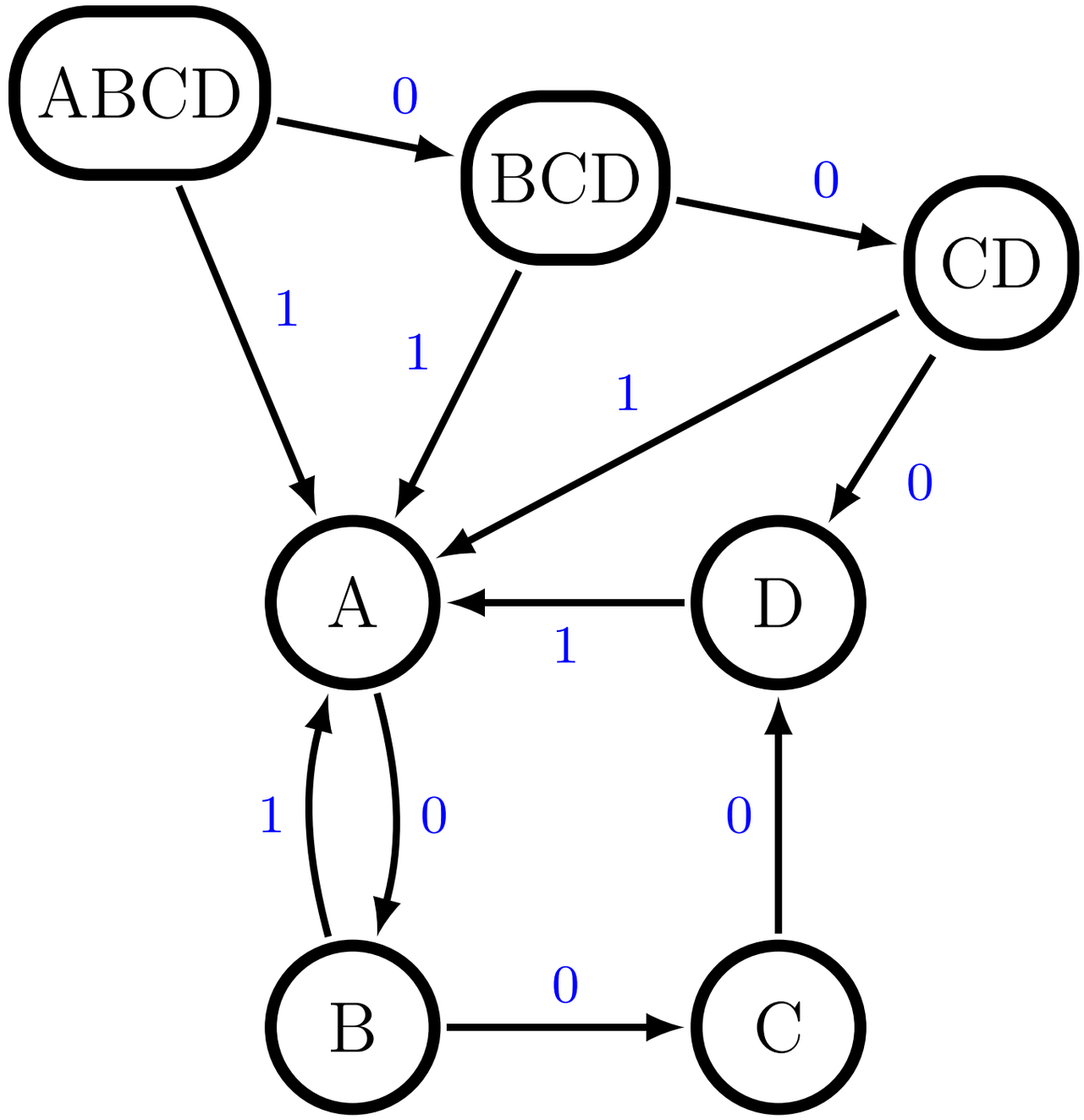

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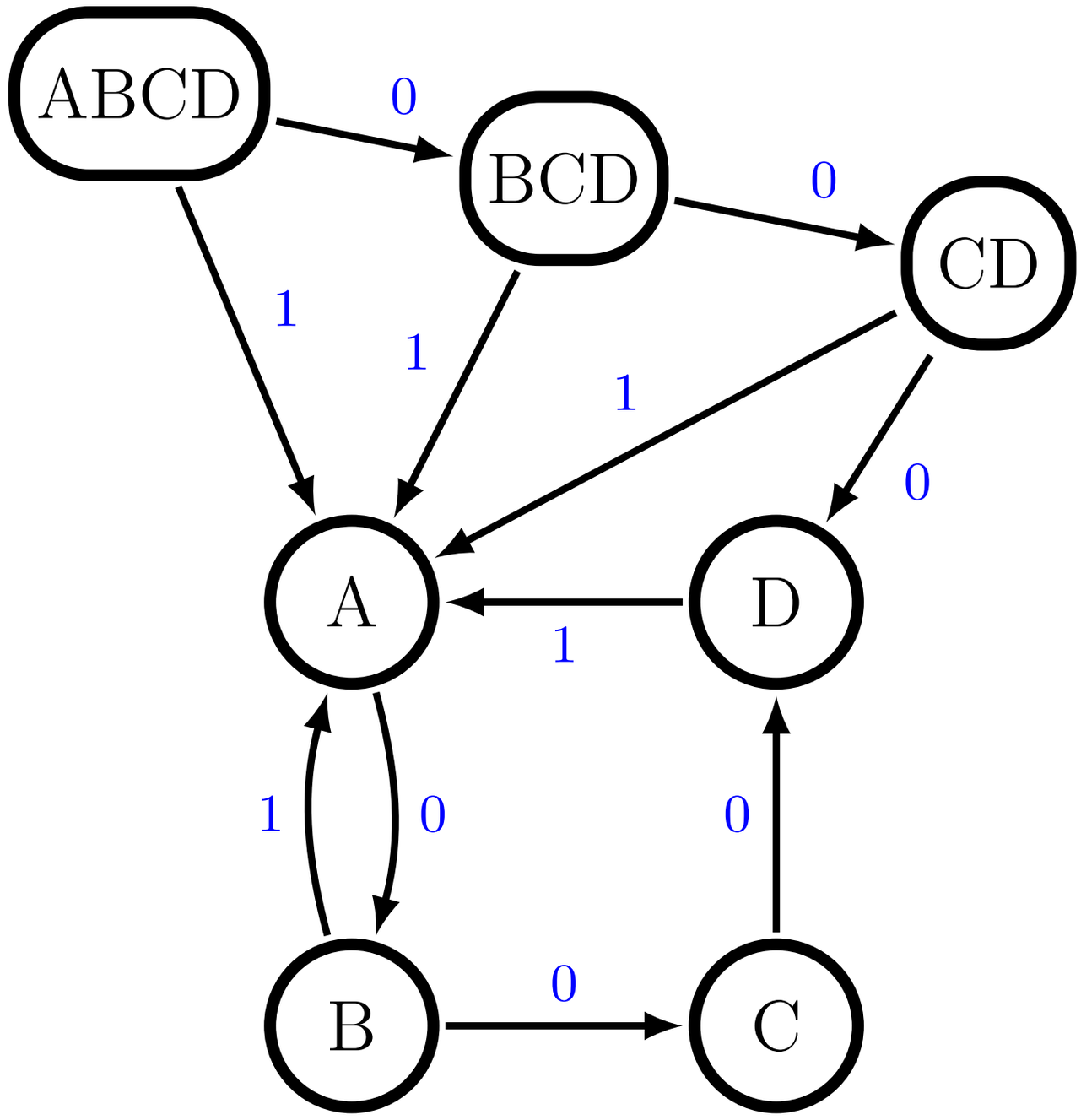

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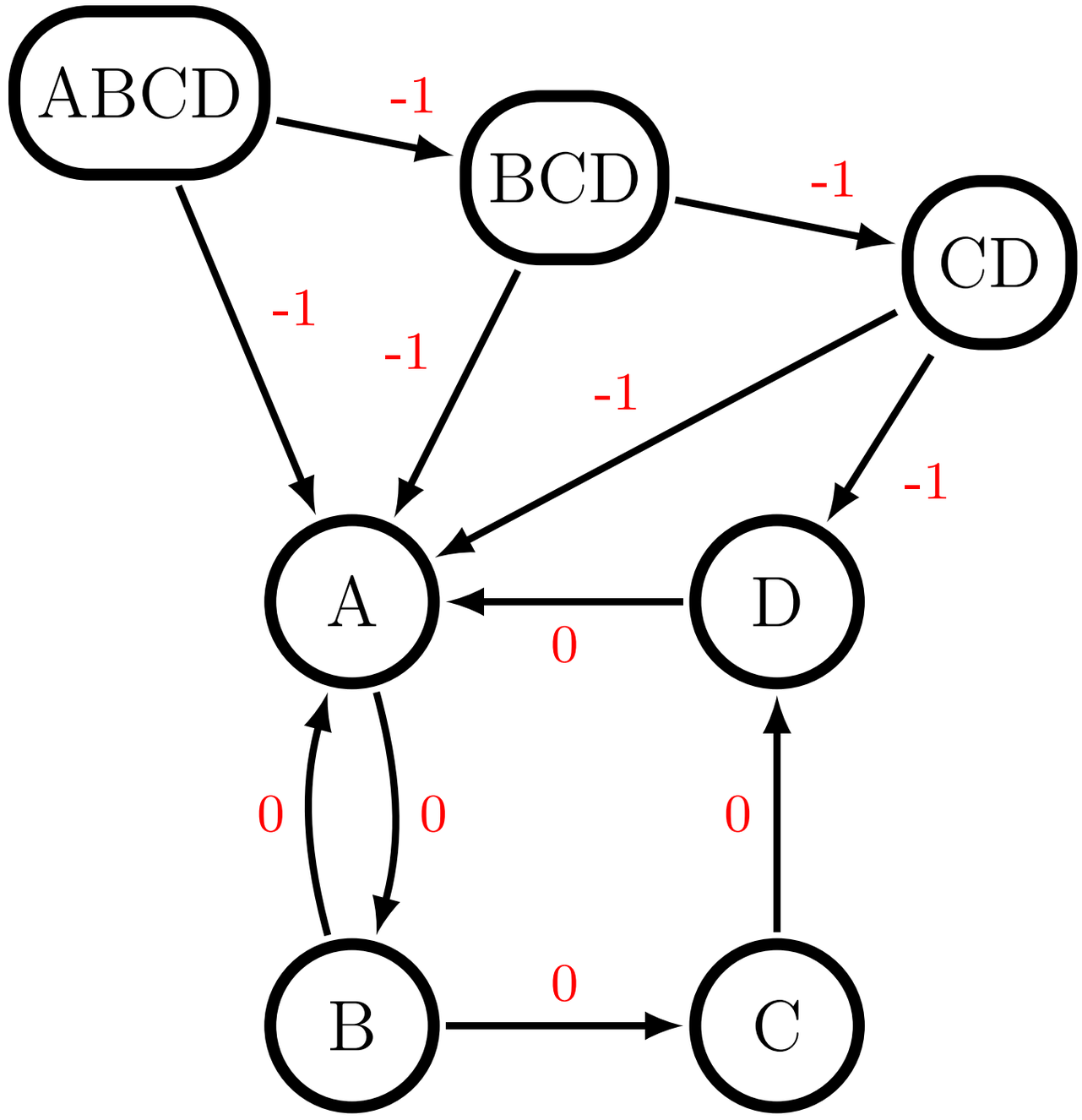

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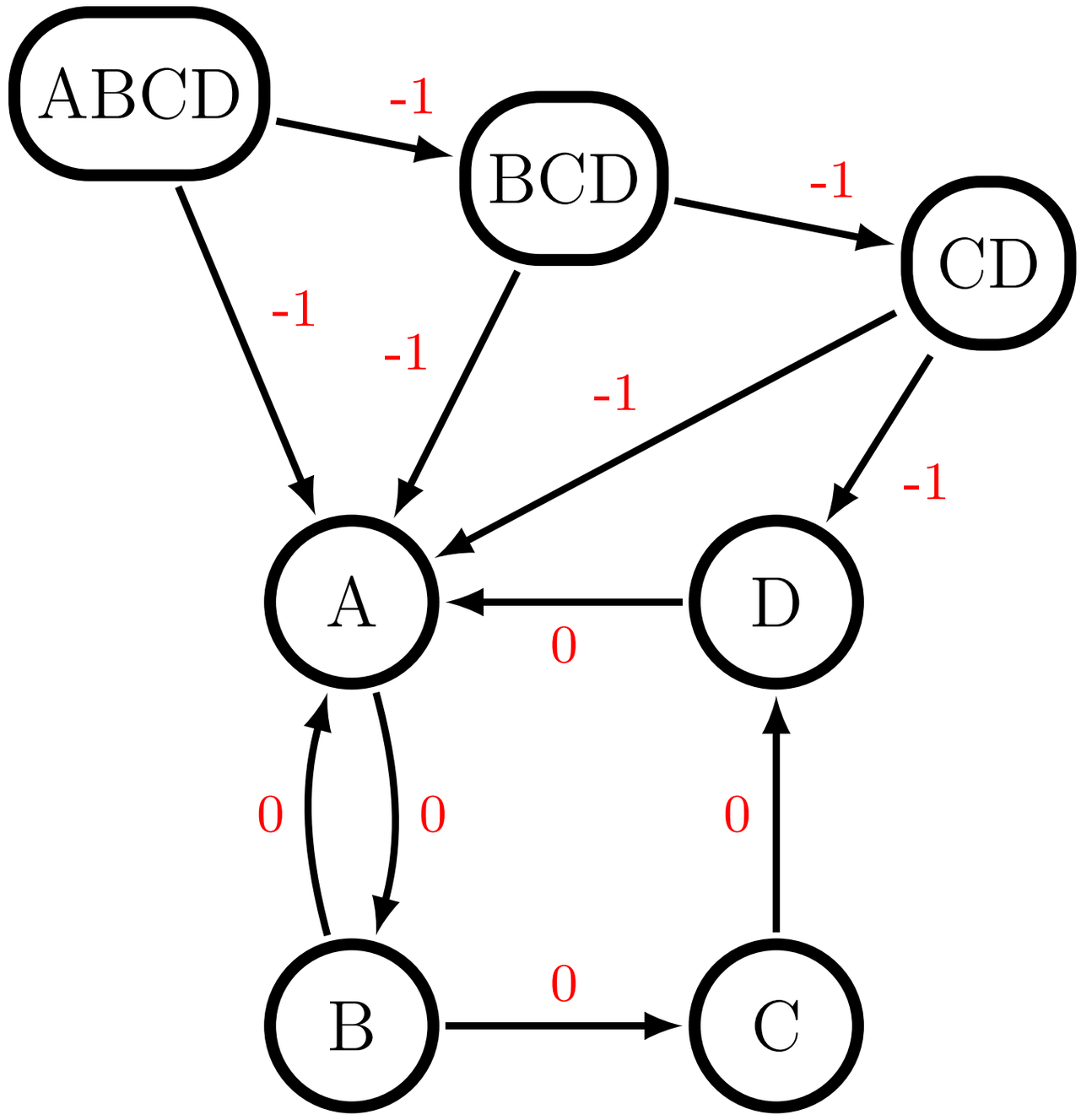
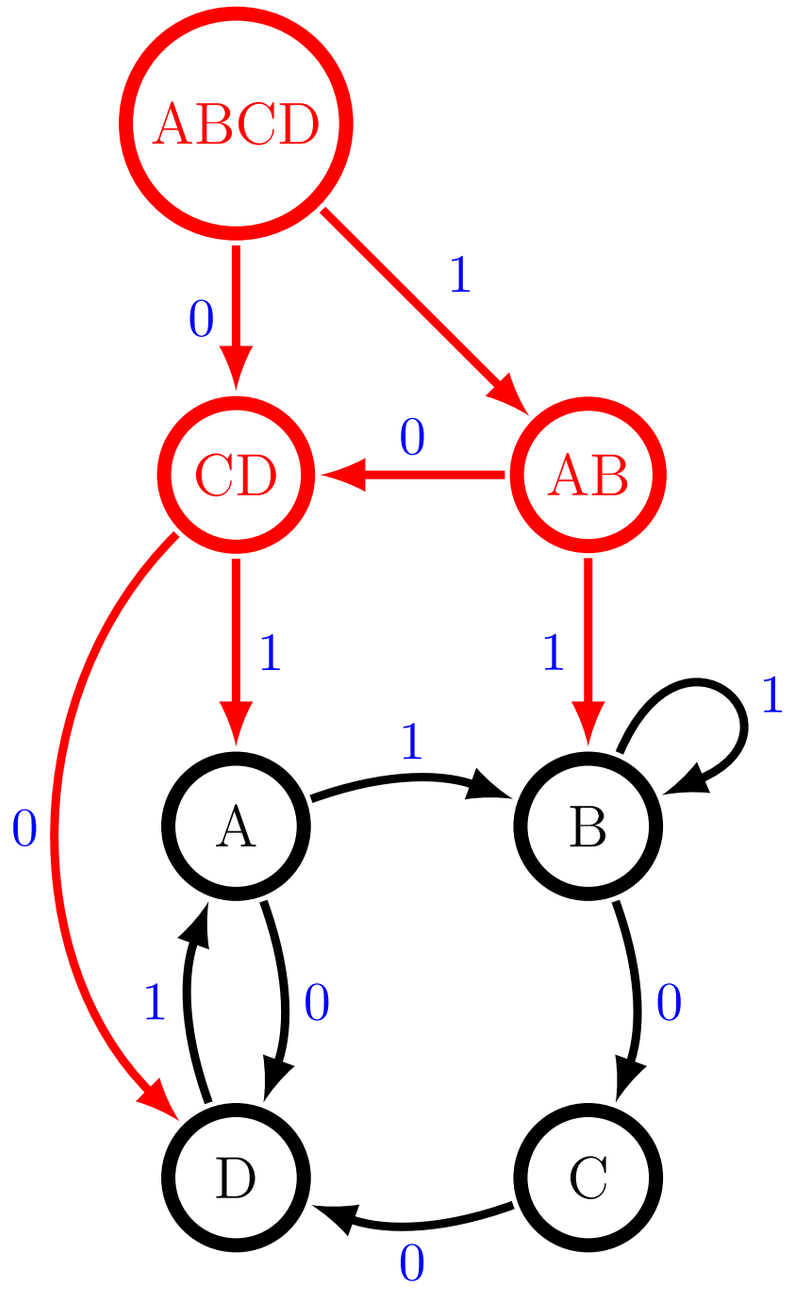
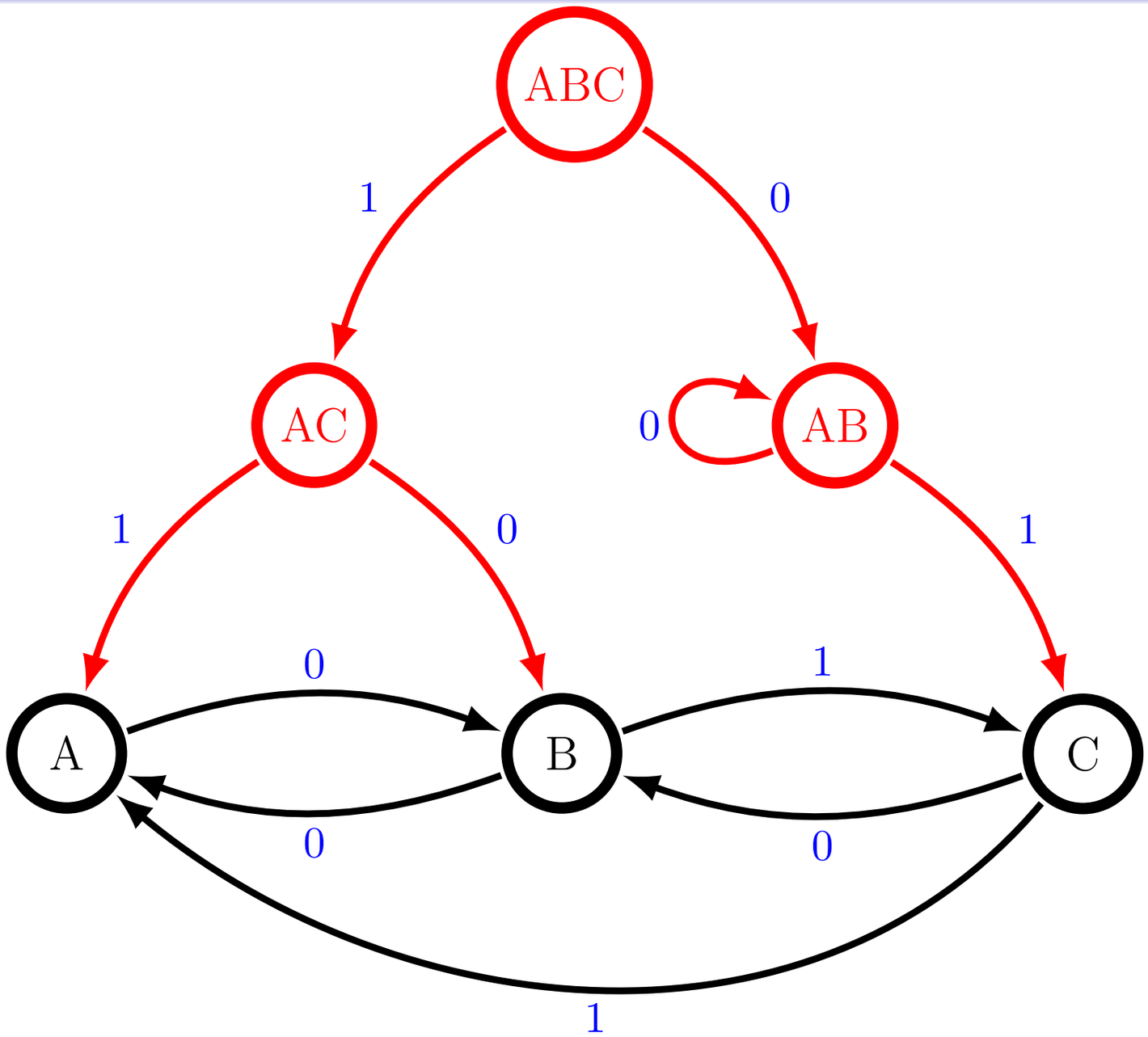
- Min-weight path from ABCD to A *is* longest path!



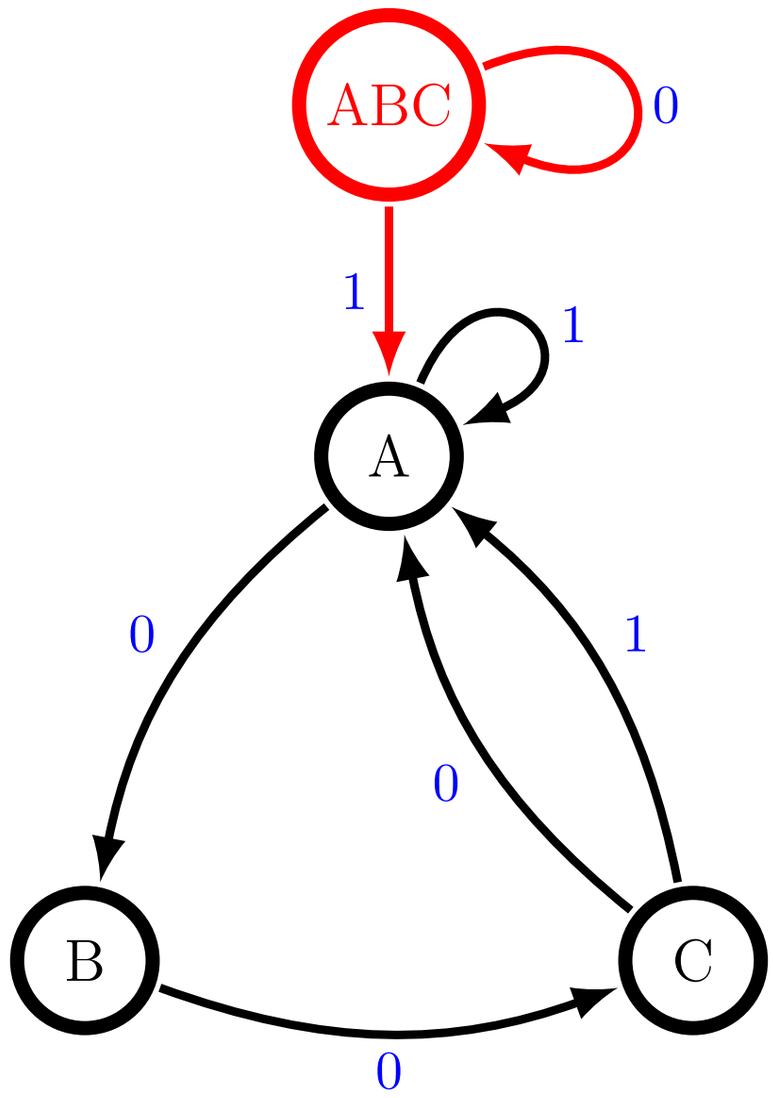
Example the First



Example the Second



Example the Third

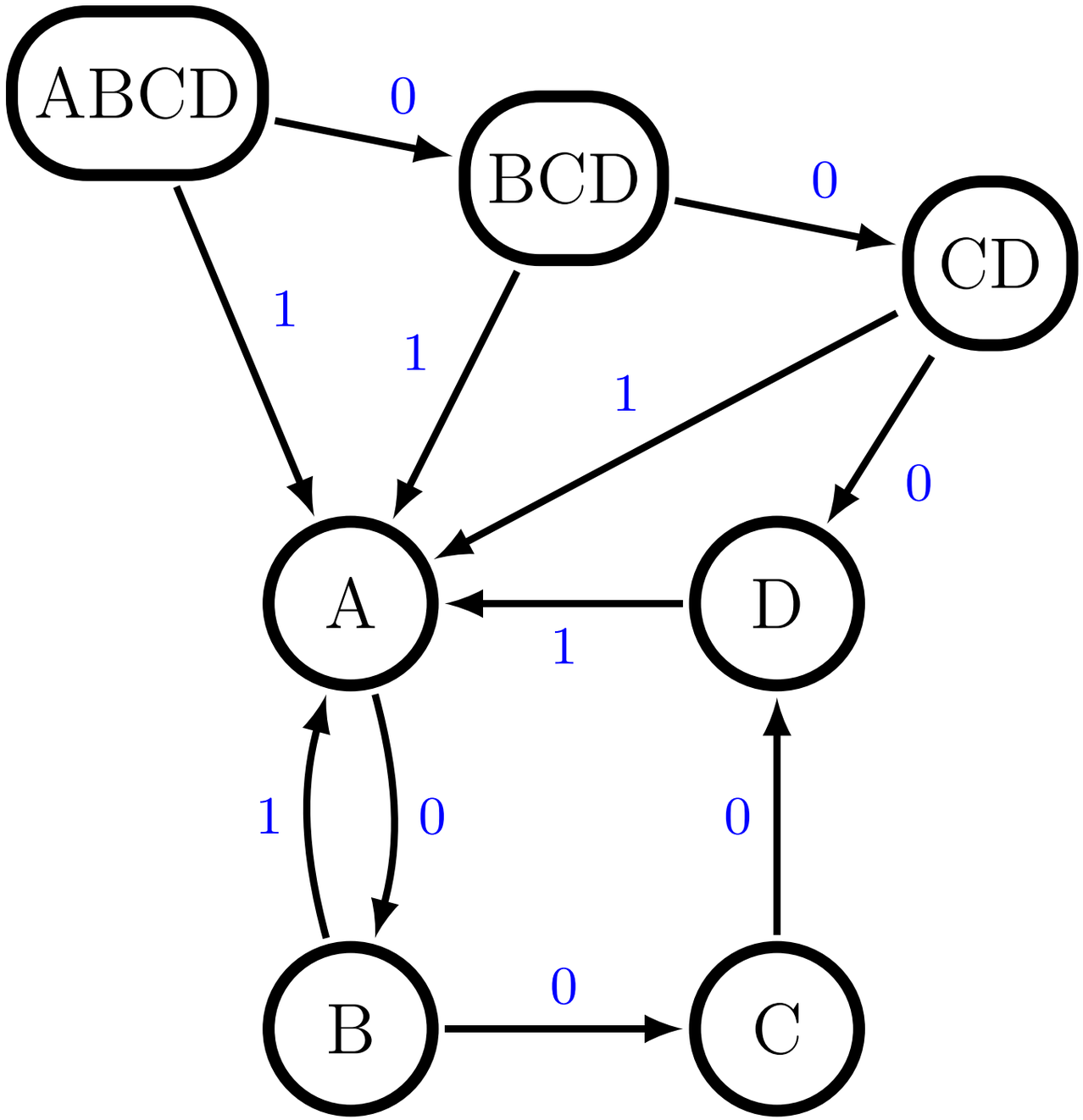


Retrodiction

Legend:

- checking
- removing
- adding
- good
- to check

Each edge: what states could have actually made that transition?



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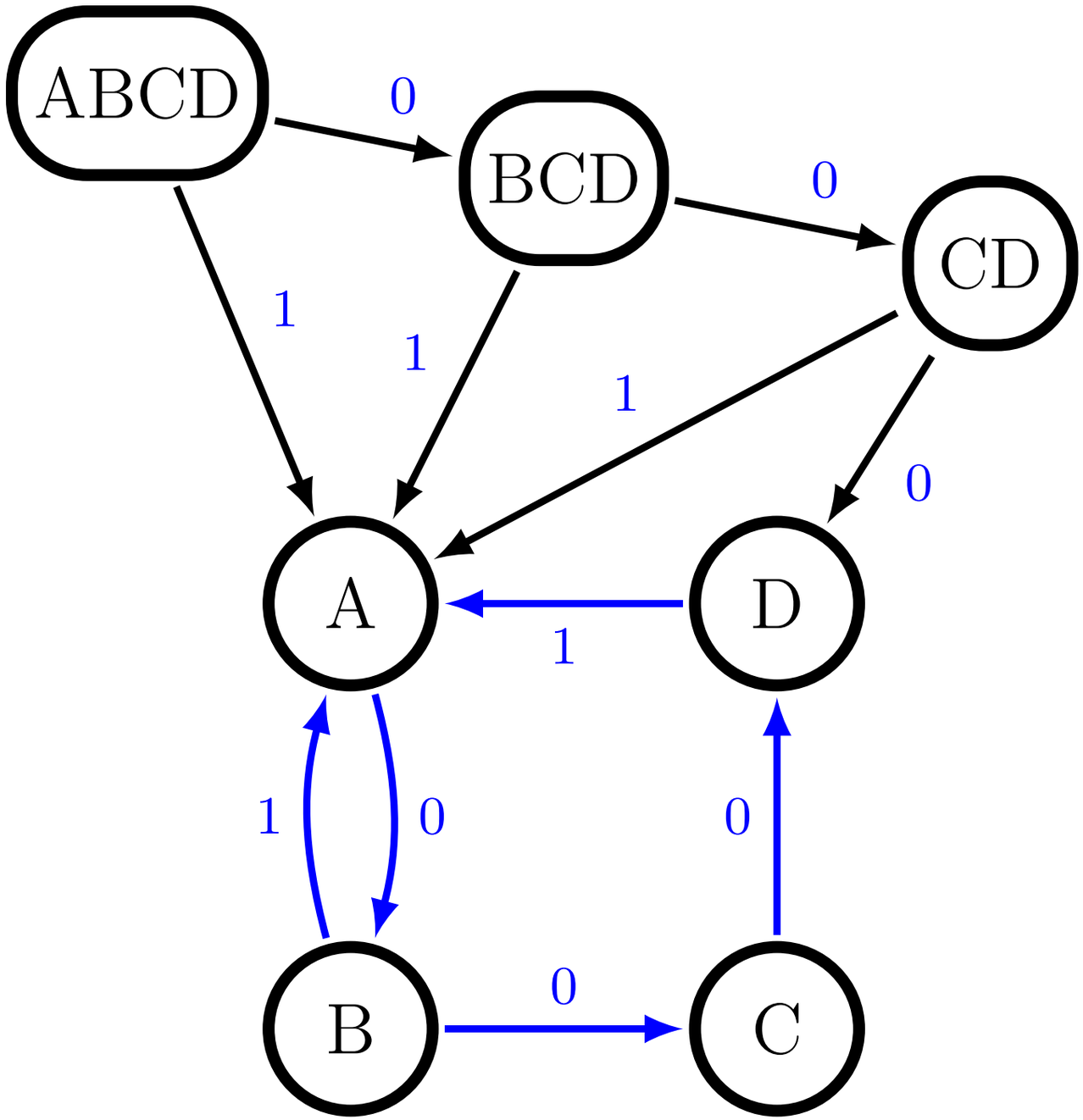

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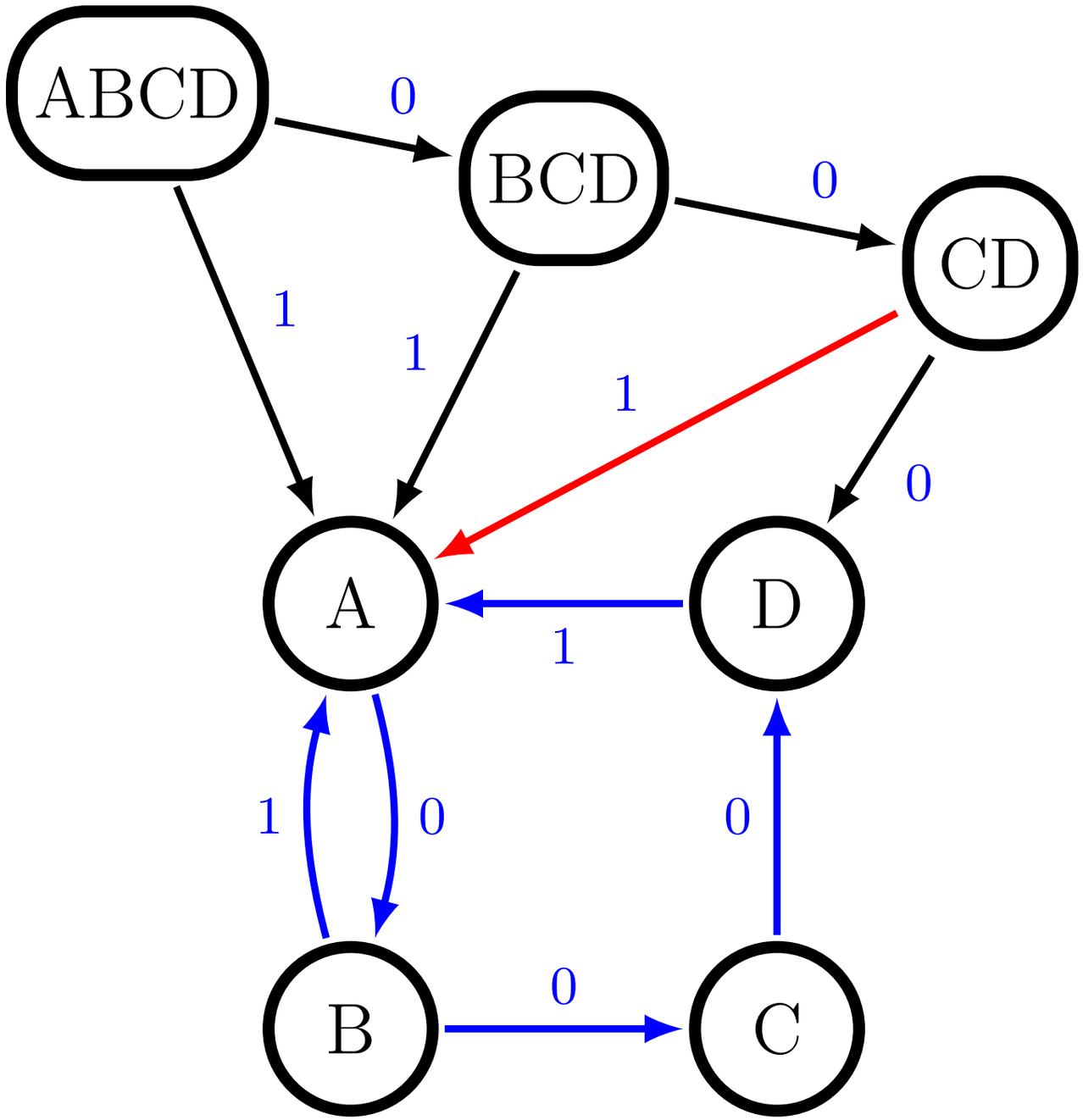

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Retrodiction

Legend:

- checking
- removing
- adding
- good
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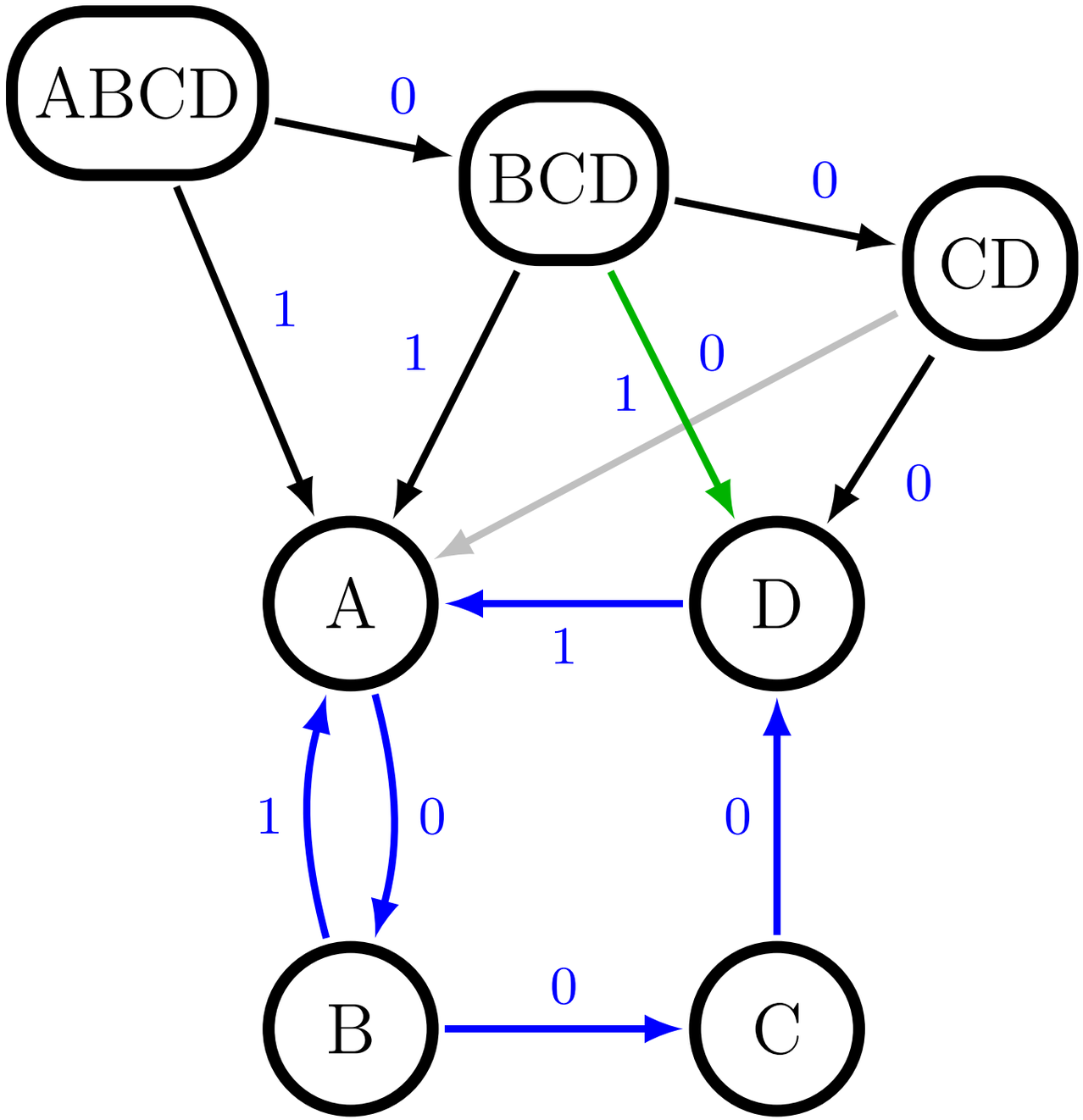

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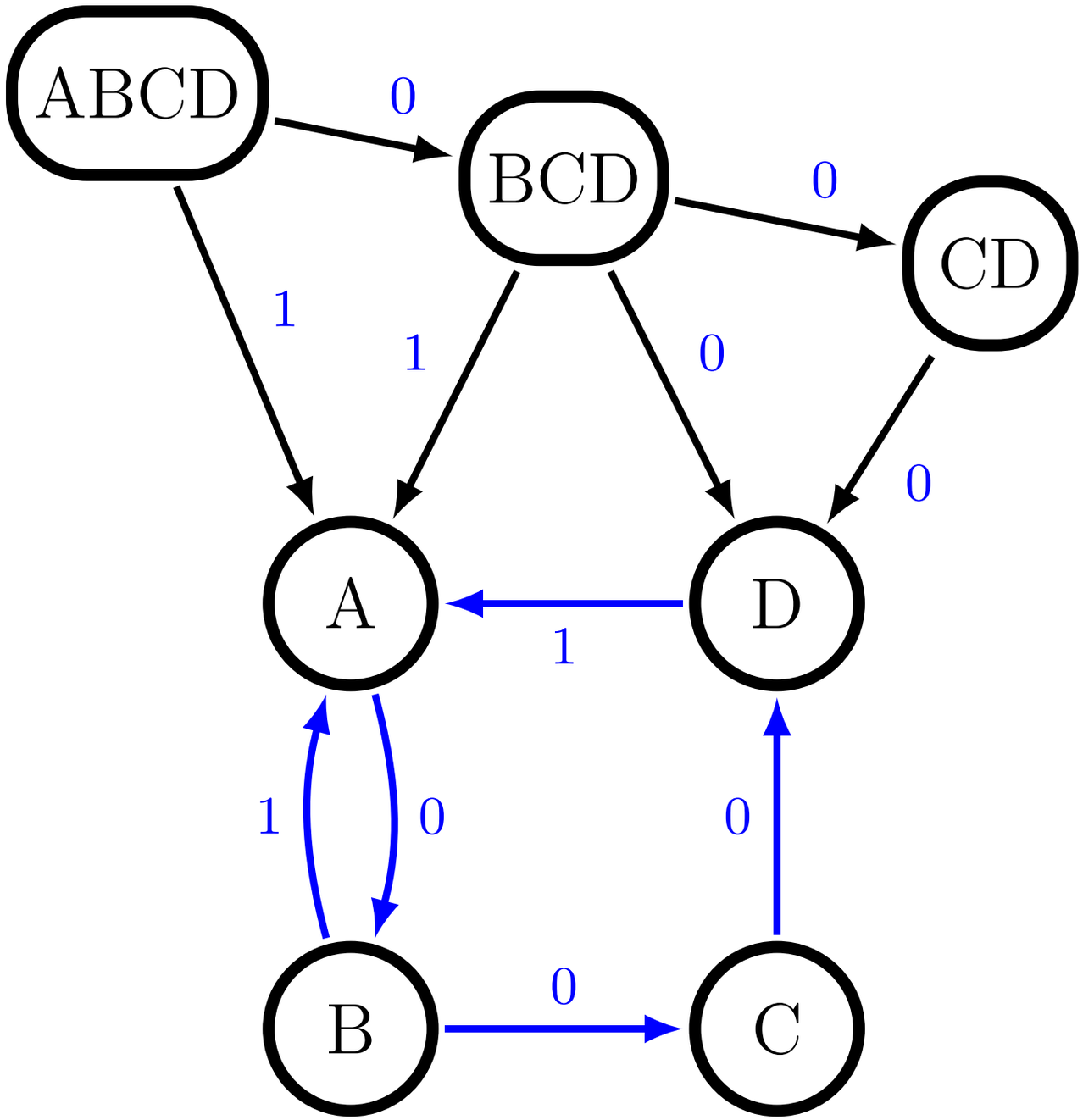

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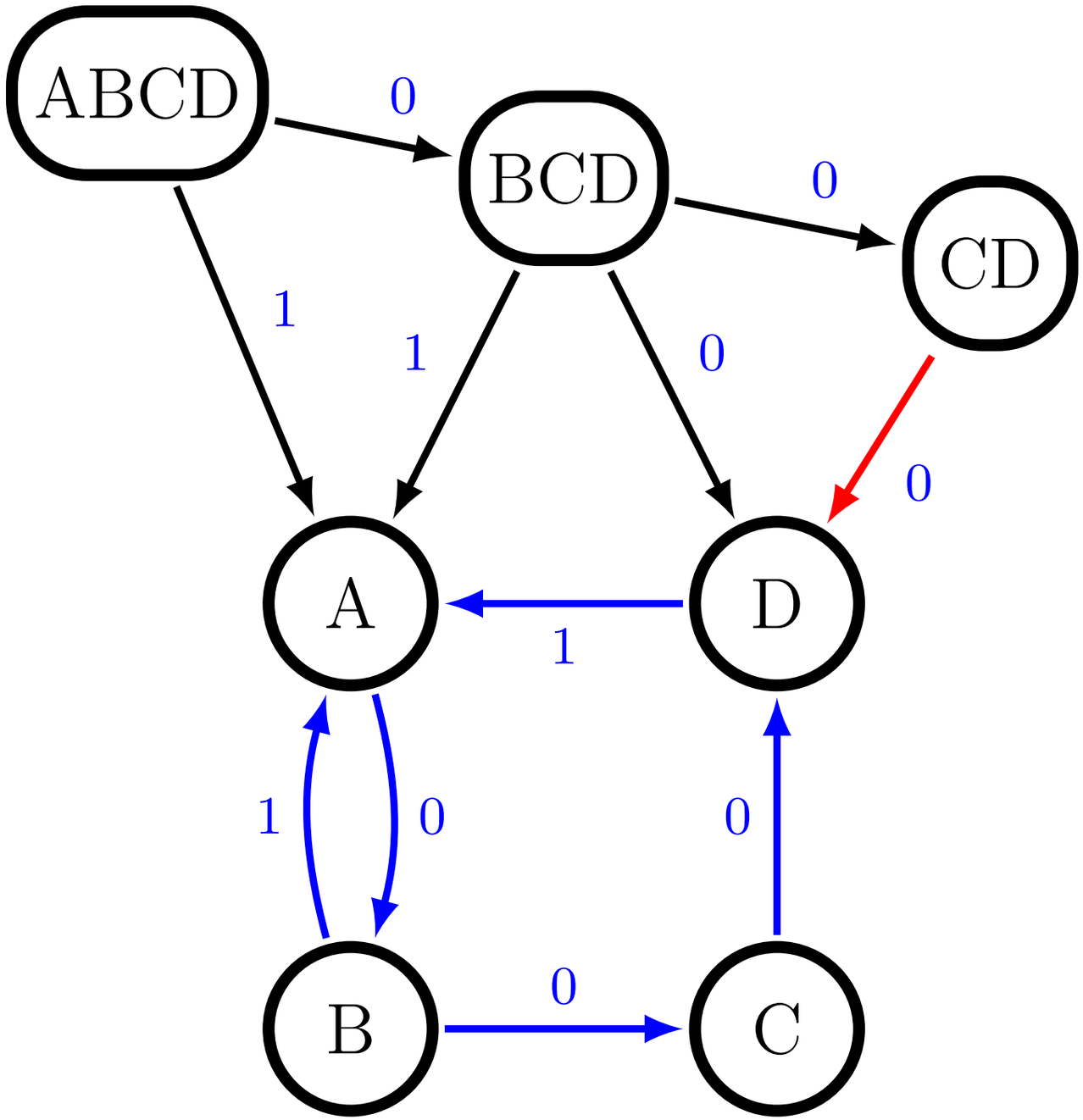

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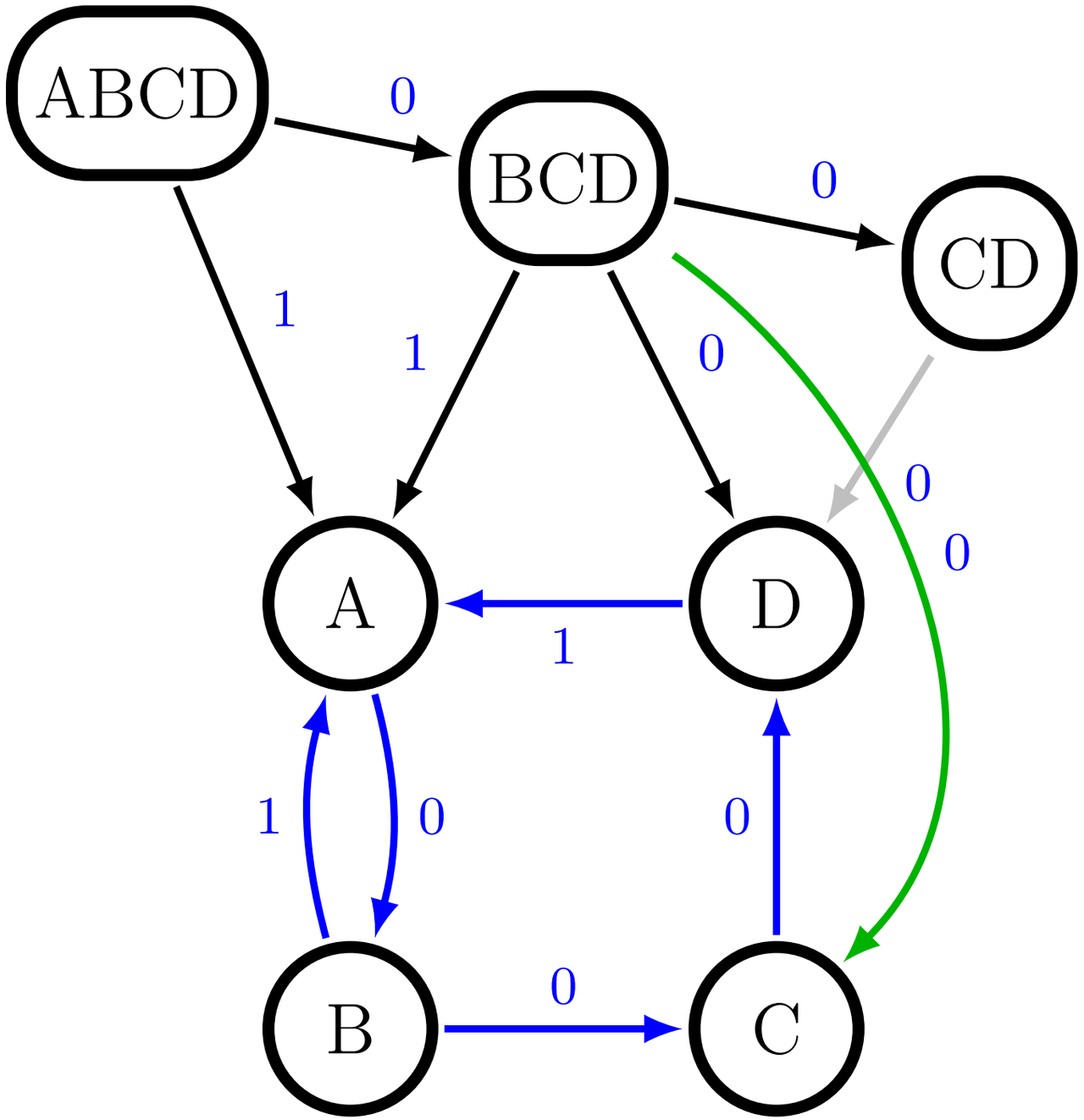

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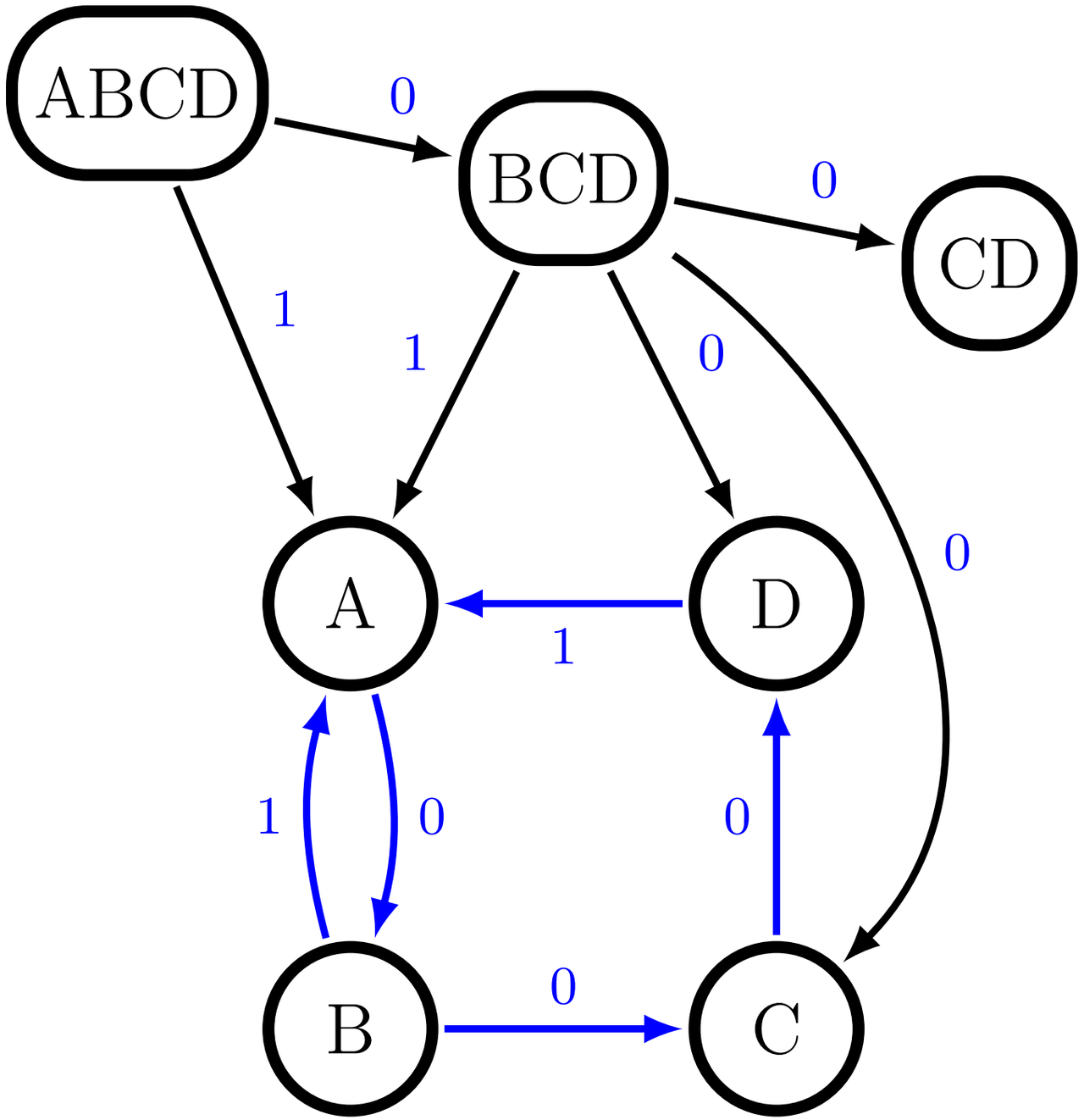

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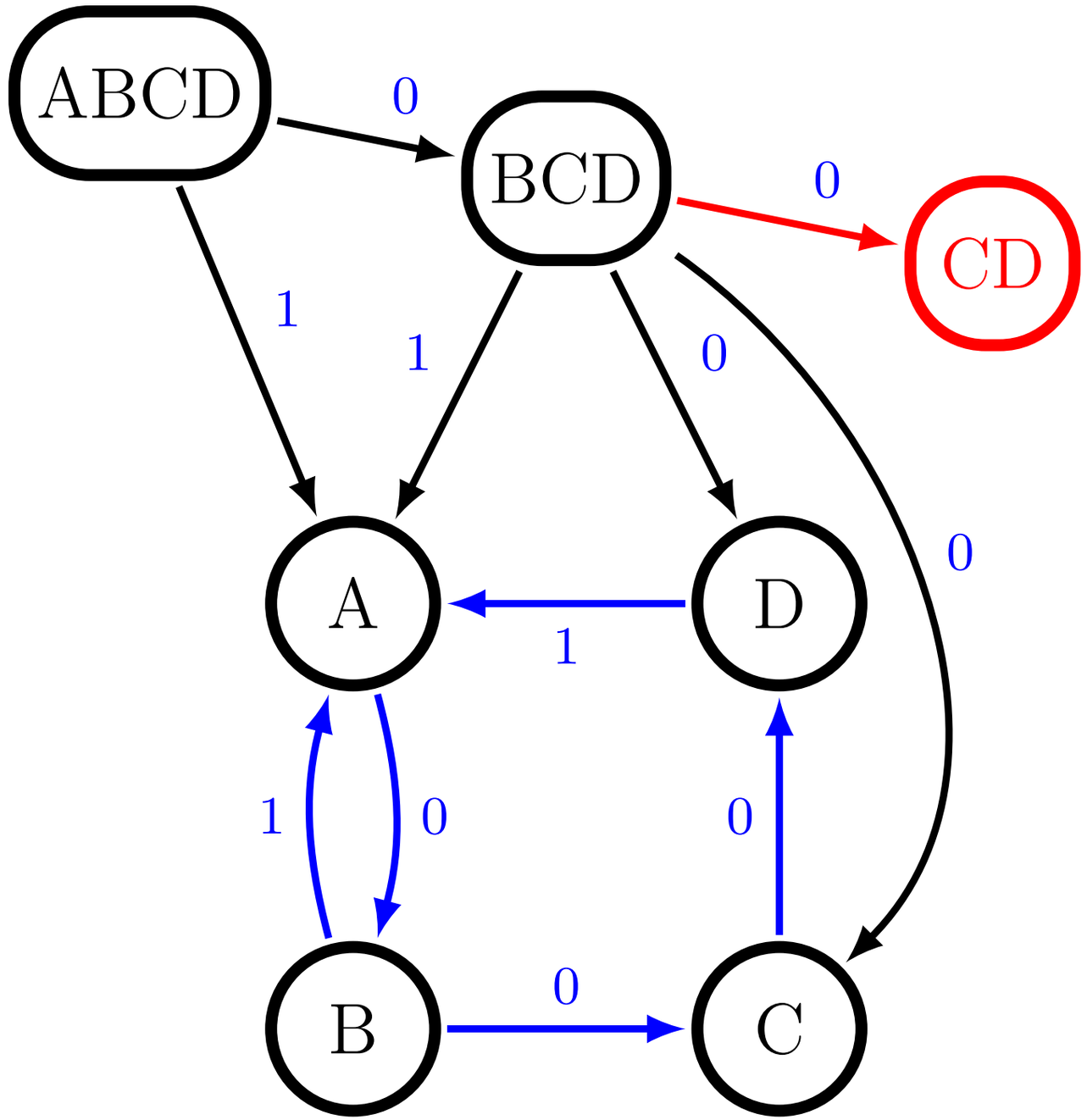

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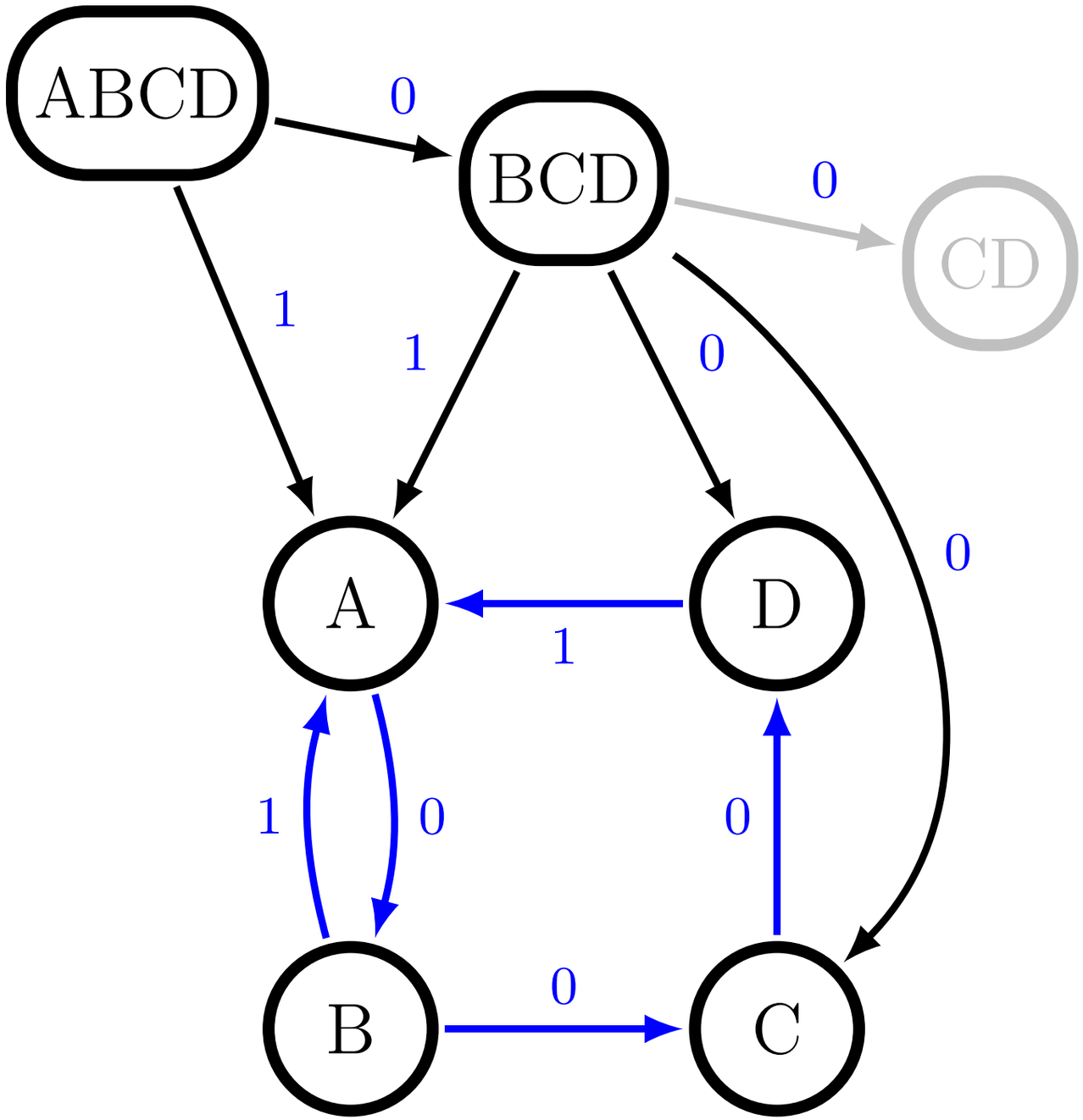

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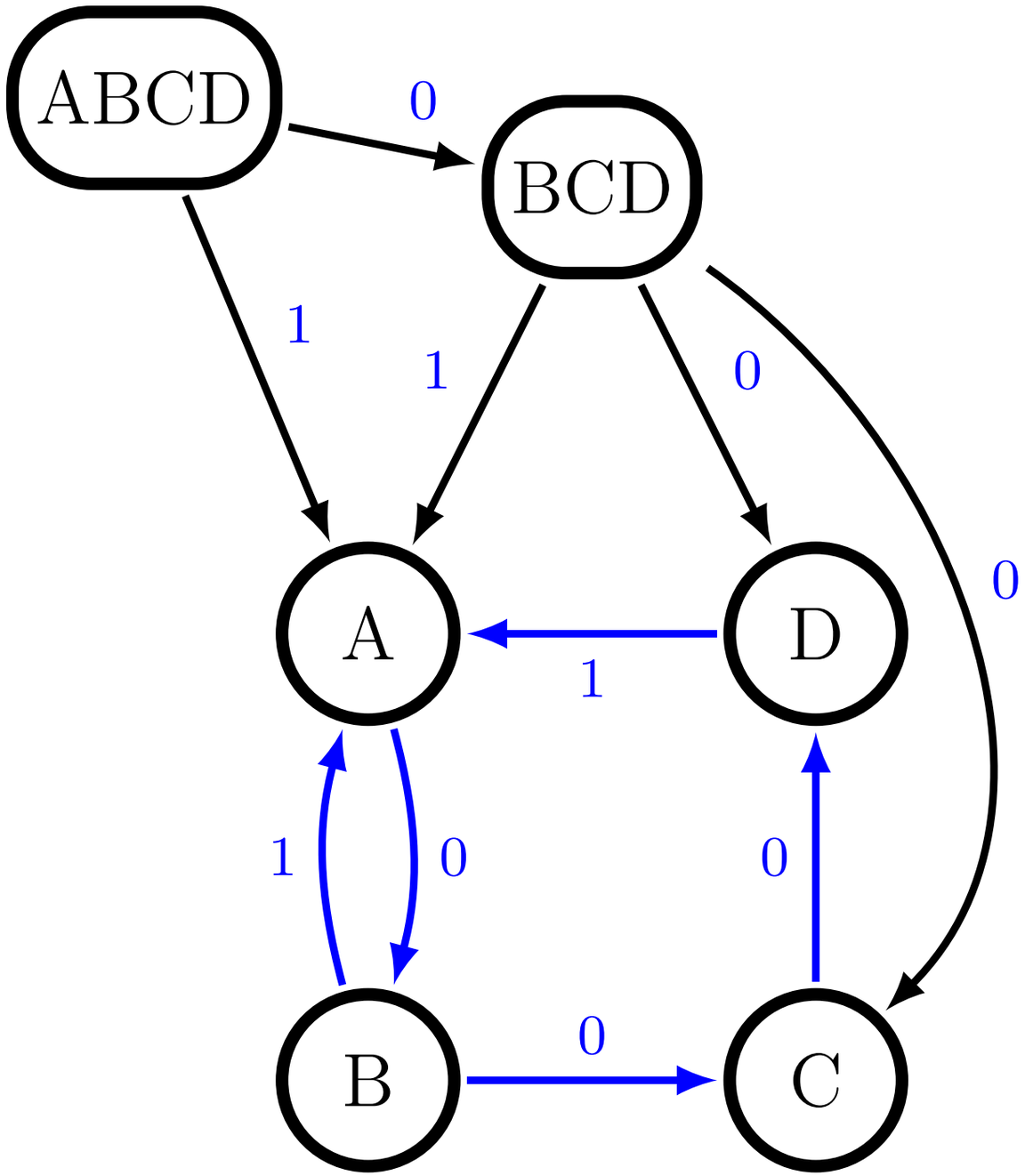

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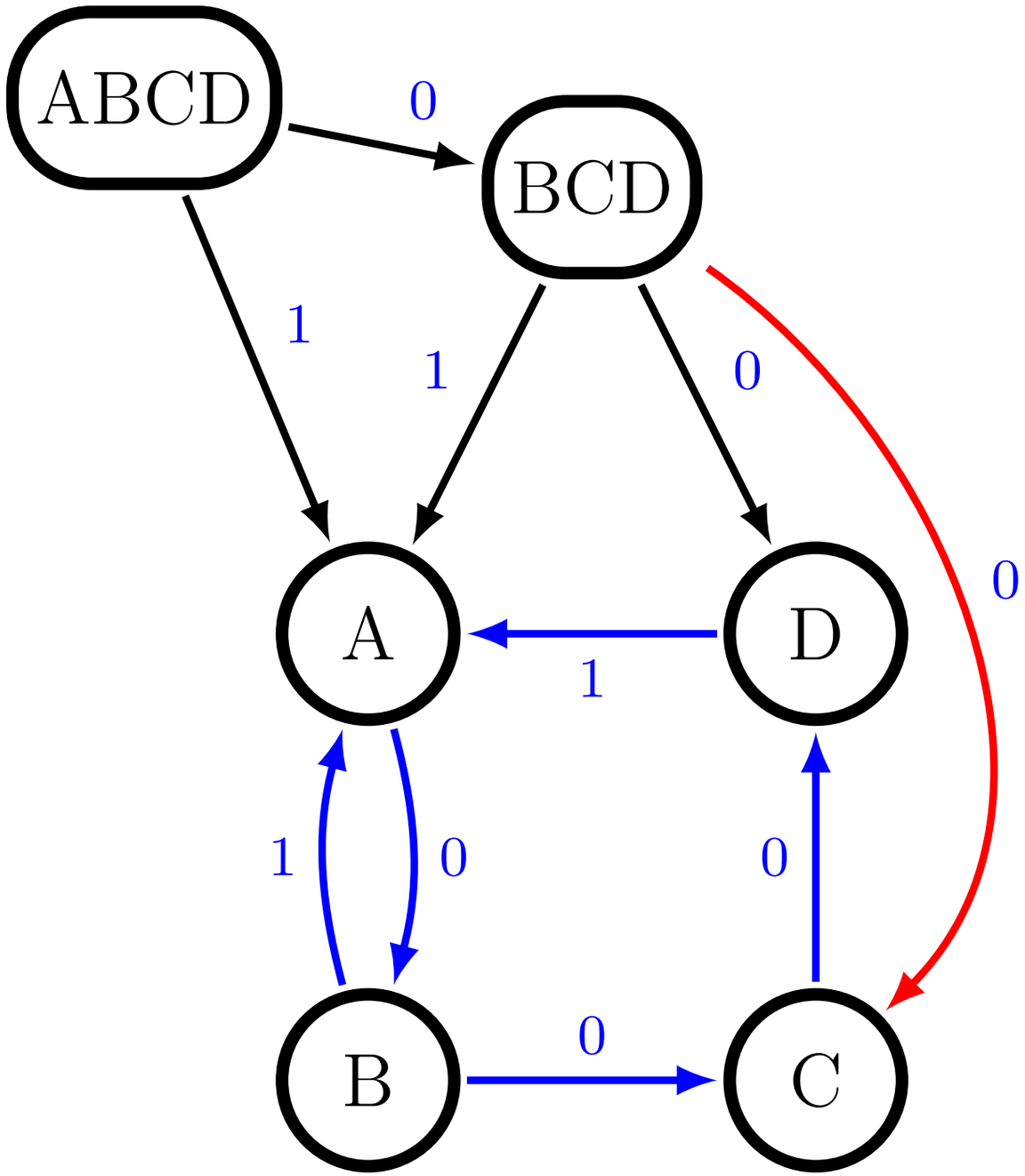

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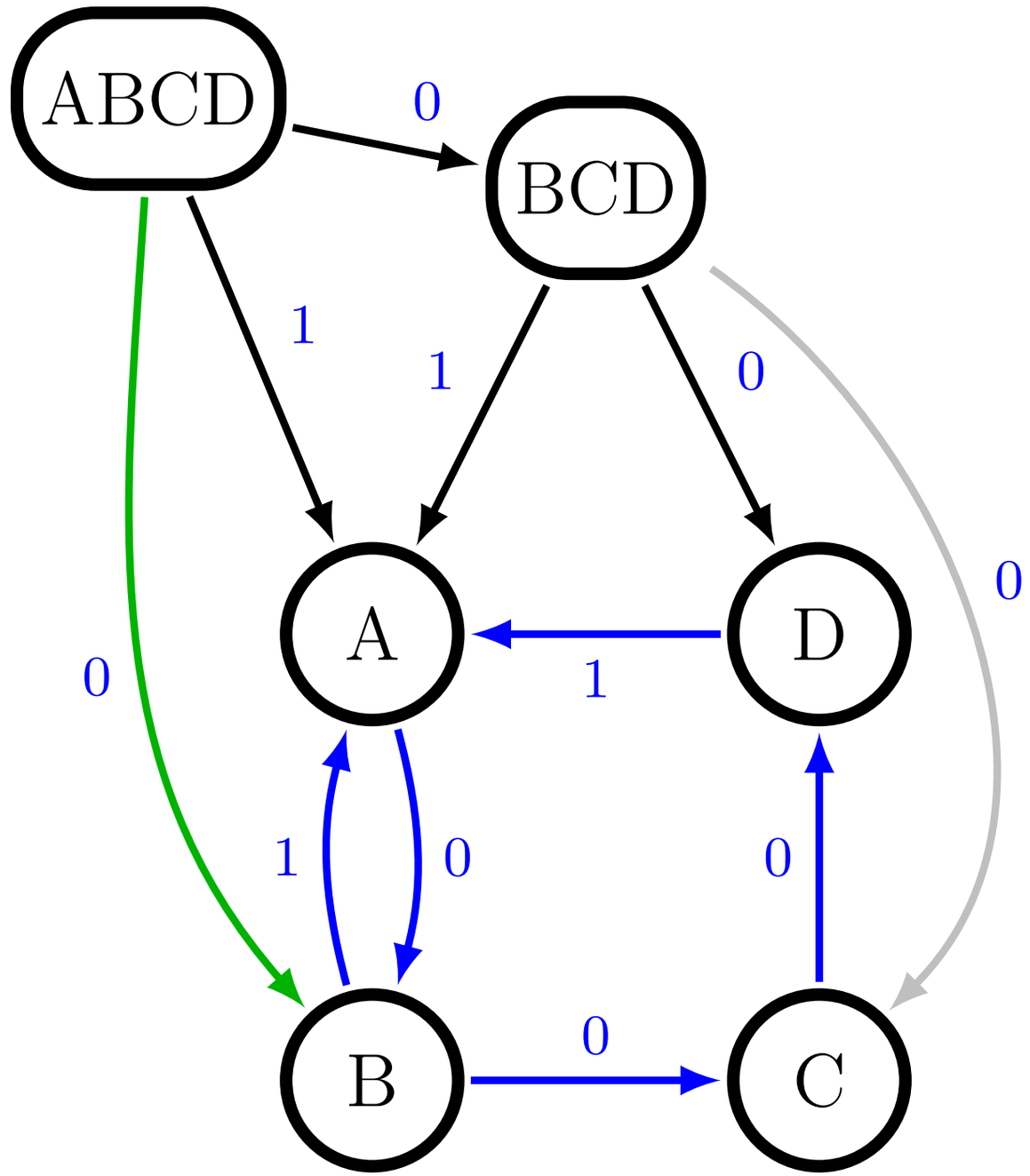

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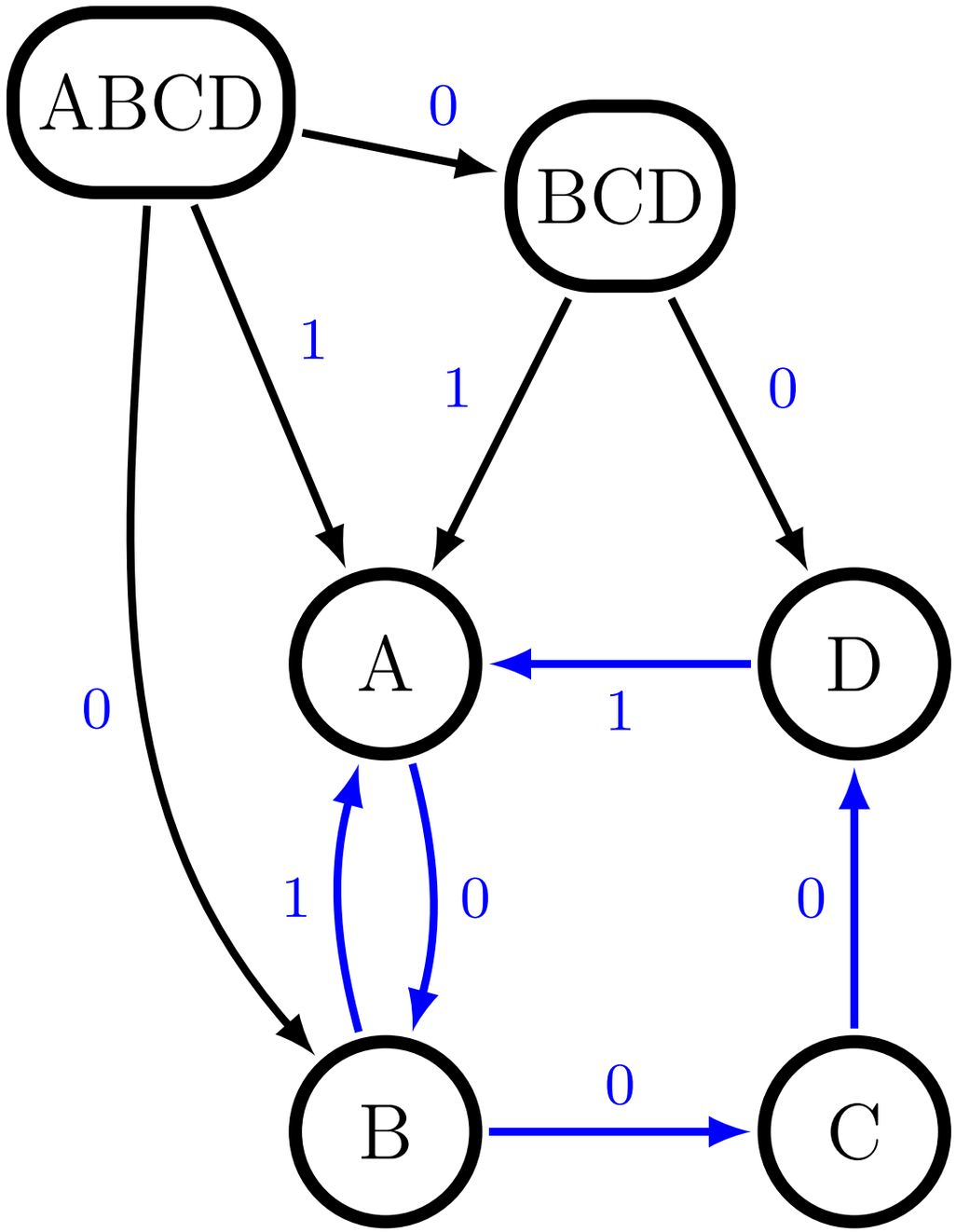

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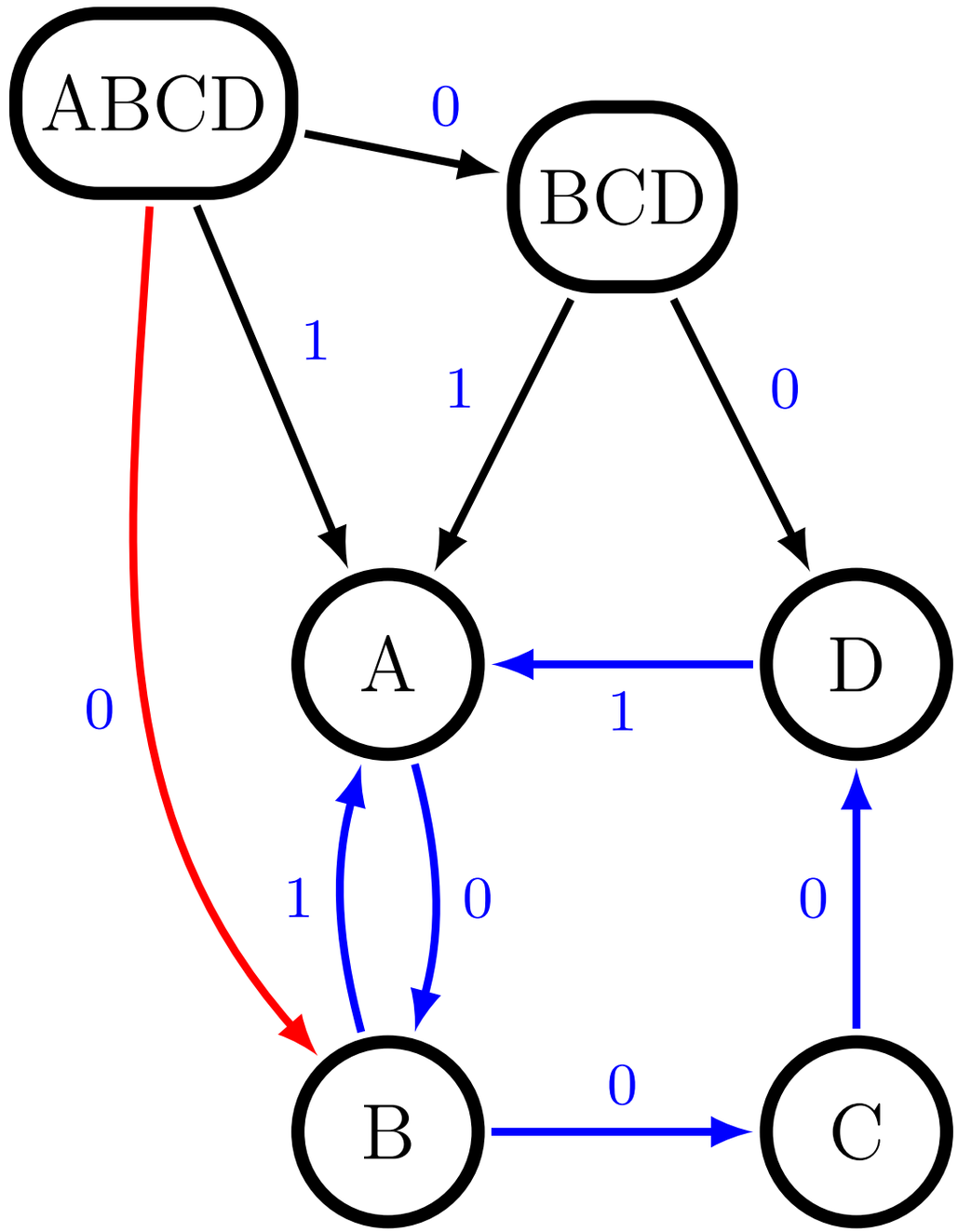

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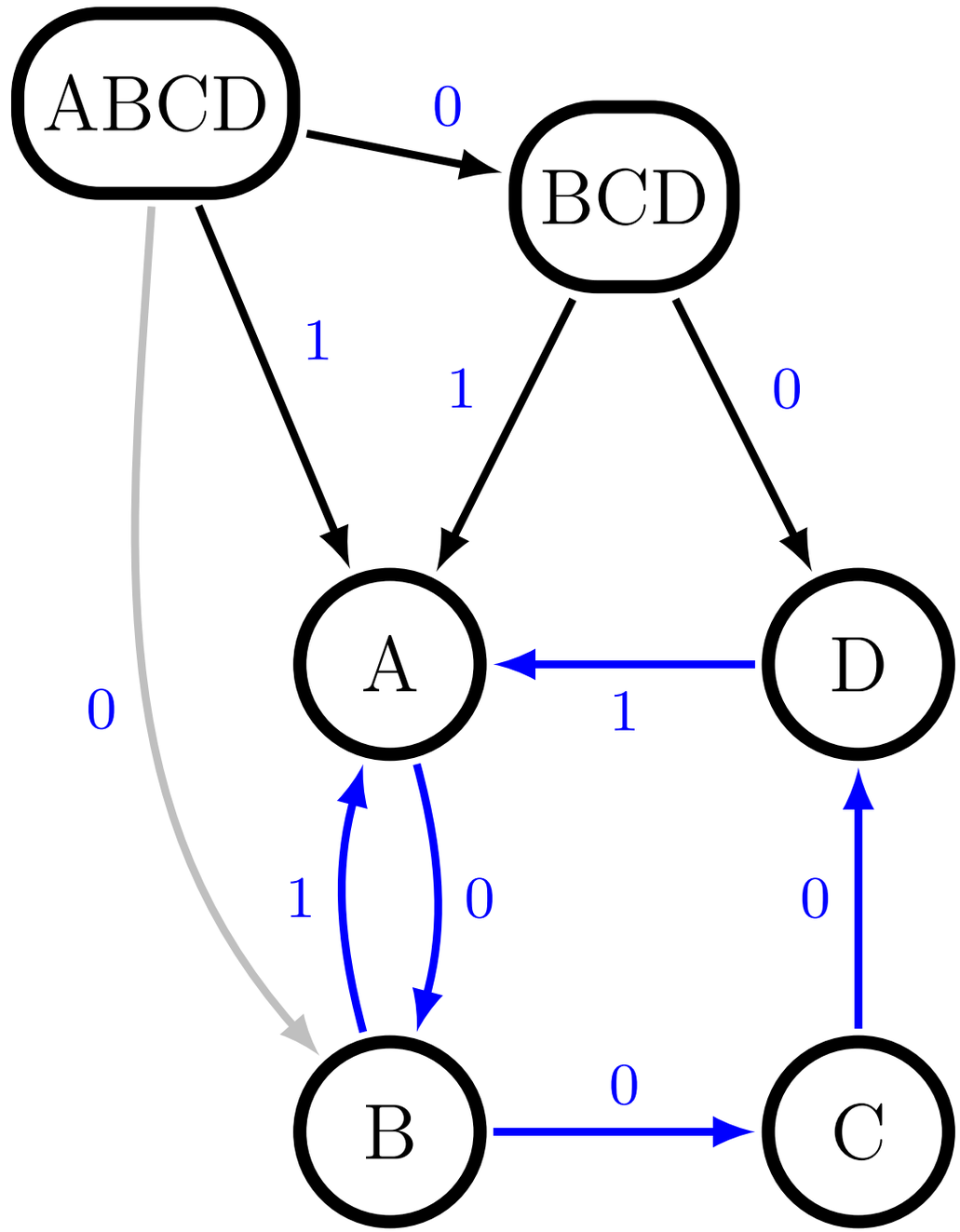

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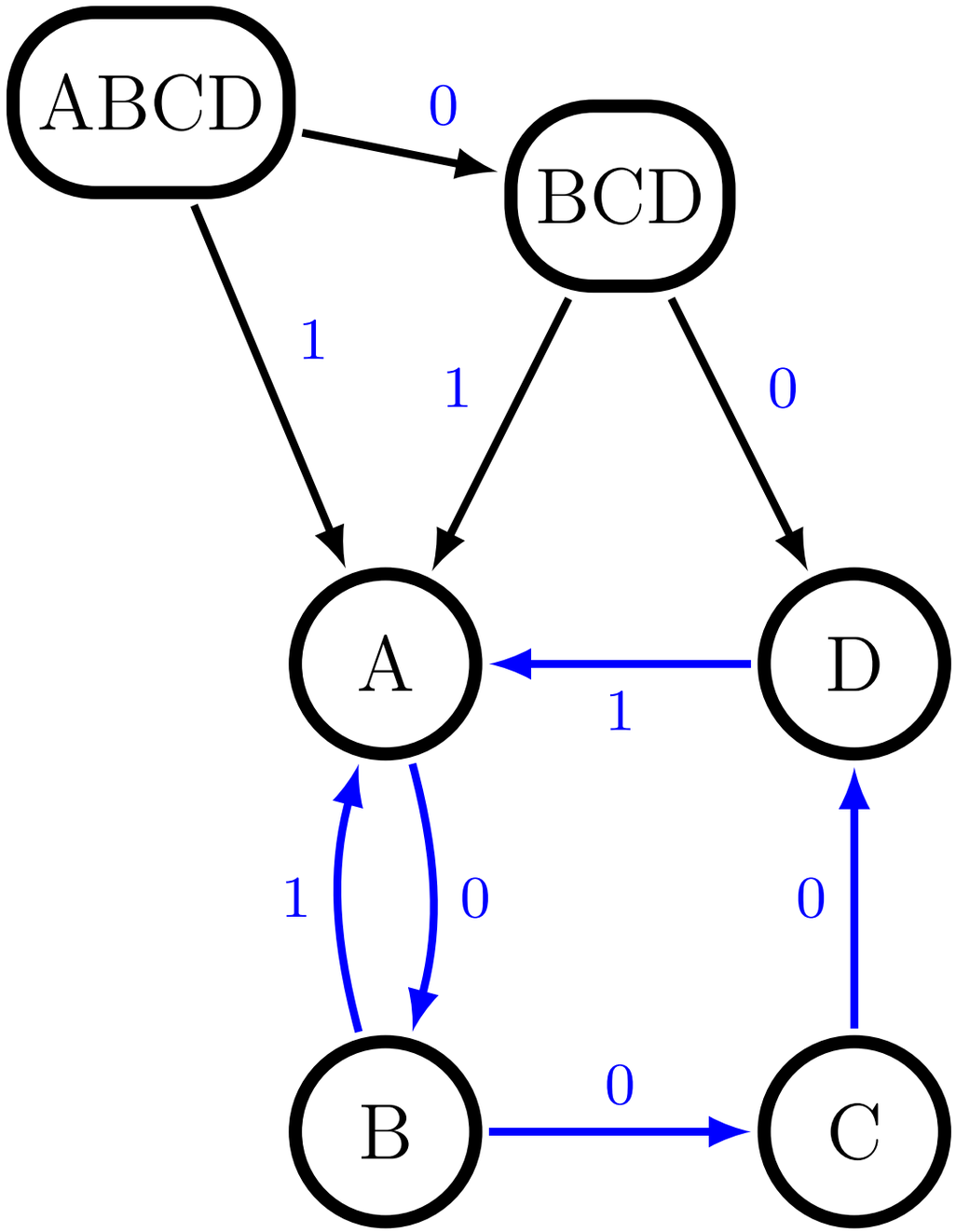

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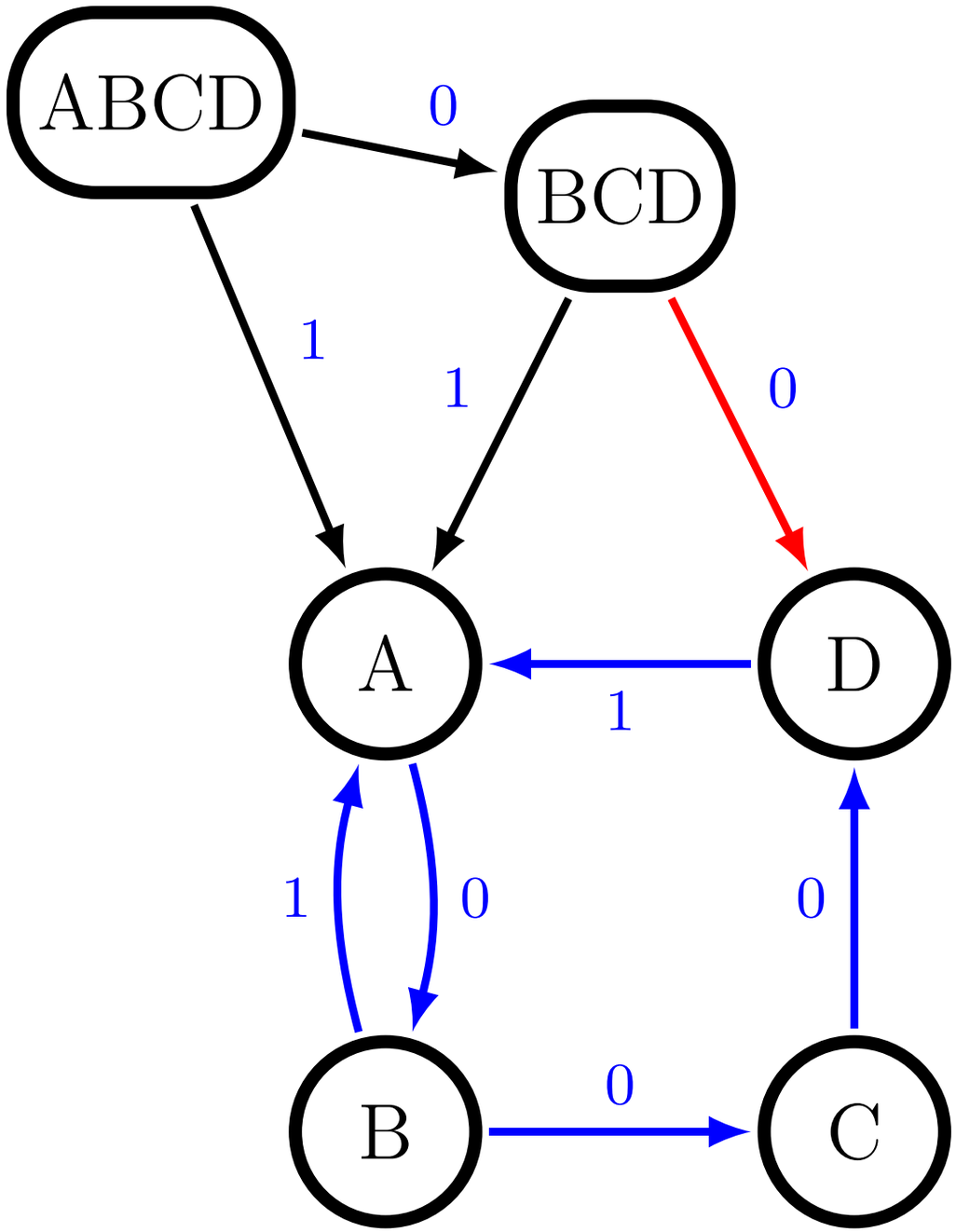

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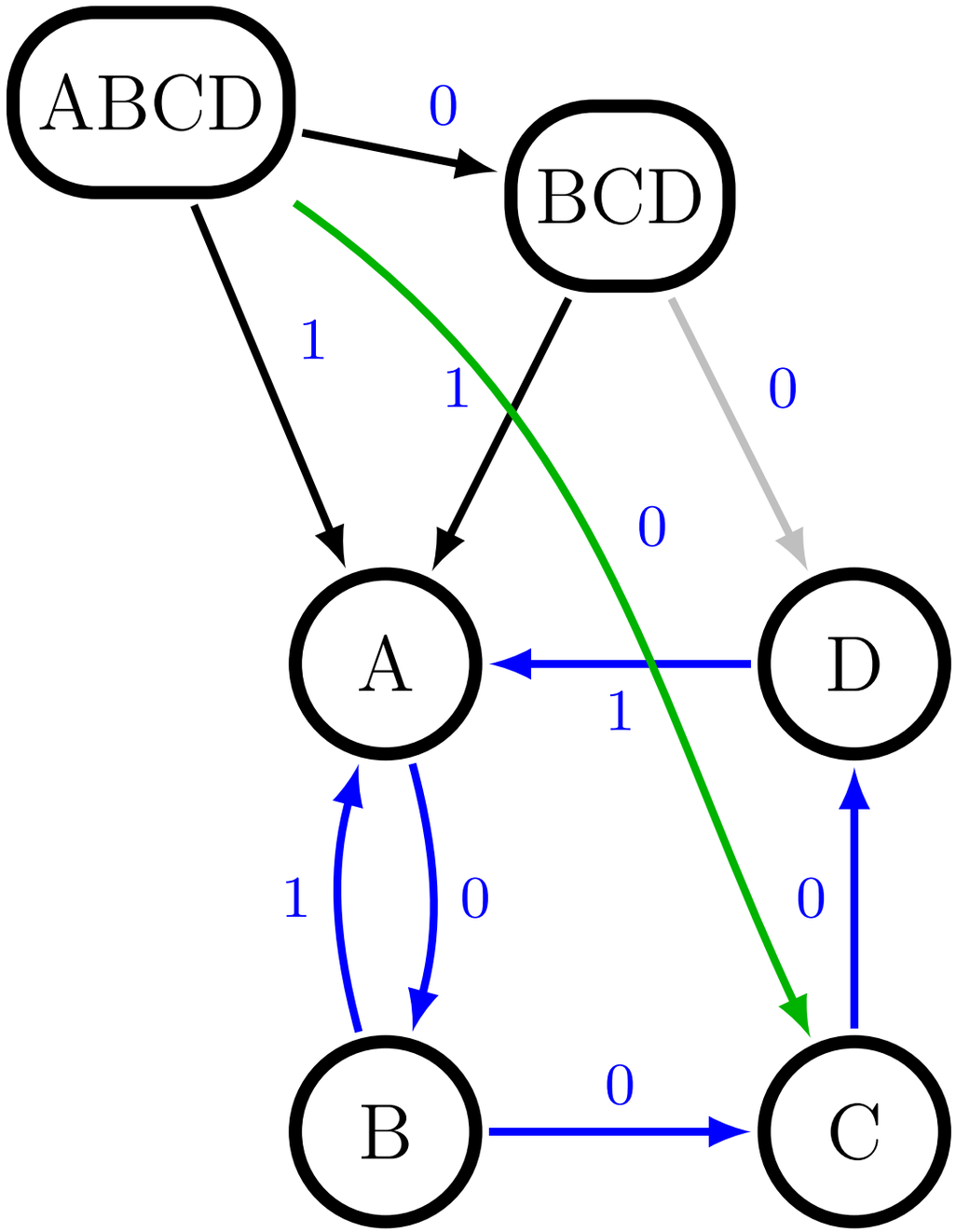

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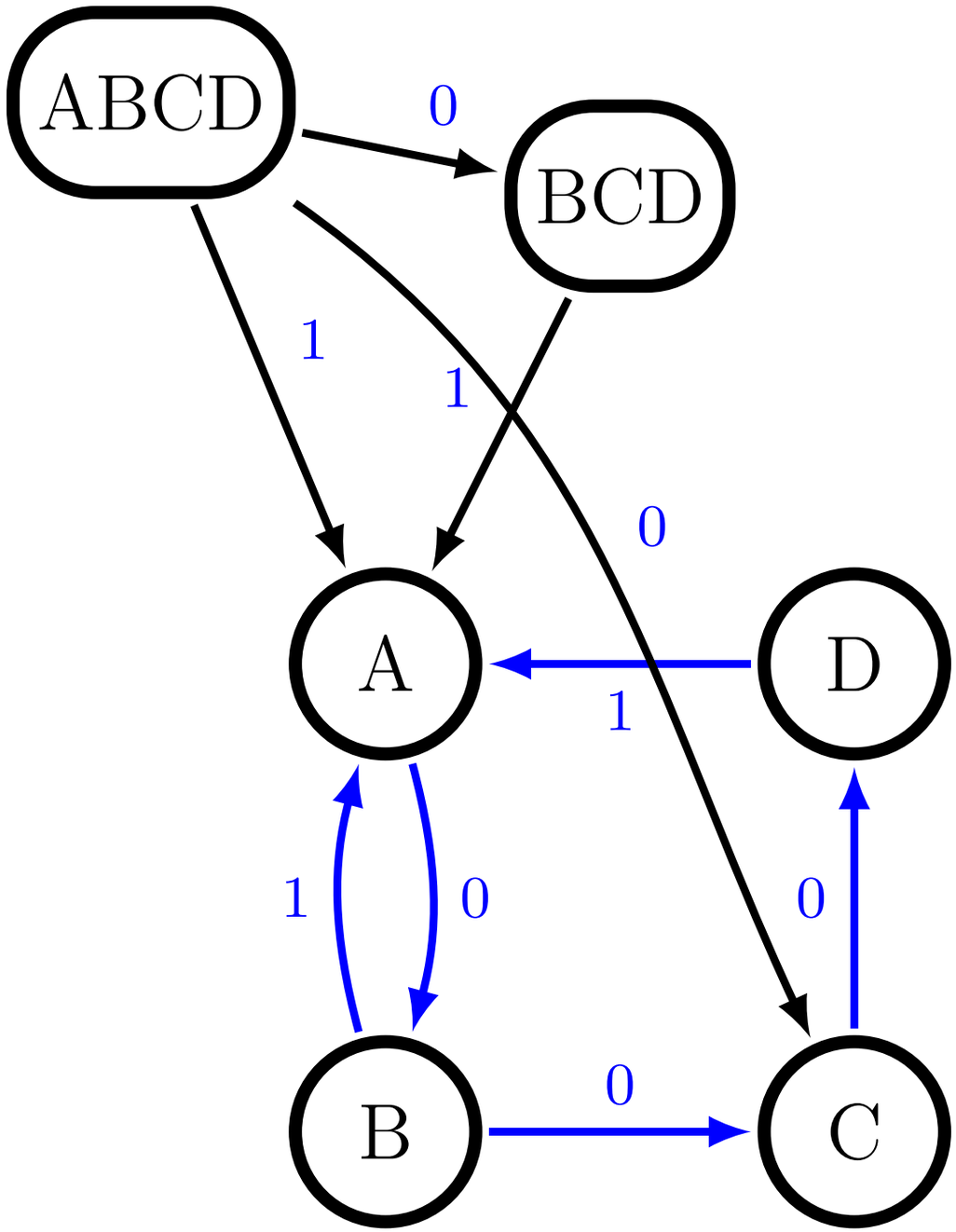

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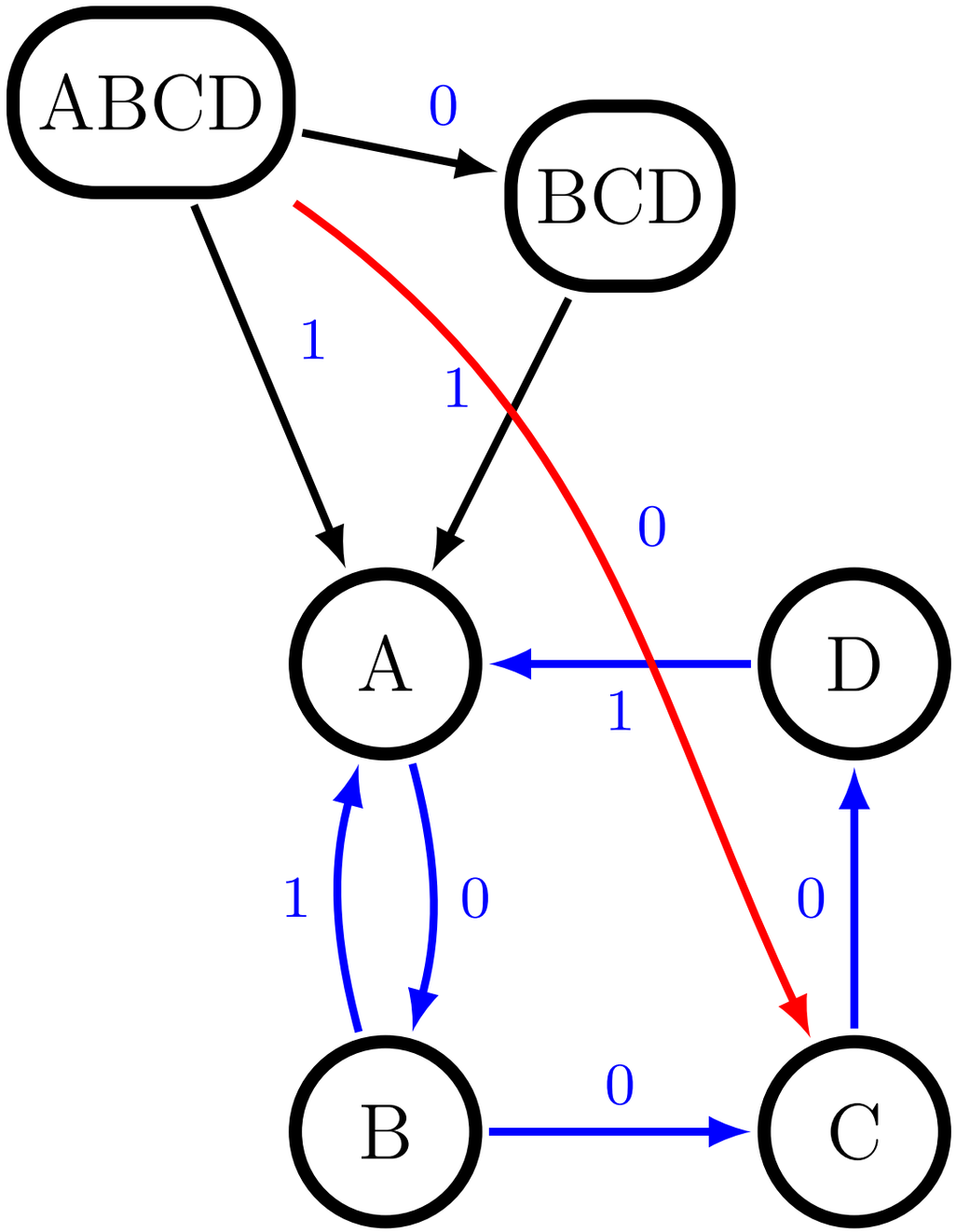

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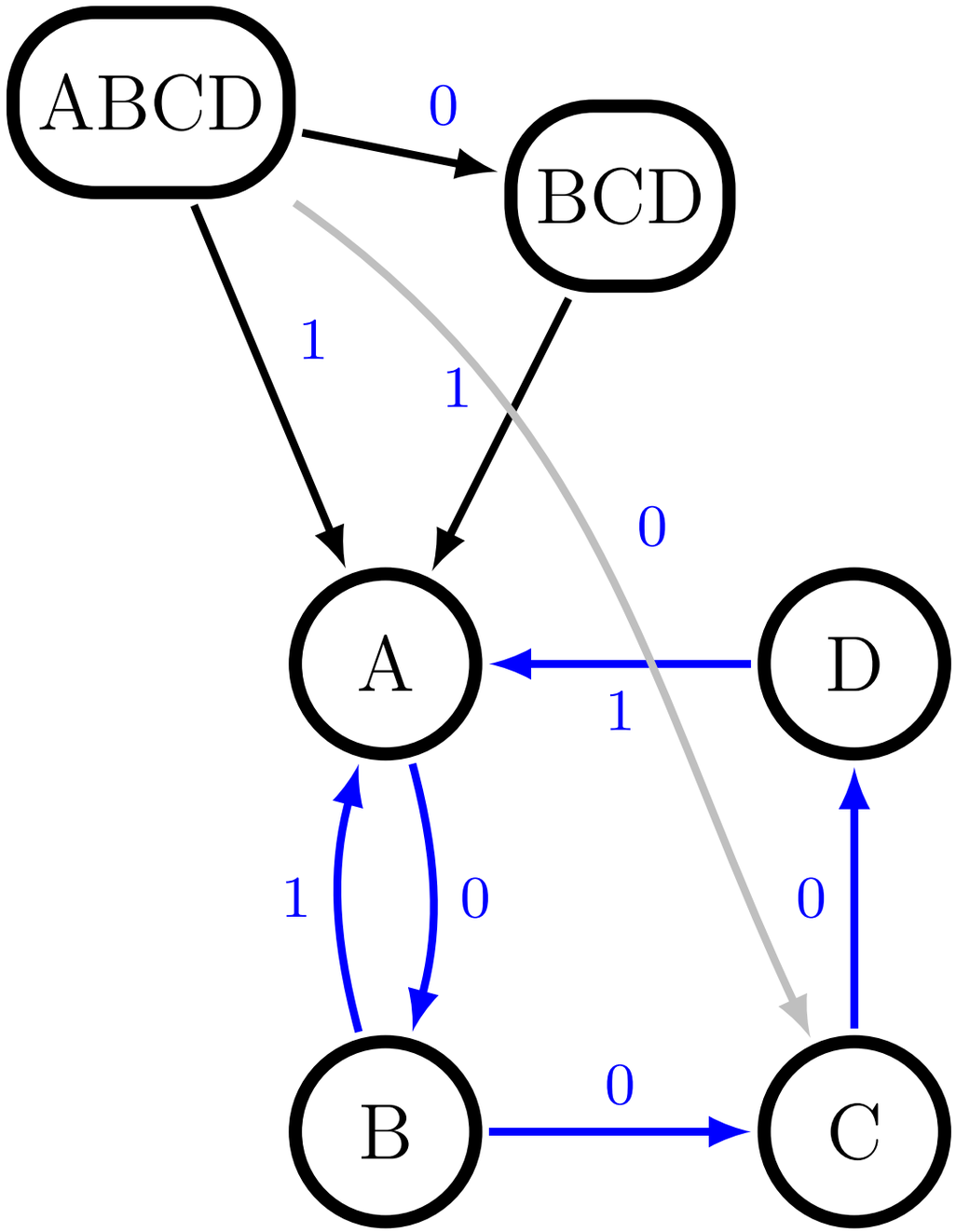

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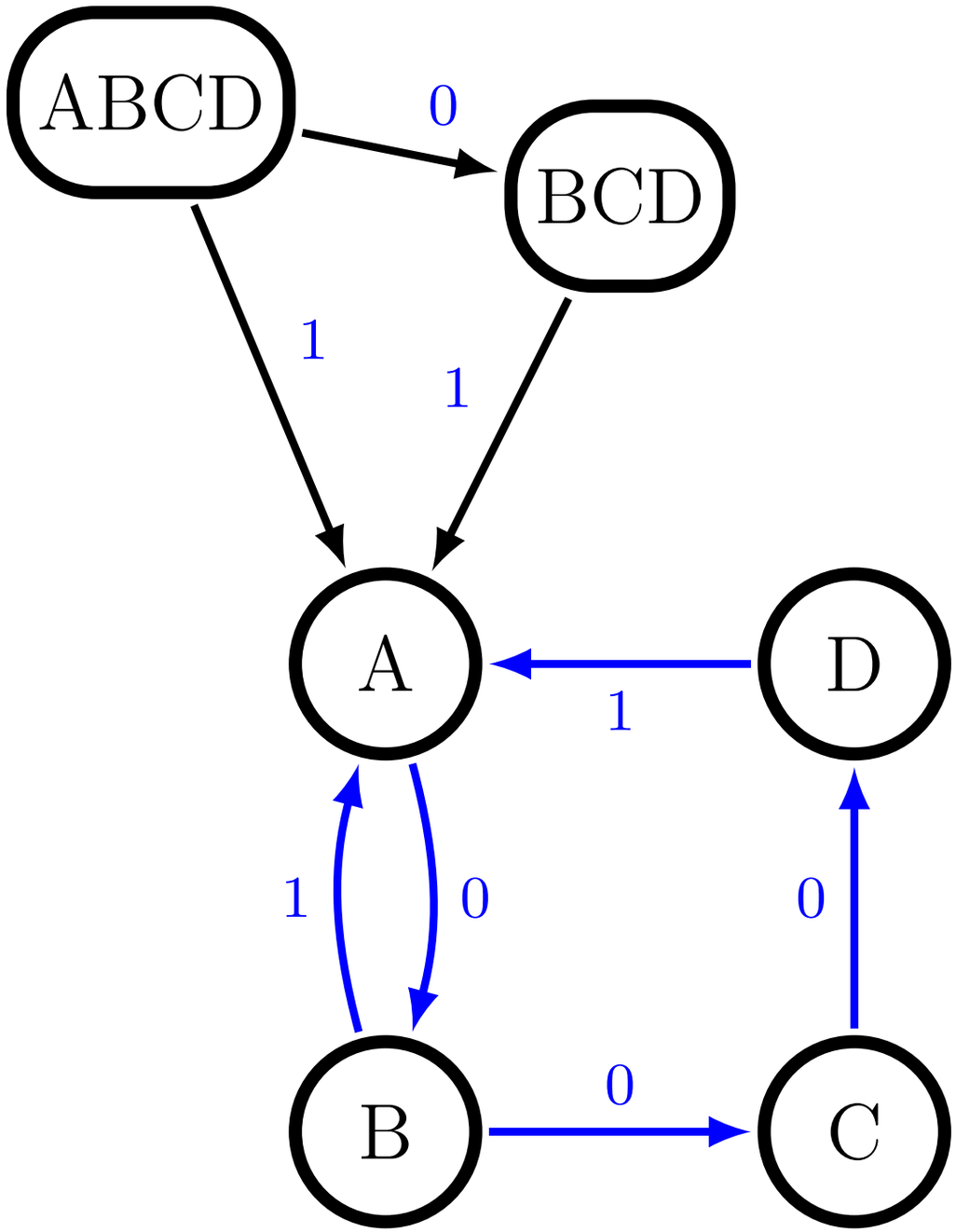

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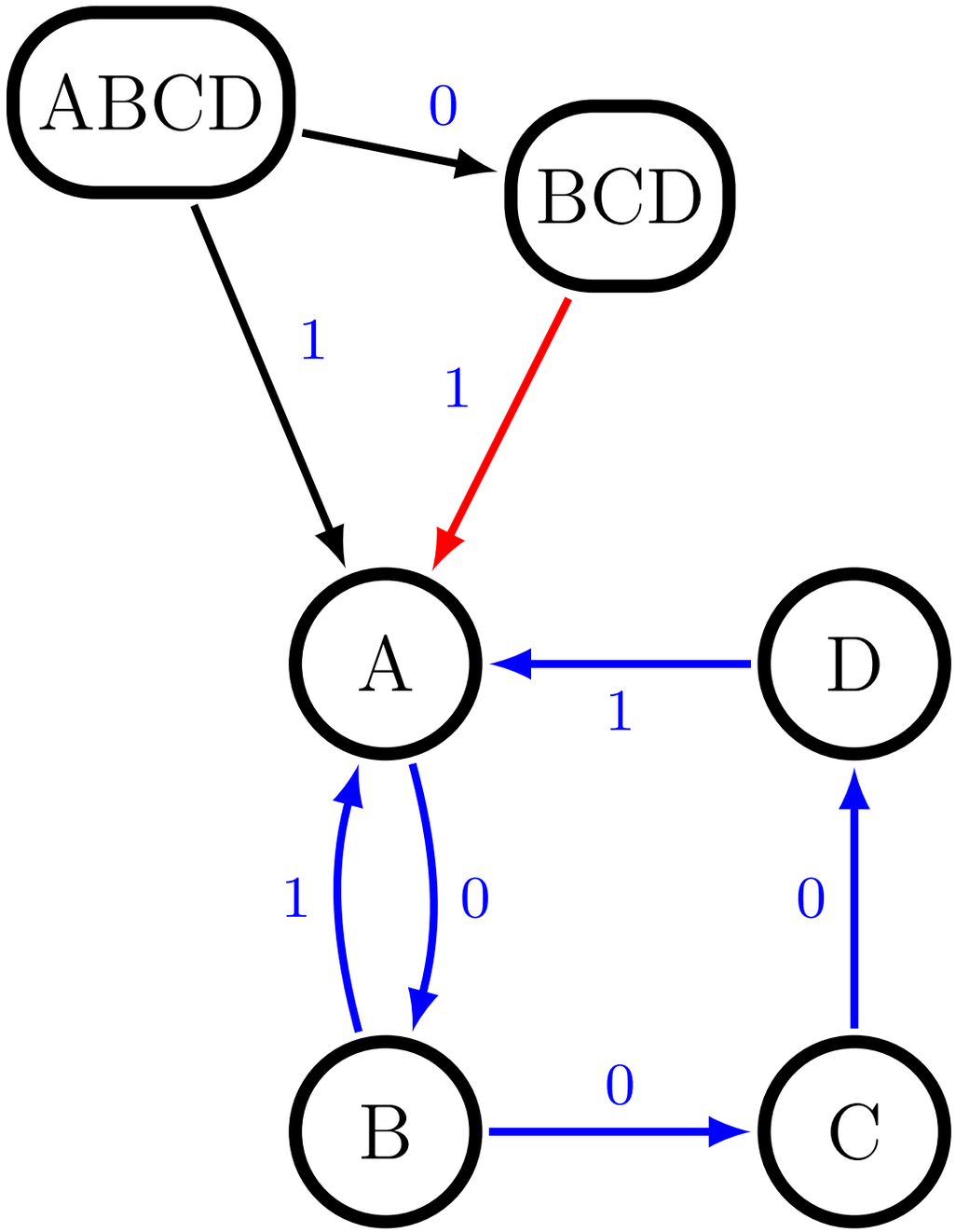

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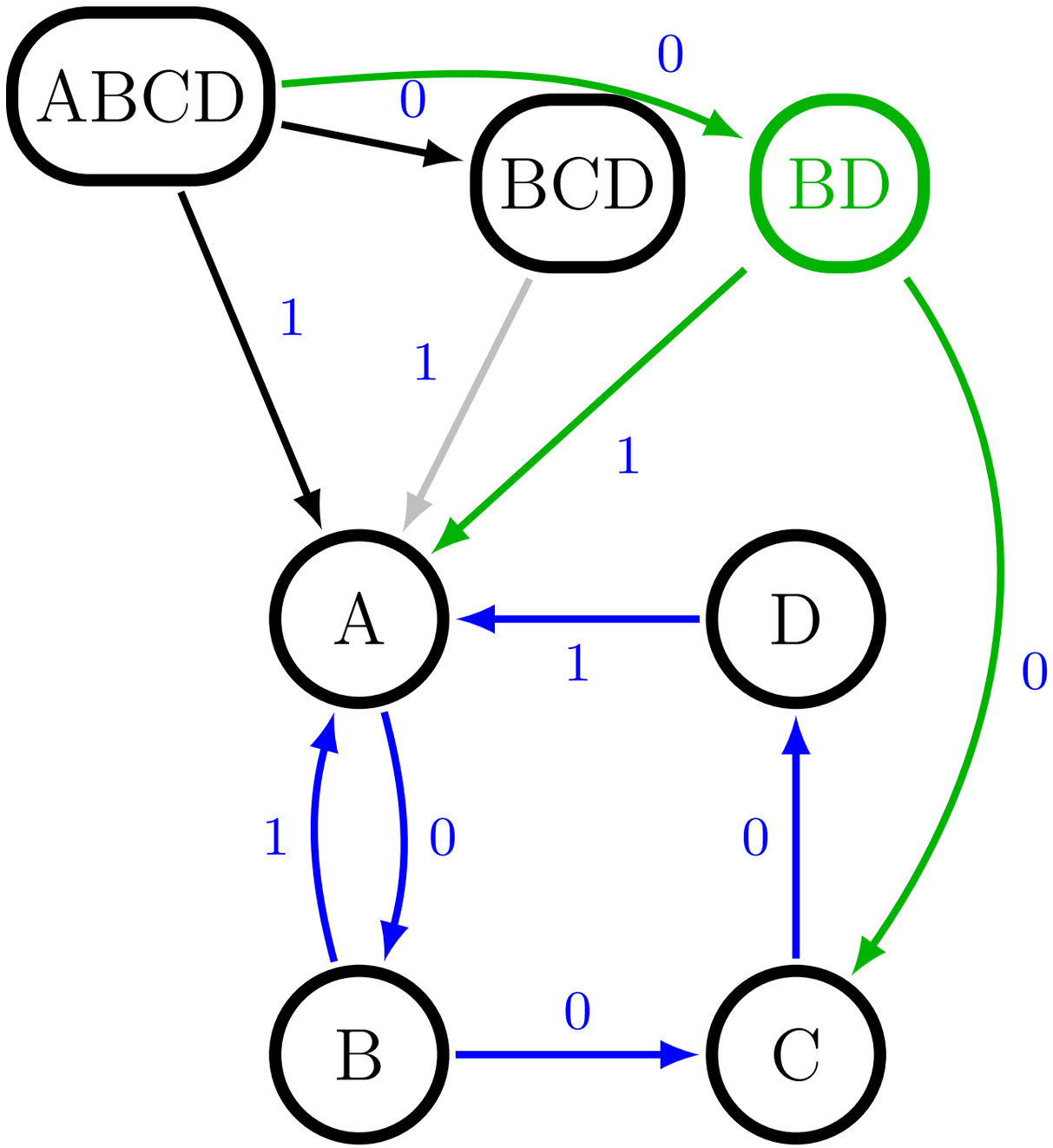

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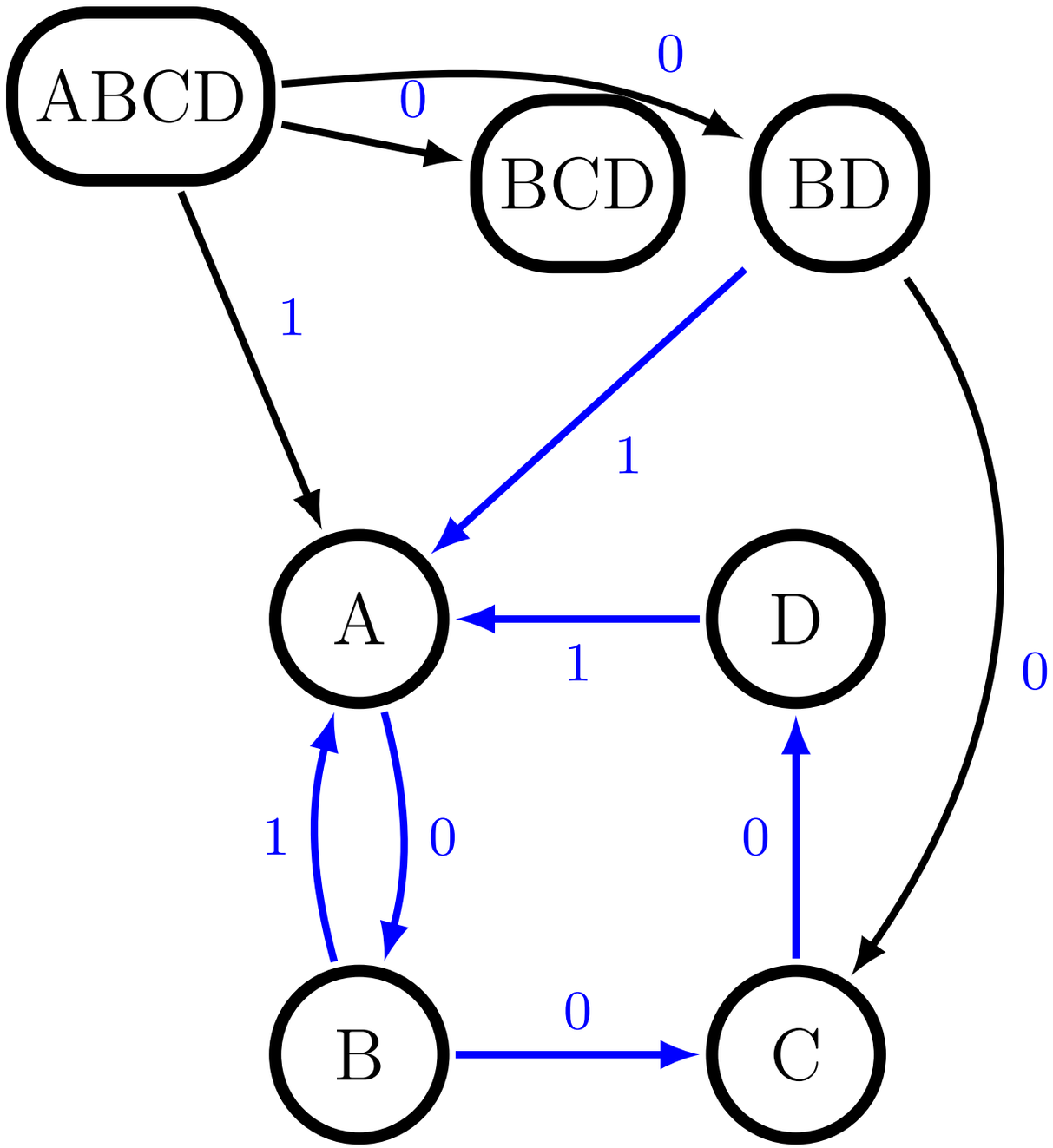

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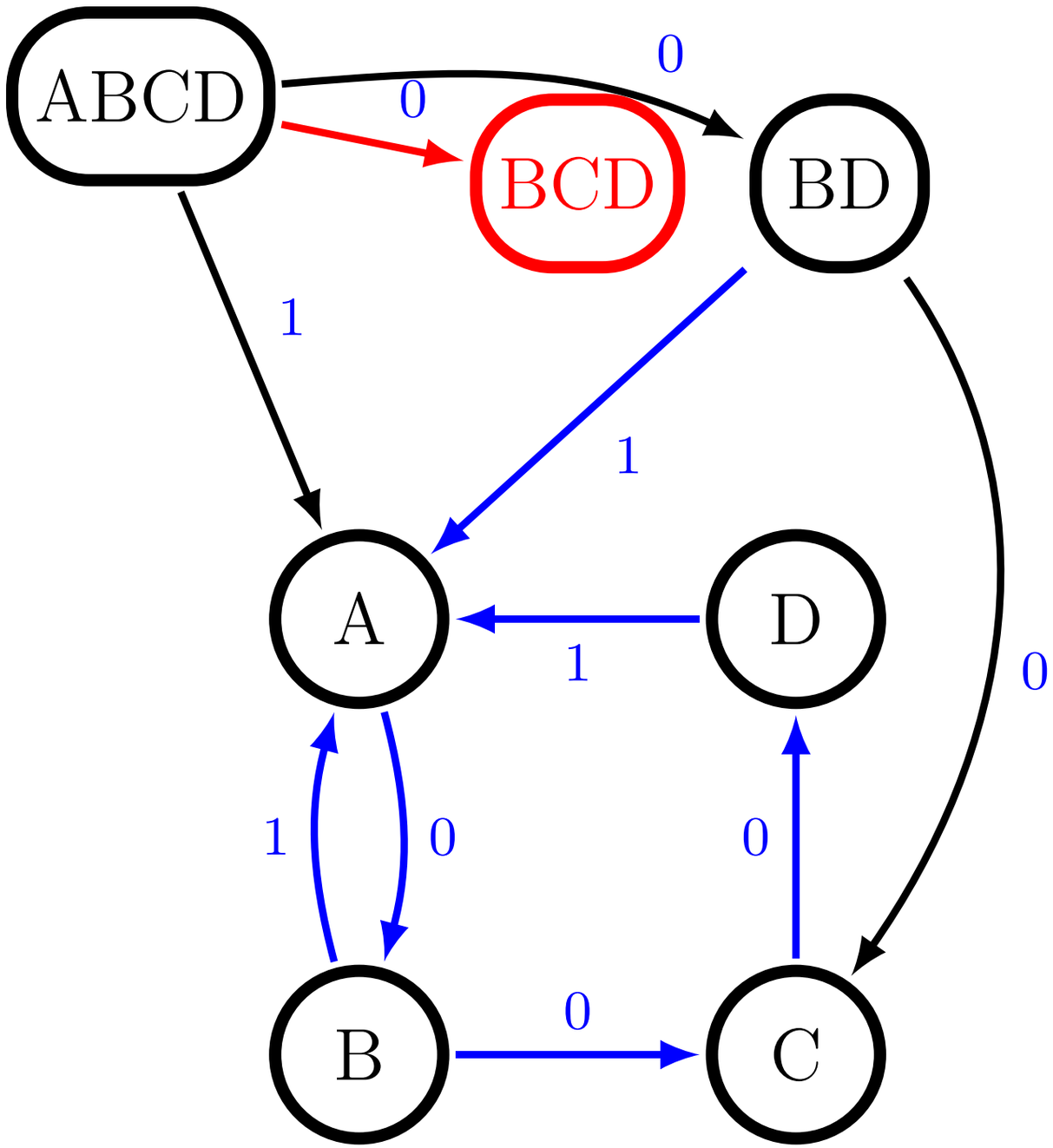

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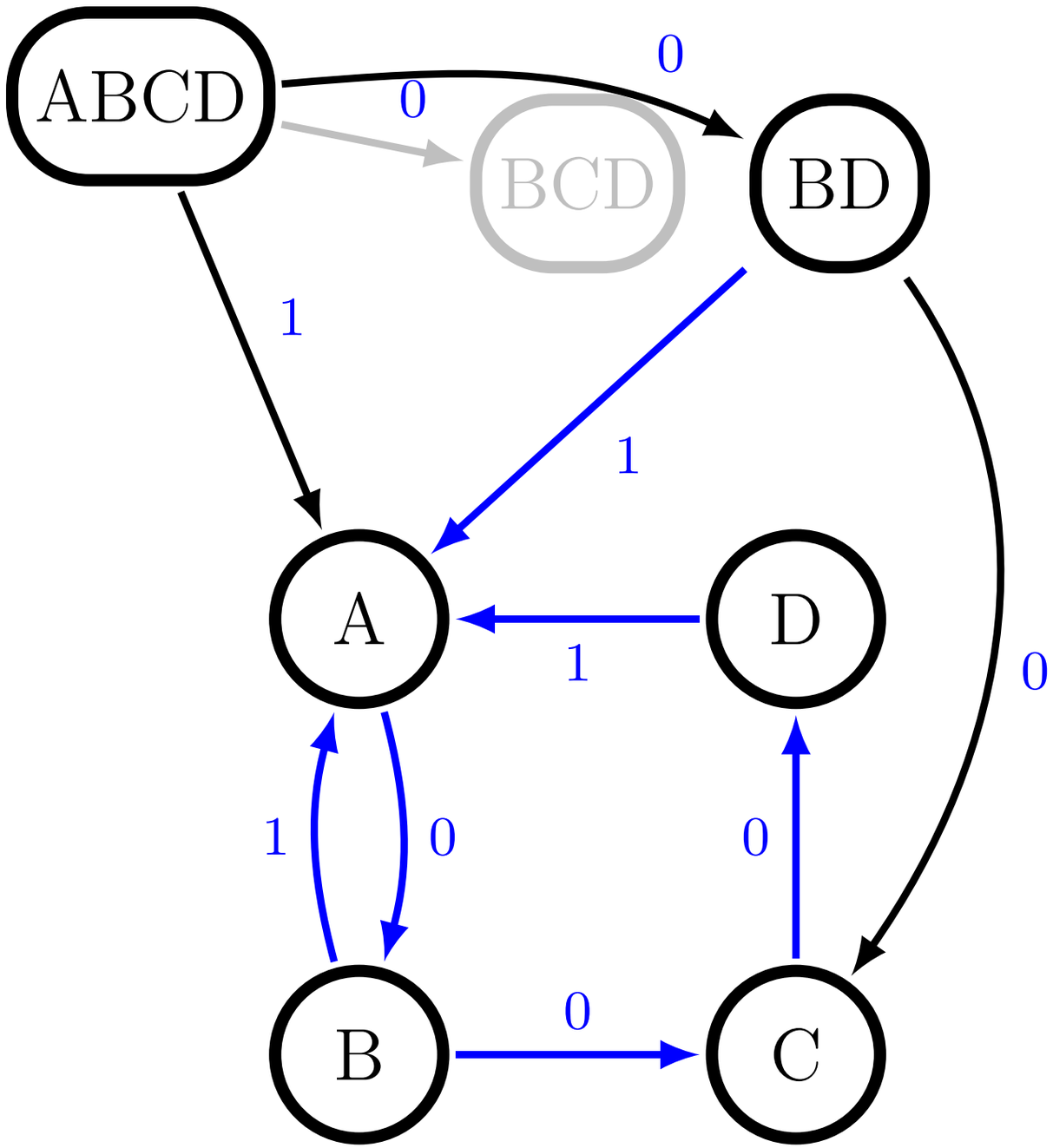

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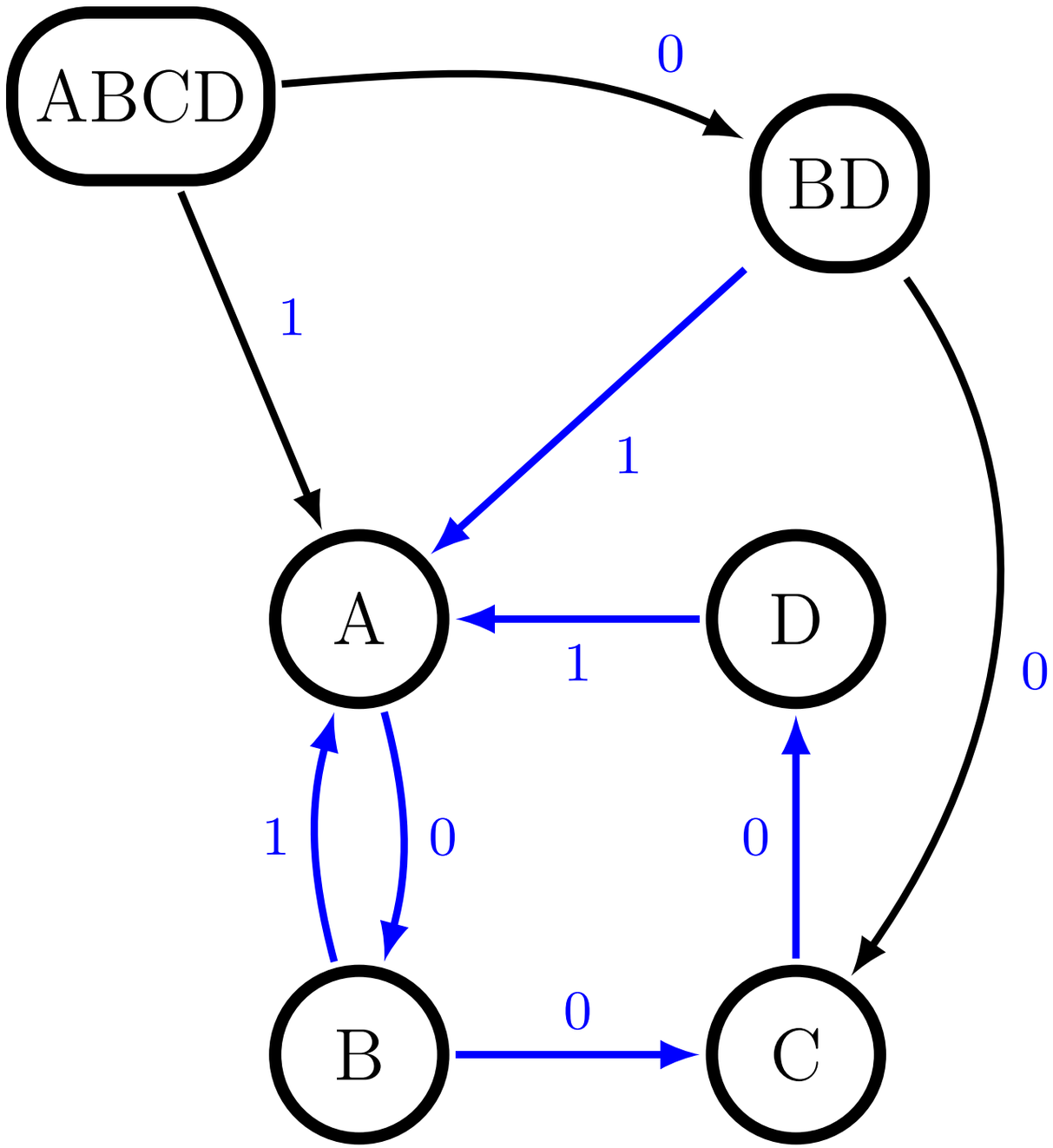

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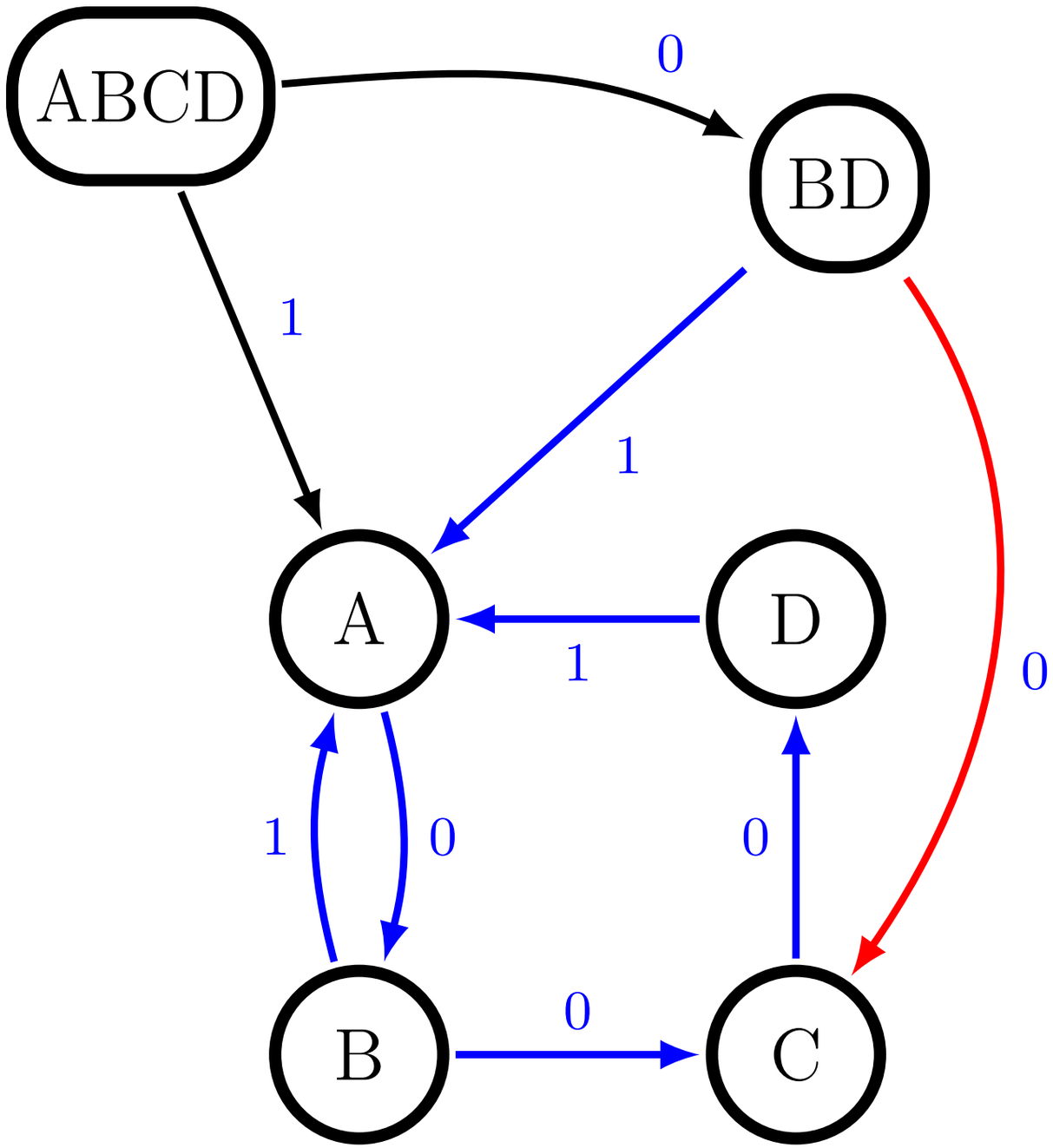

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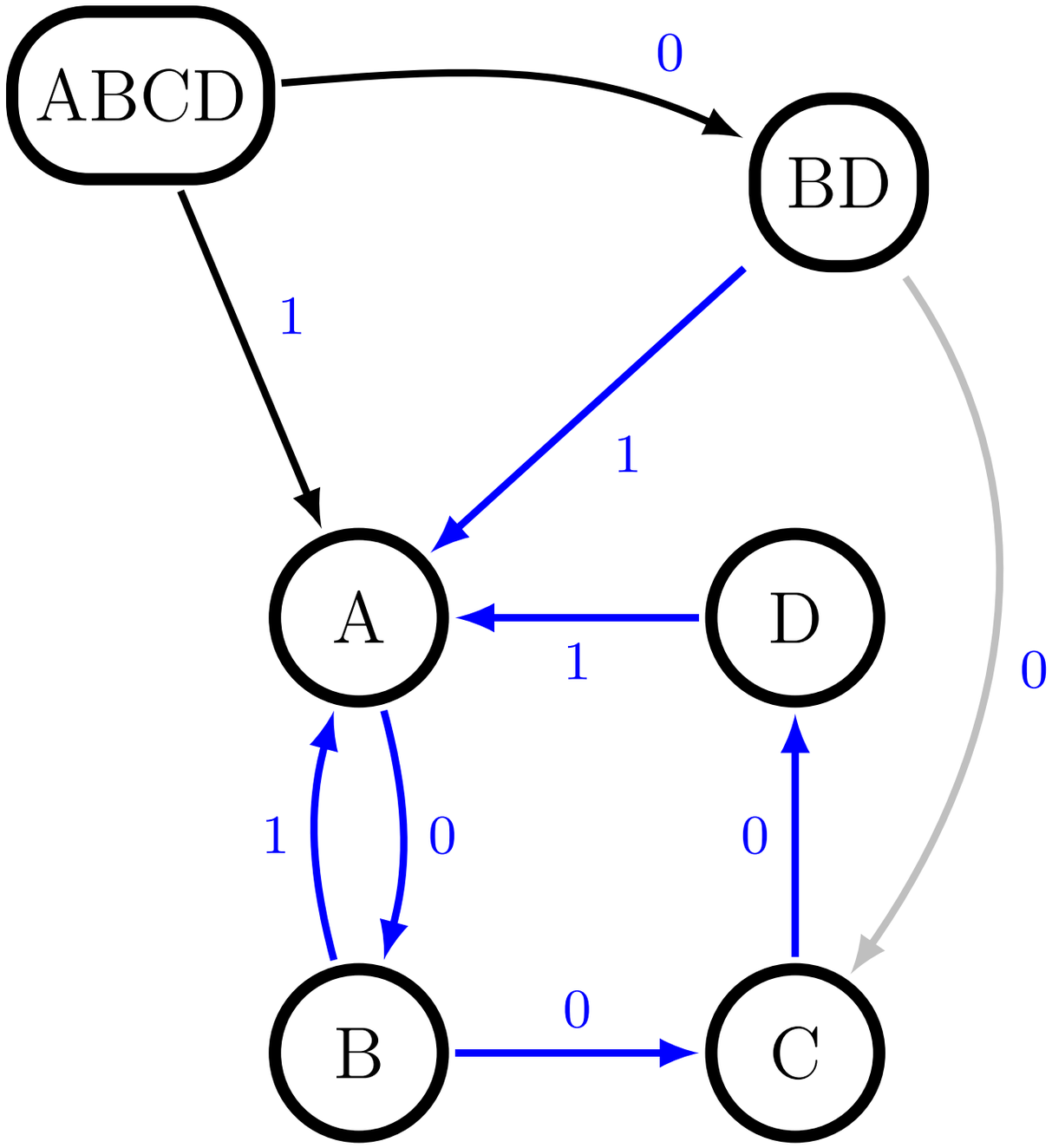

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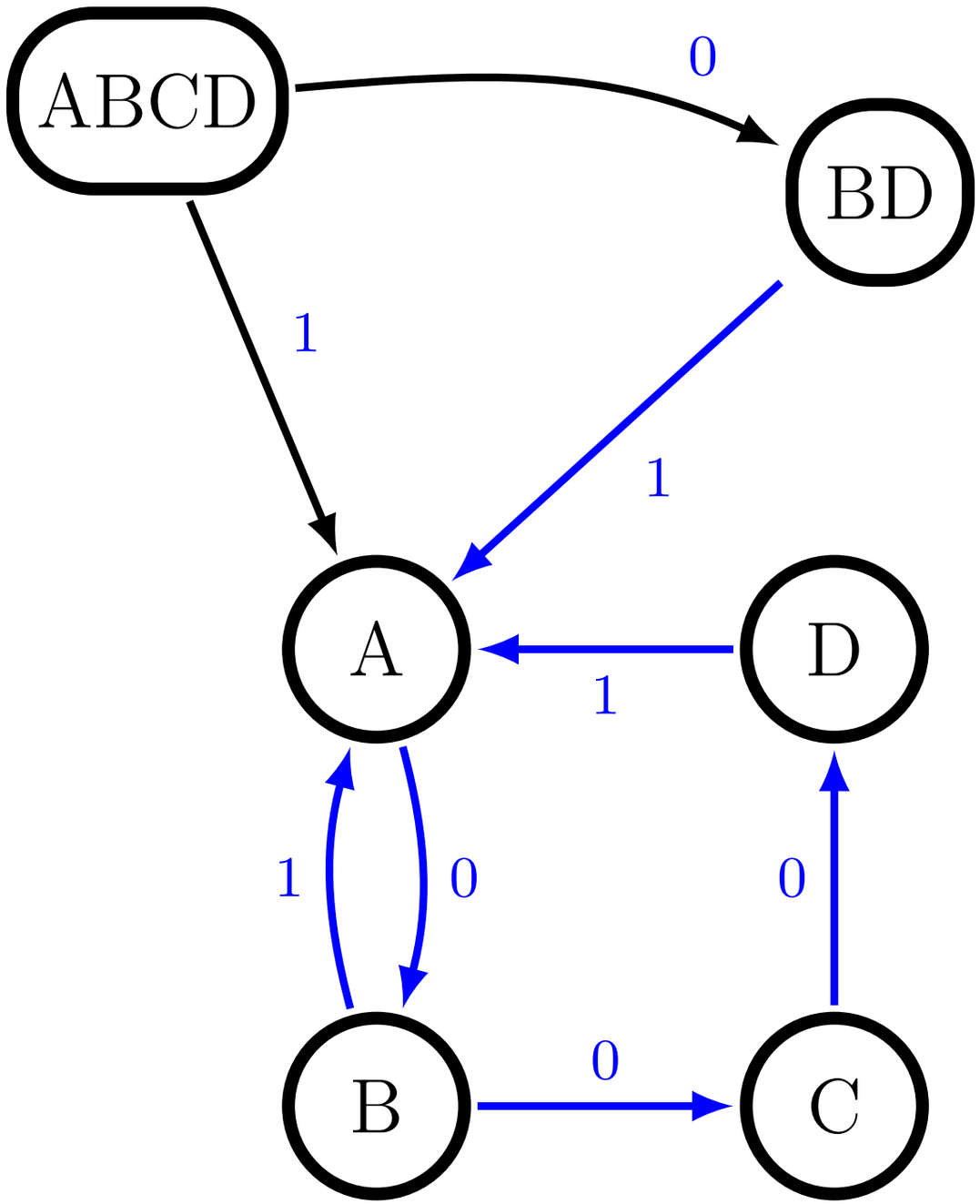

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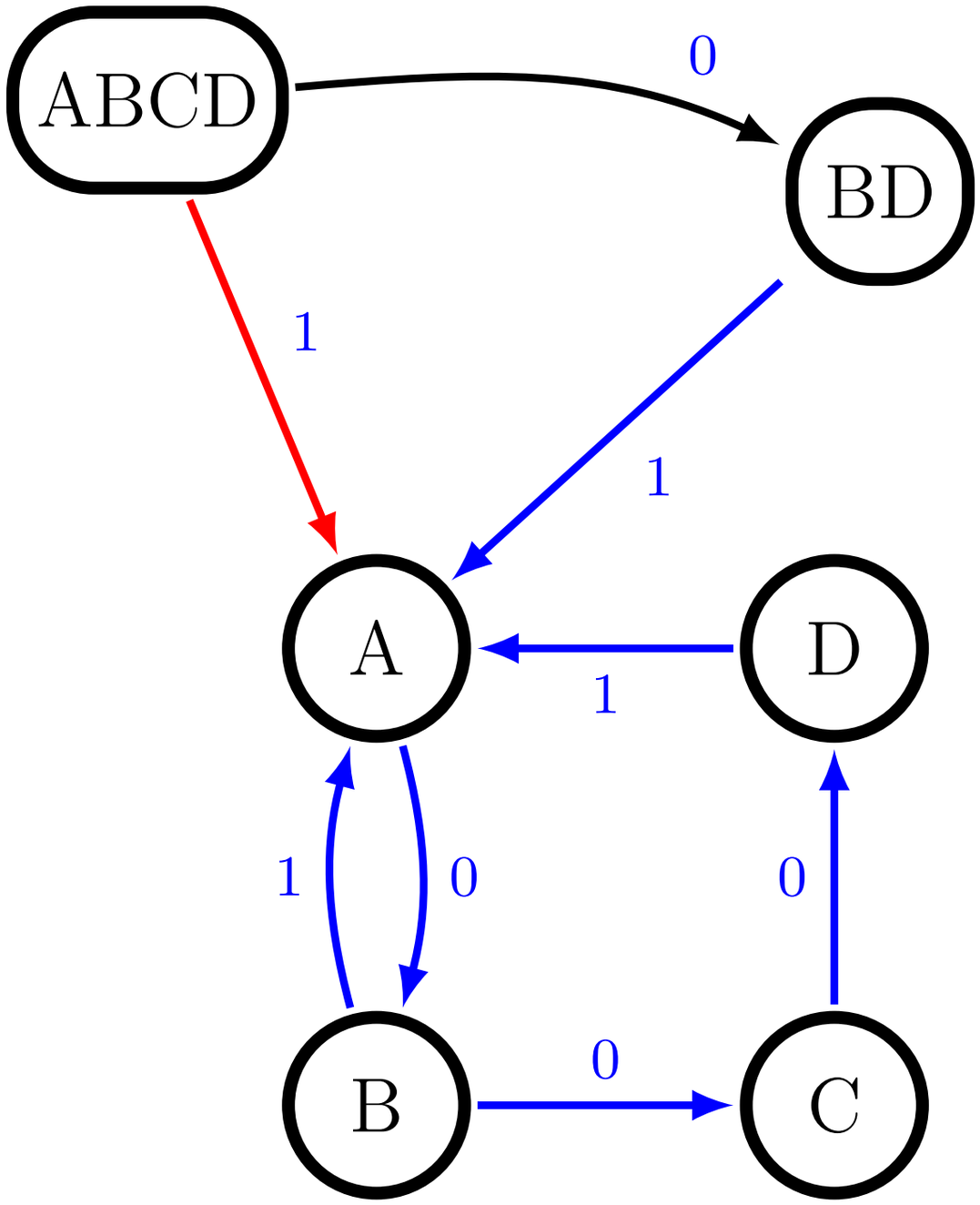

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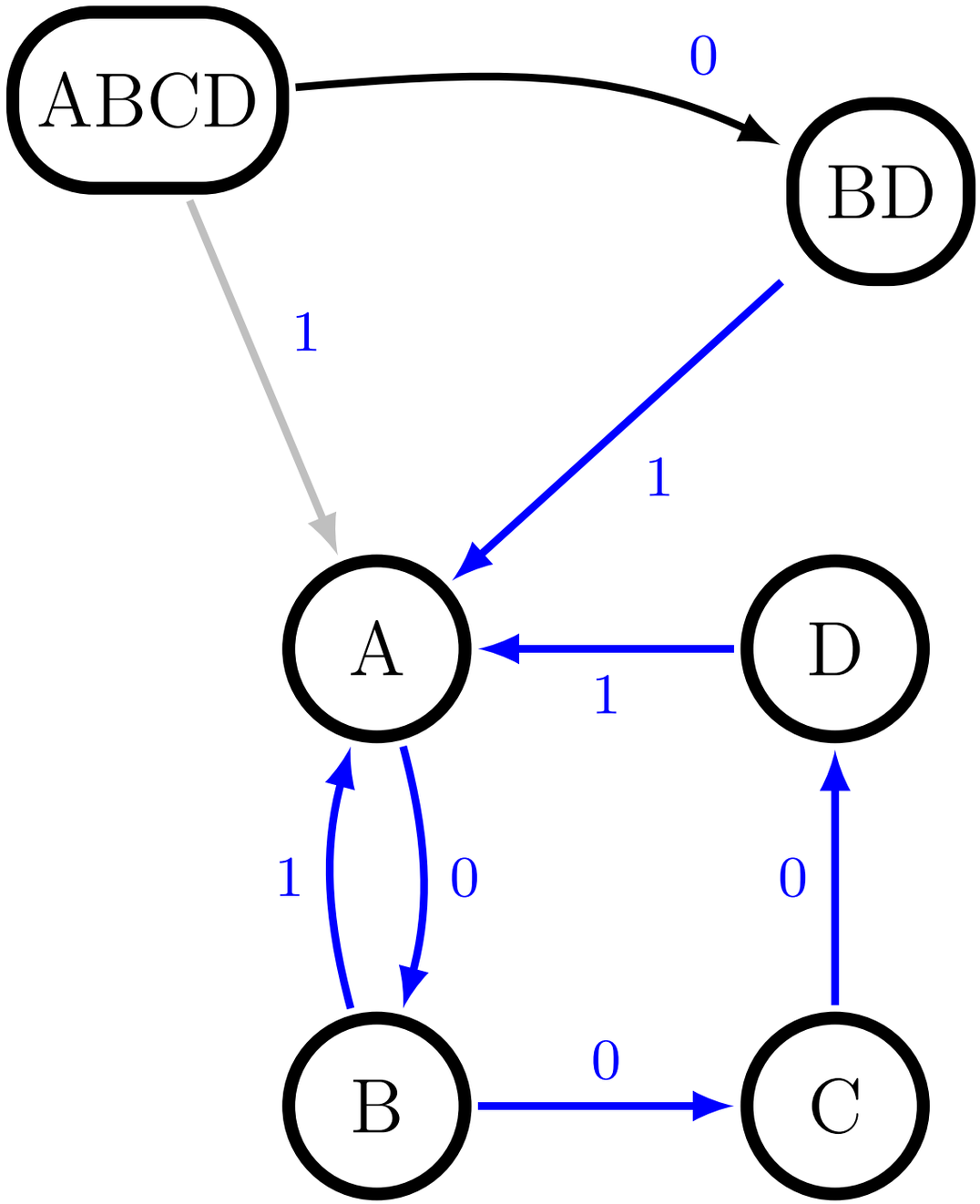

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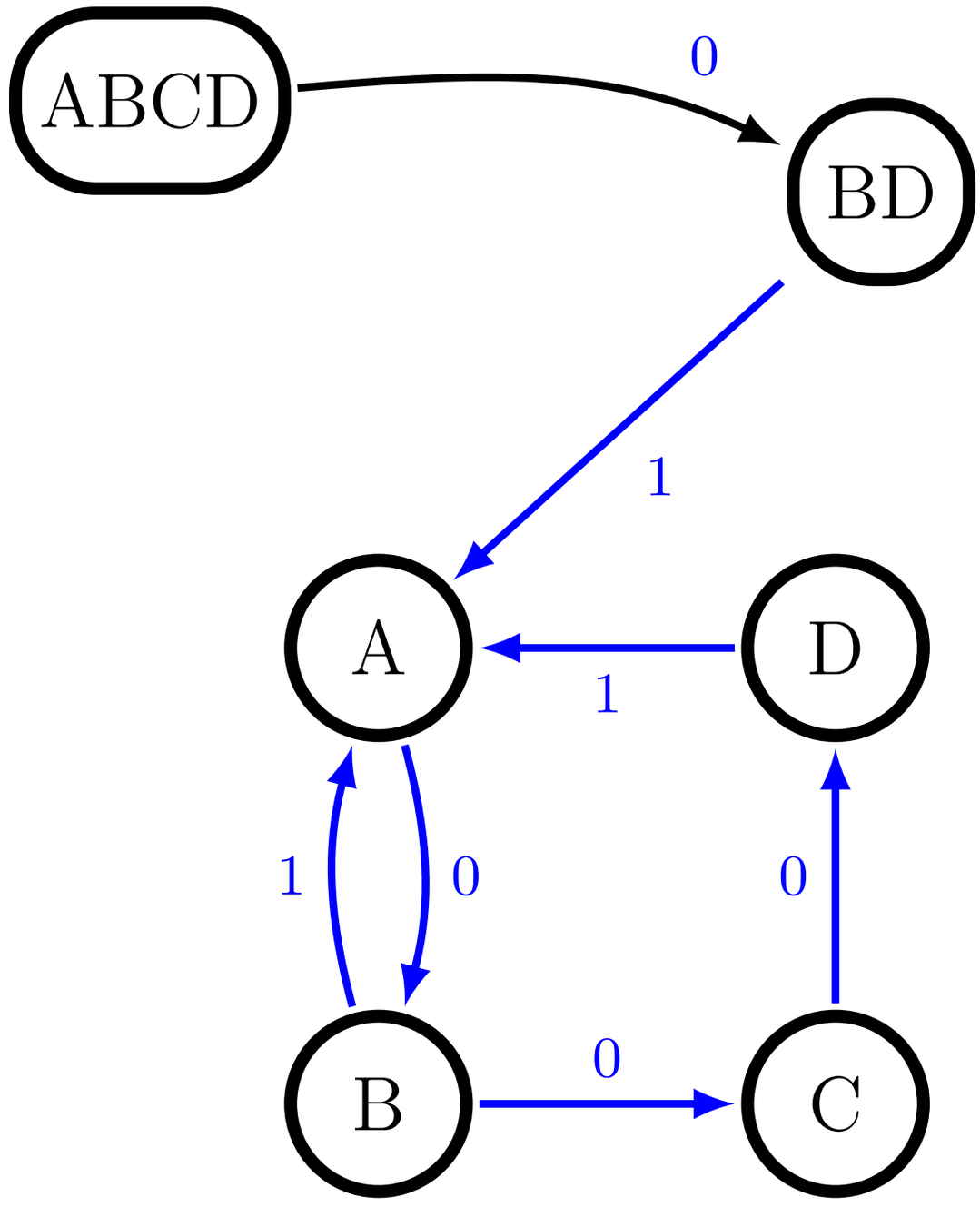

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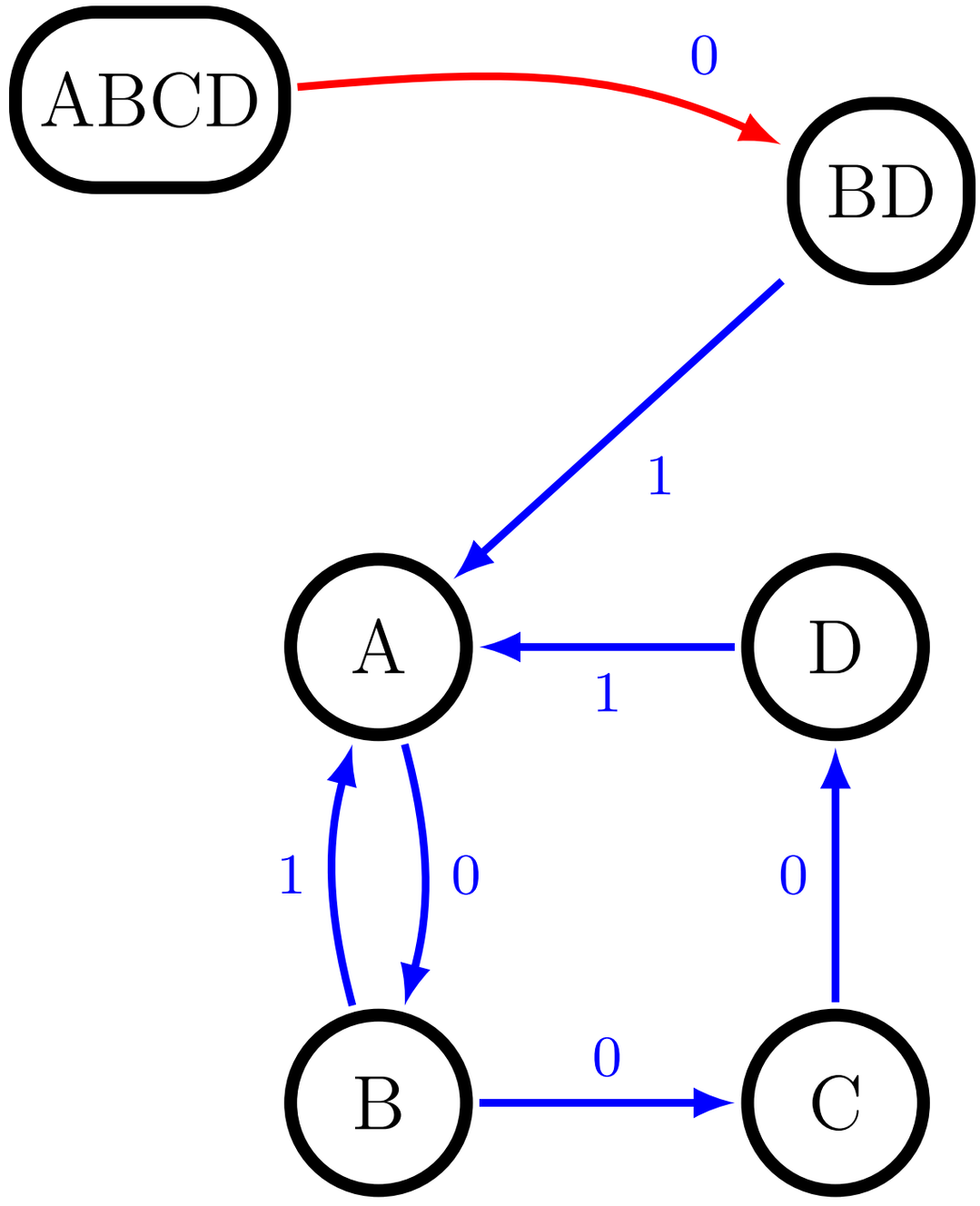

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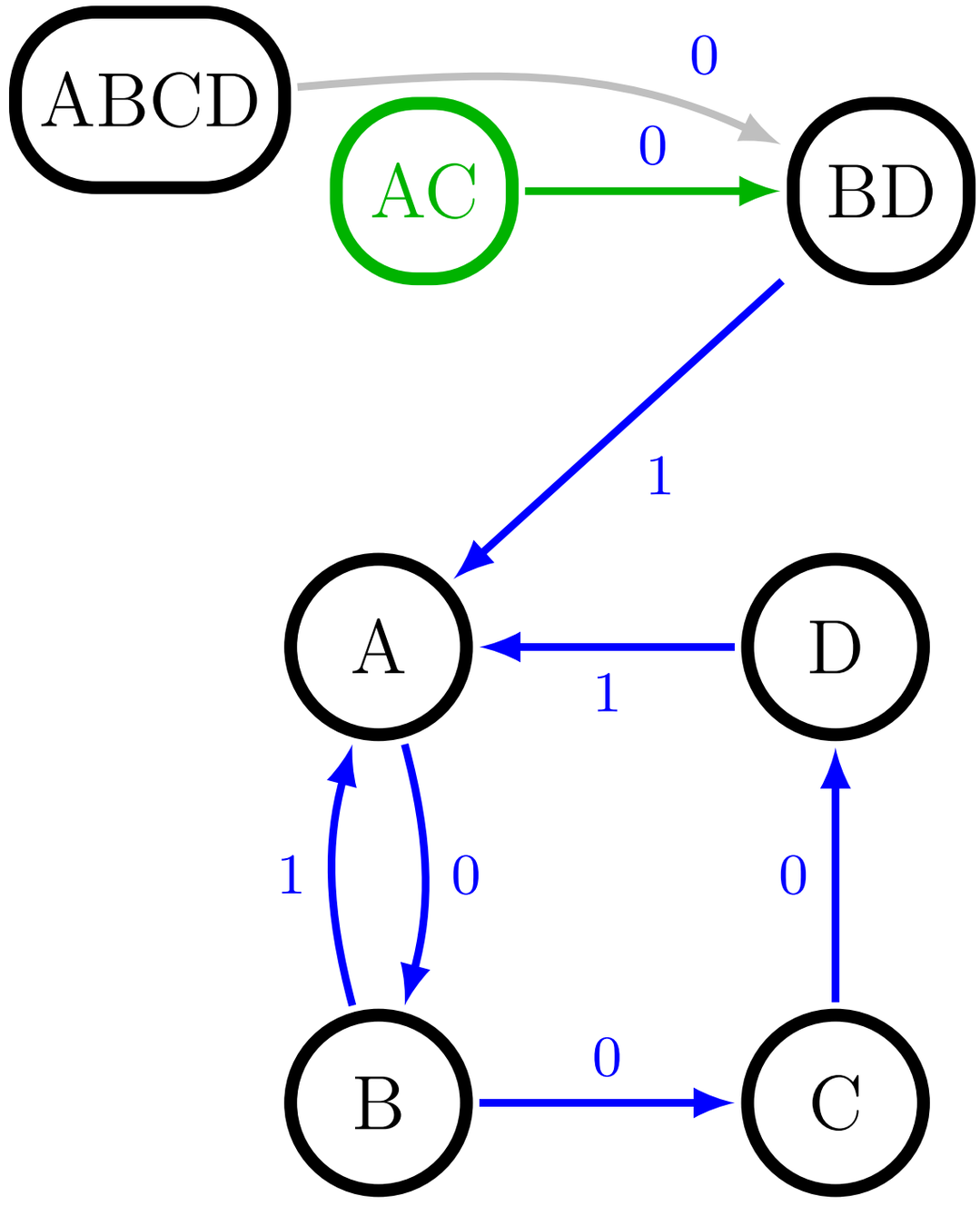

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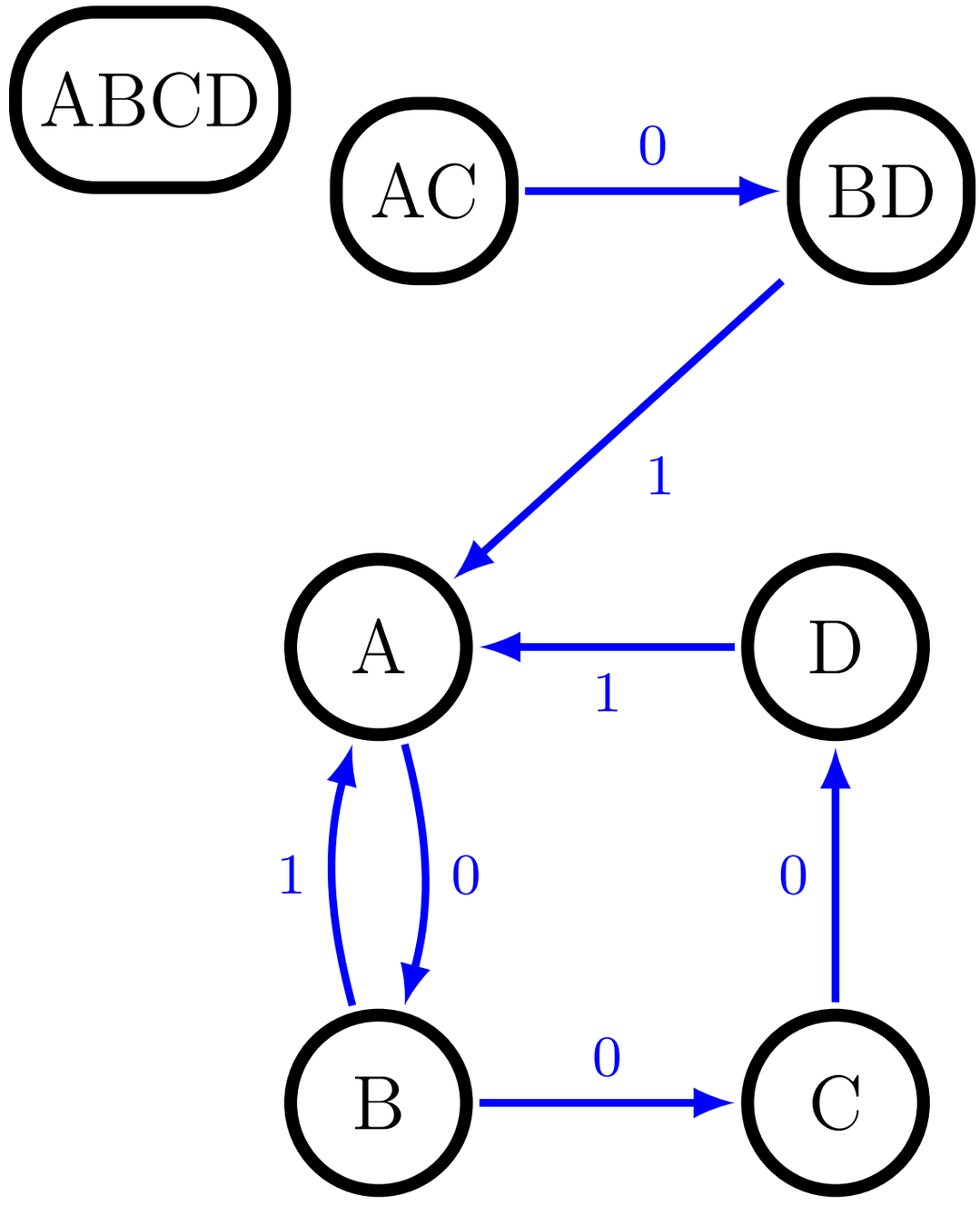

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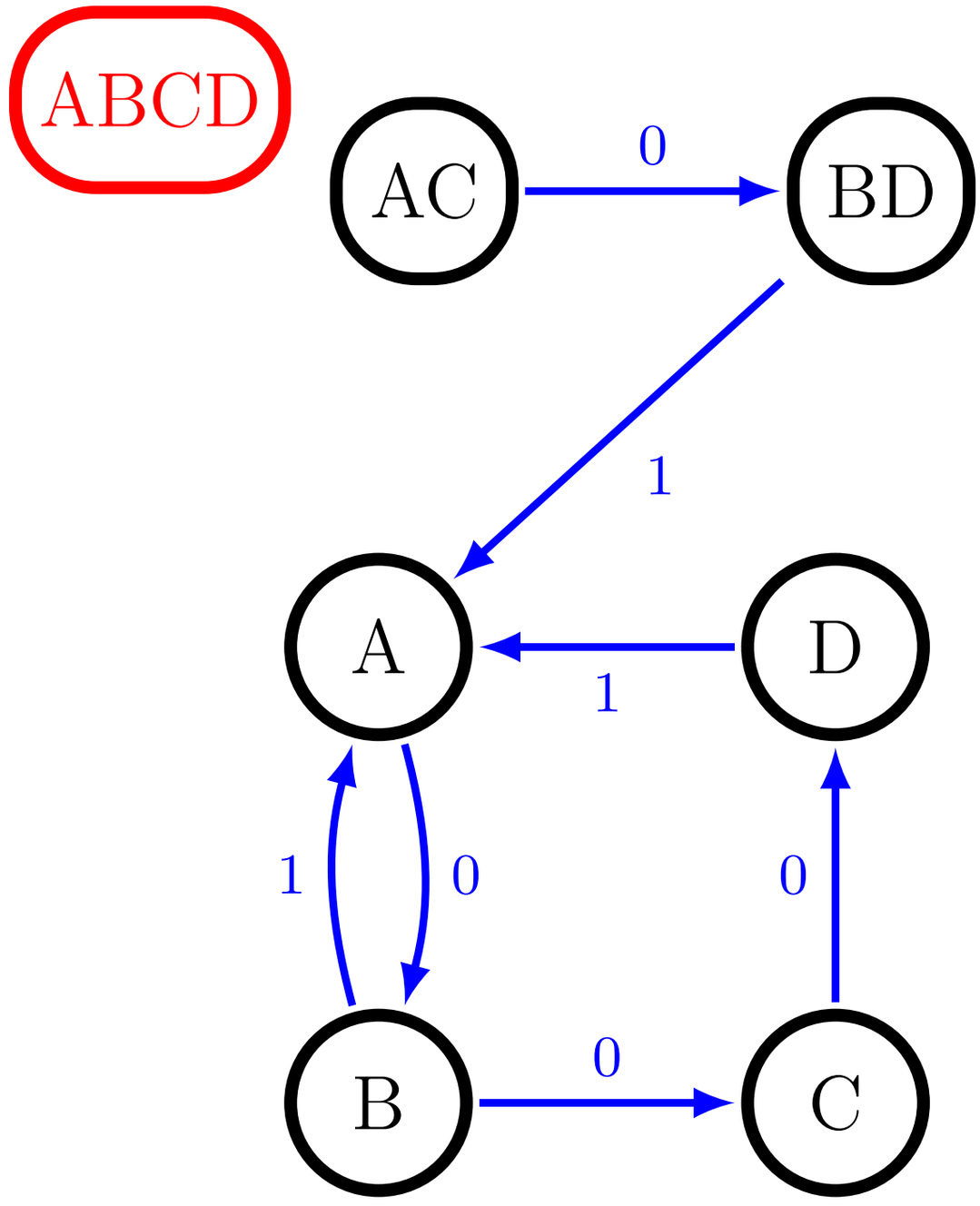

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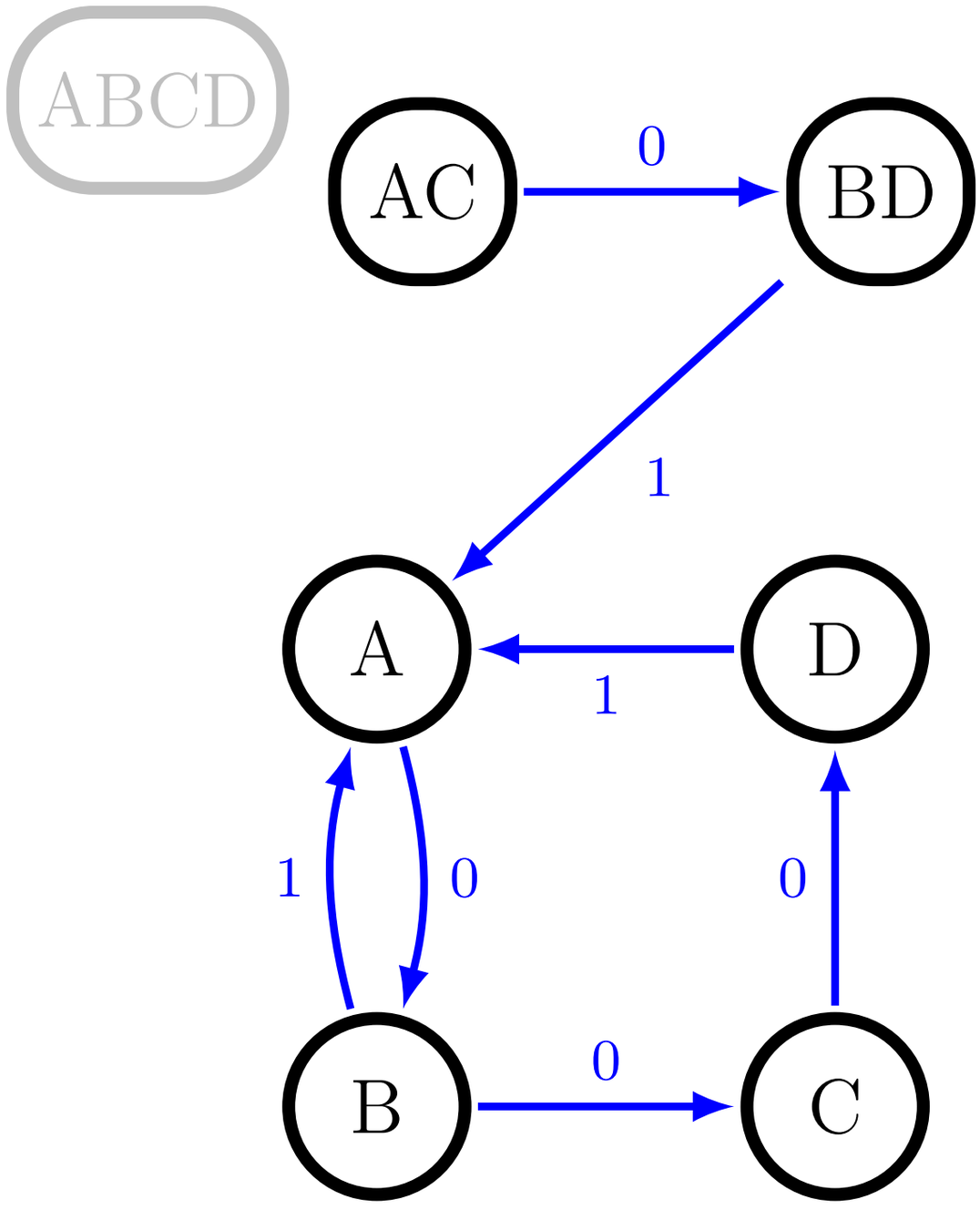

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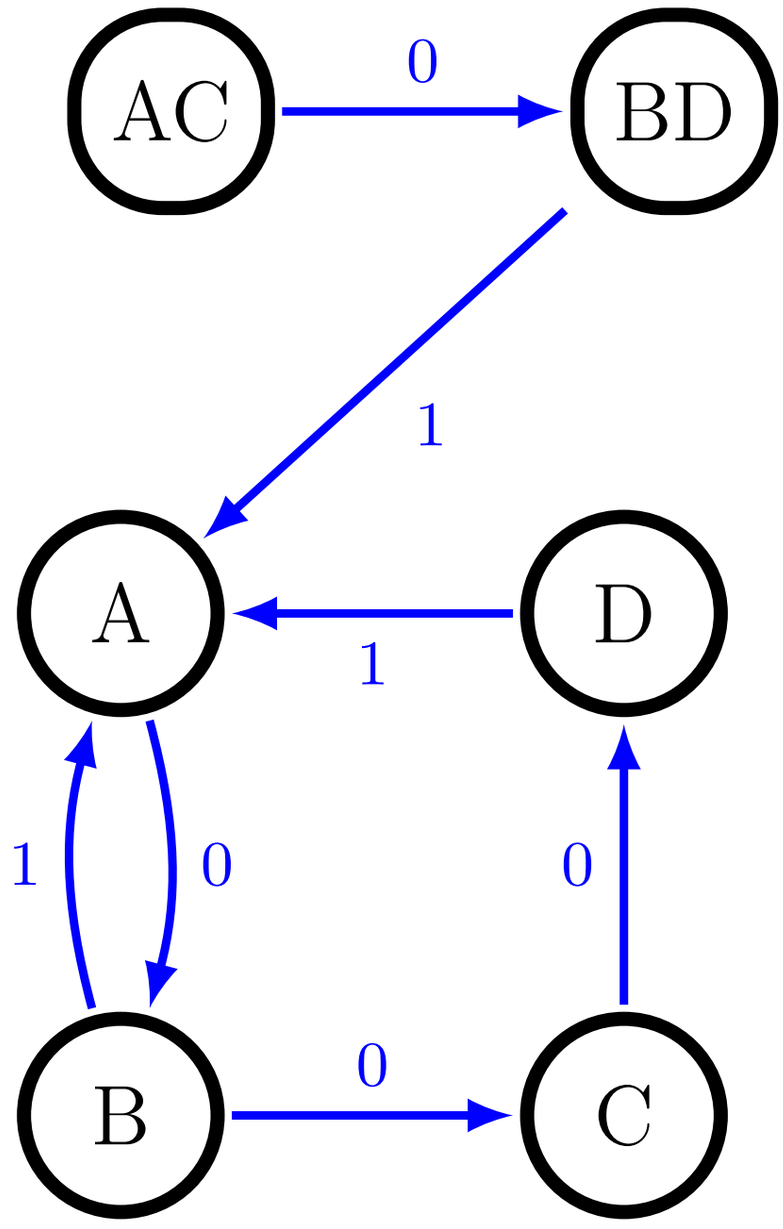

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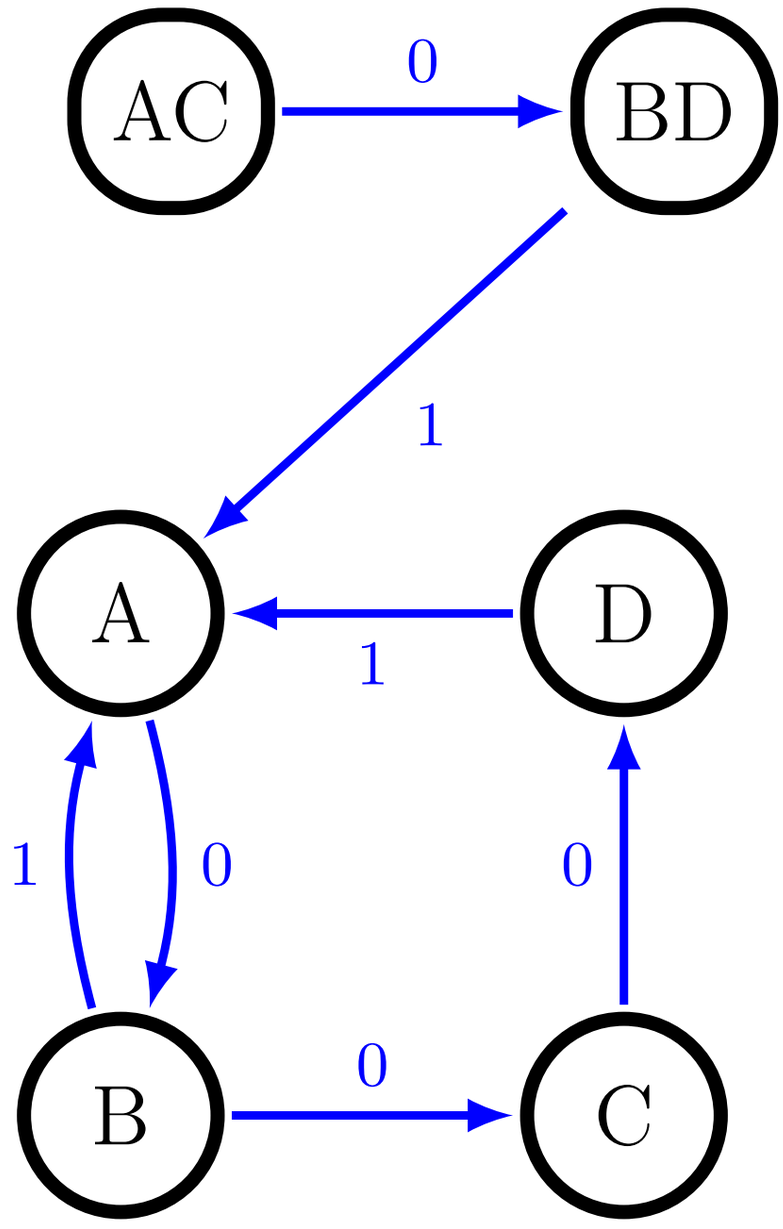
Now do Bellman-Ford again



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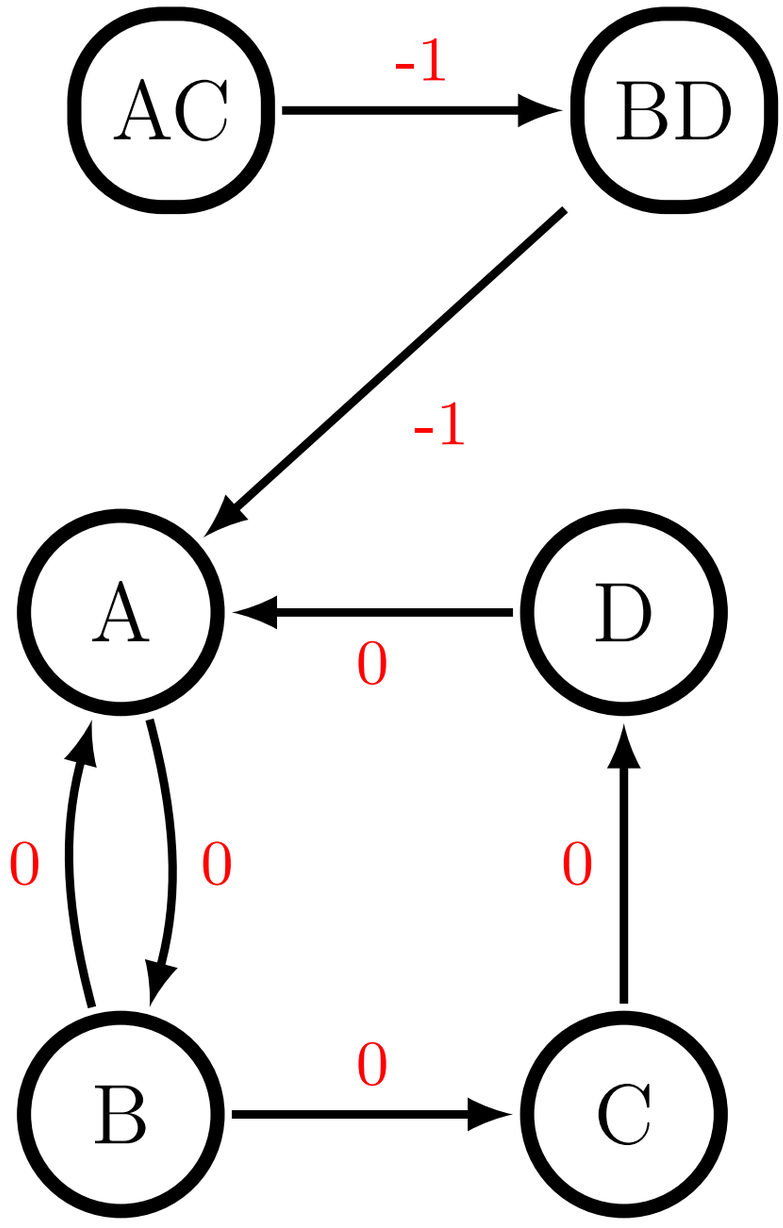

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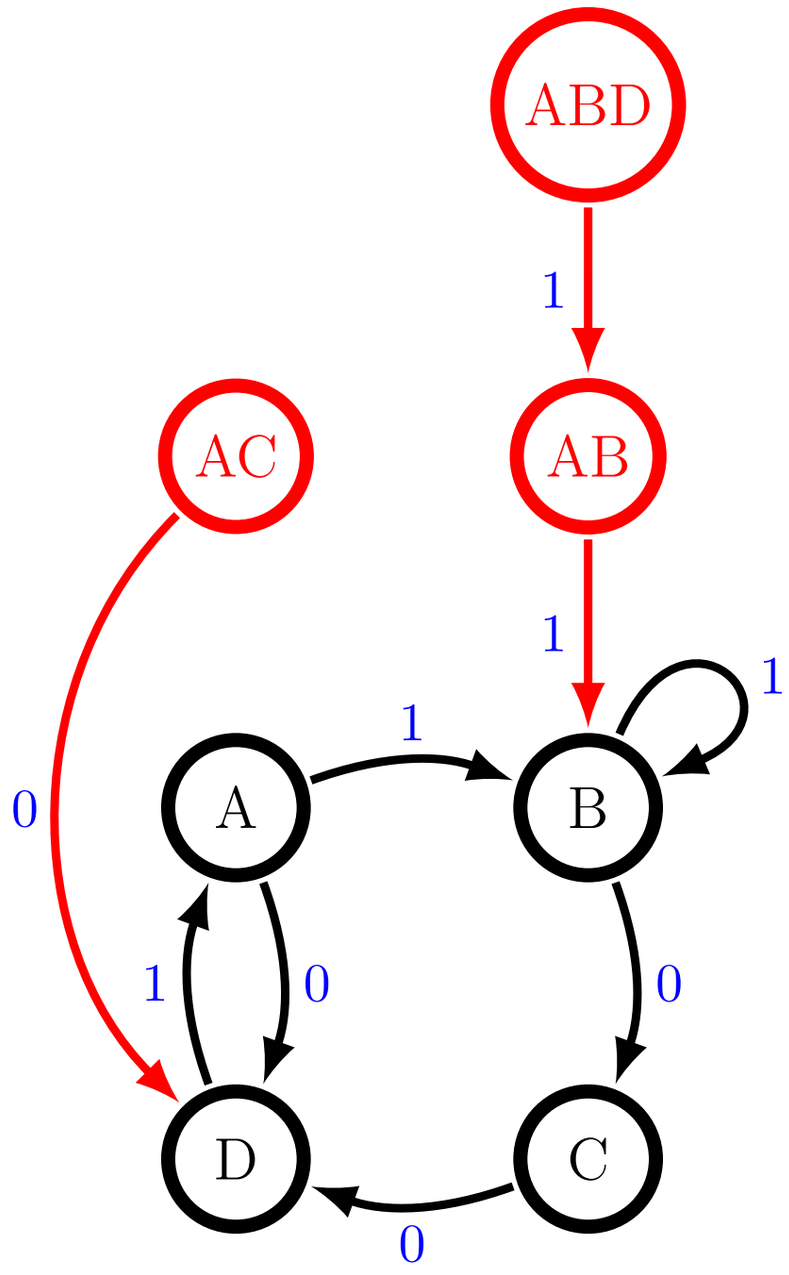
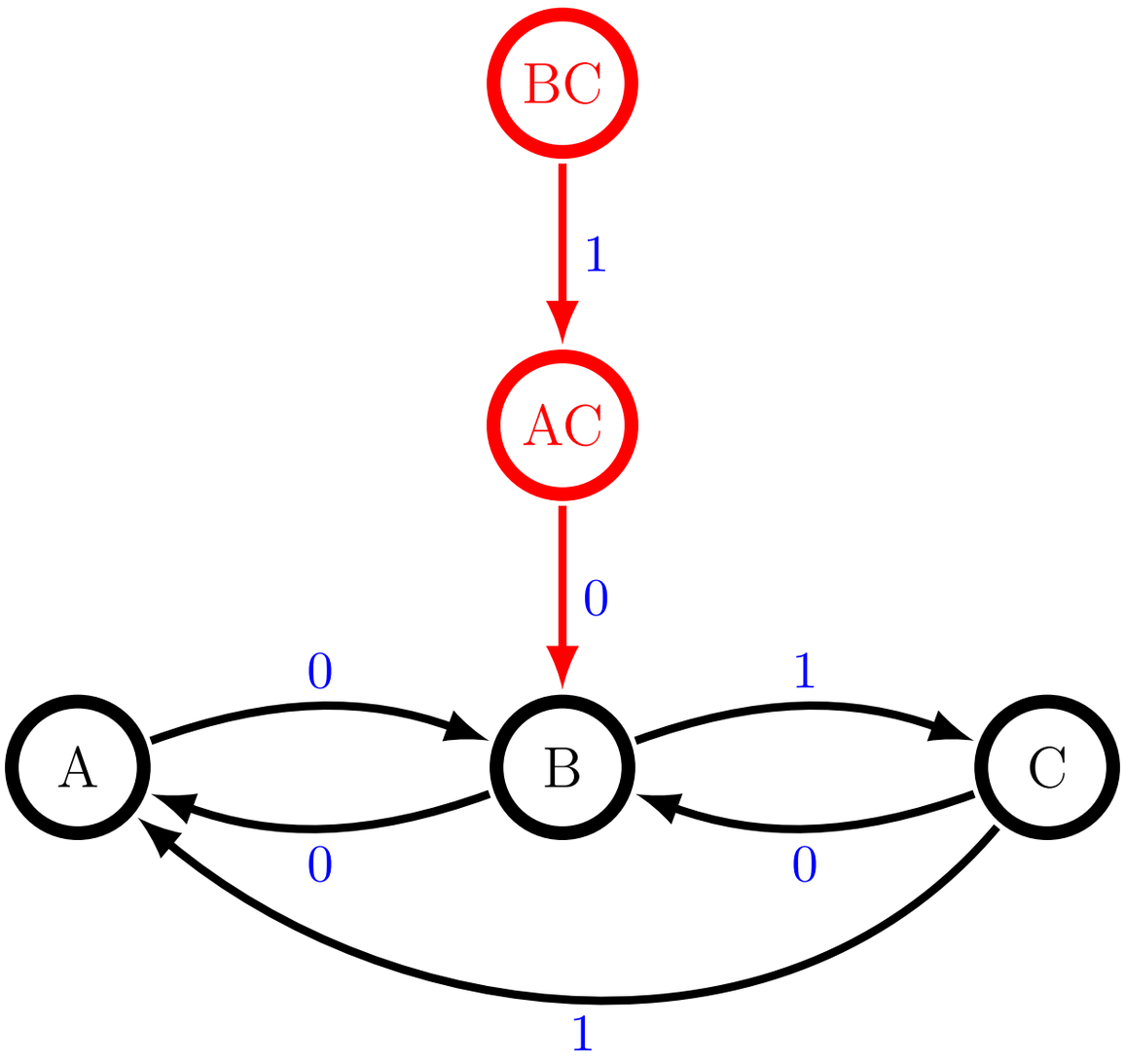
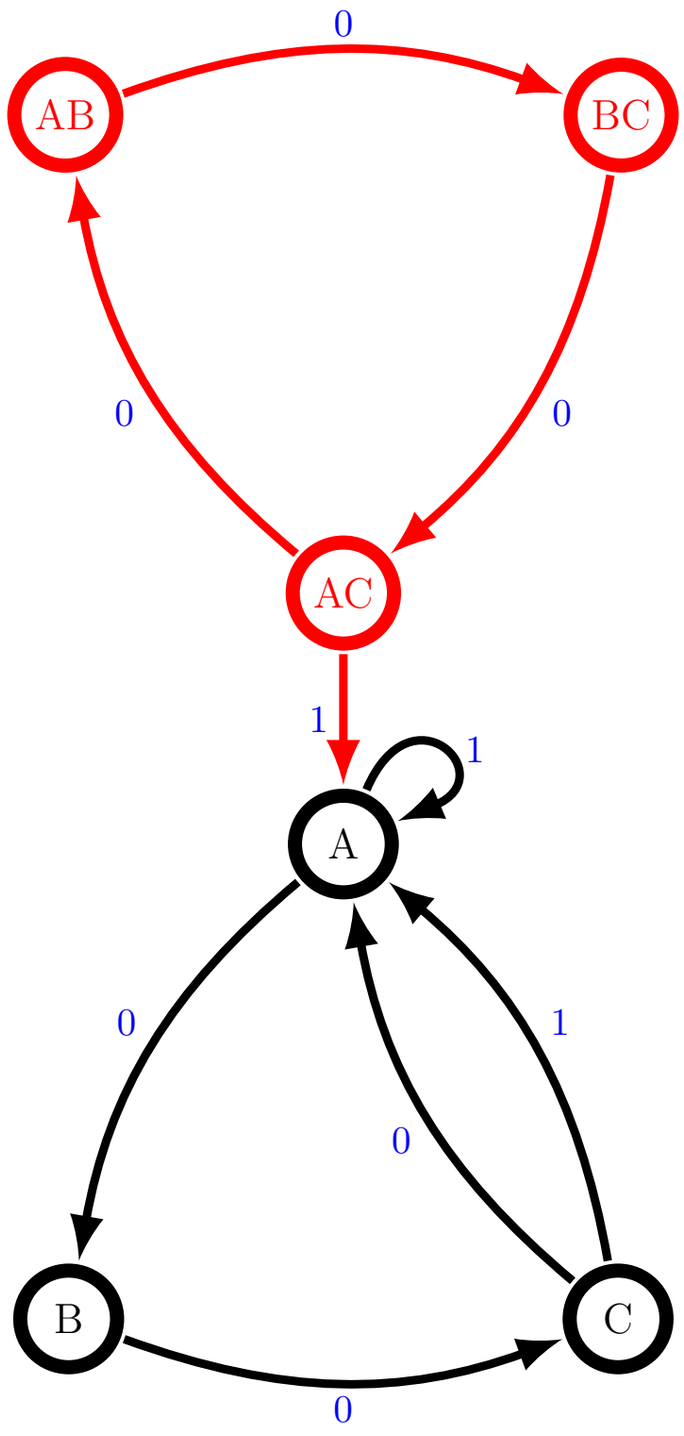
Example the First



Example the Second



Example the Third



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Further Improvements

Now where are we?

Further Improvements

Now where are we?

- Need to know the structure of the ϵ-machine, but not transition probabilities

Further Improvements

Now where are we?

- Need to know the structure of the ϵ-machine, but not transition probabilities
- Nor are any information-theoretic properties of the time series needed

Further Improvements

Now where are we?

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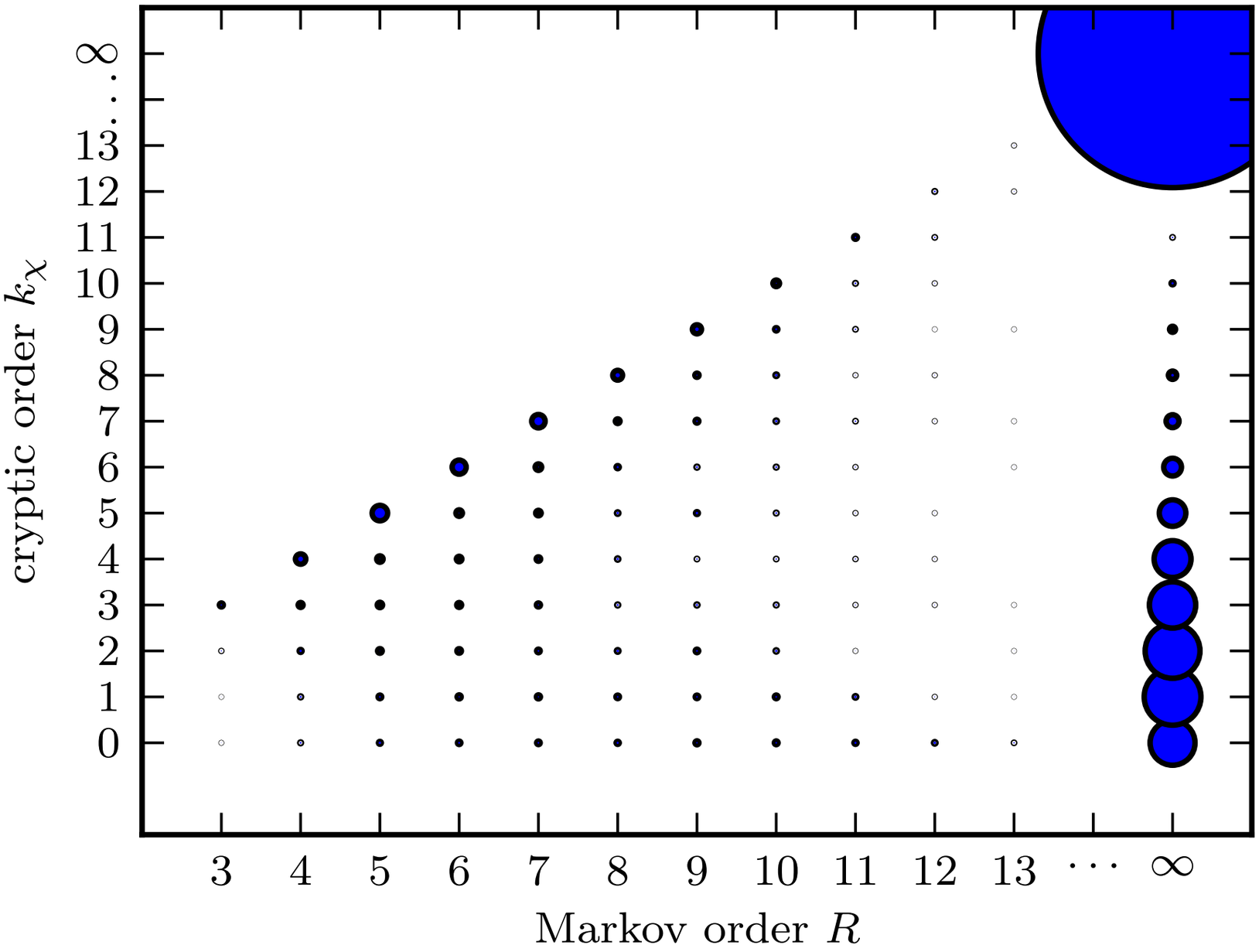
- Utilizes Bellman-Ford (Floyd-Warshall works too) to compute the longest path or detect cycles efficiently

R vs k_χ for 6-state ϵ-machines



Complexity Sciences Center
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UNIVERSITY OF CALIFORNIA