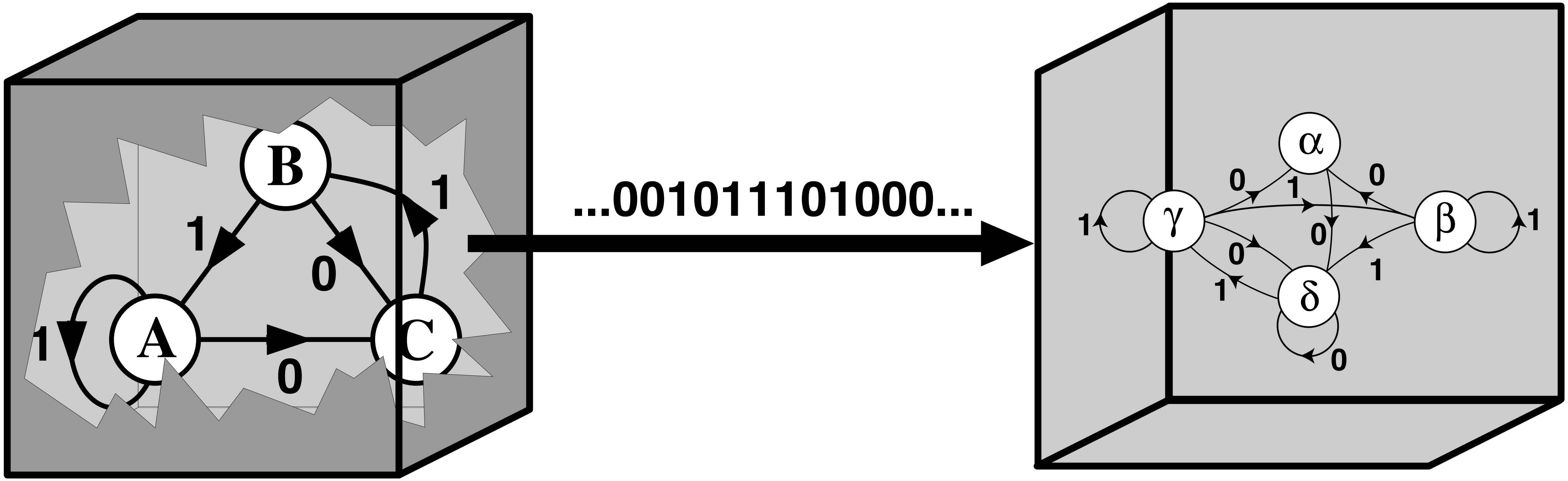
What We Didn't Cover: The Present and Future

Reading for this lecture:

Lecture Notes and cited papers.

Topics & Applications...

Course narrative:



System **Instrument** **Process** **Modeller**

Forms of Chaos:
- Deterministic sources of novelty
- Mechanisms that produce unpredictability
- Sensitive dependence on initial condition
- Sensitive dependence on parameter

Measurement Theory:
- Partitions
- Optimal Instrument:

$$\max_{\{\mathcal{P}\}} h_\mu$$

$$\min_{\{\mathcal{P}\}} C_\mu$$

How random?

$$\lambda_{\max}, H(L), h_\mu,$$
$$\mathbf{G}, \mathcal{R}$$

How structured?

$$C_\mu, \mathbf{E}, \mathbf{T}, \chi, \Xi$$

Universal model:
- $\epsilon-\mathrm{Machine}$
- Pattern defined
- Causal Architecture
- Intrinsic Computation

Topics & Applications...

What we didn't cover:

ε-Machine Enumeration
Infinite ε-Machines & Generalized Hidden Markov Models
Hierarchical ε-Machine Reconstruction
Quantum Processes & Quantum Machines
Rate Distortion Theory & Optimal Causal Inference
Cellular Automata Computational Mechanics
Spin Systems in 1D and 2D
Optimal Instrument Design
Complexity-Entropy Diagrams

Topics & Applications...

What we didn't cover ...

ε-Machine Enumeration

Every process has its ε-machine presentation

Space of processes = Space of ε-Machines

ε-Machines can be exactly enumerated

Alphabet

States

$n \backslash k$	2	3	4	5	6
1	3	7	15	31	63
2	7	141	1,873	20,925	213,997
3	78	15,598	1,658,606	136,146,590	
4	1,388	3,625,638			
5	35,186				
6	1,132,613				
7	43,997,426				
8	1,993,473,480				

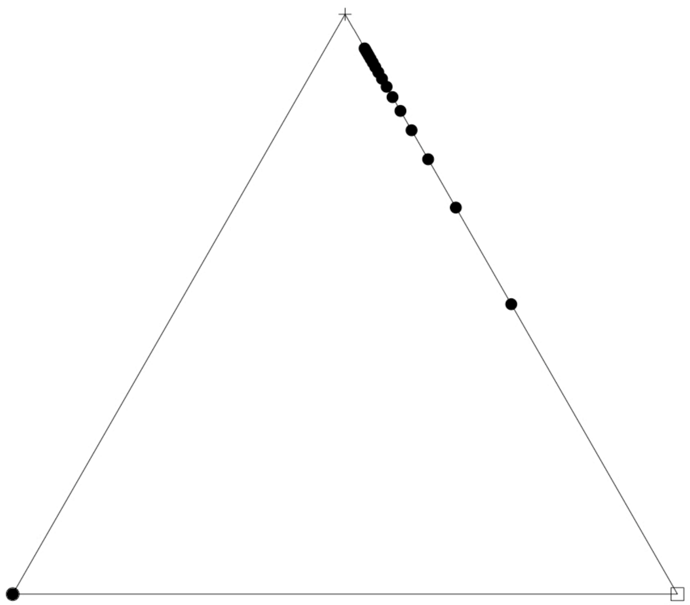
Unknown unknowns!

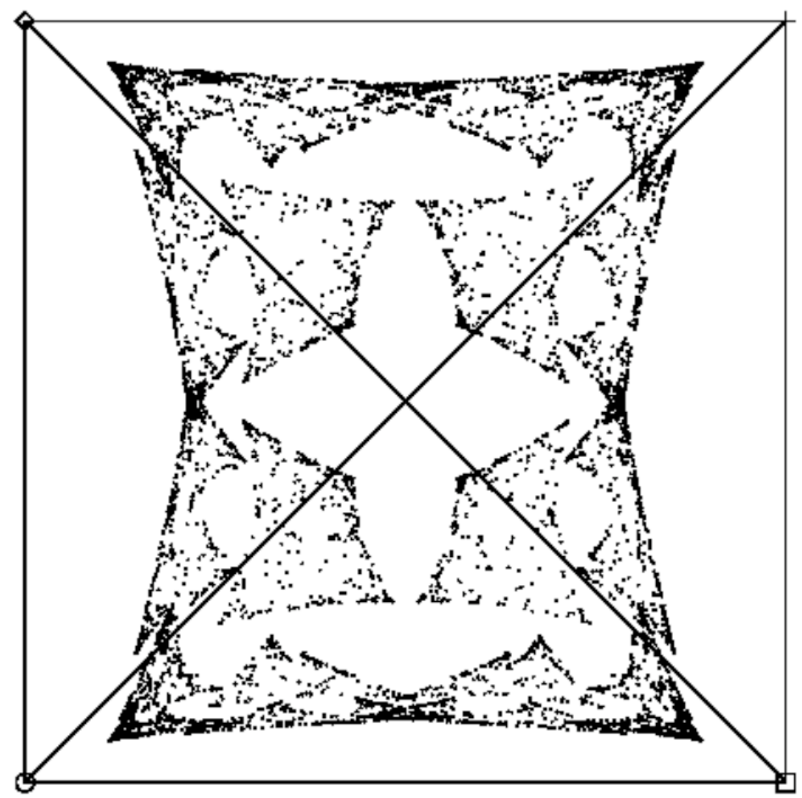
B. D. Johnson, J. P. Crutchfield, C. J. Ellison, and C. S. McTague, "Enumerating Finitary Processes", (2010) submitted.
Santa Fe Institute Working Paper 10-11-027. arXiv:1011.0036 [cs.FL].

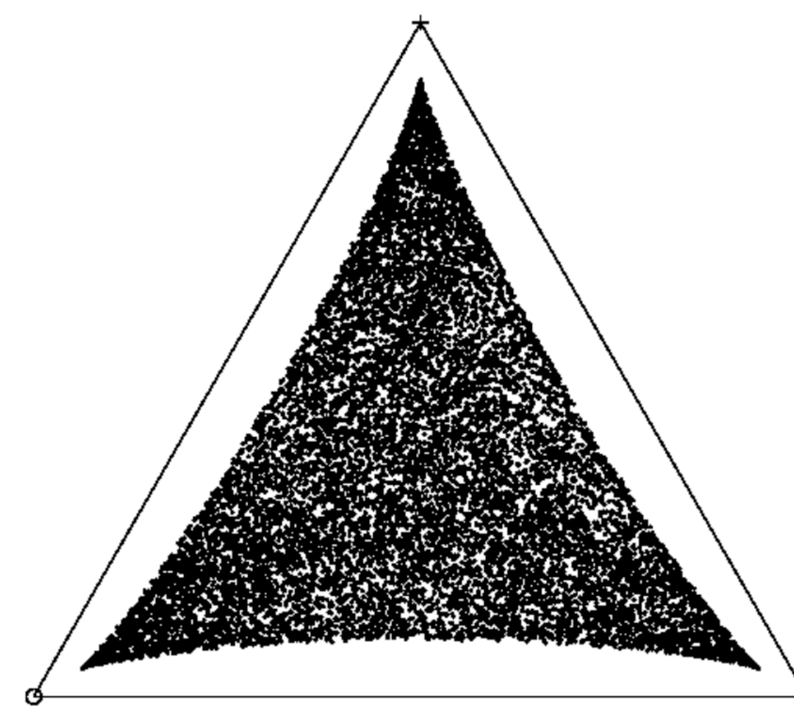
Topics & Applications...

What we didn't cover ...

Infinite ε-Machines & Generalized Hidden Markov Models





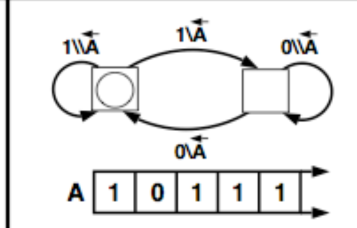
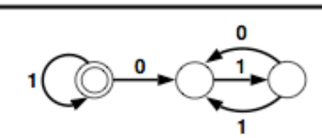
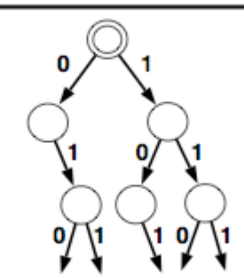
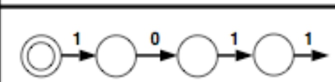


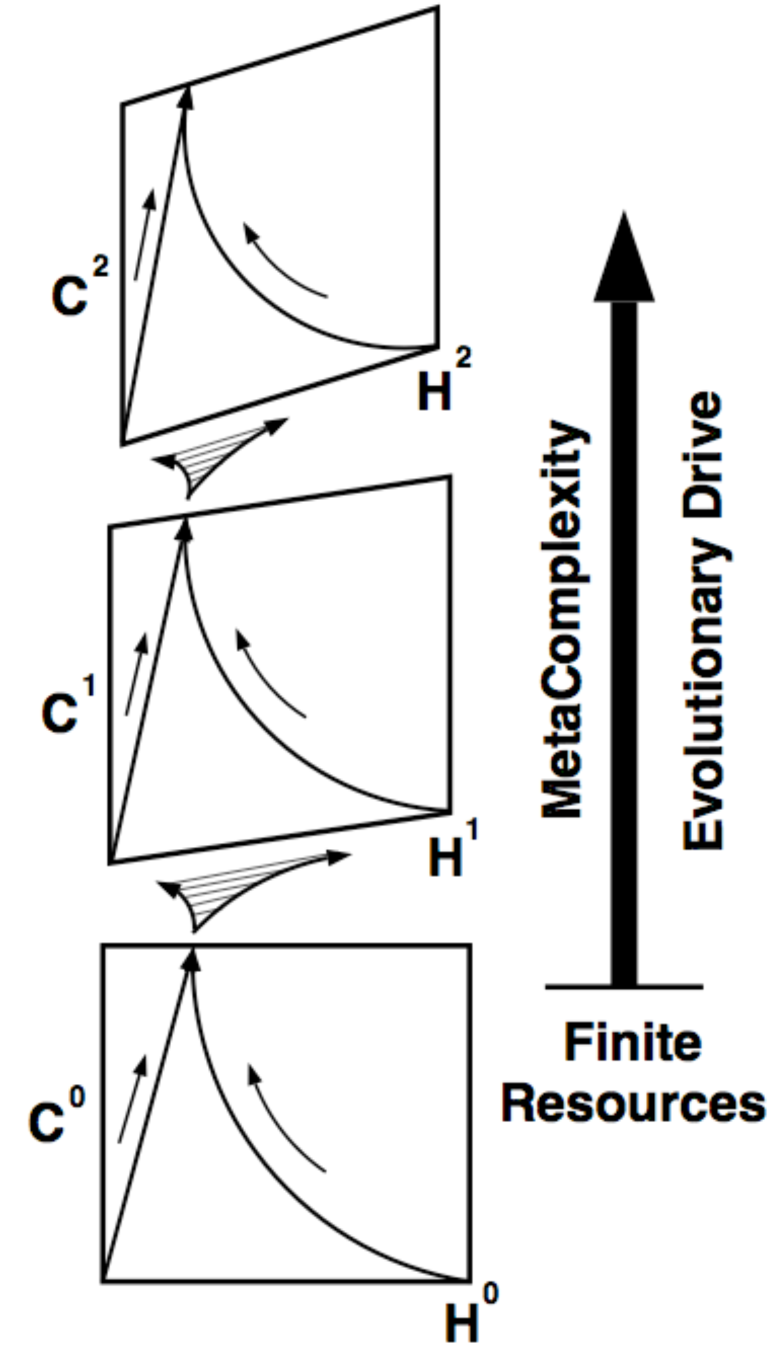
D. Upper, Ph.D. Dissertation (Mathematica, Berkeley, February 1997).
J. P. Crutchfield, "Calculi of Emergence:", Computation, Dynamics, and Induction", Physica D **75** (1994) 11-54.

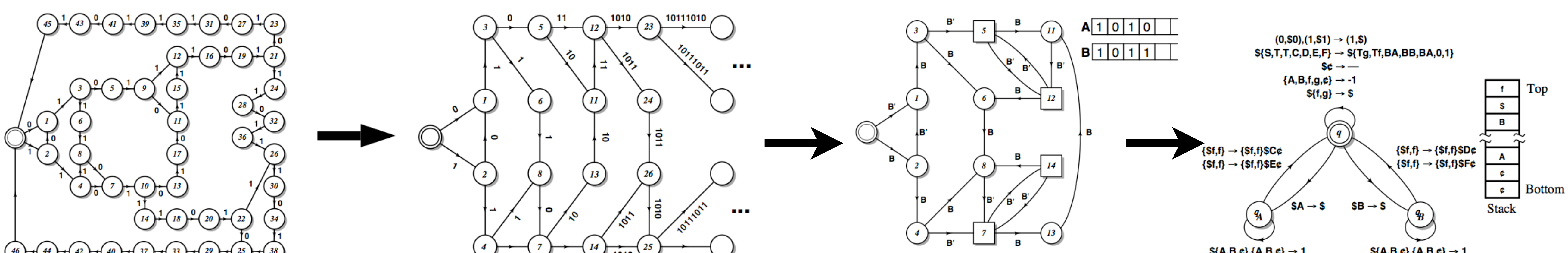
Topics & Applications...

What we didn't cover ...

Hierarchical ε-Machine Reconstruction

Level	Model Class	Machine	Model Size, if class is appropriate	Equivalence Relation
...	...	...	...	
3	String Production		$\mathcal{O}(\|V\| + \|E\| + \|P\|)$	Finitary-Recursive Conditional Independence
2	Finite Automata		$\mathcal{O}(\|V\| + \|E\|)$	Conditional Independence
1	Tree		$\mathcal{O}\left(\|\mathcal{A}\|^D\right)$	Block Independence
0	Data Stream		m	Measurement



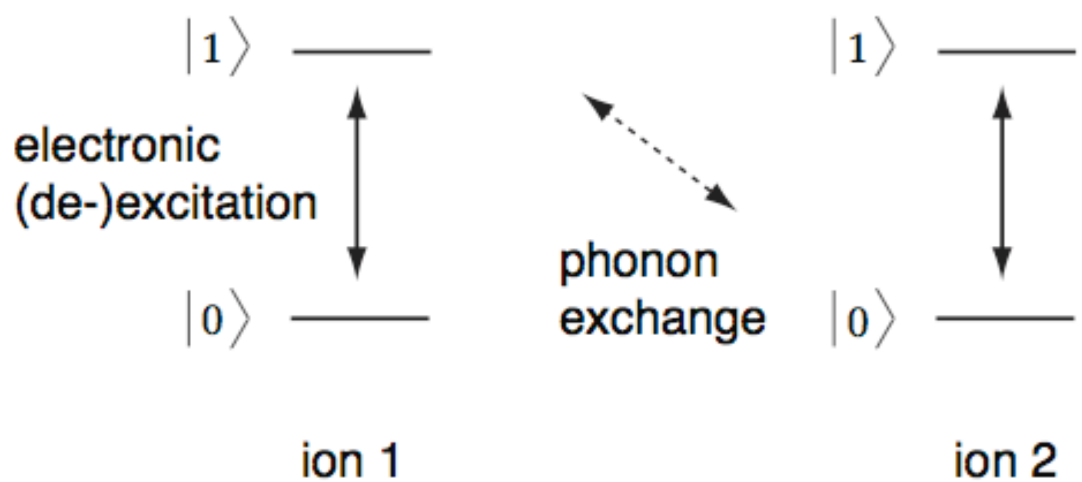


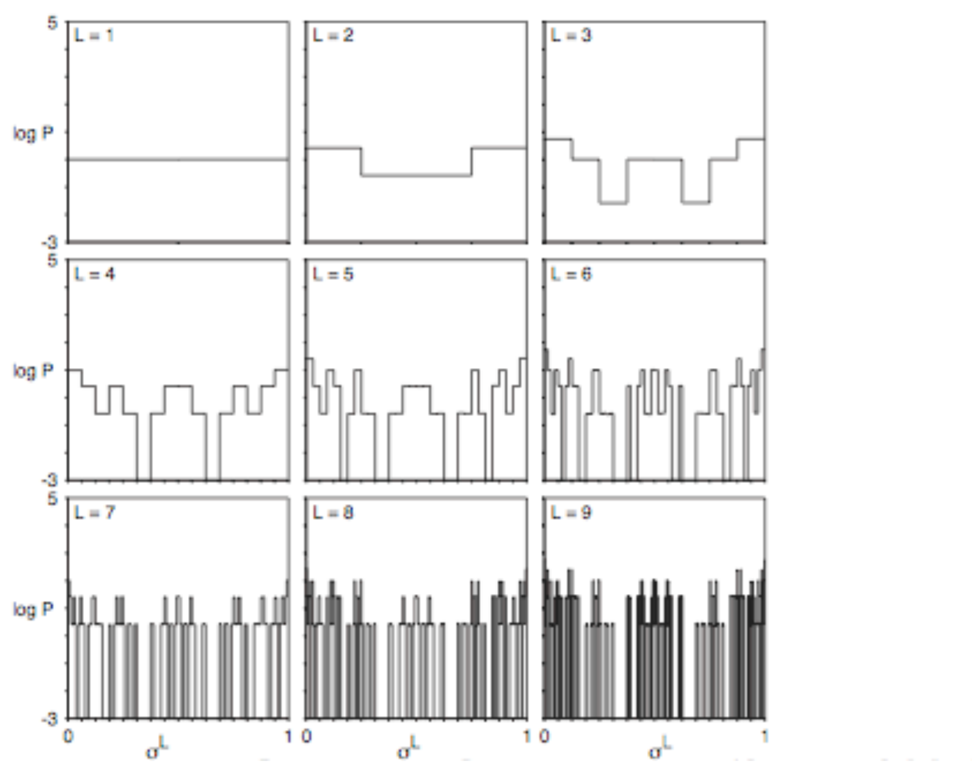
J. P. Crutchfield, "Calculi of Emergence:", Computation, Dynamics, and Induction", Physica D **75** (1994) 11-54.

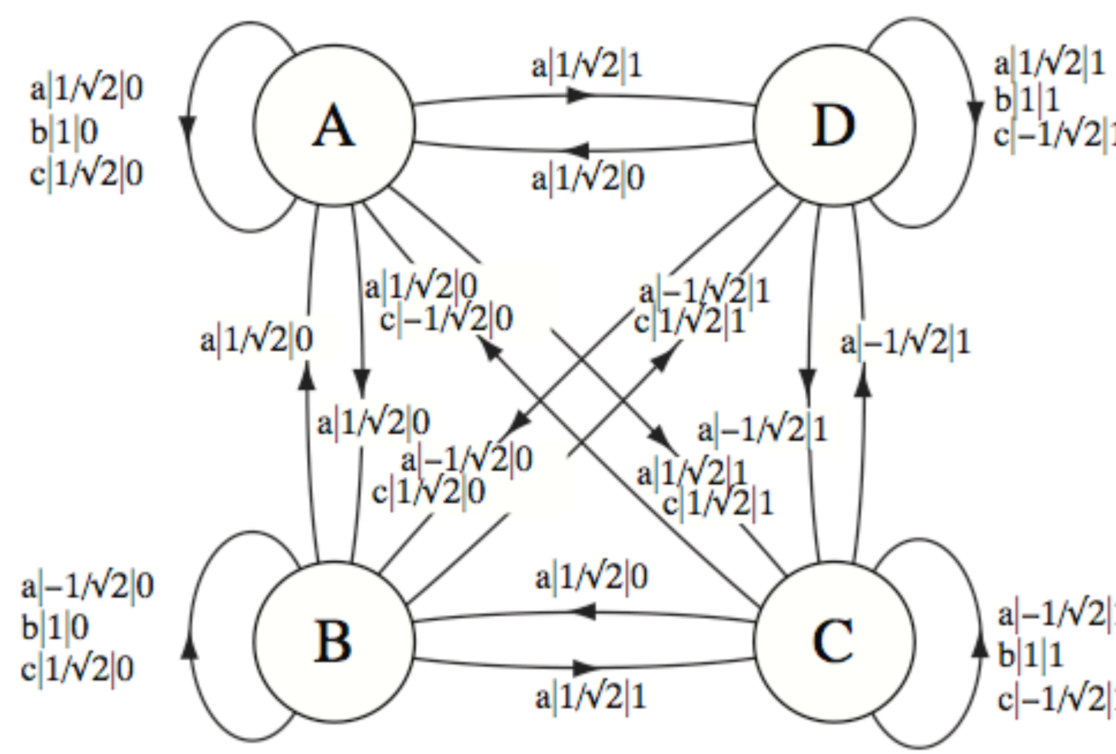
Topics & Applications...

What we didn't cover ...

Quantum Processes & Quantum Machines







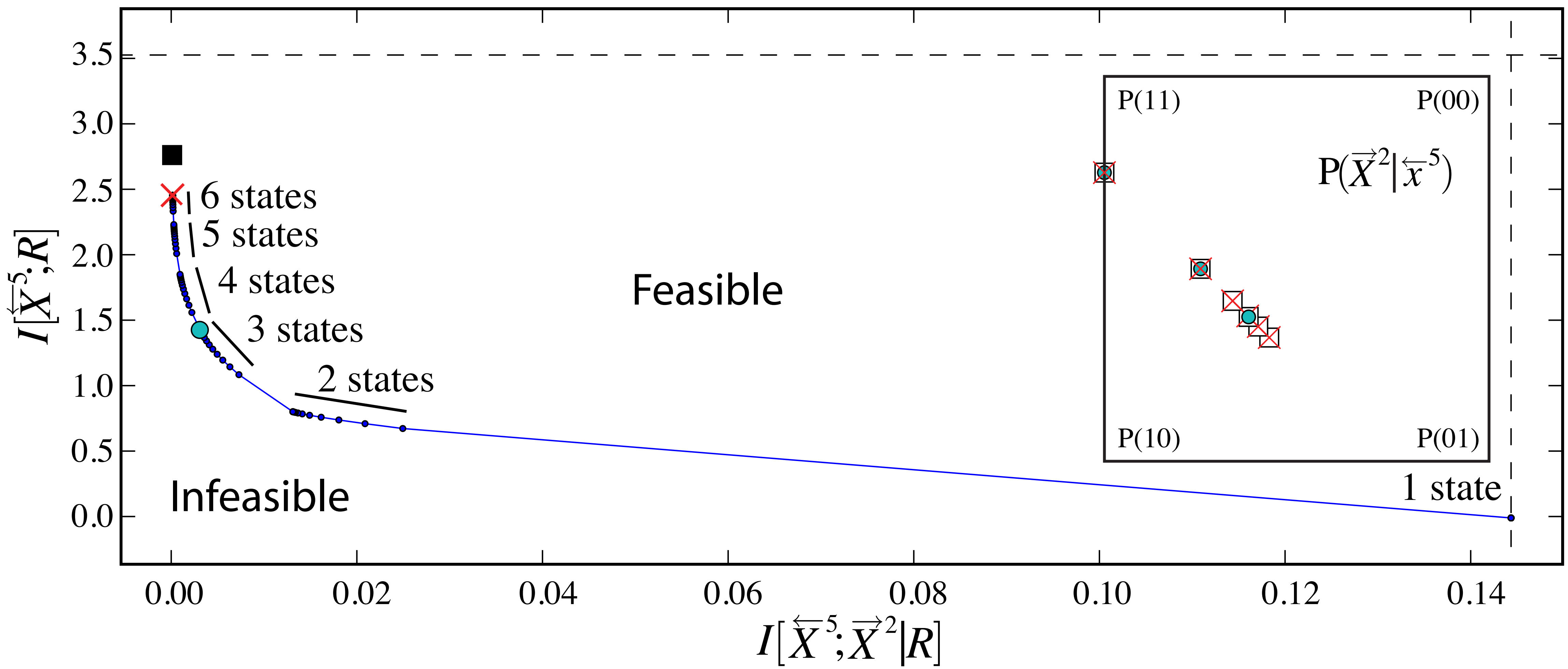
K. Wiesner and J. P. Crutchfield, "Computation in Finitary Stochastic and Quantum Processes", Physica D **237**:9 (2008) 1173-1195.

Topics & Applications...

What we didn't cover ...

Rate Distortion Theory & Optimal Causal Inference

$$\min_{\Pr(\mathcal{R}|\overleftarrow{X})} \left(I[\overleftarrow{X};\mathcal{R}] + \beta I[\overleftarrow{X};\overrightarrow{X}|\mathcal{R}] \right)$$

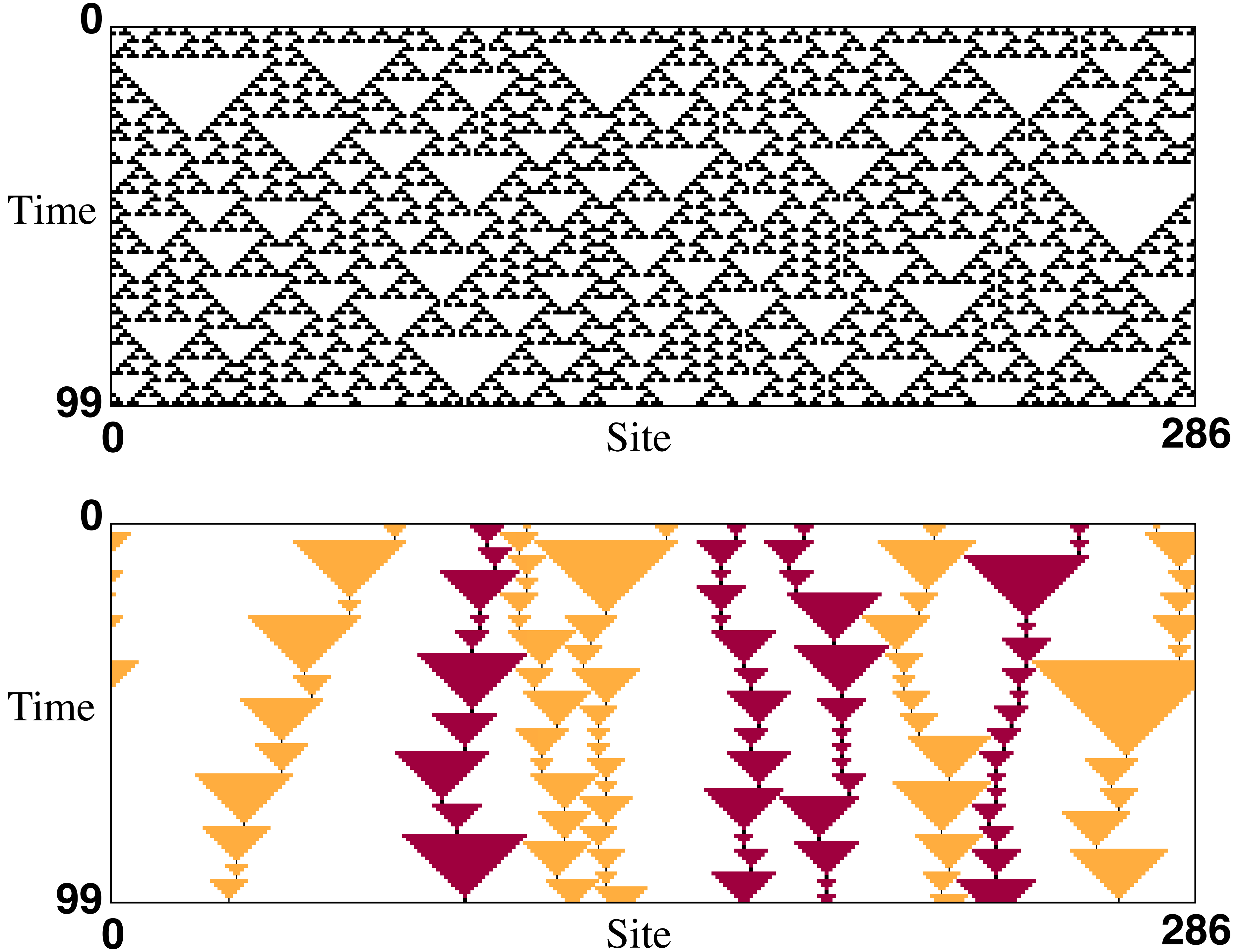


S. Still, J. P. Crutchfield, and C. J. Ellison "Optimal Causal Inference: Estimating Stored Information and Approximating Causal Architecture", CHAOS **20**:3 (2010) 037111.

Topics & Applications...

What we didn't cover ...

Cellular Automata Computational Mechanics:

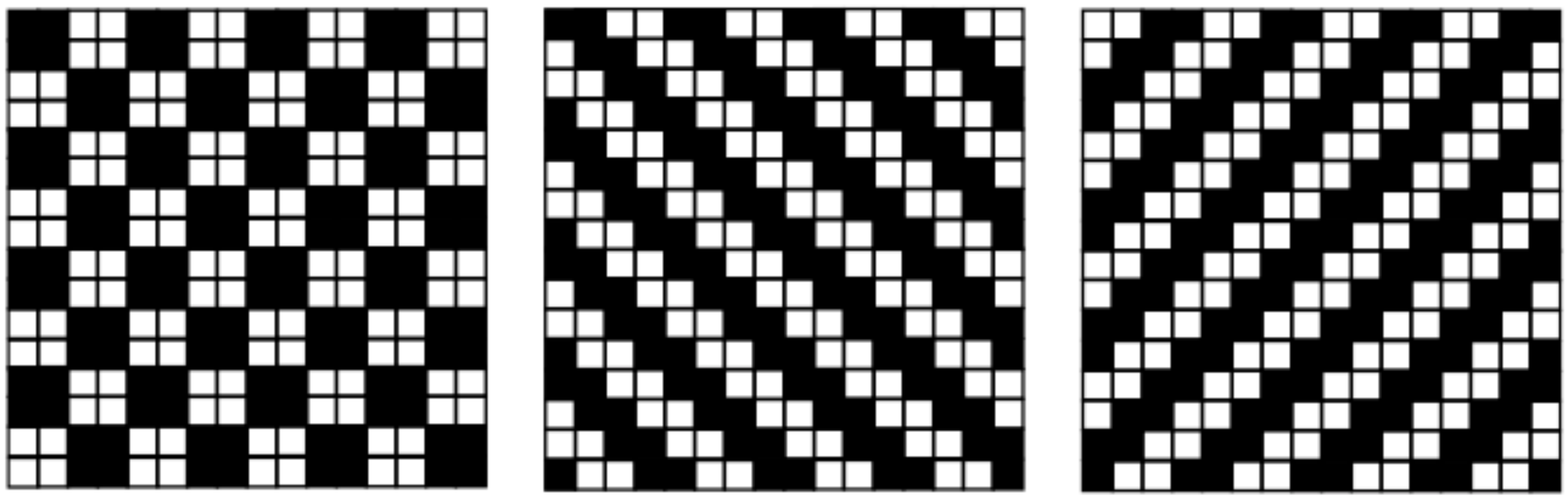


J. E. Hanson and J. P. Crutchfield, "Computational Mechanics of Cellular Automata: An Example", Physica D 103 (1997) 169-189.

Topics & Applications...

What we didn't cover ...

Spin Systems in 1D and 2D

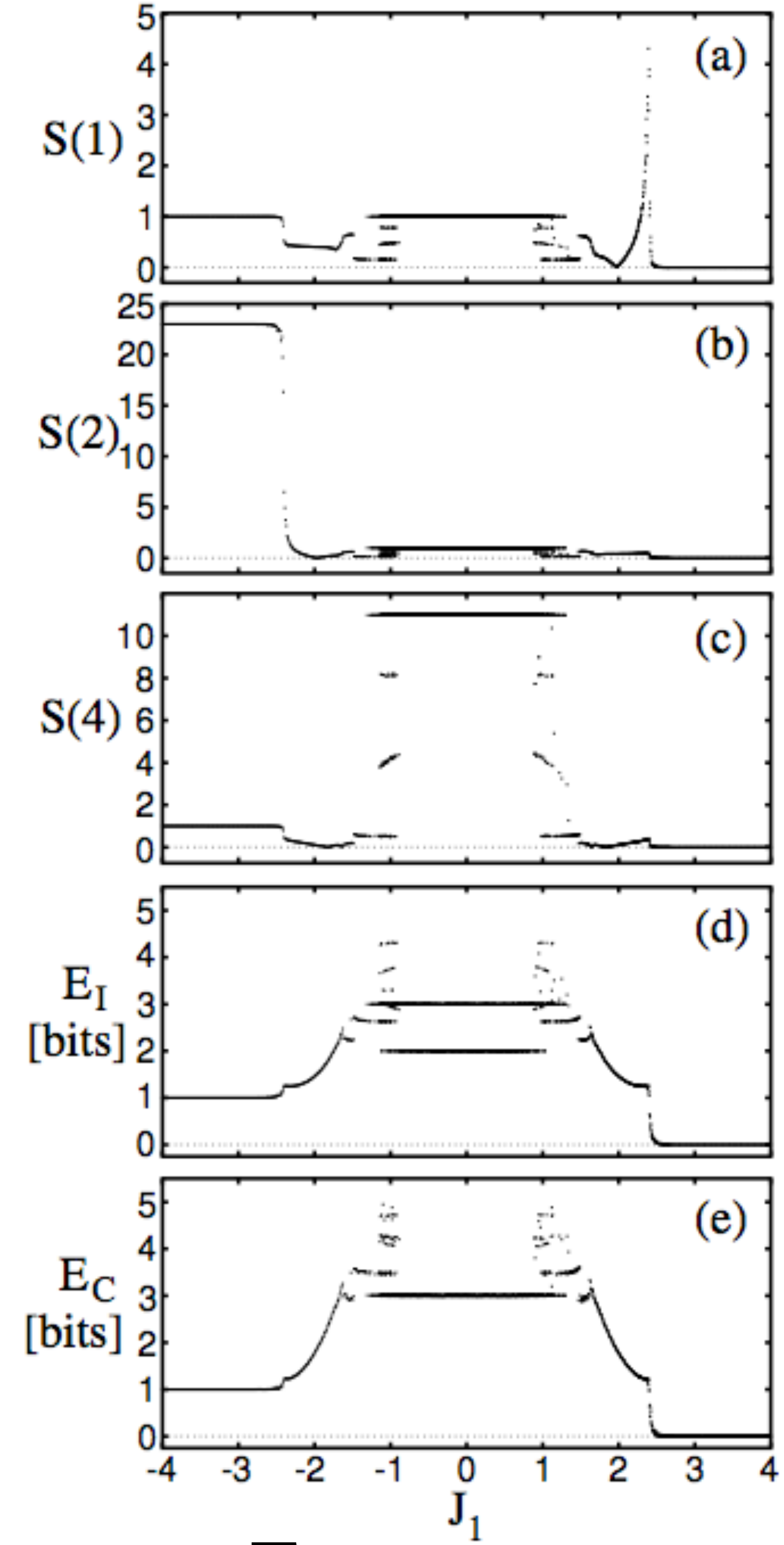


$$\mathbf{E}_{\mathrm{I}} \equiv \lim_{M,N\to\infty} I\left[\ \updownarrow N \ \boxed{} \ ; \ \boxed{} \ \updownarrow N\ \right]$$

$$H(M,N) = H\left[\ \overset{\longleftarrow M \longrightarrow}{\boxed{}} \ \updownarrow N\ \right]$$

$$\sim \ \mathbf{E}_{\mathrm{S}} + \mathbf{E}^x_{\mathrm{S}} M + \mathbf{E}^y_{\mathrm{S}} N + h_\mu M N$$

D. P. Feldman and J. P. Crutchfield, "Structural Information in Two-Dimensional Patterns: Entropy Convergence and Excess Entropy", Physical Review E **67** (2003) 051104.

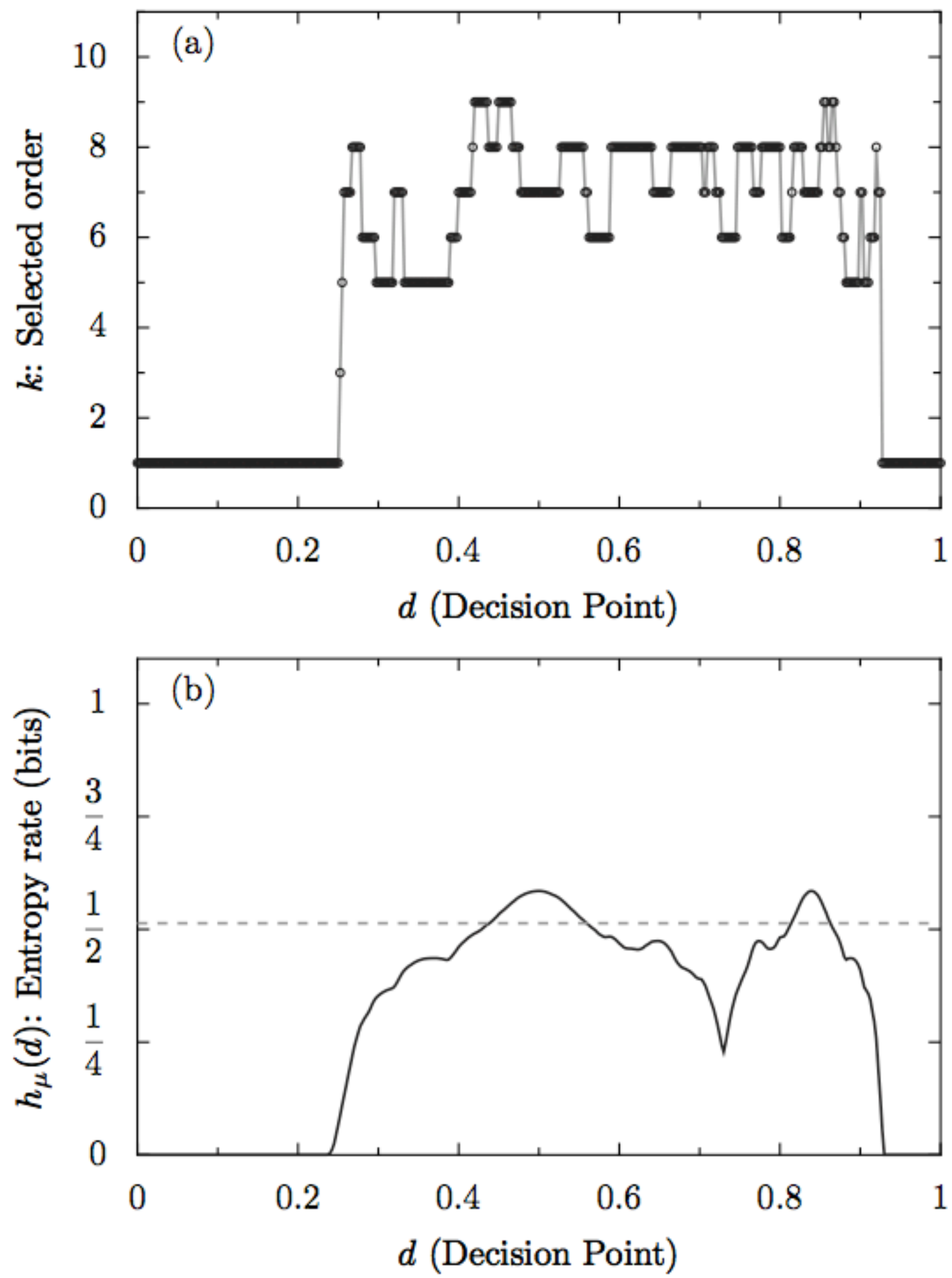


E: Generalized Order Parameter!

Topics & Applications...

What we didn't cover ...

Optimal Instrument Design



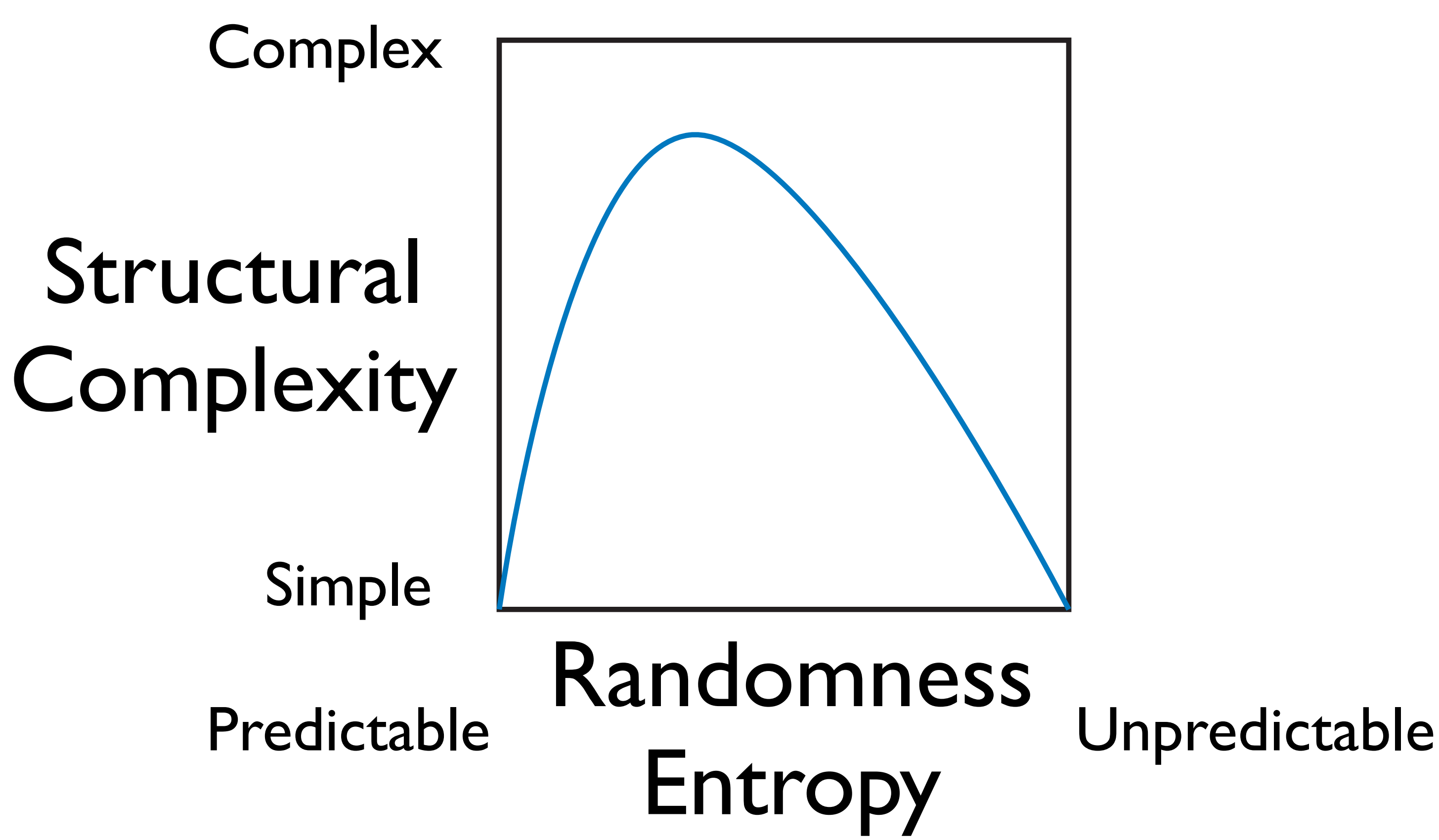
C. C. Strelioff and J. P. Crutchfield, "Optimal Instruments and Models for Noisy Chaos", CHAOS 17 (2007) 043127.

Topics & Applications...

What we didn't cover ...

Complexity-Entropy Diagrams:

Analyze a class of processes



Analogous to Thermodynamic Phase Diagram (gas, liquid, solid).

But uses only intrinsic computation properties.

A wide diversity of Complexity-Entropy Diagrams.

D. P. Feldman, Carl S. McTague, and J. P. Crutchfield, "The Organization of Intrinsic Computation: Complexity-Entropy Diagrams and the Diversity of Natural Information Processing", CHAOS 18:4 (2008) 53-73.

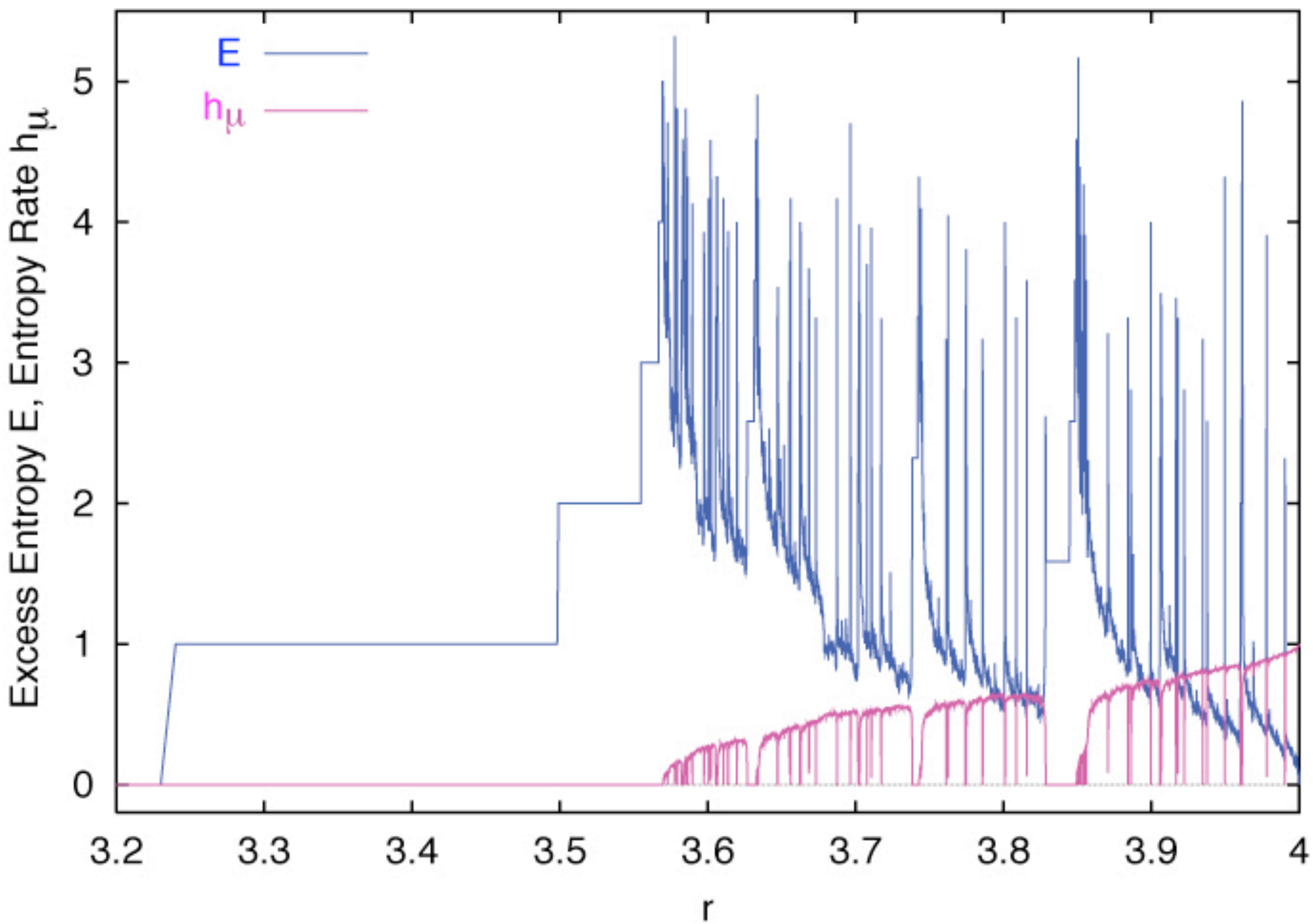
Topics & Applications...

What we didn't cover ...

Complexity-Entropy Diagrams:

Analyze a class of processes

Logistic Map



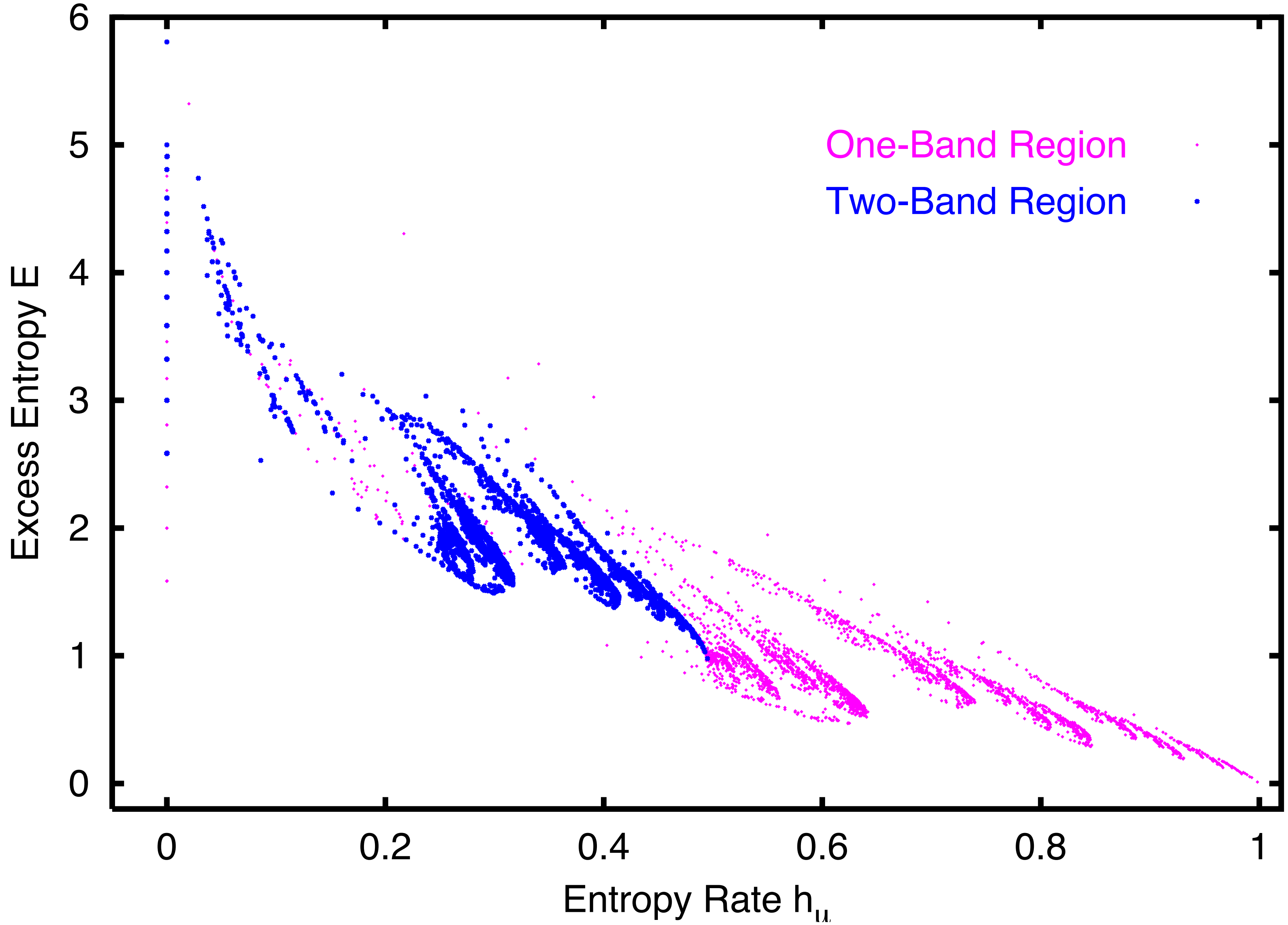
Topics & Applications...

What we didn't cover ...

Complexity-Entropy Diagrams:

Analyze a class of processes

Logistic Map



Last Lecture: Natural Computation & Self-Organization, Physics 256 (Spring 2014); Jim Crutchfield

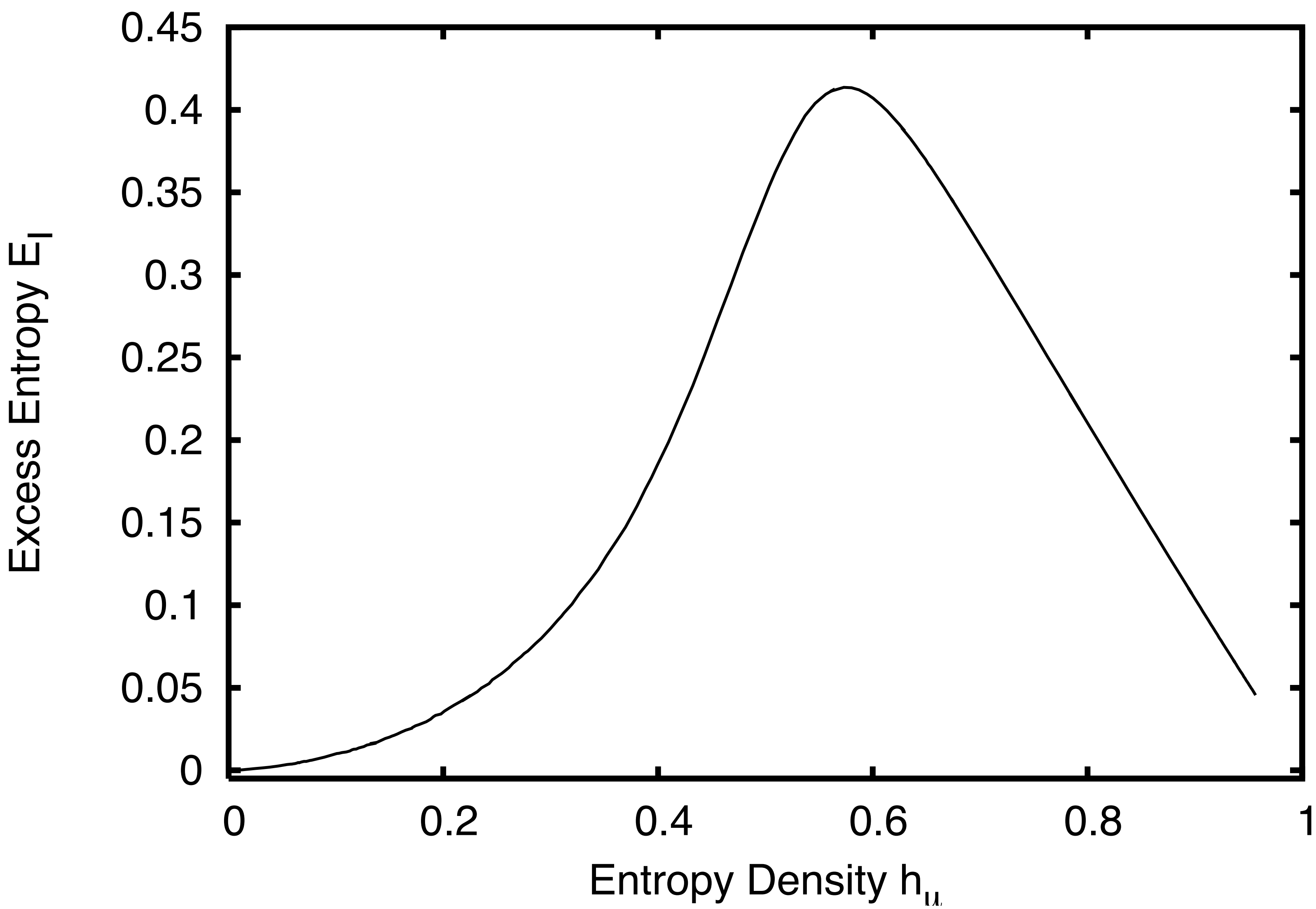
Topics & Applications...

What we didn't cover ...

Complexity-Entropy Diagrams:

Analyze a class of processes

2D Ising Spin System

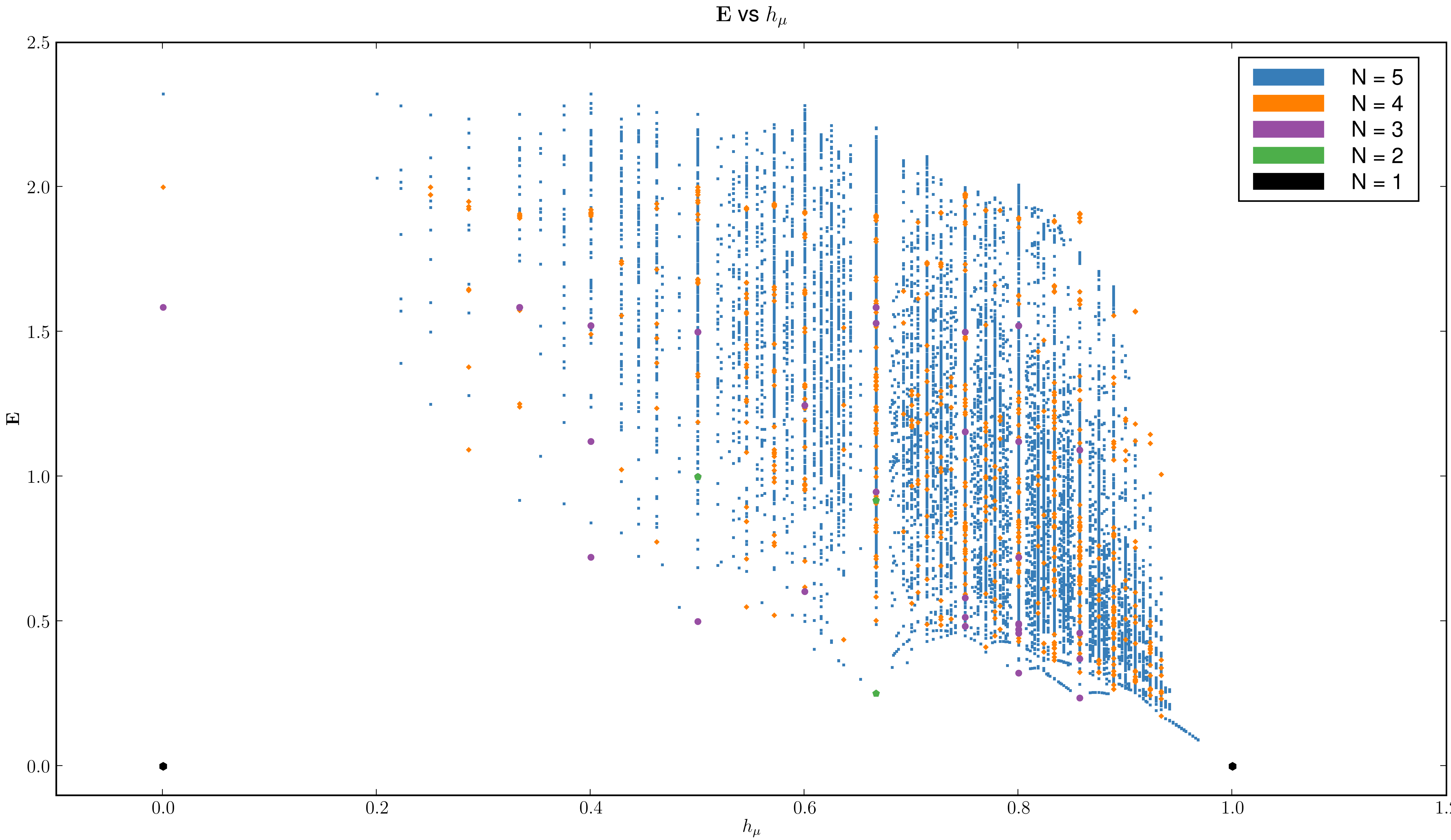


Topics & Applications...

Complexity-Entropy Diagram:

Analyze a class of processes ...

ε-Machines



Topics & Applications...

Computational Mechanics Applications by Others:

- Chaotic Dynamical Systems
- Symbolic Dynamics
- Statistical Mechanical Models: Spin systems (Ising, glasses, ...)
- Cellular Automata
- Hidden Markov Models
- Chaotic Crystallography
- Hydrodynamics: Dripping faucet, turbulence
- Quantum Dynamical Systems
- Single Molecule Dynamics

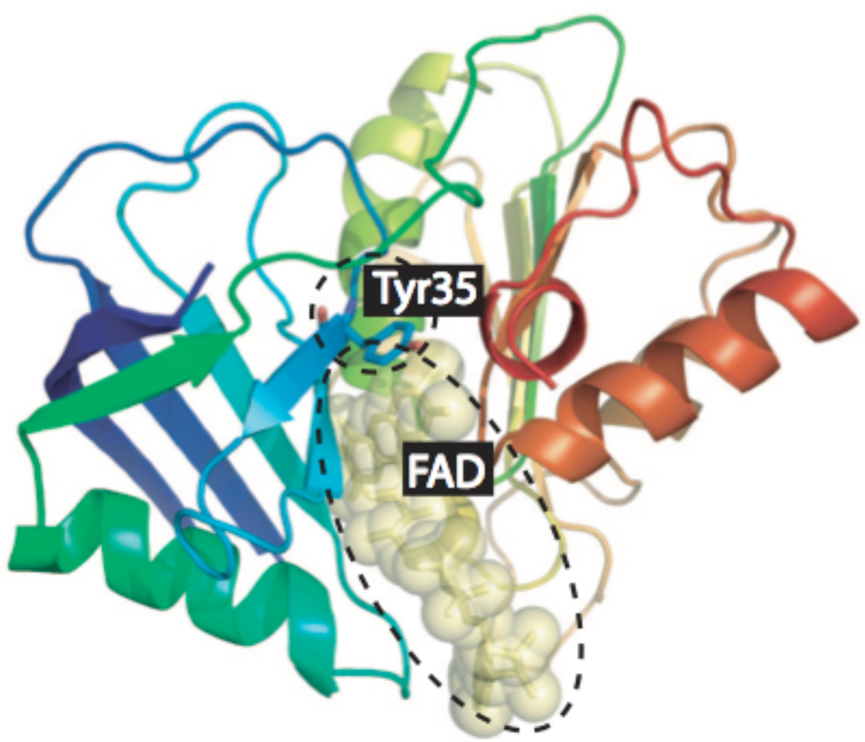
Topics & Applications...

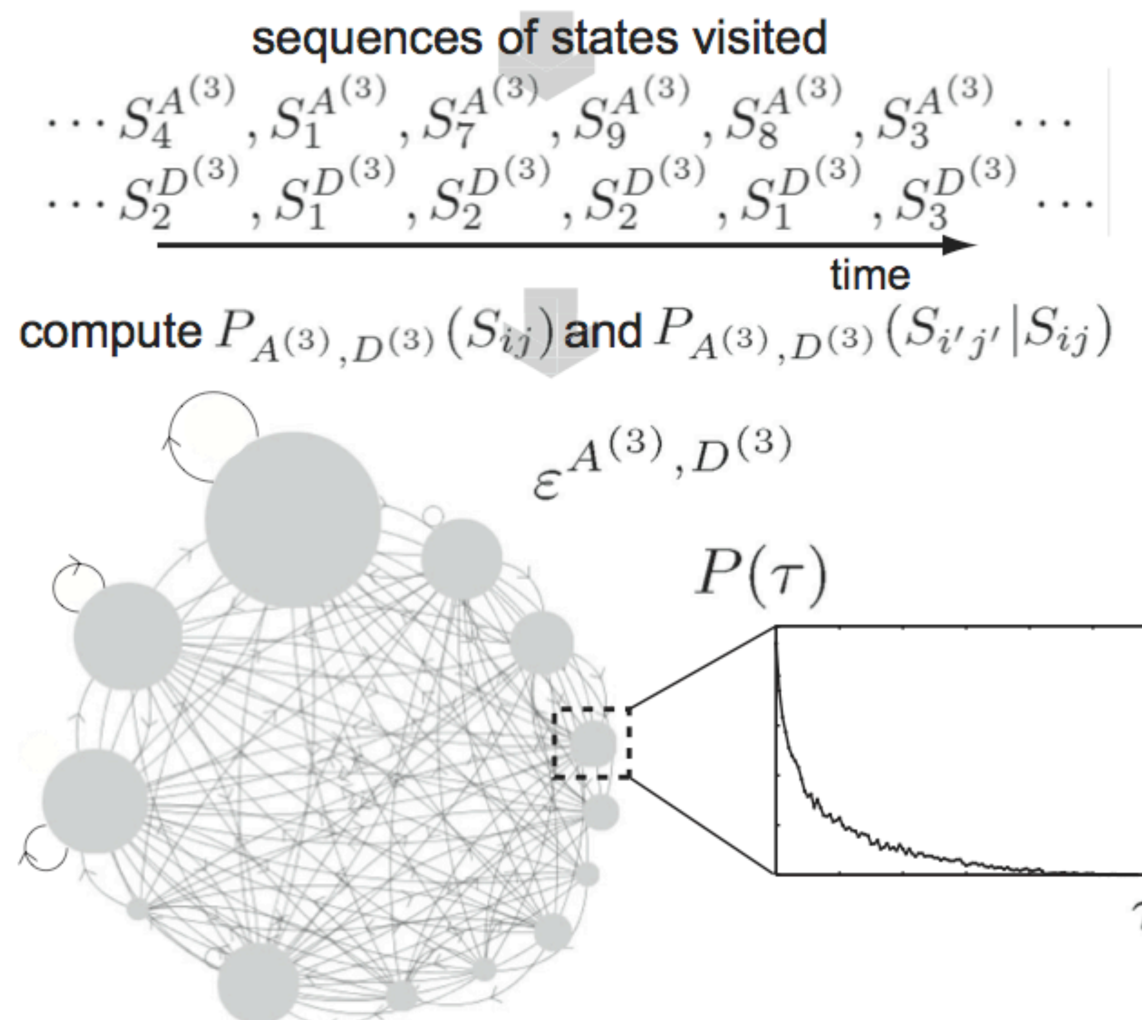
Molecular Dynamics Spectroscopy:

Multiscale complex network of protein conformational fluctuations in single-molecule time series

Chun-Biu Li*†‡, Haw Yang§¶, and Tamiki Komatsuzaki*†‡||

*Nonlinear Sciences Laboratory, Department of Earth and Planetary Sciences, Faculty of Science, Kobe University, Nada, Kobe 657-8501, Japan; †Core Research for Evolutional Science and Technology (CREST), Japan Science and Technology Agency (JST), Kawaguchi, Saitama 332-0012, Japan; §Department of Chemistry, University of California, Berkeley, CA 94720; and ¶Physical Biosciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720





C.-B. Li, H. Yang, & T. Komatsuzaki, Proc. Natl. Acad. Sci USA **105**:2 (2008) 536–541.

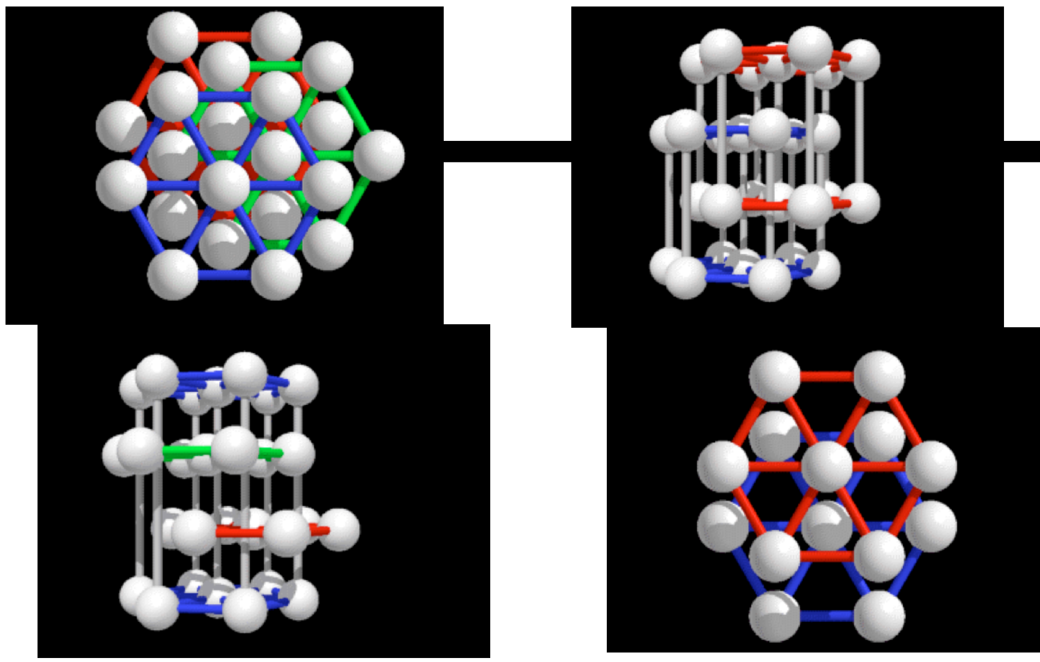
Topics & Applications...

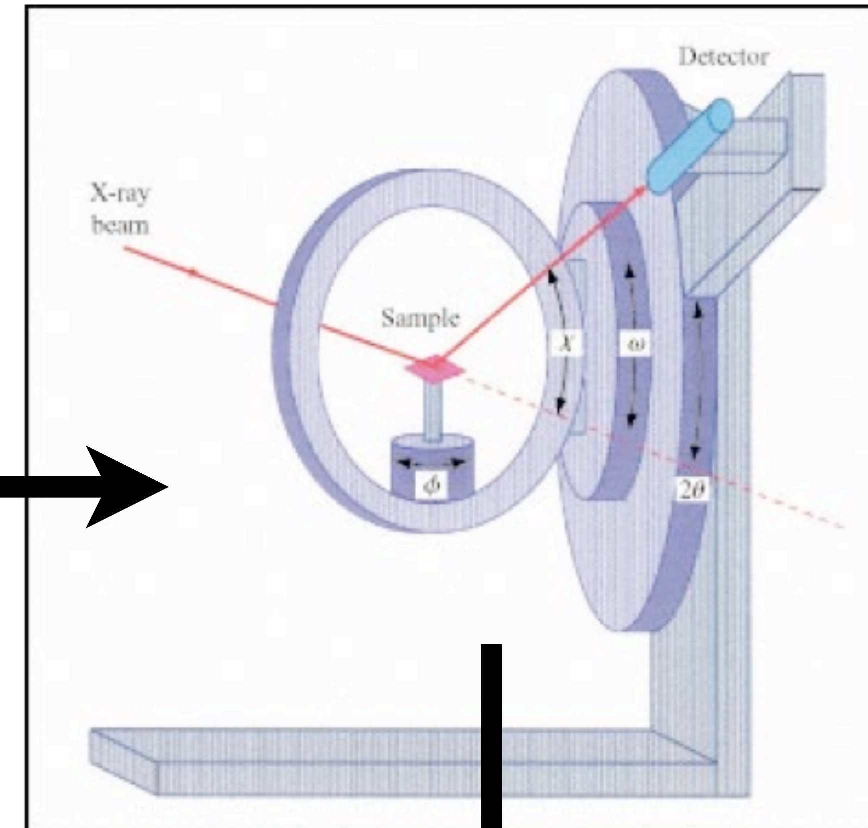
Computational Mechanics Research, Ongoing:

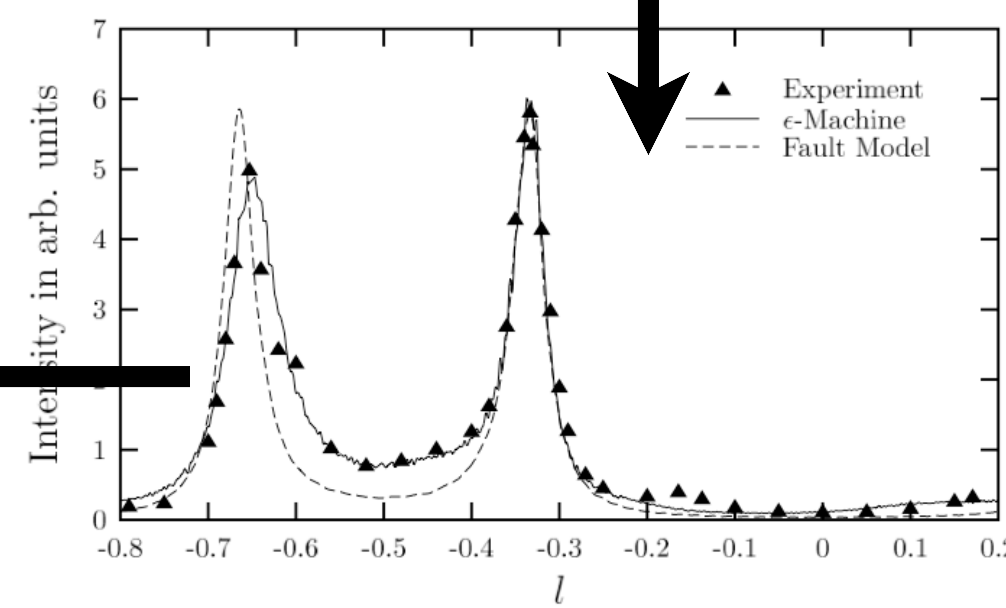
Novel materials
Quantum chaotic dynamics & measurement effects
Continuous processes
Spatiotemporal processes
Interactive learning
Network dynamics
Neurobiological processes
Multiagent systems
Distributed robotics
Evolutionary dynamics
...

Topics & Applications...

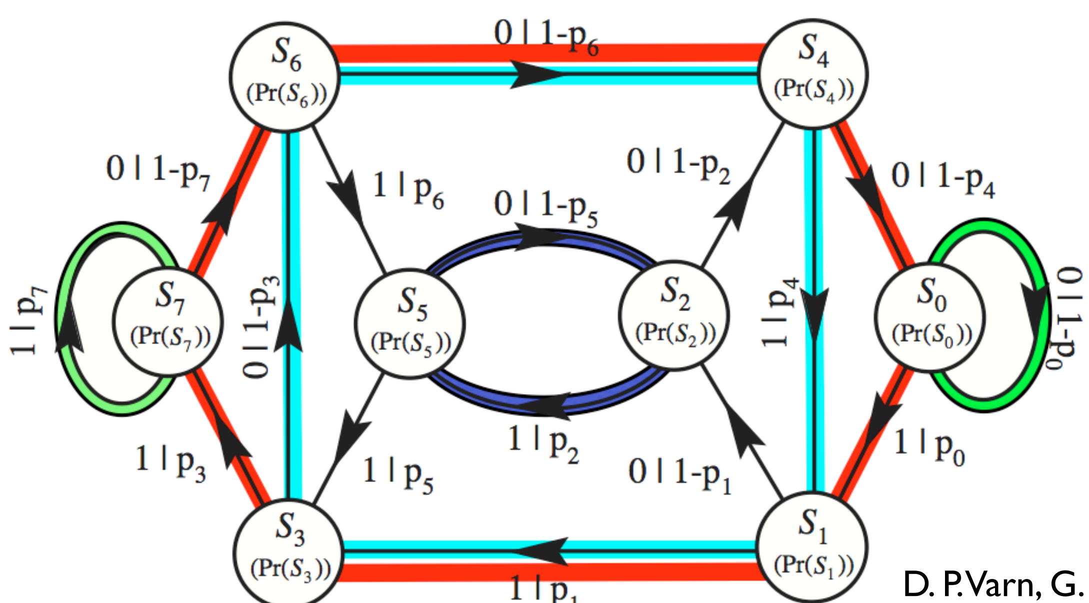
Computational Mechanics Research ...
Complex Materials &
ε-Machine Spectral Reconstruction







ε-MSR



D. P. Varn, G. S. Canright, and J. P. Crutchfield, "ε-Machine spectral reconstruction theory: A direct method for inferring planar disorder and structure from X-ray diffraction studies", Acta Cryst. Sec. A **69**:2 (2013) 197-206.

Topics & Applications...

Computational Mechanics Research ...

Novel materials:
Use eMSR to design
Run eMSR backwards

Hypothesis: Structure key to properties.
But we can now analyze structure.

E.g., Design polytypes = Exotic semiconductors

Topics & Applications...

Computational Mechanics Research ...

Spectral Decomposition of Intrinsic Computation:

Exact, closed-form expression of all information measures using ε-machine presentation.

How?

Mixed states + Eigenspectrum of ε-machine.

J. P. Crutchfield, C. J. Ellison, P. Riechers in preparation.

Topics & Applications...

Computational Mechanics Research ...

Spectral decomposition of intrinsic computation ...

Recall mixed state expression:

$$h_\mu(L) = H\big[X_{L-1} | X_0^{L-1}, S_0 \sim \mu_0(\lambda)\big]$$
$$= H\big[X_{L-1} | \big(R_{L-1} | R_0 = \mu_0(\lambda)\big)\big]$$

Need only track evolution of mixed states via T^L:

Track analytically using spectral decomposition:

$$T^L = \sum_{\lambda \in \Lambda_T} \lambda^L T_\lambda$$

Eigenvalues: $\Lambda_T \equiv \{\lambda \in \mathbb{C} : \det(\lambda I - T) = 0\}$

Projection operators: $T_\lambda = \prod_{\substack{\zeta \in \Lambda_T \\ \zeta \neq \lambda}} \frac{T - \zeta I}{\lambda - \zeta}$

J. P. Crutchfield, C. J. Ellison, P. Riechers in preparation.

Topics & Applications...

Computational Mechanics Research ...

Spectral decomposition of intrinsic computation ...

Excess entropy: $$\mathbf{E} \equiv \sum_{L=1}^{\infty} [h_\mu(L) - h_\mu]$$

$$= \sum_{\substack{\lambda \in \Lambda_T \\ |\lambda| < 1}} \frac{1}{1-\lambda} \langle \delta_\pi | T_\lambda | H(\{T^{(s)}\}) \rangle$$

Synchronization information:

$$\mathbf{S} = \begin{cases} \sum_{\substack{\lambda \in \Lambda_T \\ |\lambda| < 1}} \frac{1}{1-\lambda} \langle \delta_\pi | T_\lambda | H[\eta] \rangle & \text{if } \langle \pi_T | H[\eta] \rangle = 0 \\ \infty & \text{otherwise} \end{cases}$$

Transient information: $$\mathbf{T} \equiv \sum_{L=1}^{\infty} L \, [h_\mu(L) - h_\mu]$$

$$= \sum_{\substack{\lambda \in \Lambda_T \\ |\lambda| < 1}} \frac{1}{(1-\lambda)^2} \langle \delta_\pi | T_\lambda | H(\{T^{(s)}\}) \rangle$$

J. P. Crutchfield, C. J. Ellison, P. Riechers in preparation.

Topics & Applications...

Computational Mechanics Research ...

ε-Transducers: Input-output functions!

- ε-Machines: Behaviors and distributions over them
- Now, mappings on stochastic behaviors
- Optimal, minimal, unique presentation of mappings

Noisy communication channel:

Input process

$$\ldots, X_{-1}, X_0, X_1, \ldots$$

Output process

$$\ldots, Y_{-1}, Y_0, Y_1, \ldots$$

Joint process

$$\ldots, (X,Y)_{-1}, (X,Y)_0, (X,Y)_1, \ldots$$

N. Barnett, C. J. Ellison, J. P. Crutchfield, in preparation.

Topics & Applications...

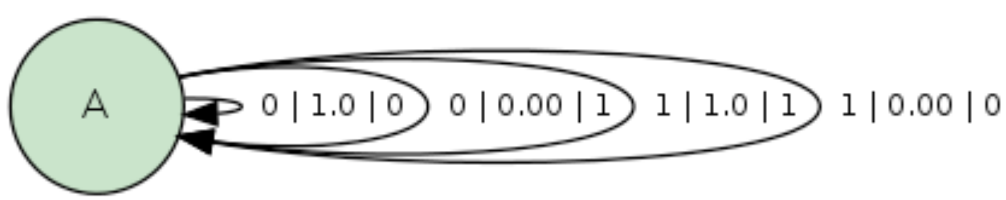
Computational Mechanics Research ...

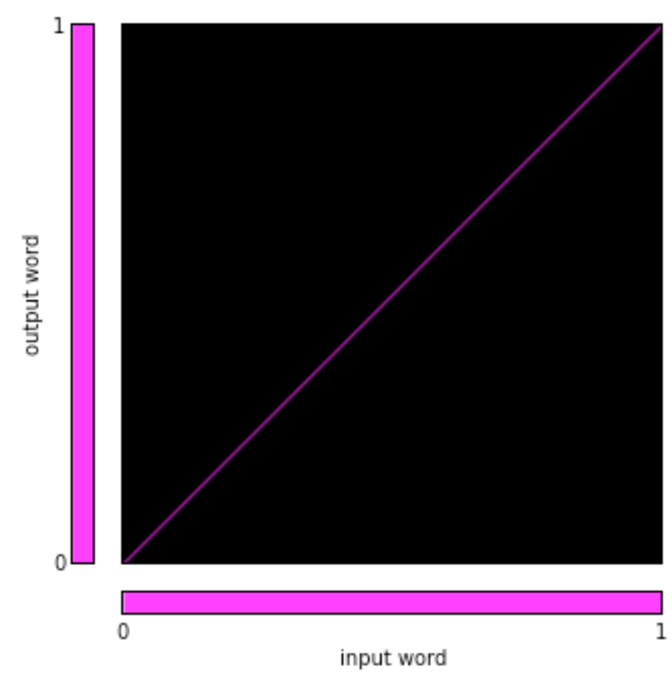
ε-Transducers ...

Transitions: x|p|y
- x is input symbol,
- y is output symbol, and
- p is probability

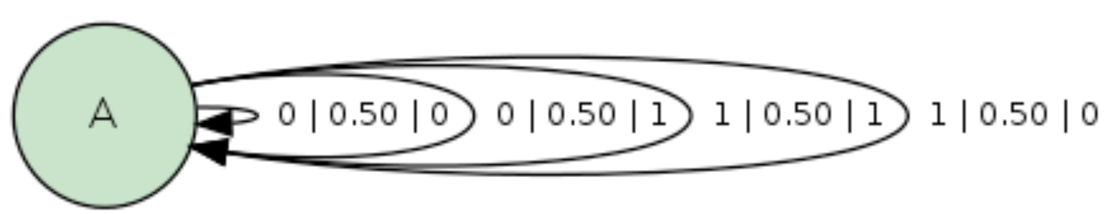
Memoryless channels:

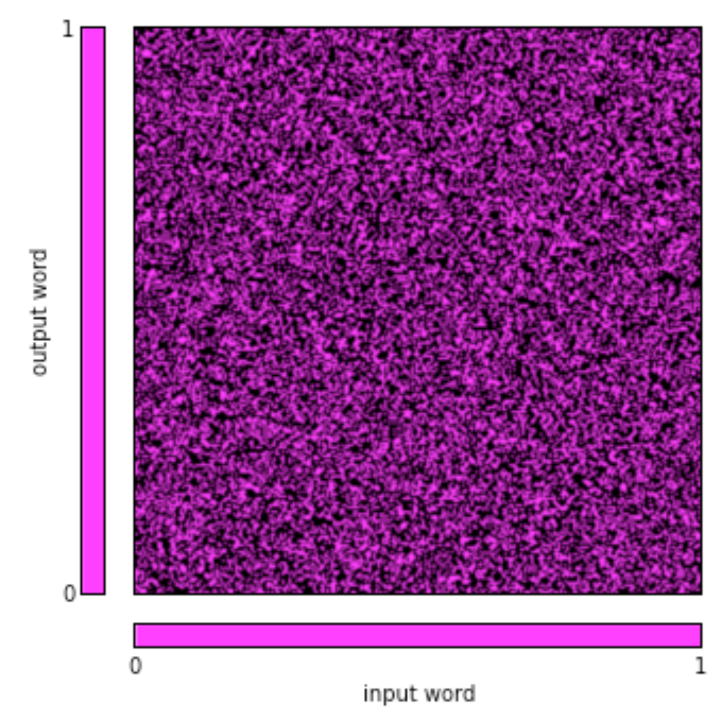
Identity channel.





All-to-fair channel.





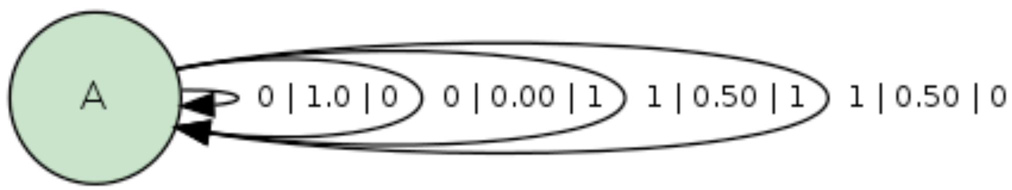
Topics & Applications...

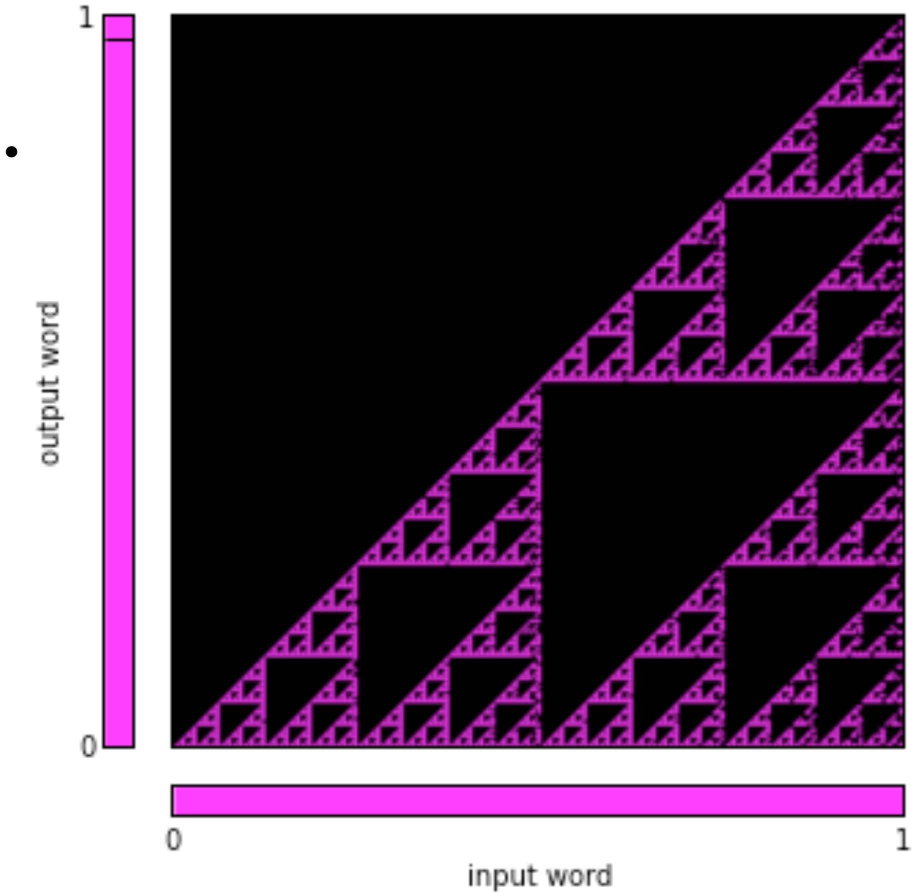
Computational Mechanics Research ...

ε-Transducers ...

Memoryless channels ...

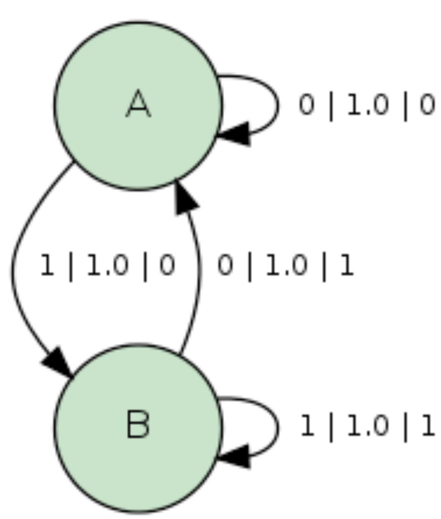
Z Channel: Fuzz out ones, transmit zeros.

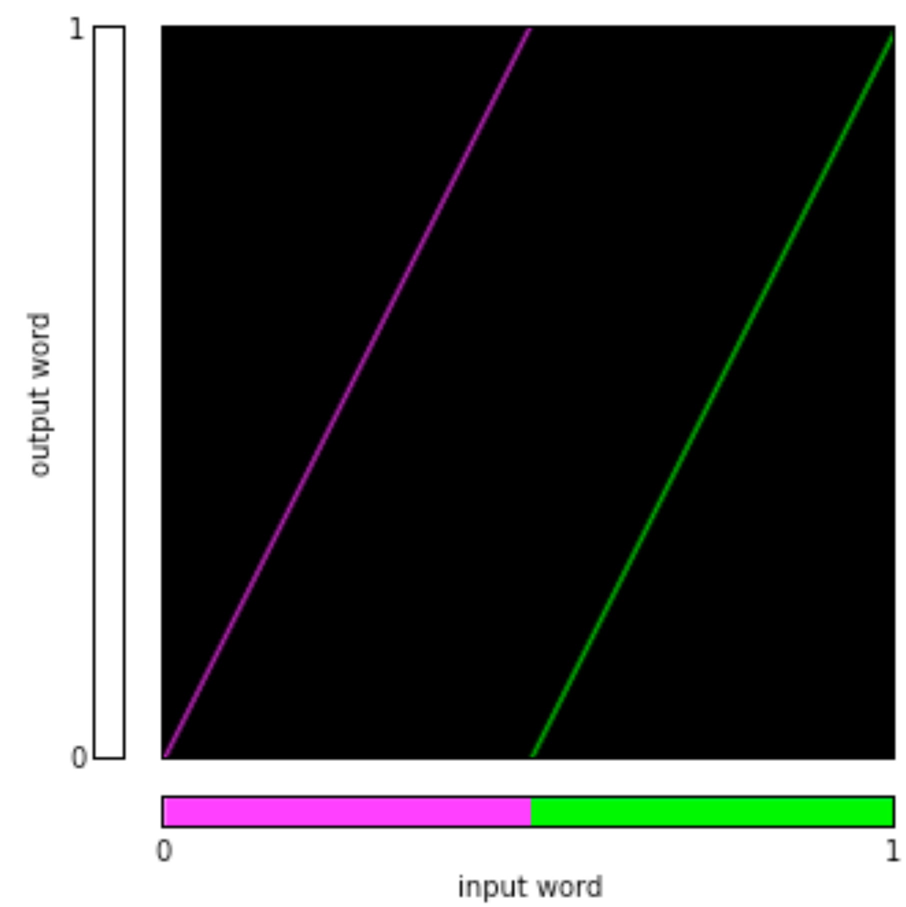




Memoryful Channel:

Delay-by-1 Channel:
Input is order-1 Markov, output nonsynchronizing.





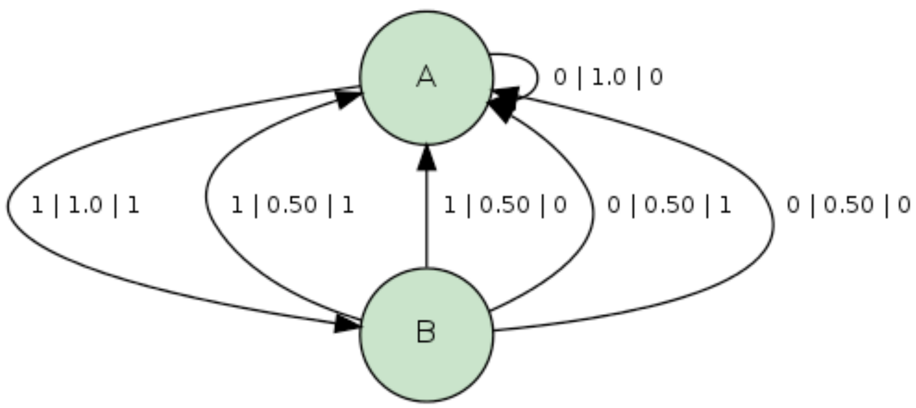
Topics & Applications...

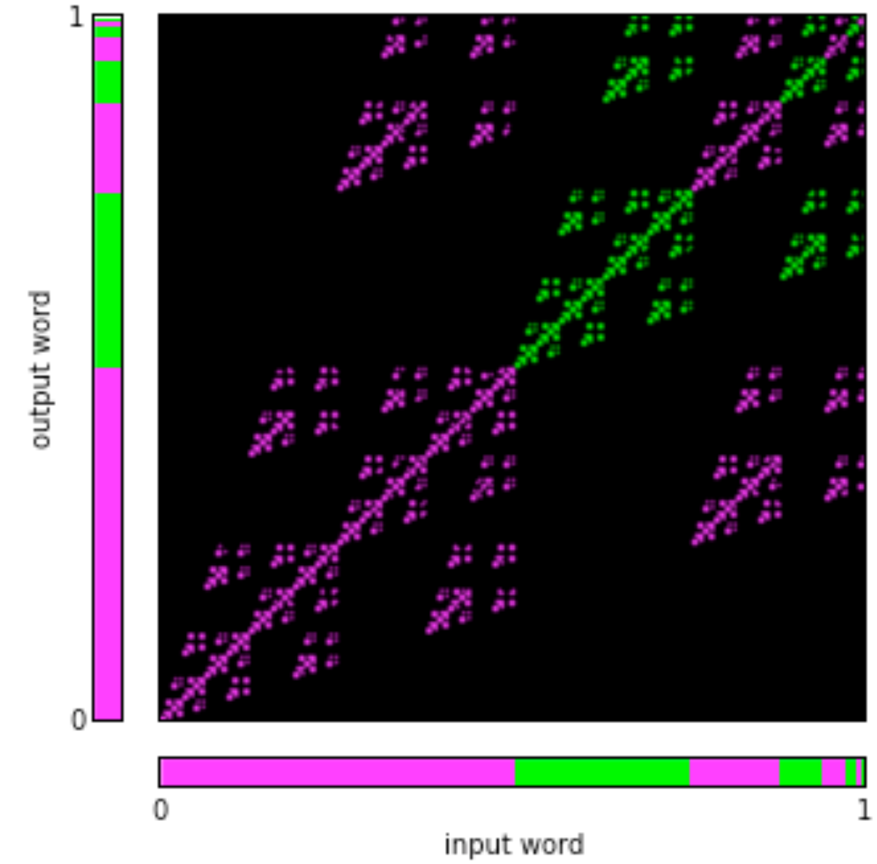
Computational Mechanics Research ...

ε-Transducers ...

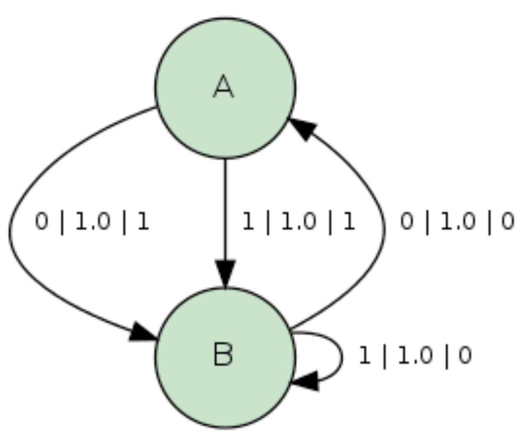
Memoryful channels ...

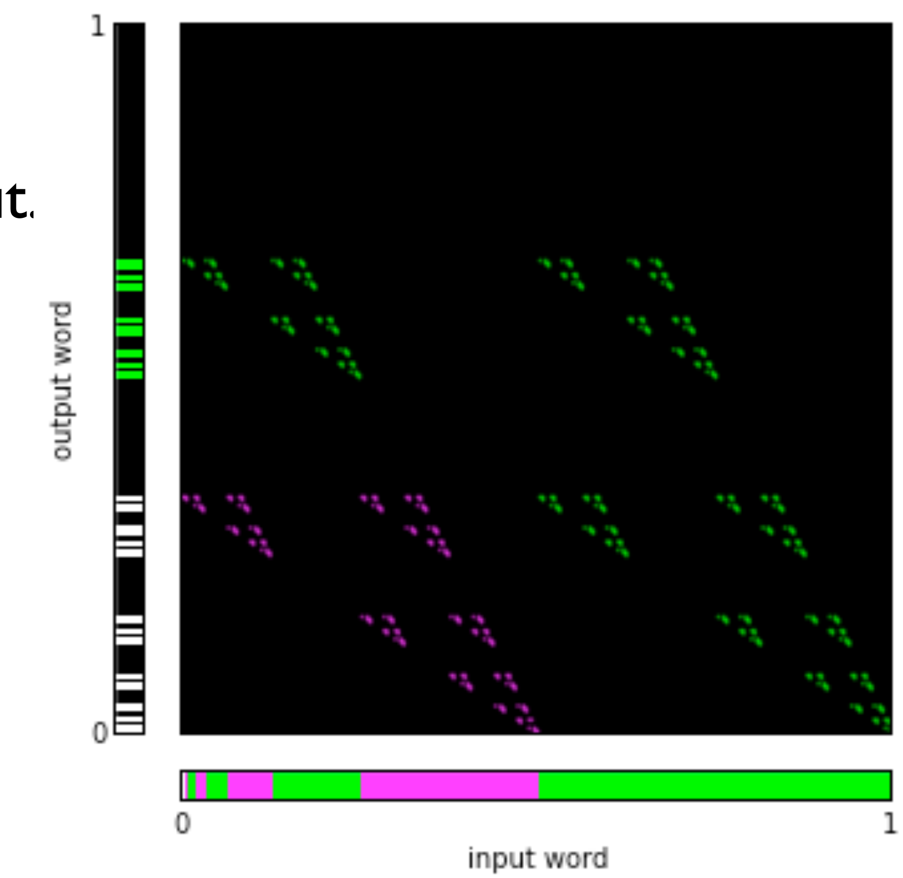
Even-to-Fair: Map Even Process to Fair Coin.
Random bits from Even Process, but fuzz out pairs of 1s.





Feedback-NOR: NOR of the previous output & current input.
Strictly sophic on input, nonsynchronizing on output, but order-1 Markov on joint symbols.



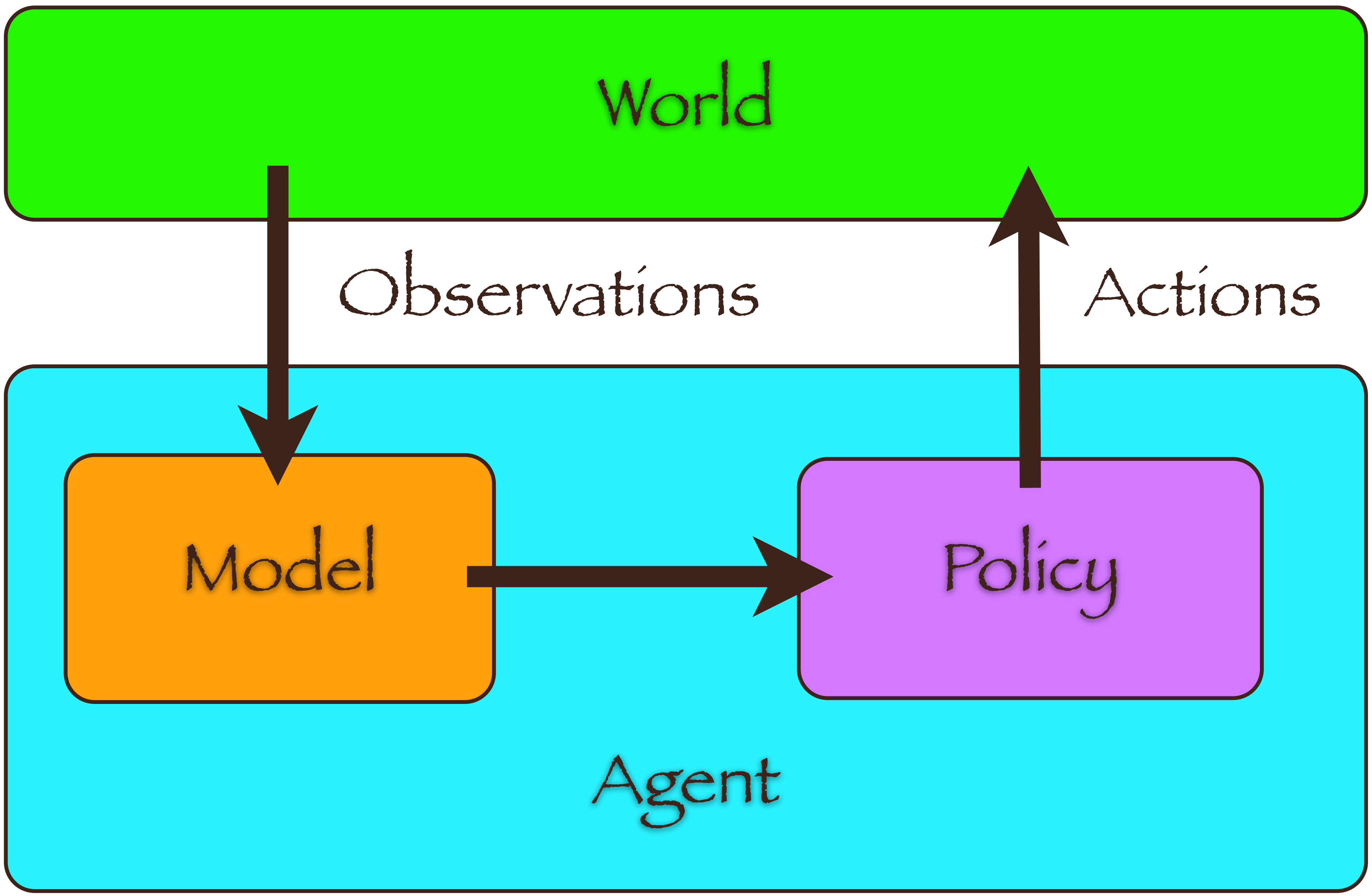


Topics & Applications...

Computational Mechanics Research ...

Interactive Learning:

The Feedback Loop



Topics & Applications...

Computational Mechanics Research ...

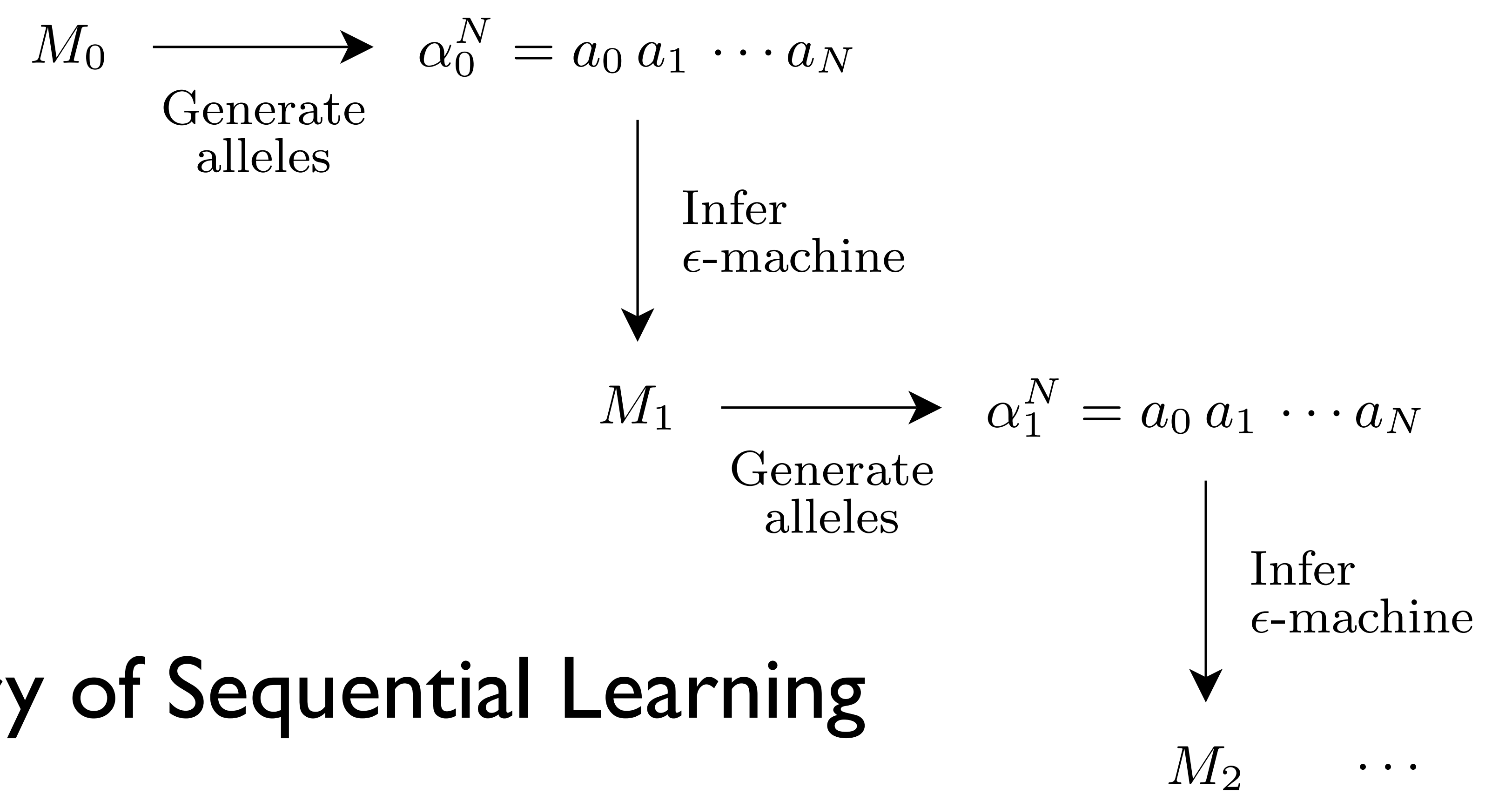
The Evolution of Language:

Heard over radio during WWI:

“Send Three- and Four-Pence, We’re Going to a Dance”

Intended message:

“Send reinforcements we’re going to advance”



Game of Telephone:

Mathematical Theory of Sequential Learning

J. P. Crutchfield and S. Whalen, "Structural Drift: The Population Dynamics of Sequential Learning", PLoS Computational Biology **8**:6 (2012) e1002510.

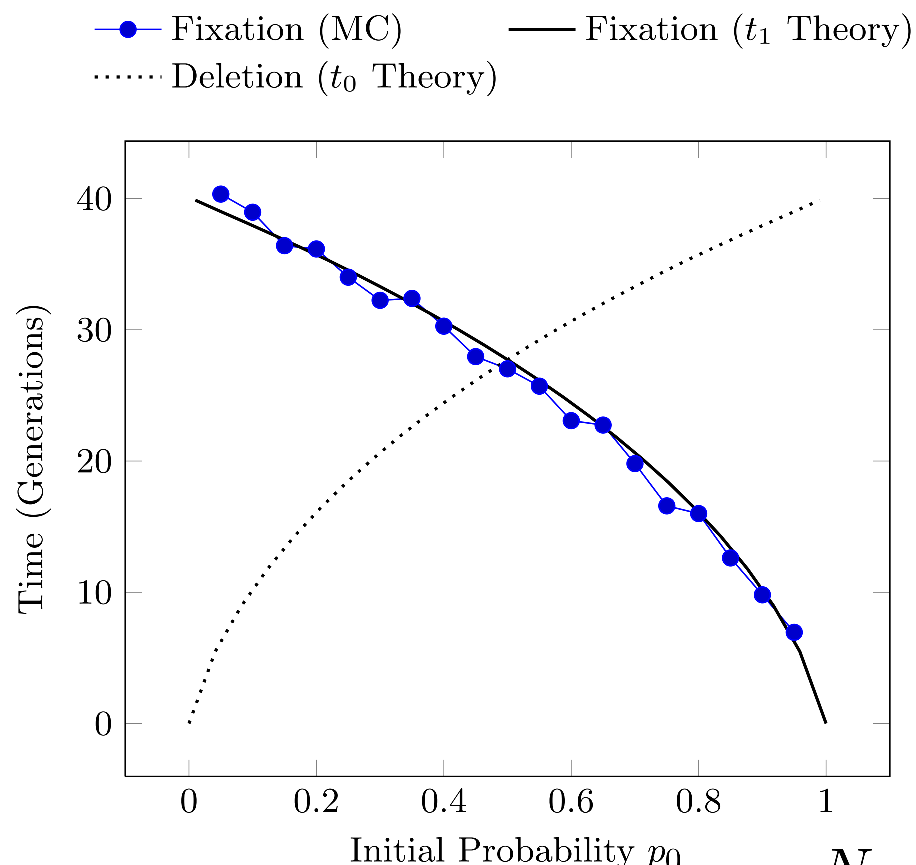
Topics & Applications...

Computational Mechanics Research ...

The Evolution of Language ...

Mathematical Theory of Sequential Learning

Genetic Drift



$N = 10$

(Theory: Kimura)

$$t_1(p_0) = -\frac{1}{p_0}\left[4N_e(1-p_0)\log(1-p_0)\right]$$

J. P. Crutchfield and S. Whalen, "Structural Drift: The Population Dynamics of Sequential Learning", PLoS Computational Biology **8**:6 (2012) e1002510.

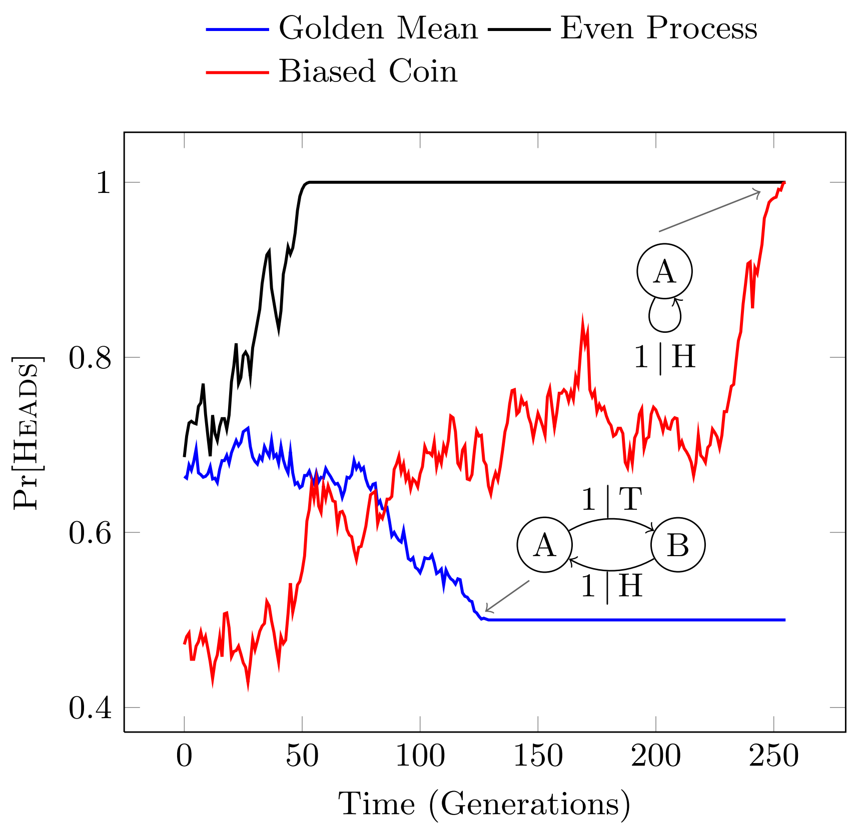
Topics & Applications...

Computational Mechanics Research:

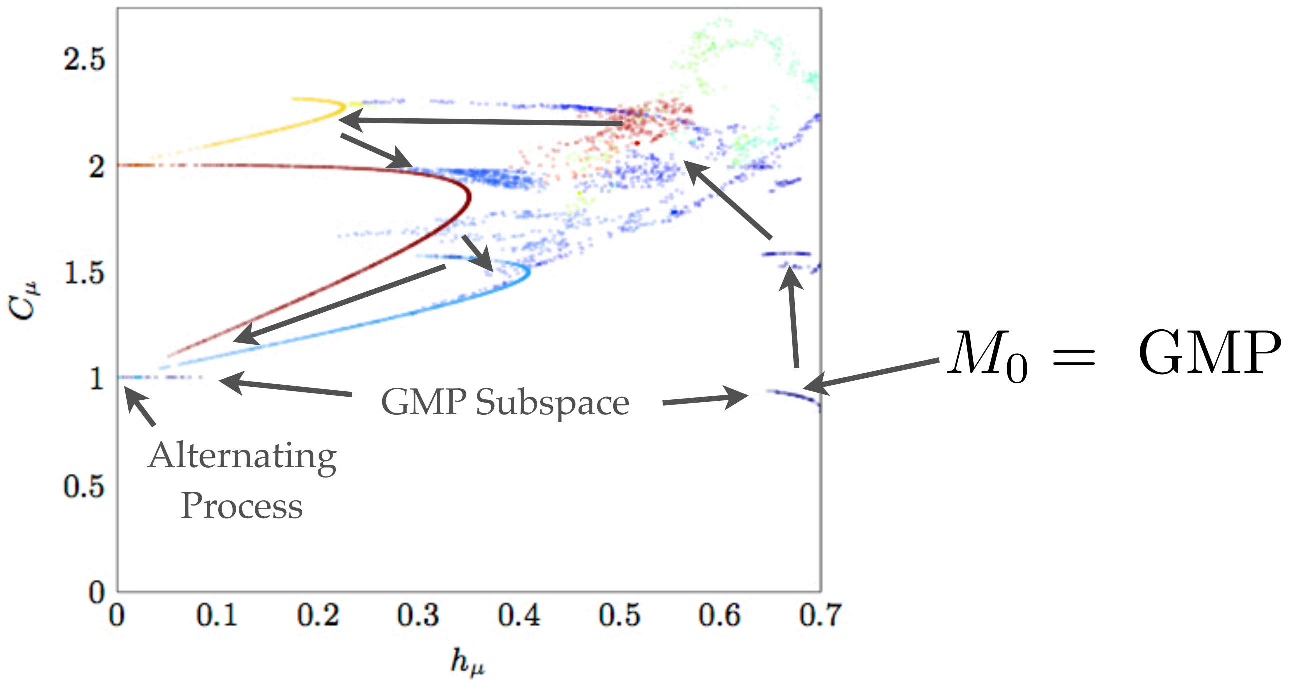
The Evolution of Language ...

Sequential learning as a diffusion in space of ε-Machines

Structural Drift



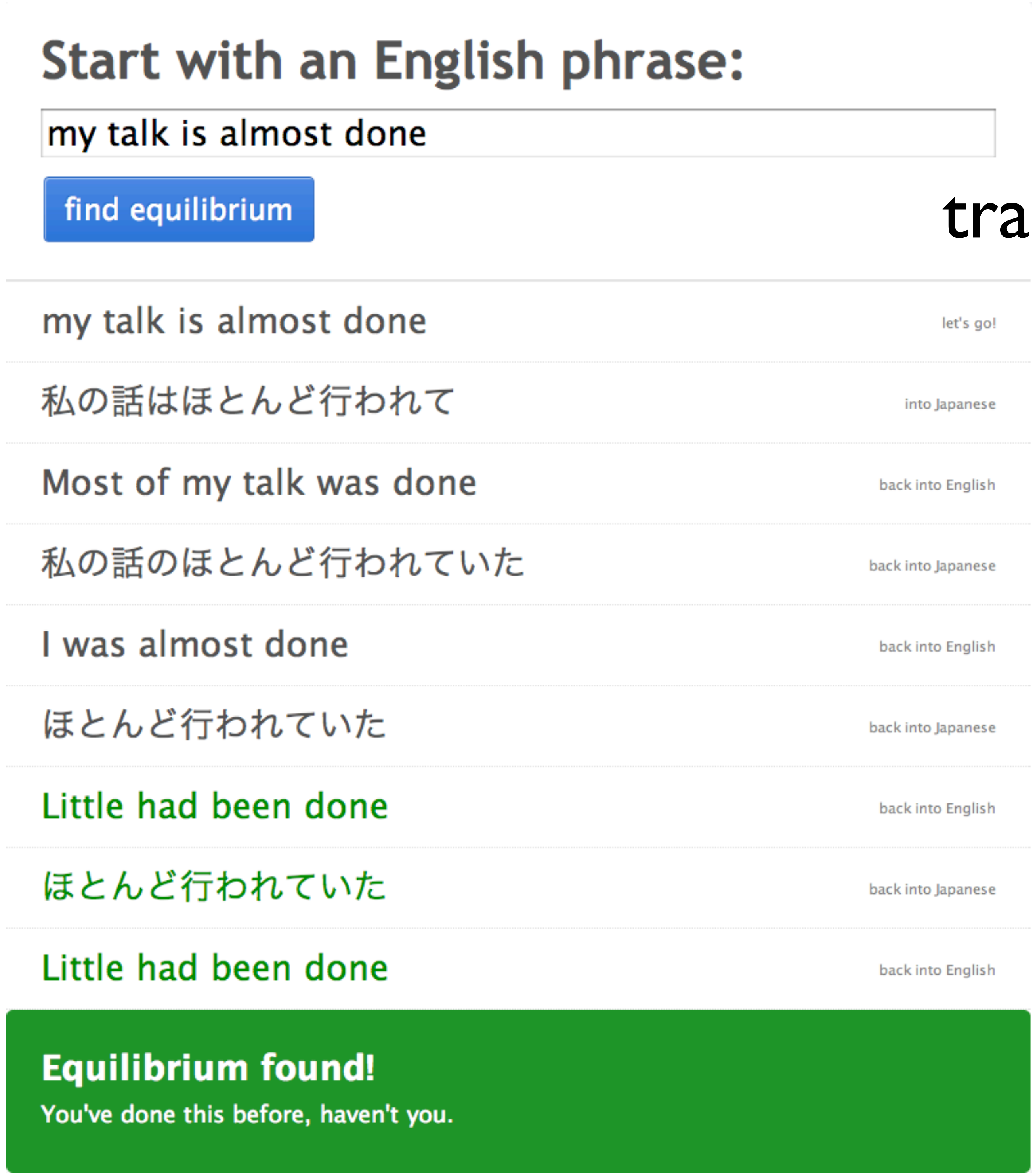
Semantic Creation



Topics & Applications...

Computational Mechanics Research:

The Evolution of Language ...



translationparty.com

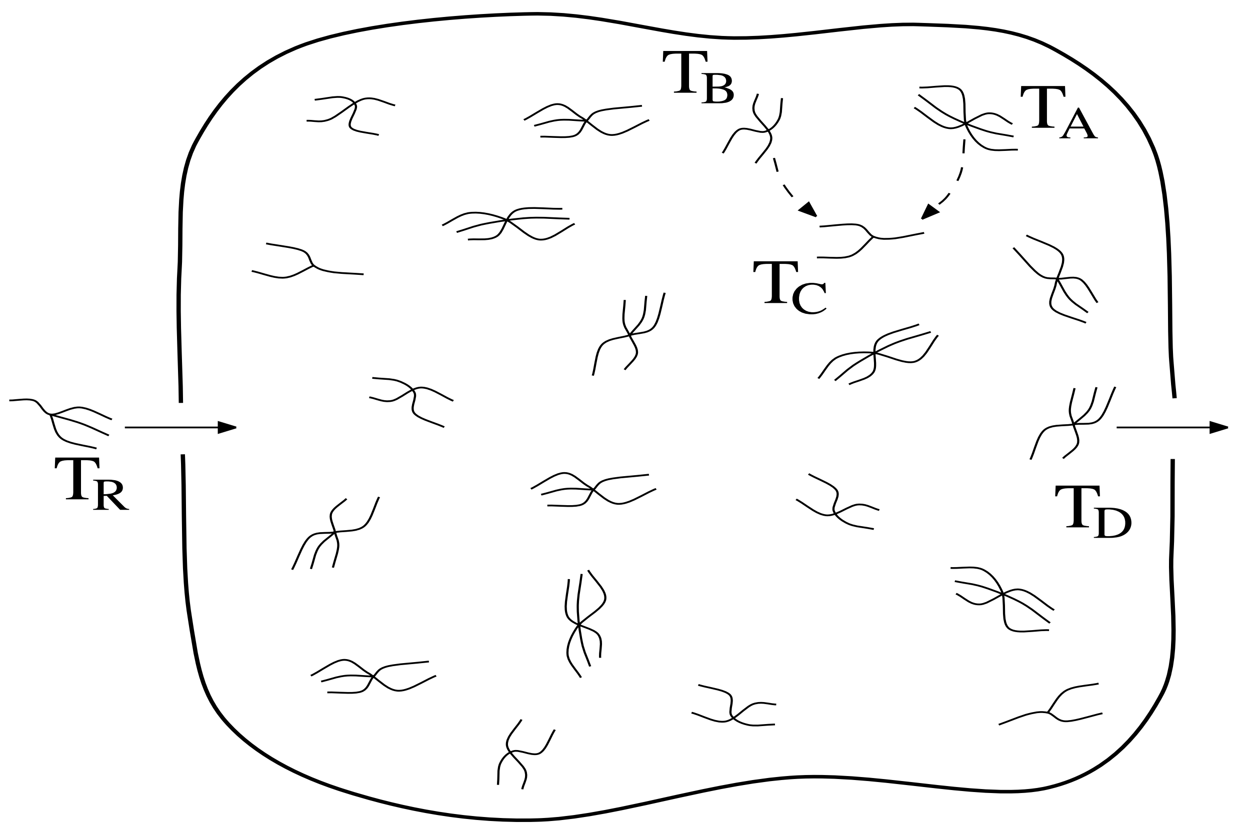
Topics & Applications...

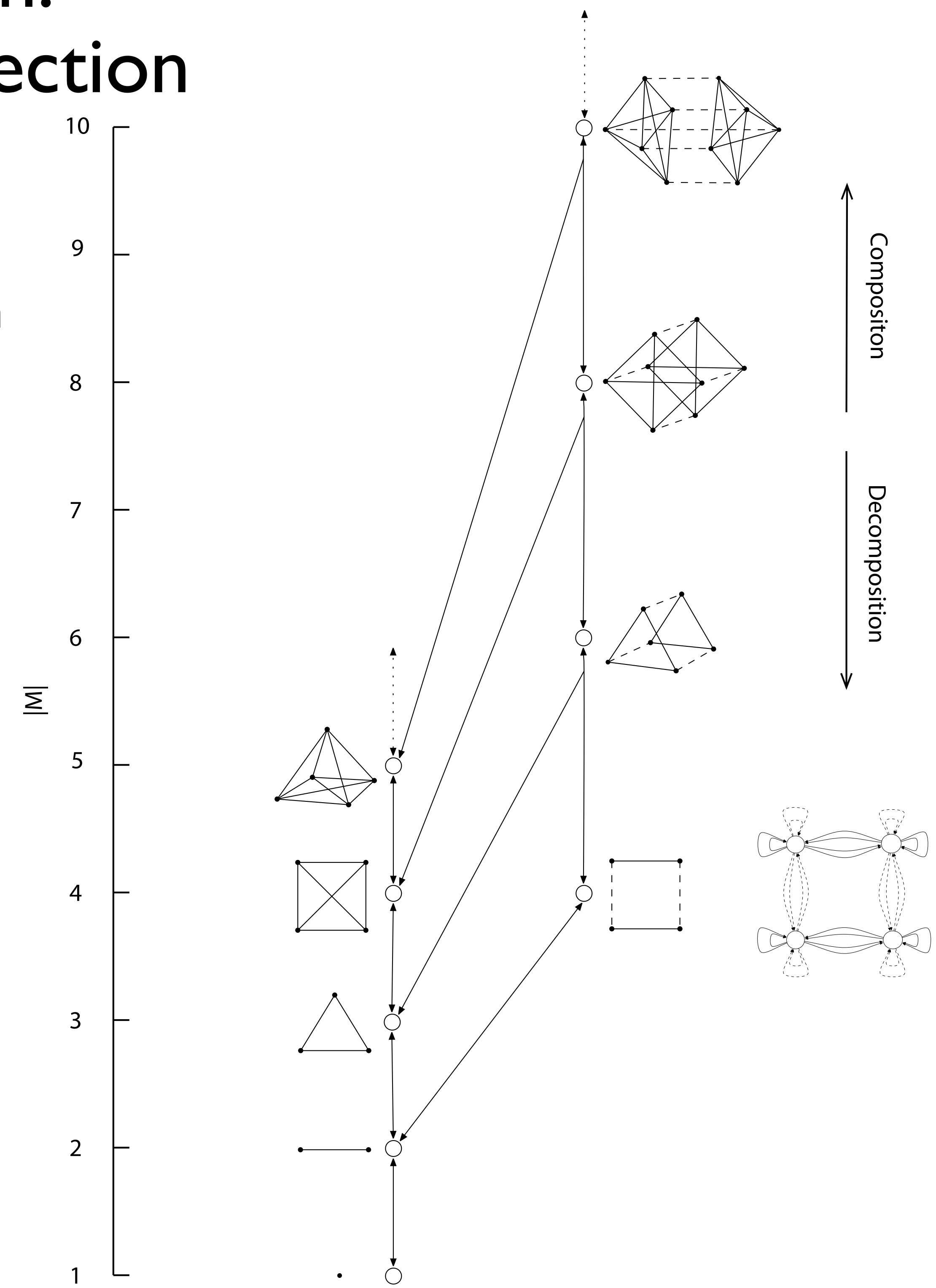
Computational Mechanics Research: Emergence of Evolutionary Selection

Self-Organizing Networks

Evolutionary Theory:

From the Primordial Soup to Natural Selection

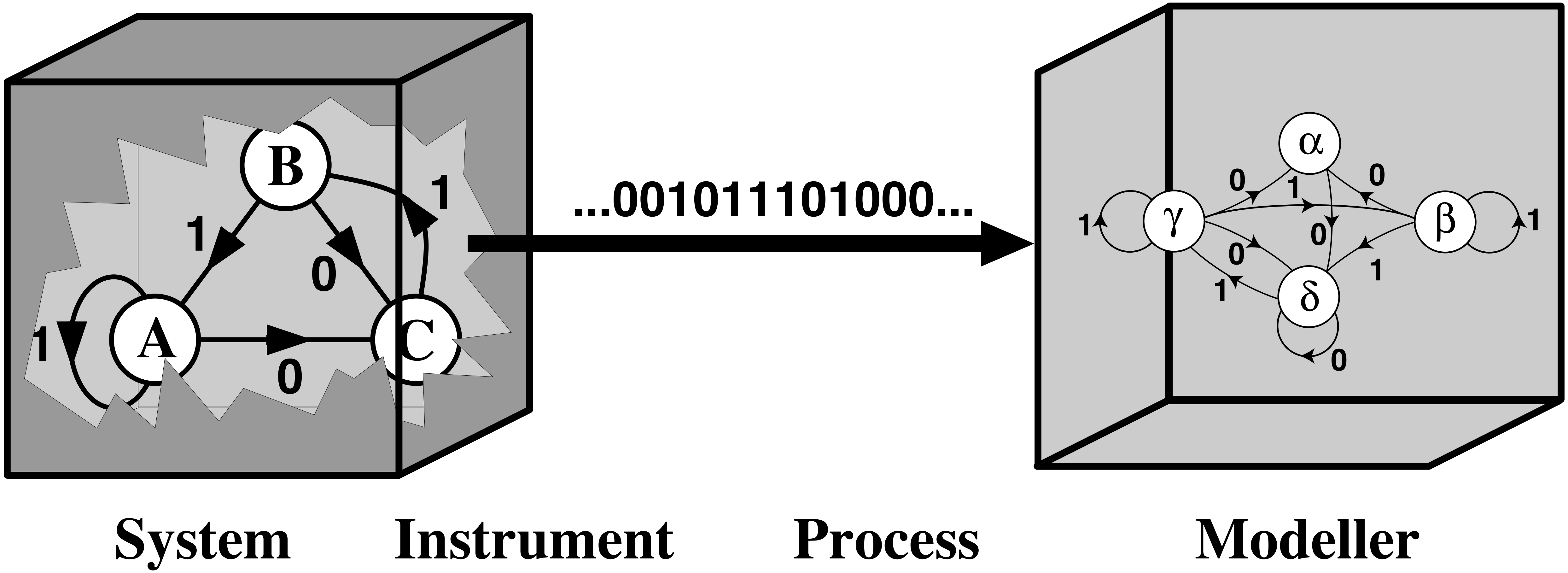




J. P. Crutchfield and O. Goernerup, "Objects That Make Objects: The Population Dynamics of Structural Complexity", J. Royal Society Interface **3** (2006) 345-349.

Topics & Applications...

The Message:



How Nature is Organized
is How Nature Computes

Topics & Applications...

Further:

Computational Mechanics Archive:
http://csc.ucdavis.edu/~cmg/

Dynamics of Learning Research Group Mail List:
http://lists.csc.ucdavis.edu/mailman/listinfo/dynlearn