Yun Tao
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Decision Dynamics via Coupled 2-D Maps

Introduction

Ever since the onset of population biology decades ago, the use of mathematical models and computational simulations have served as indispensable tools for rigorous and in-depth understanding of the ways which individuals interact and emergent phenomena form. Now widely studied equations such as the logistic model and the Lotka-Volterra predator-prey systems linked the once descriptive to the quantitative disciplines of behavior and ecology. While the former discover the parametric dependency of deterministic chaos, the latter and its various versions aimed to implement the findings within a naturally realistic framework. Yet, even with the unprecedented emphasis on macroscopic pattern and mechanistic detail, majority of the generated models fail to capture and predict the living world as intended. Though the problem has often been attributed to the presence of environmental and demographic stochasticity, the fact remains that many governing components of behavior, ecology, and evolution are absent from traditional models. In addition, due to the widespread conviction that much of nature's events can be inherently unpredictable – as exemplified by non-linear dynamical systems – little effort are put forth in validating that claim for the problems at hand. Such fatalistic ideology obviously does not benefit the advancement of this field nor propel effective management strategies. As a result, the model showcased here is developed upon a different foundation, namely, the search for order in an otherwise random system. As population regulation is, neglecting abiotic factors, essentially done via agent-level behavior, it is then reasonable to begin one's formulation from that angle.

Biological Properties

When dealing with the subtleties of behavior, the central element is the decision and how it temporally transform due to agent's internal state and potentially other governing forces. Here, decision denotes a preference for a particular option, whether it'd be a reward (resource, shelter, mate) or behavioral strategy (forage, rest, migrate). Trade-offs are inextricably woven into the fabric of decision dynamics in the forms of commitment cost/opportunity loss versus guaranteed and immediate benefit. For instance, carnivorous predators need to sacrifice then chance of capturing bigger prey when they utilize the time to hunt down and handle lesser options. Preys themselves often need to choose between starving gradually and risking their survival to venture into a resource-rich yet dangerous habitat. As mentioned earlier, the problem boils down to a sliding scale on the spectrum of preference. This thus is the primary variable to consider in the following model.

Empirical Background

Experimental studies regarding the process of decision-making are surprisingly sparse. A series of such research conducted by the Behavioral Ecology Group of Oxford University is consulted here, particularly those done on European starlings. In short, the birds are starved or motivated to work prior to gaining food bits of equal size and content. The objective was to see whether a preference would be generated solely based on the animal's 'state' (or 'activeness', as it will be referred as in this paper) and memory when the rewards are experienced. The equivalent intrinsic value of the food bits eliminates external biases. The findings are the following: 1) As the animal's activeness decreases over time with no replenishment, its fitness declines accordingly. The function exhibits a convexity that can be characterized by a sinusoidal wave from the radian range of 0 to 2π . Therefore, the magnitude of the increase in fitness upon consuming a food option, ν, is proportional to both the state of the subject at the time of consumption and the value of the food itself. In this case of identical food values, the hungrier or exhausted the starlings are, the more greater the jump in its fitness upon replenishment. 2) During the learning phase – period in which the birds are acquainted with the experimental protocol – the food option that yields the greater fitness increase will be more preferred upon a later date regardless of the subject's activeness at the time. In other words, if option x is experienced and produces the more apparent effect in memory, despite its nutritional equivalence or even inferiority compared to others, it will be preferred when the birds are later presented with all the options under both hungry and satiated states. 3) The skewness of the preference neutralizes and eventually inverts as the delay in obtaining the preferred option increases. Hence, longer the wait for the preferred option to be available, the less desirable it becomes. Conversely, the preference for the alternatives gradually rises.

Mathematical Model

$\omega:$ *intrinsic option value*
$\nu:$ *fitness gain*
$E:$ *agent weariness*
$S:$ *agent satiation / activeness*
$\delta:$ *metabolic decay*
$\epsilon:$ *chance of occurrence*
$\beta:$ *suboptimal preference*
$\varphi:$ *impatience exponent*

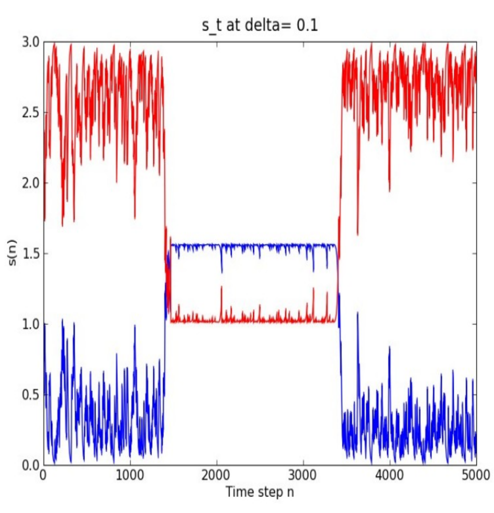
$$\text{if } \omega_t = \omega_t^p$$
$$E_{t+1} = E_t e^{\delta(1-E_t)} - \epsilon_t \omega_t$$
$$\beta_{t+1} \approx 0$$

$$\text{else}$$
$$E_{t+1} = E_t e^{\delta(1-E_t)} - \epsilon_t \omega_t \beta_t$$
$$\beta_{t+1} = \beta_t e^{\varphi}$$

The above 2-dimensional Maps coupled by the intrinsic option value encapsulates the general relationship between preference and activeness. Since the encounter of events that shape option preference is a discrete process, it is modeled under an exponential distribution. The agent's activeness or similarly, weariness, is represented by a logistic curve as a function of time. This assumption is made by evaluating multiple such hypotheses along with their effects in terms of metabolic trajectory and evolutionary optimality. More questionably however, though the preference for the alternative option is model to increase exponentially, empirical data does not strong distinguish between it and a linear increase. Furthermore, an important assumption is made regarding the agent's expectation of its commitment. Due to the instant favoritism towards the preferred option and the gradually rising preference for the alternative from near zero, the individual must then be interpreted to commit unreservedly, hence expecting high reward to risk ratio, at each time step despite the short-lived benefit of the options. This in turn denotes a total lack of reasoning and predictive capability on the decision maker.

$$
\begin{aligned}
&For\ each\ \epsilon_t = 1, \\
&\quad \omega_t = pareto.rv(k_t)\zeta,\ k_t = f(E_t) \\
&\quad \{\omega_t\}_{t\geq 0} \rightarrow storage \\
&\quad \nu_t = \sin(S_t + \omega_t) - \sin(S_t),\ S_t = \frac{\pi}{2} - E_t \\
&\quad \{\nu_t\}_{t\geq 0} \rightarrow storage \\
&\quad \omega_t^p = \{\omega_t : \nu_t = max\{\nu_t\}_{t\geq 0}\}
\end{aligned}
$$

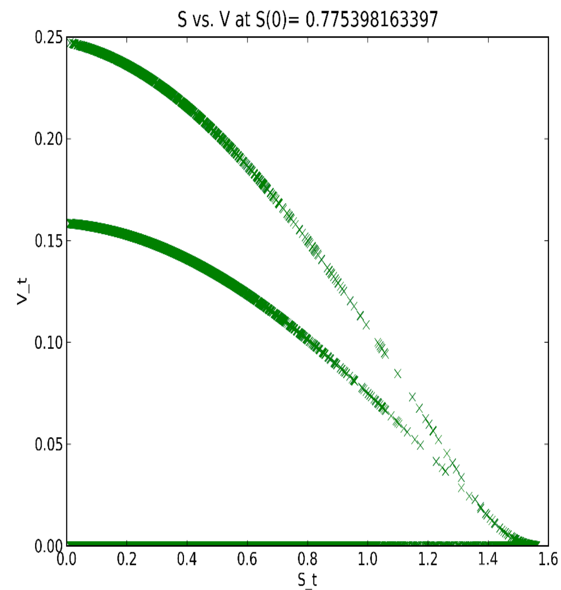
The set of equations above constitutes the “memory box”, where the past information is stored and, via the coupled 2-D maps, influences the agent's behavior. Here, a Pareto, or Bradsford distribution is assumed to characterize the likelihood of encountering specific options. In short, the more valuable an option is, the smaller the probability of finding it as a result of mere scarcity and competition. The distribution is dependent upon the parameter *k* which indirectly correlates to the chance of encountering more valuable resources. It is established as a linear function of the individual's activeness, the reasoning being that the less weary the agent is, the more it will work, seek out, and likely capture the rarer resources. The encountered options themselves are stored in memory, along with their matching boost in fitness. The option has yields the highest fitness increase will be preferred. Obviously, it will be regularly updated until it asymptotes.



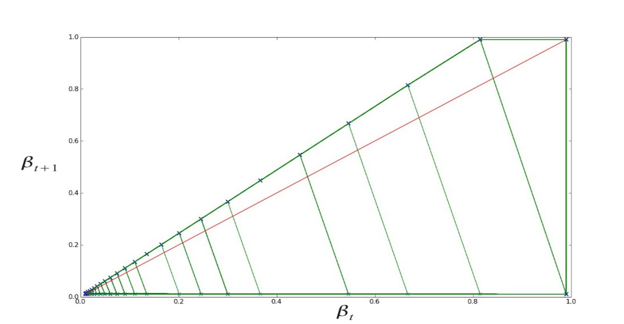
Simulations

Using Python to simulate the dynamics of this system, many interesting features appeared. The graph to the left shows a rather typical temporal trajectory of both the agent's activeness (blue) and the *k* parameter value (red). As can be seen, occasionally these variables become temporarily trapped near one boundary yet will eventually escape it and fluctuate wildly. This likely contributes to the resulting decisional dynamics.

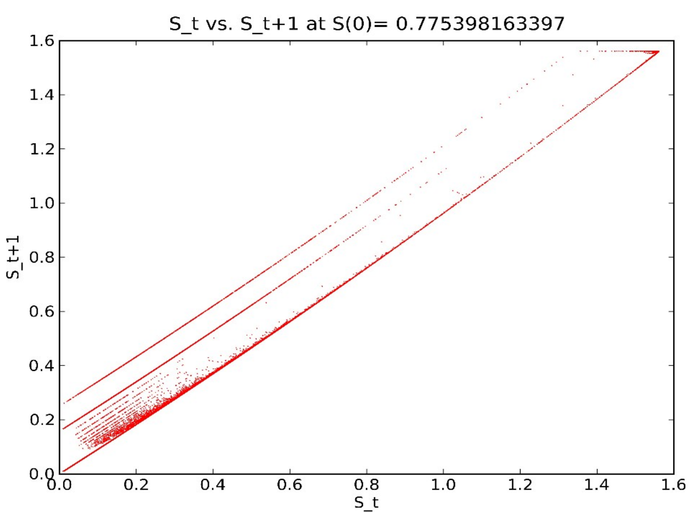
The figure to the right intuitively illustrates the relationship between fitness increase, options, and the respective preference for them. As the agent is active, little fitness gain can be achieved and whichever option is selected generates the same result. As the agent become more weary, however, the intake of disparate options begin to have more dramatic effect on the rise in fitness. Their differences likewise expand. In conclusion, it is when the agent is least active when the contrast between the options is the greatest hence, when the decision-made is most optimal.



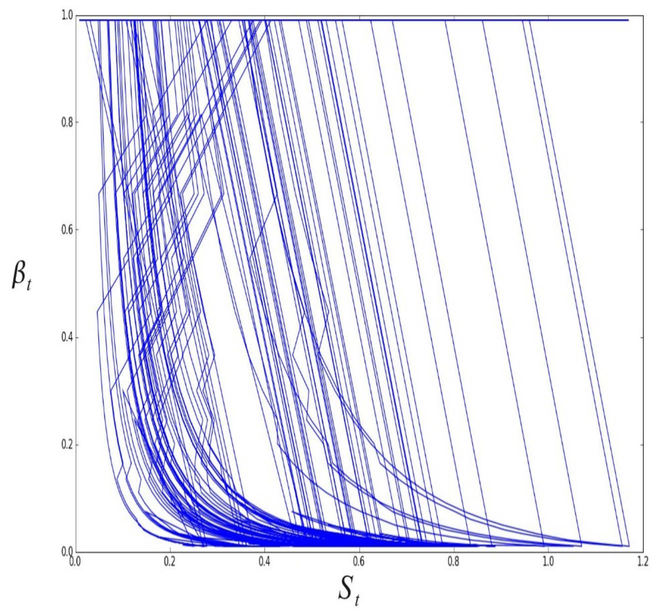
Plotting the preference for the alternative option against itself at the next time interval reveals a fundamental pattern in discrete decisional dynamics, namely, that at each denoted instance, the agent either encounters the preferred option or remain content about the existing one. Although, the time at which all decisional possibilities are experienced is relatively random as it's governed by two components of stochasticity, the event occurrence rate and the Pareto-distributed encounter rate.



A more revealing pattern can be detected in the same graph of the agent's activeness. Exemplified by the near fractal shape below, the agent's internal state becomes more predictable in the higher active regime.



When the two central variables are plotted against each other, a semi-ordered phenomenon emerges despite the embedded stochastic components. Within these aperiodic cycles, a high density of state variables reside on the bottom edge of the graph. This refers to the "stalling" behavior in which the individual refuses encountered options and waits for the preferred one. As the trajectories ascend, the behavior tends towards "compromise". The path to complete switch of preference may differ each time, depending upon the individual's then activeness, memory, and encounter probability. As the preference for the alternative option reaches 1, the trajectories travels along the upper boundary until the preferred option becomes available, causing them to rain back down to the "stalling" phase.



Future Works

Much more behavioral traits will be incorporated into future models: individual rationality, continuous time and options where the preference relies on them being similar rather than identical, benefit delay, choice-locking, more complex memory box, etc.

Appendix: Codes

```
# Import plotting routines
from pylab import *
import math as m
import numpy as np

# Generating stochasticity in epsilon
mean = 5  # average number of times for 1 success
lam = 1.0/mean
def randd(n):
        for i in range(n):
                a = np.random.rand()
                if a < 1 - m.exp(-1.0*lam): a = 1.0
                else: a = 0.0
        return a
```

```
# Simulation parameters

# Control parameter of the map:
delta = 0.1
c = 3.0
kslope = (c-1)/(pi/2)
eta = (sin(0.01 + 1.0/4) - sin(0.01)) / 200    # eta = Vmax/(100*2) = V(Smin, Wmax)/200
phi = 0.2

# Set up an array of iterates and set the initital condition
elist = [0.01+pi/4]
slist = [pi/2-0.01-pi/4]
klist = [c - slist[0]*kslope]

betalist = [0.01]

wlist = []
vlist = []
vplist = []
wplist = []
# The number of iterations to generate
N = 50000

# The main loop that generates iterates and stores them
for n in xrange(0,N):

  epsilon = randd(1)

# Memory Box
  w = np.random.pareto(klist[n]) + 1
  if w <= 2.0: w = 1/(2.0*pi)
  elif w > 2.0: w = 1.0/4
  wlist.append(w)

  if epsilon == 0.0: w_actualized = 0.0
  elif epsilon == 1.0: w_actualized = w

  v = sin(slist[n]+w_actualized)-sin(slist[n])
  if slist[n]+w_actualized >= pi/2: v = 1 - sin(slist[n])
```

```
  vlist.append(v)

  vp = max(vlist)
  vplist.append(vp)

  if v >= vp-eta and v <= vp+eta: wp = w_actualized
  if max(vlist[0:n+1]) > 0.0 and v == 0.0: wp = wplist[-1]
  wplist.append(wp)

# Coupled 2-D Map

  if w_actualized == wp:
        e = elist[n]*exp(delta*(pi/2-elist[n])/(pi/2)) - w_actualized
        beta = 0.01
  elif w_actualized != wp:
        e = elist[n]*exp(delta*(pi/2-elist[n])/(pi/2)) - w_actualized * betalist[n]
        beta = betalist[n]*exp(phi)

  if e <= 0.01: e = 0.01
  elif e >= pi/2-0.01: e = pi/2-0.01
  elist.append(e)

  if beta <= 0.01: beta = 0.01
  elif beta >= 0.99: beta = 0.99
  betalist.append(beta)

  s = pi/2-e
  slist.append(s)

  k = c - s*kslope
  if k <= 1.0: k = 1.0
  elif k >= c: k = c
  klist.append(k)

#  print 't:', n+1, 'eps', epsilon, 's:', slist[n+1], 'w:', w, 'w_act:', w_actualized, 'v:', v, 'v_choice:', vp,
'w_choice:', wp, 'beta:', betalist[n+1]

# Setup the plot
xlabel('Time step n') # set x-axis label
```

```
ylabel('s(n)') # set y-axis label
title('s_t at delta= ' + str(delta)) # set plot title
# Plot the time series: once with circles, once with lines
#plot(slist , 'b')
plot(slist, 'b')
plot(klist, 'r')
# Use command below to save figure
#savefig('LogisticMap', dpi=600)
#print 'u', u, '\nk', k
# Display the plot in a window
show()
```

Reference:

Kacelnik, A. & **Marsh, B**. 2002. Cost can increase preference in starlings. *Animal Behaviour*, **63**, 245-250.

Pompilio, L. & **Kacelnik, A**. 2005. State-dependent learning and suboptimal choice: when starlings prefer long over short delays to food. *Animal Behaviour*, **70**, 571-578.