Single Electron Tunneling Junctions

Paul Riechers

Modeling an isolated SETJ

$$+ V_b$$

$$R$$

SETJ
(Single Electron
Tunneling Junction)

$$C \quad v_C$$

Pump

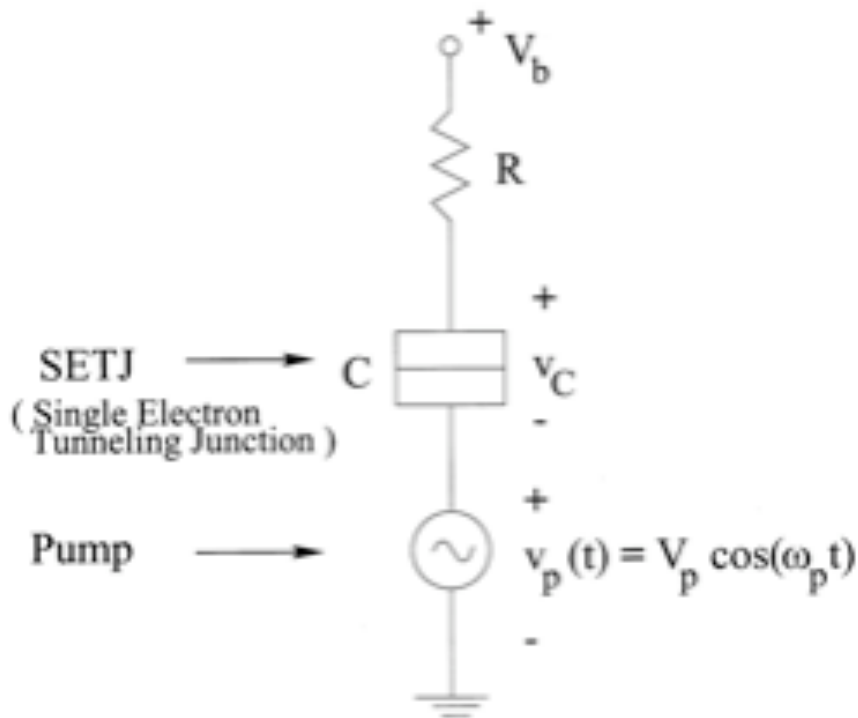$$v_p(t) = V_p \cos(\omega_p t)$$

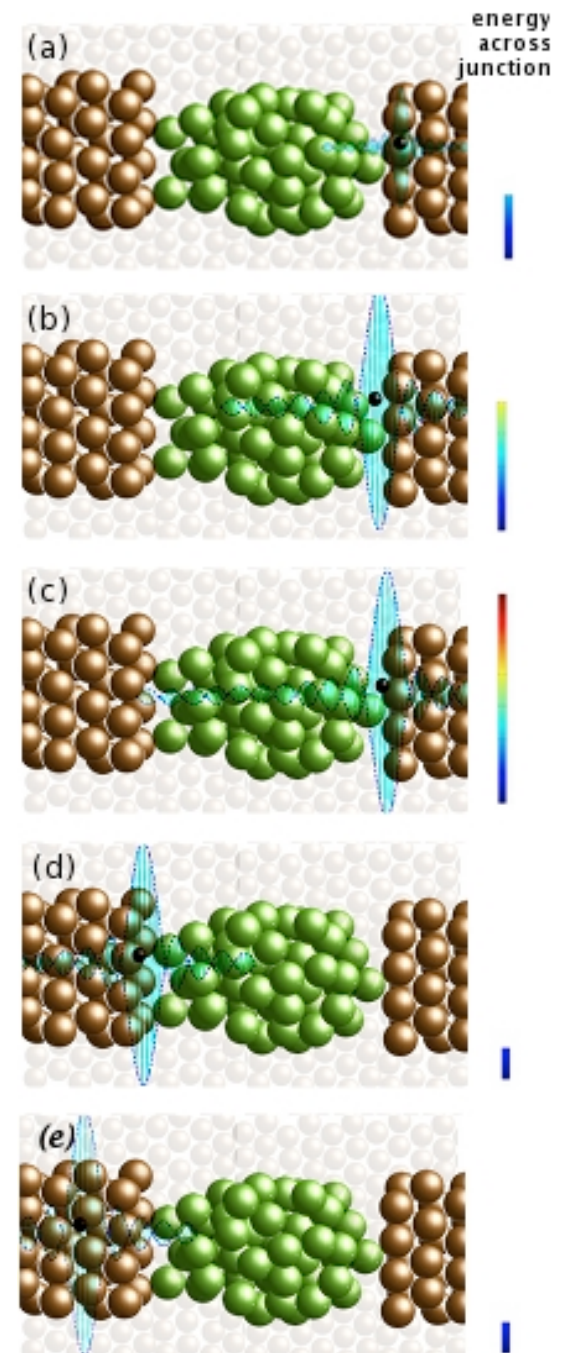Fig. 1.   An isolated driven single-electron tunneling junction circuit.

$$\frac{dv_C(t)}{dt} = \frac{1}{RC}(V_b - v_C(t)$$

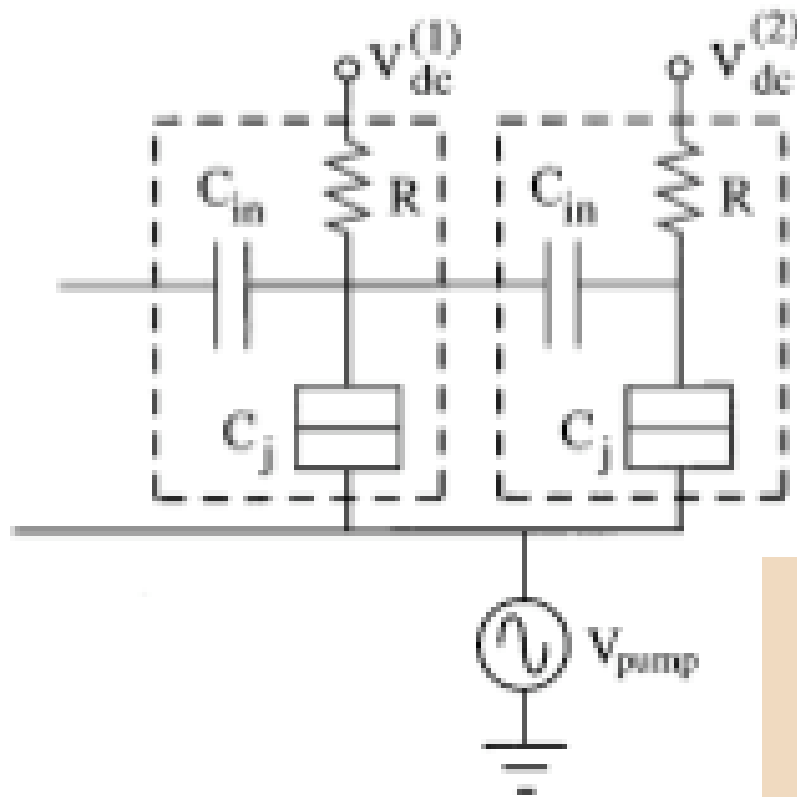$$- V_p \cos(\omega_p t)), \quad \text{if } |v_C(t)| < V_T,$$

$$\Delta v_C(t_i) \triangleq v_c(t_i^+) - v_c(t_i)$$

$$= \begin{cases} -2V_T, & v_c(t_i) \geq V_T, \\ 2V_T, & v_c(t_i) \leq -V_T, \end{cases}$$

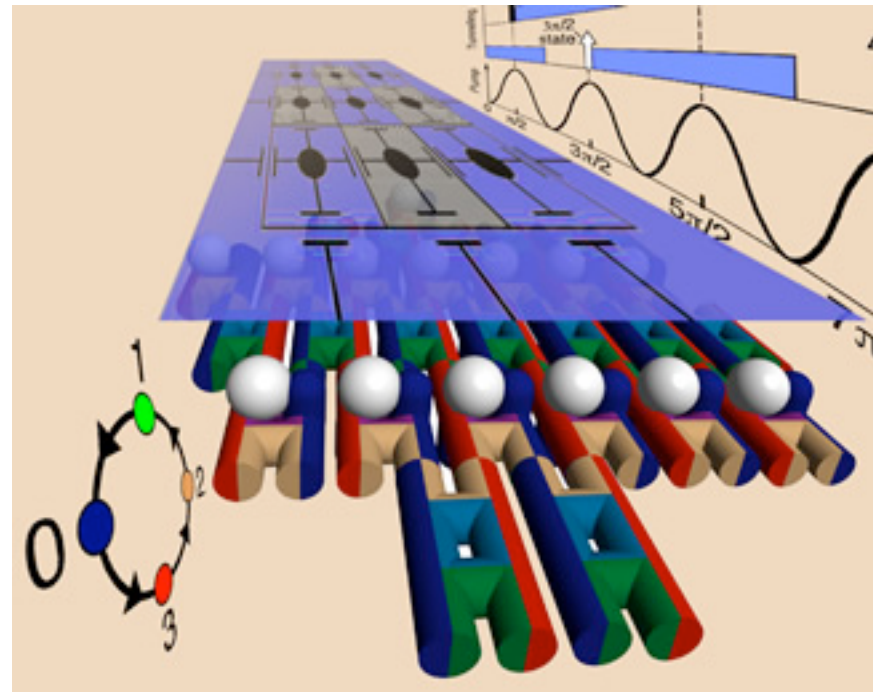$$\frac{d\theta}{dt} = -\frac{\gamma}{\pi}\theta + \gamma(a - b\cos t), \quad t \neq \tau_i,$$

$$\Delta\theta(\tau_i) = -2\pi, \quad t = \tau_i,$$

$$|\theta(0)| < \pi,$$

Stages of Tunneling in SETJ

energy
across
junction

(a)

(b)

(c)

(d)

(e)

From isolated SETJ circuits to arrays

Now, back to the isolated SETJ circuit...

$+ \; V_b$

R

SETJ
(Single Electron
Tunneling Junction)

C

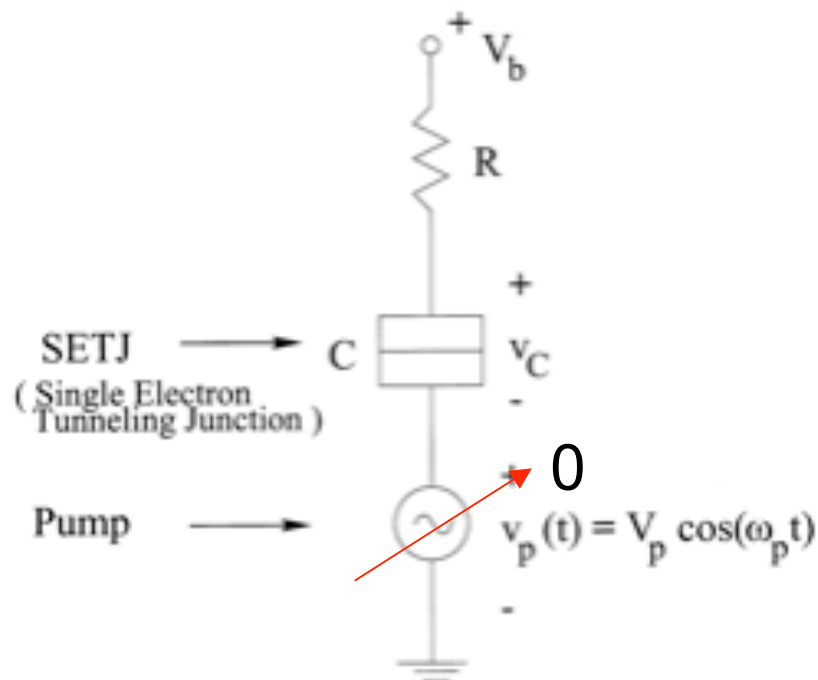$+ \; v_C \; -$

0

Pump

$v_p(t) = V_p \cos(\omega_p t)$

Fig. 1.   An isolated driven single-electron tunneling junction circuit.

To get a feeling for the behavior of an isolated SETJ, let's first turn off the pump voltage. The behavior then resembles a relaxation oscillator.

Now, back to the isolated SETJ circuit...
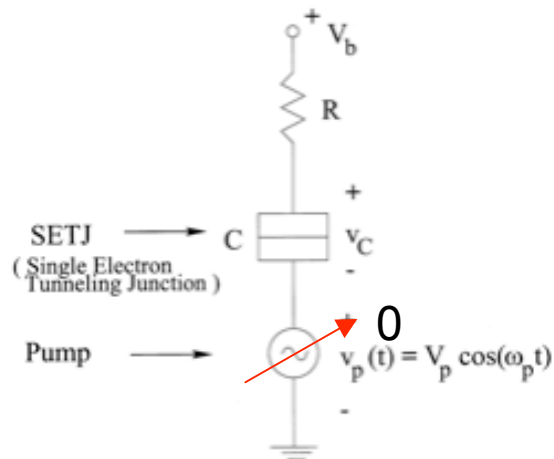
To get a feeling for the behavior of an isolated SETJ, let's first turn off the pump voltage. The behavior then resembles a relaxation oscillator.

isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsiv
a = 1.77, b = 0, gamma = 0.333333

1092   T. Yang & L. O. Chua

V_b

R

SETJ $\longrightarrow$ C $\quad v_C$
(Single Electron
Tunneling Junction)

0

Pump $\longrightarrow$ $v_p(t) = V_p \cos(\omega_p t)$

Fig. 1. An isolated driven single-electron tunneling junction circuit.

junction voltage for theta_0 = 0.3
pump voltage

theta = 2 pi C v_C / q

tau = omega_p t

Isn't it relaxing?

Now, back to the isolated SETJ circuit...

To get a feeling for the behavior of an isolated SETJ, let's first turn off the pump voltage. The behavior then resembles a relaxation oscillator.
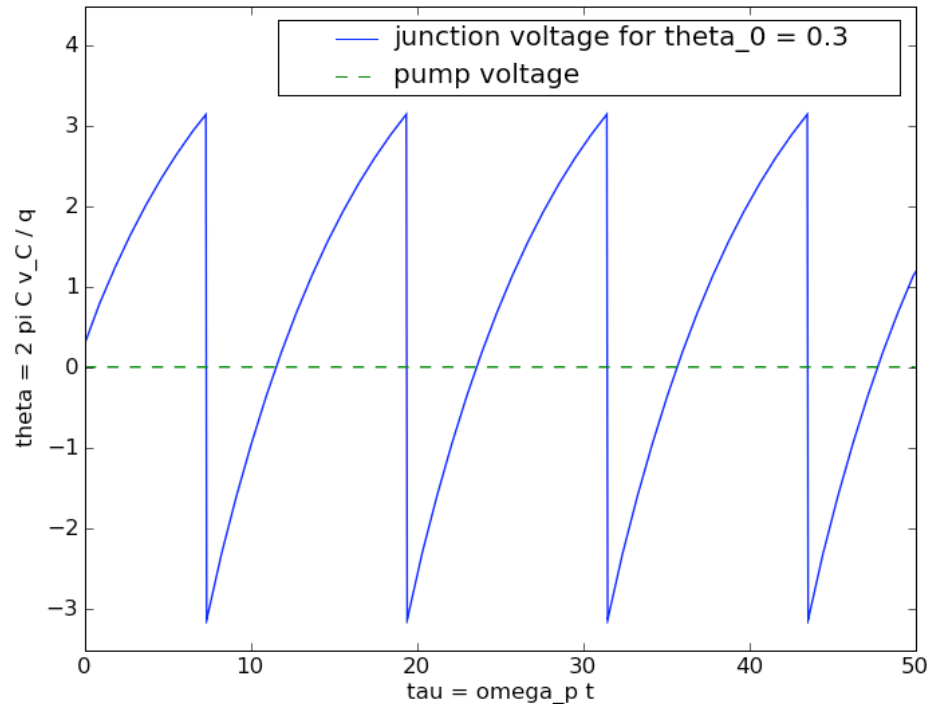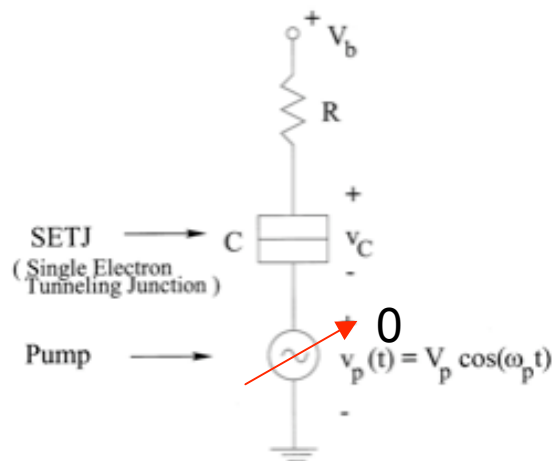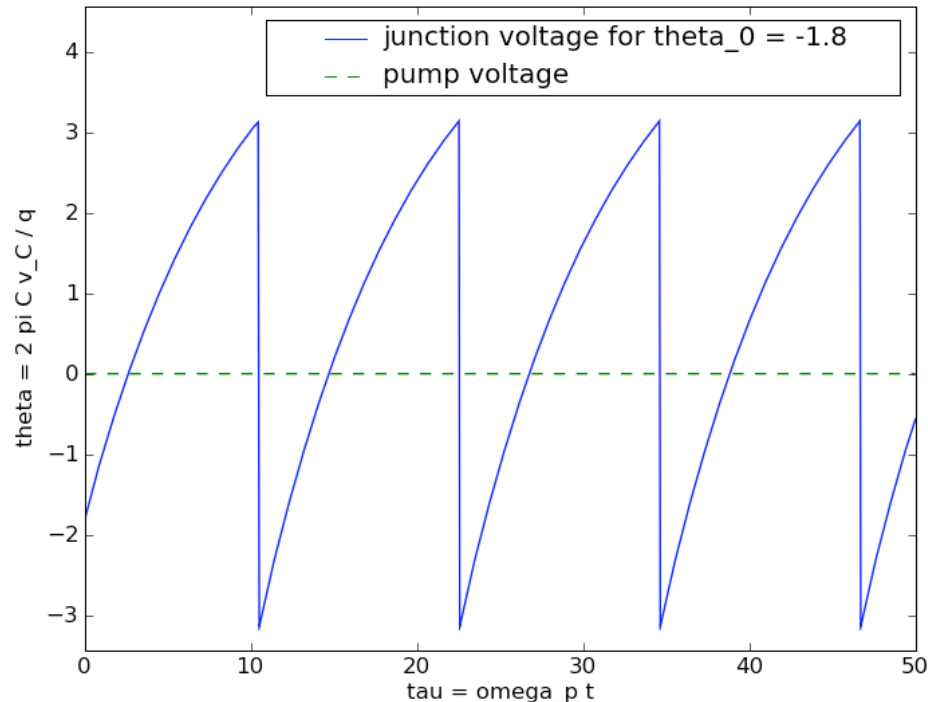
isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsiv
a = 1.77, b = 0, gamma = 0.333333

1092   T. Yang & L. O. Chua

SETJ ⟶ C
(Single Electron
Tunneling Junction)

Pump ⟶ $v_p(t) = V_p \cos(\omega_p t)$

0

Fig. 1.   An isolated driven single-electron tunneling junction circuit.

Notice that there is no transient behavior for this simple setup. Also, note that changing theta_0 yields the same waveform, only shifted in time.

The isolated SETJ circuit...

$+ V_b$

R

SETJ $\longrightarrow$ C v_C
(Single Electron
Tunneling Junction)

$+$

$-$

$+$

Pump $\longrightarrow$ $v_p(t) = V_p \cos(\omega_p t)$
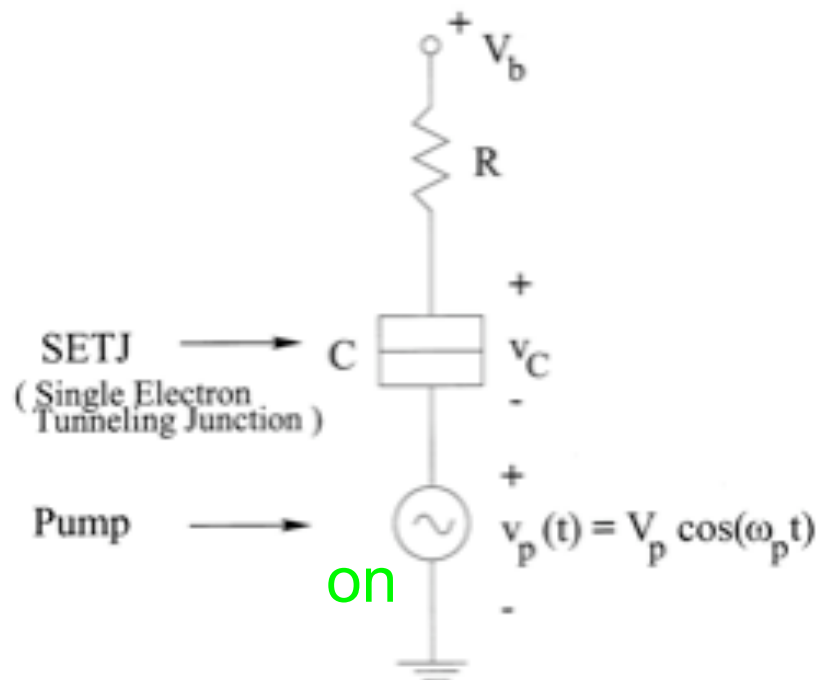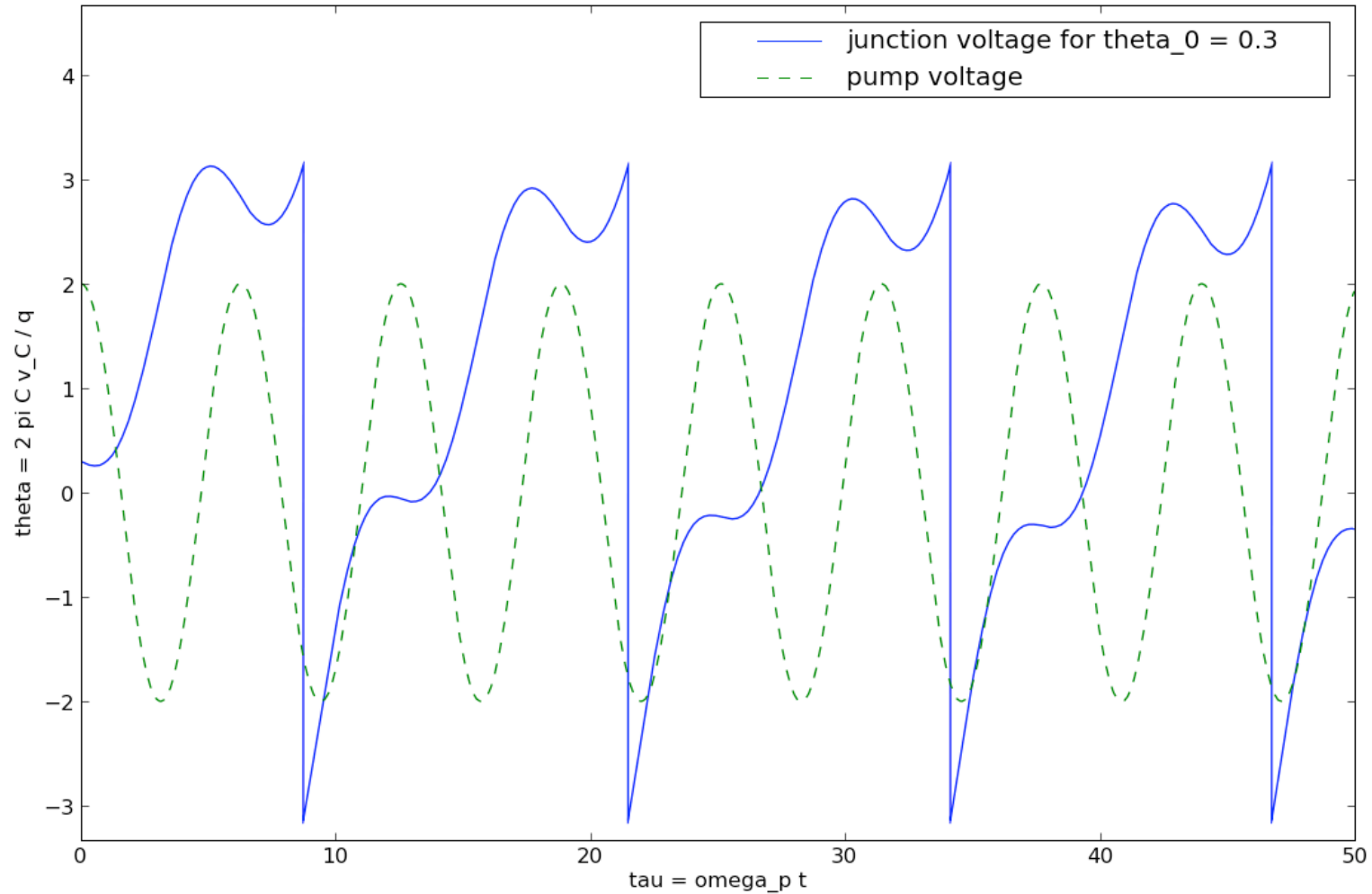
on

$-$

Fig. 1.   An isolated driven single-electron tunneling junction circuit.

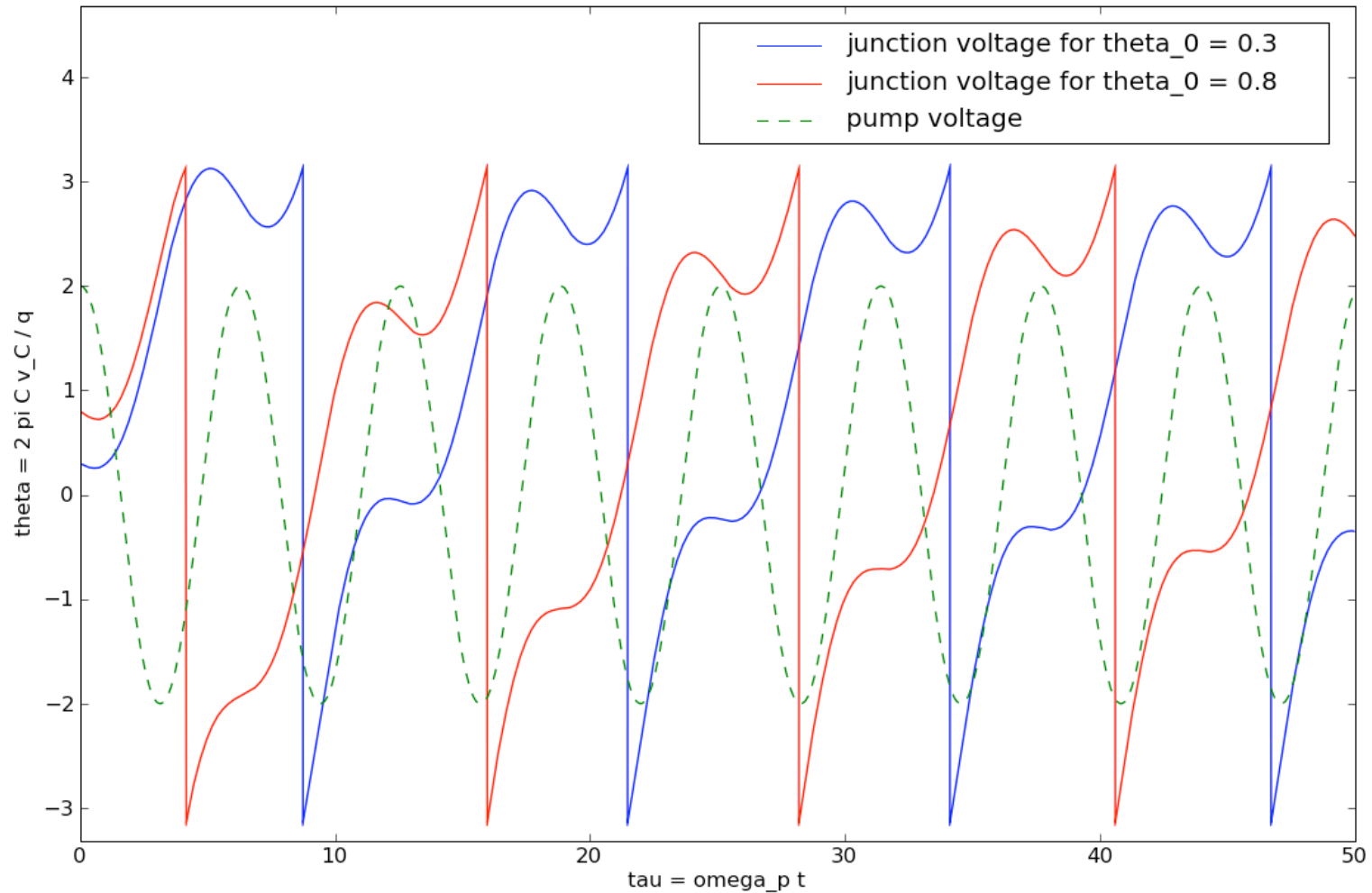So, what happens when we turn the pump on (with a sinusoidal waveform)?

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
a = 1.77, b = 2, gamma = 0.333333

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
a = 1.77, b = 2, gamma = 0.333333

The isolated SETJ circuit...

So, what happens when
we turn the pump on (with
a sinusoidal waveform)?

1092 T. Yang & L. O. Chua

$+$ V_b

R

SETJ → C V_C
(Single Electron
Tunneling Junction)

$+$

$-$

$+$

Pump → on $v_p(t) = V_p \cos(\omega_p t)$

$-$

Fig. 1. An isolated driven single-electron tunneling junction
circuit.

phase portrait for isolated SETJ

theta_0 = 0.3

pump voltage

junction voltage

The isolated SETJ circuit...

So, what happens when we turn the pump on (with a sinusoidal waveform)?

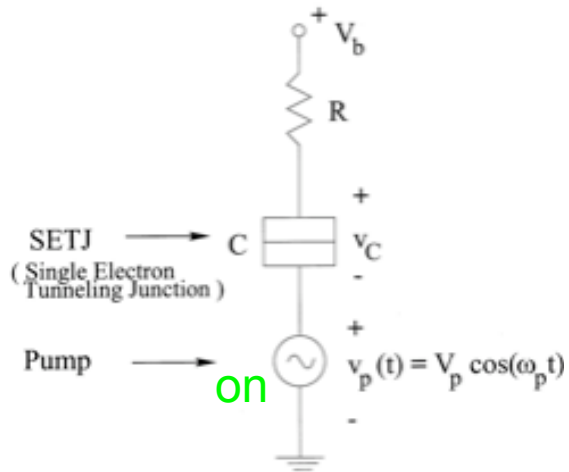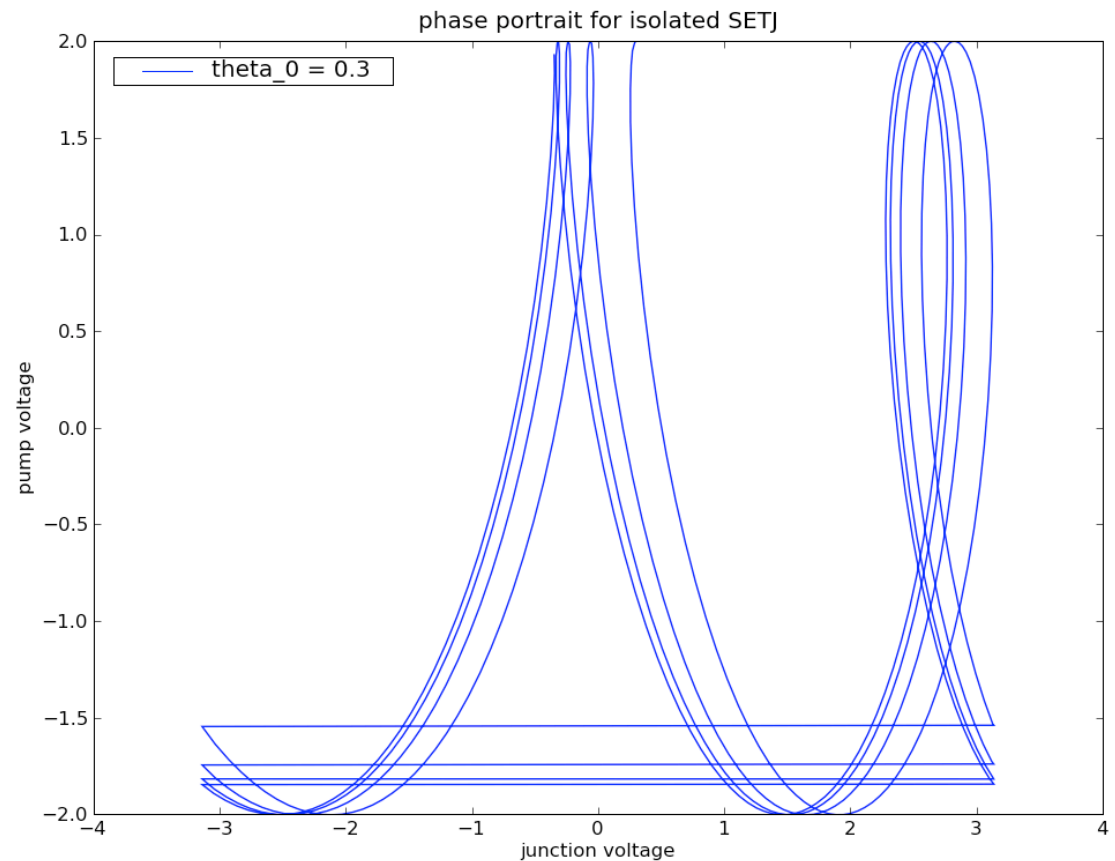1092   T. Yang & L. O. Chua

Fig. 1.   An isolated driven single-electron tunneling junction circuit.

The isolated SETJ circuit...

So, what happens when we turn the pump on (with a sinusoidal waveform)?
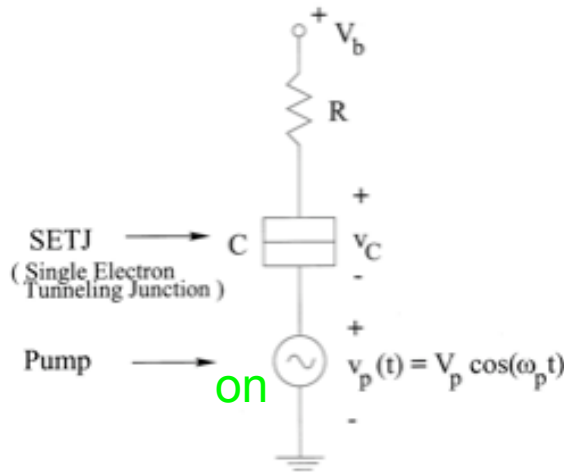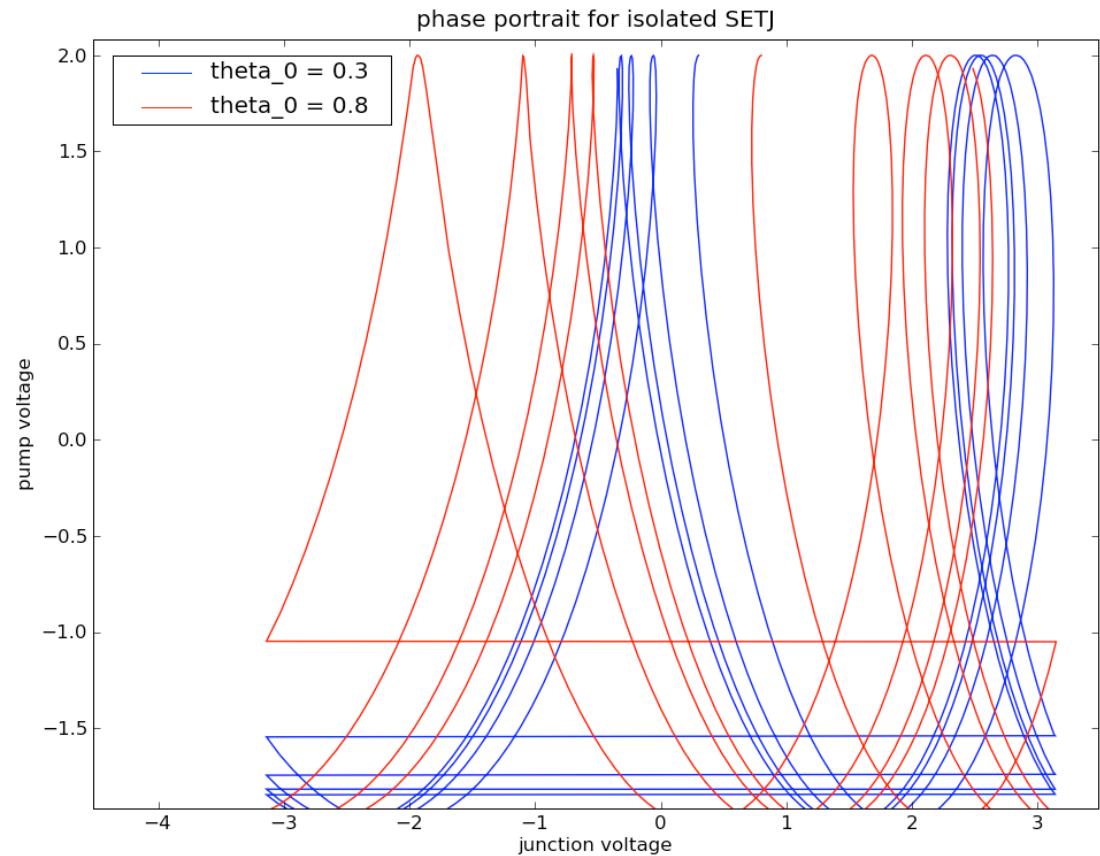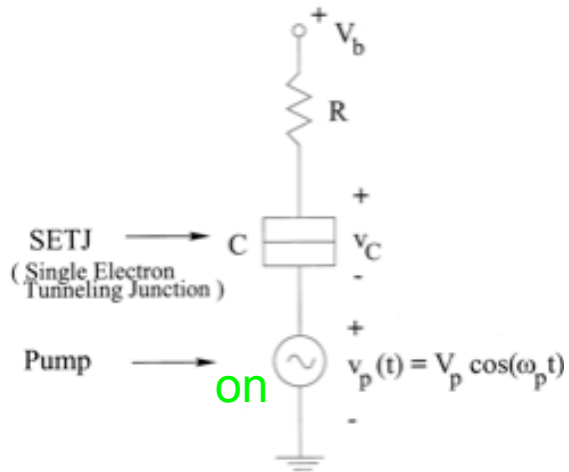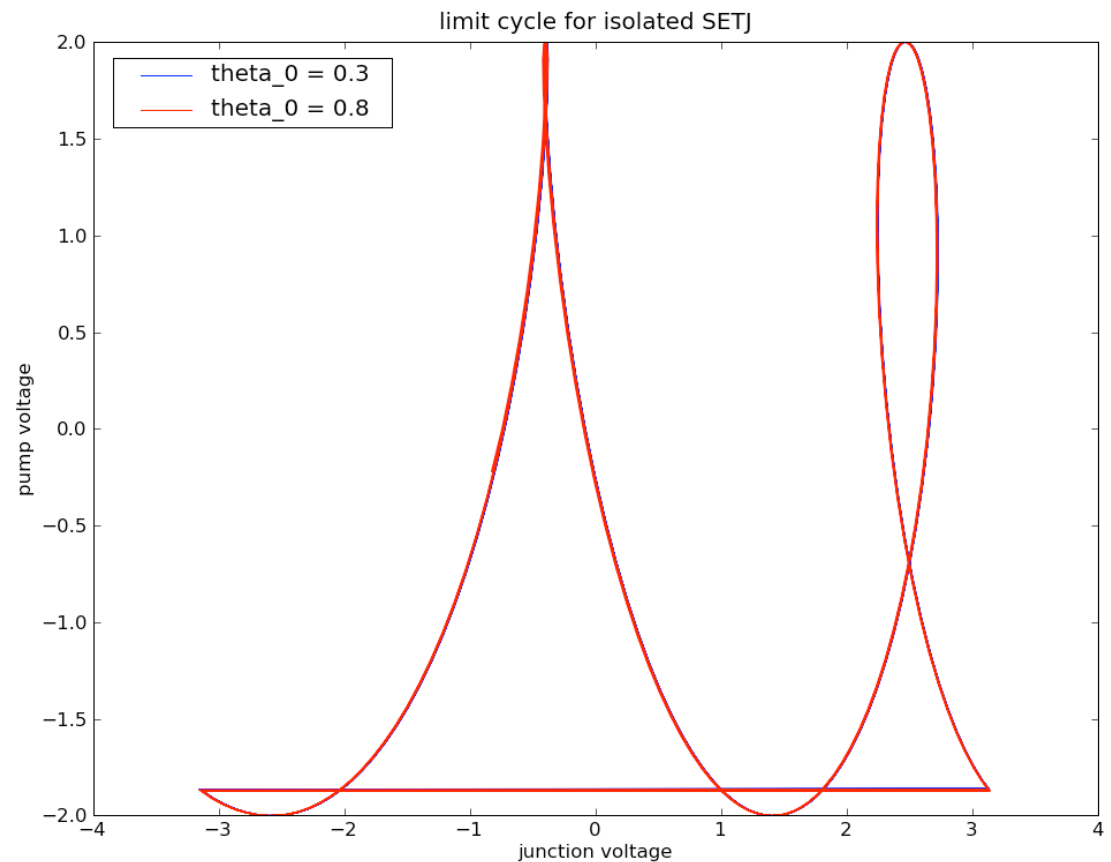
1092   T. Yang & L. O. Chua

V_b

R

SETJ
(Single Electron
Tunneling Junction)   C V_C

Pump   on $v_p(t) = V_p \cos(\omega_p t)$

Fig. 1.   An isolated driven single-electron tunneling junction circuit.

limit cycle for isolated SETJ

- theta_0 = 0.3
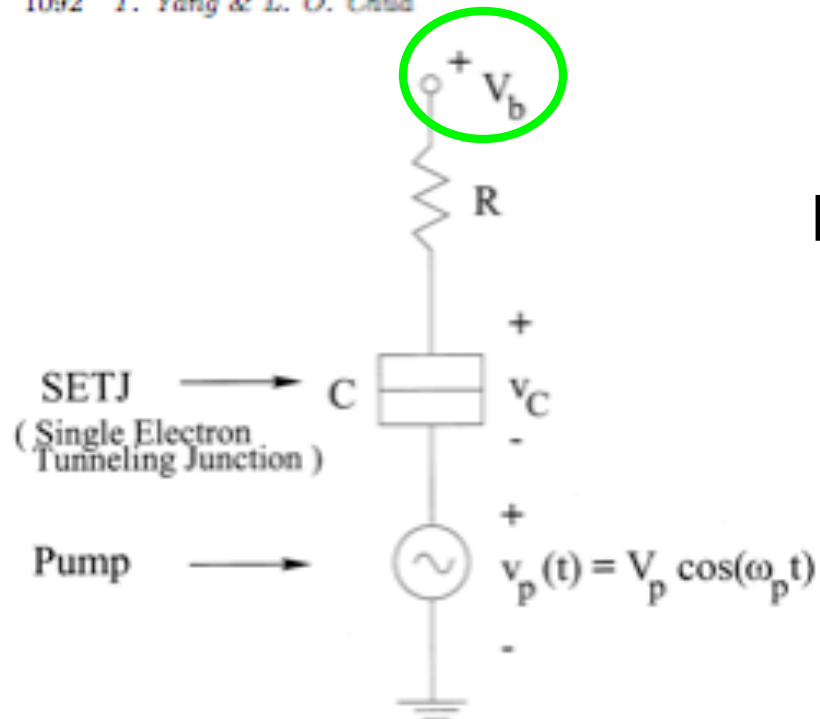- theta_0 = 0.8

pump voltage

junction voltage

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
a = 1.77, b = 2, gamma = 0.333333

junction voltage for theta_0 = 0.3
junction voltage for theta_0 = 0.8
pump voltage

theta = 2 pi C v_C / q

tau = omega_p t

The isolated SETJ circuit...

$+$ V_b

R

$+$

SETJ $\longrightarrow$ C v_C
(Single Electron
Tunneling Junction)
$-$

$+$

Pump $\longrightarrow$ $v_p(t) = V_p \cos(\omega_p t)$

$-$

Fig. 1.   An isolated driven single-electron tunneling junction circuit.
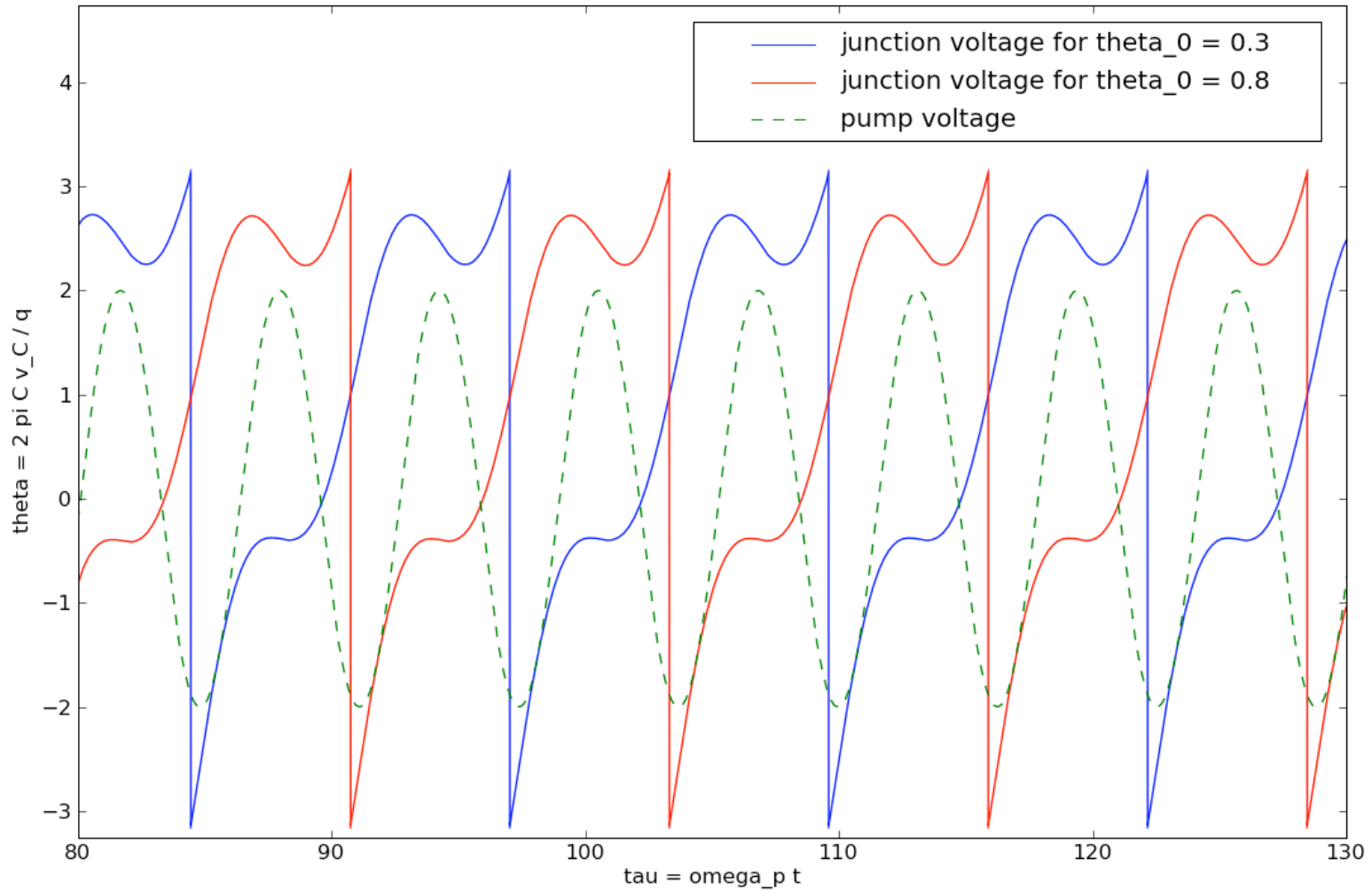
Next parameter to mess with:

$$\alpha = V_b \, / \, V_t$$

$$(V_t = e \, / \, 2C)$$

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
a = 1.95, b = 2, gamma = 0.333333

Legend:
- junction voltage for theta_0 = -0.3
- junction voltage for theta_0 = 0.8
- pump voltage

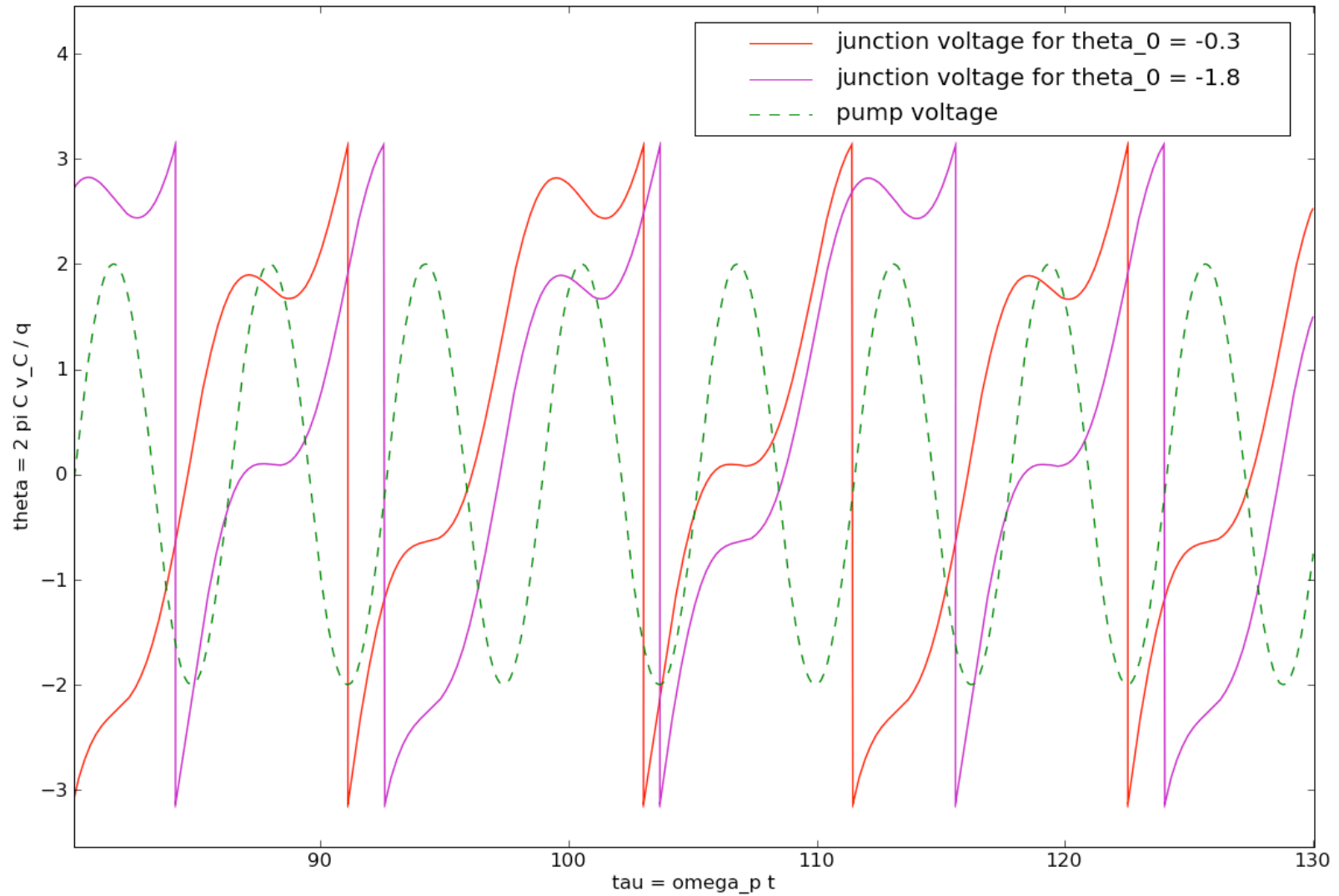y-axis: theta = 2 pi C v_C / q
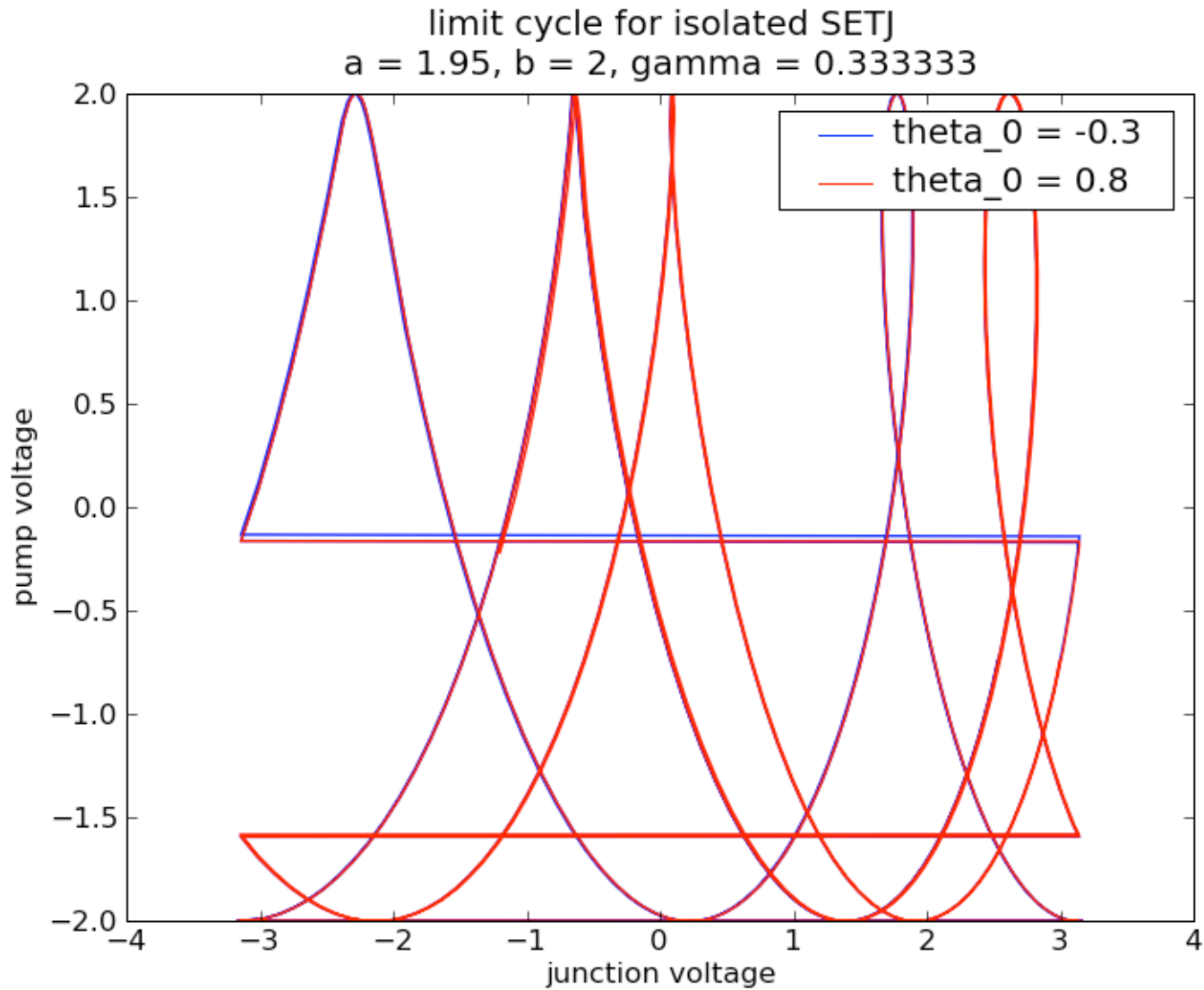
x-axis: tau = omega_p t

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
a = 1.95, b = 2, gamma = 0.333333

Legend:
- junction voltage for theta_0 = -0.3
- junction voltage for theta_0 = -1.8
- pump voltage

y-axis: theta = 2 pi C v_C / q

x-axis: tau = omega_p t

The isolated SETJ circuit...

limit cycle for isolated SETJ
a = 1.95, b = 2, gamma = 0.333333

The isolated SETJ circuit...

time series for isolated SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eqns
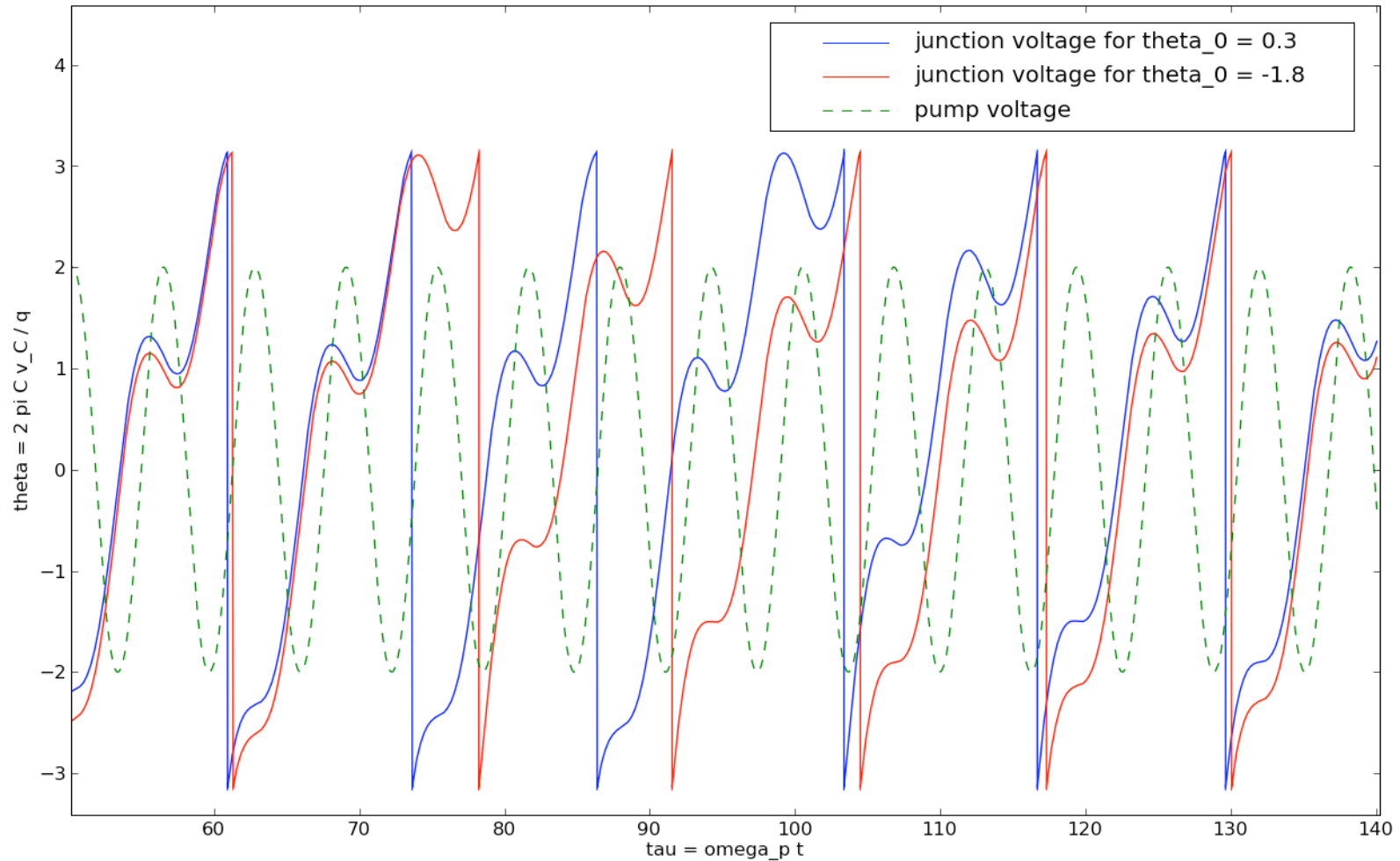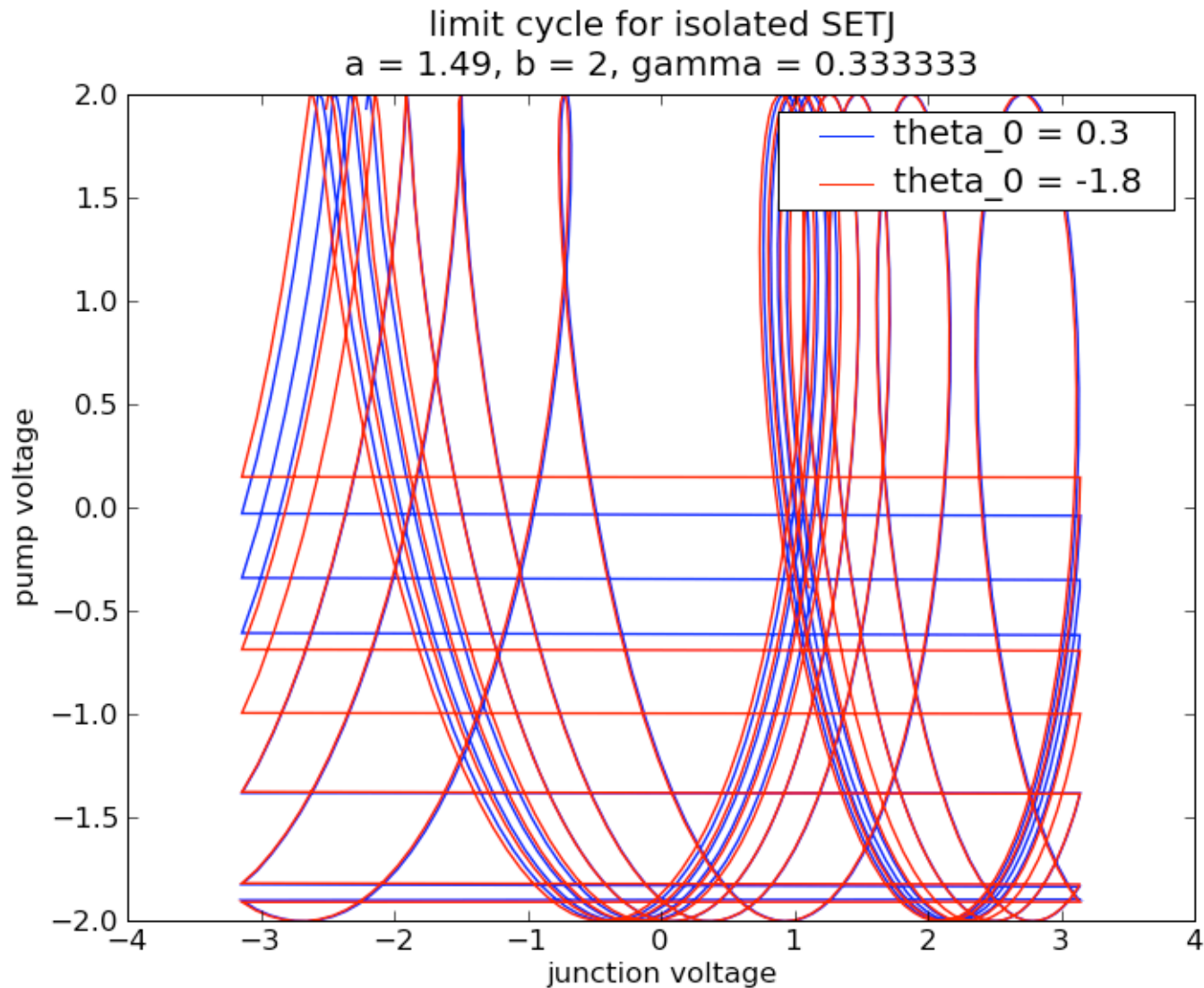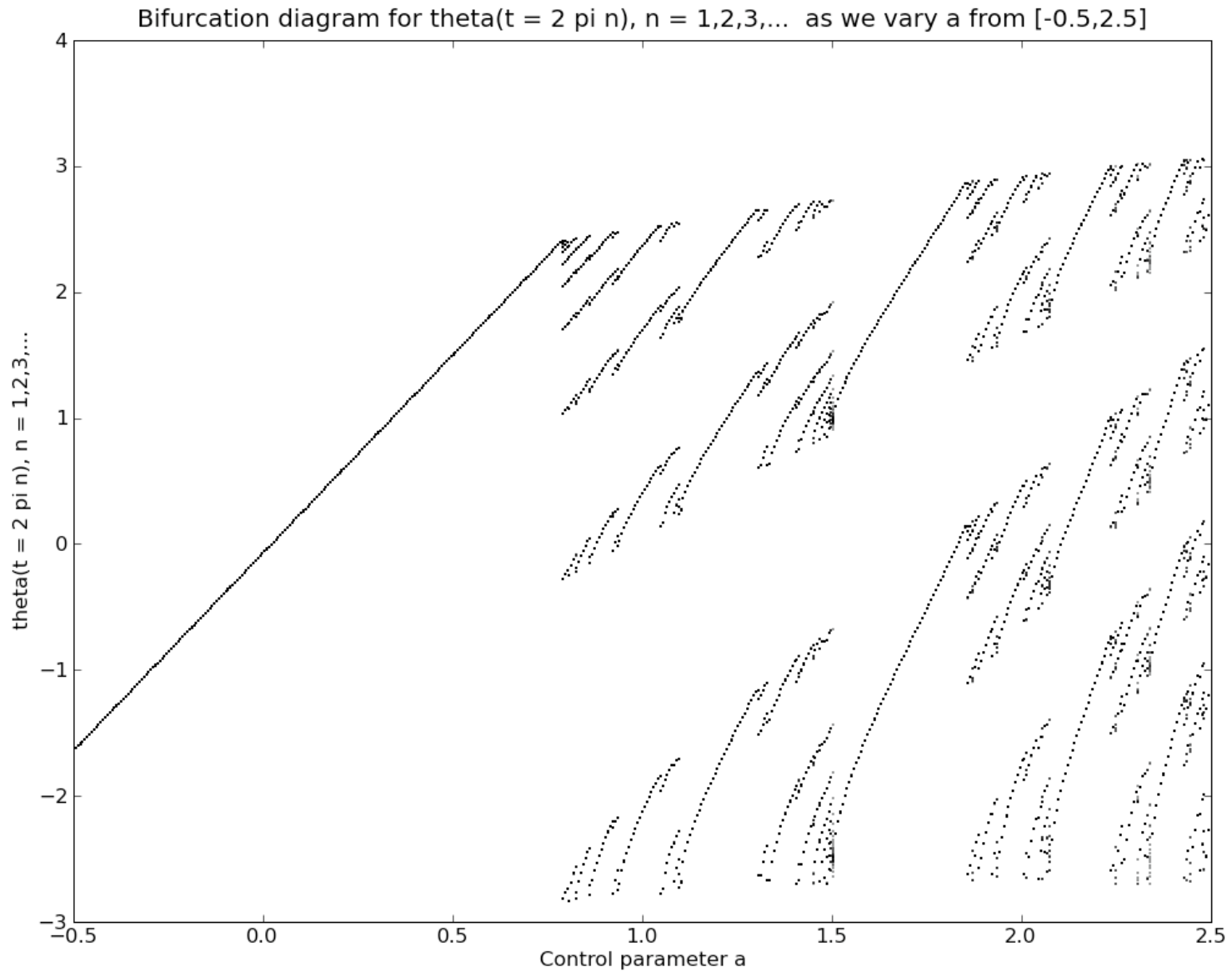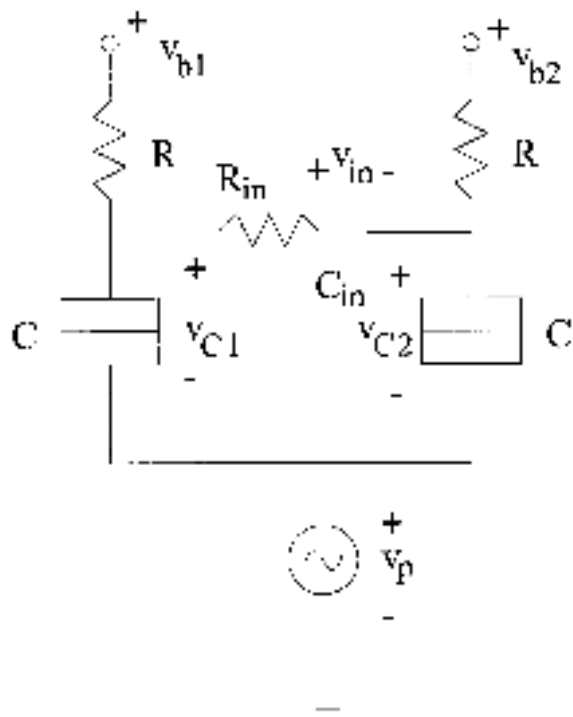a = 1.49, b = 2, gamma = 0.333333

Legend:
- junction voltage for theta_0 = 0.3
- junction voltage for theta_0 = -1.8
- pump voltage

y-axis: theta = 2 pi C v_C / q

x-axis: tau = omega_p t

The isolated SETJ circuit...

limit cycle for isolated SETJ
a = 1.49, b = 2, gamma = 0.333333

The isolated SETJ circuit...

Bifurcation diagram for theta(t = 2 pi n), n = 1,2,3,... as we vary a from [-0.5,2.5]
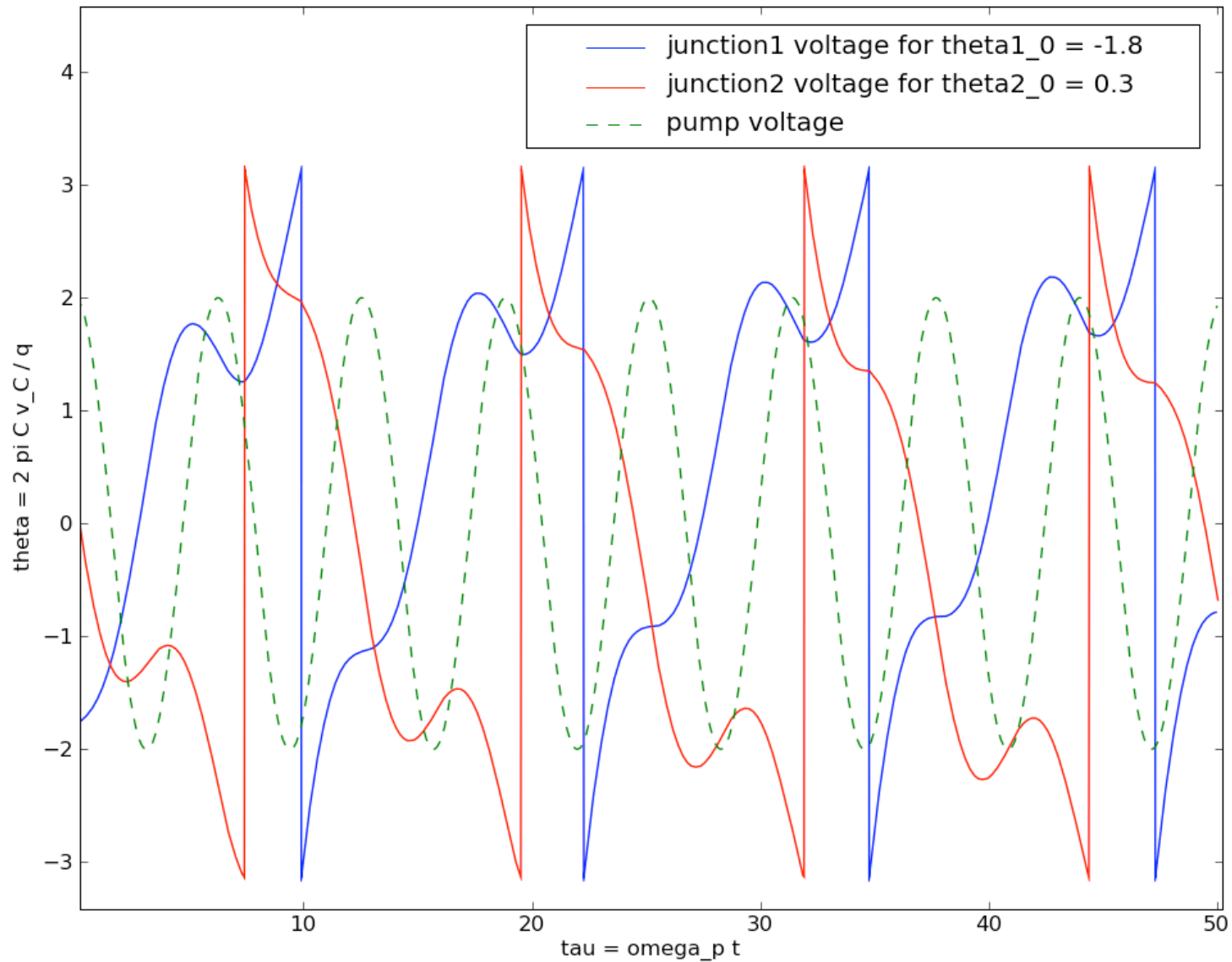
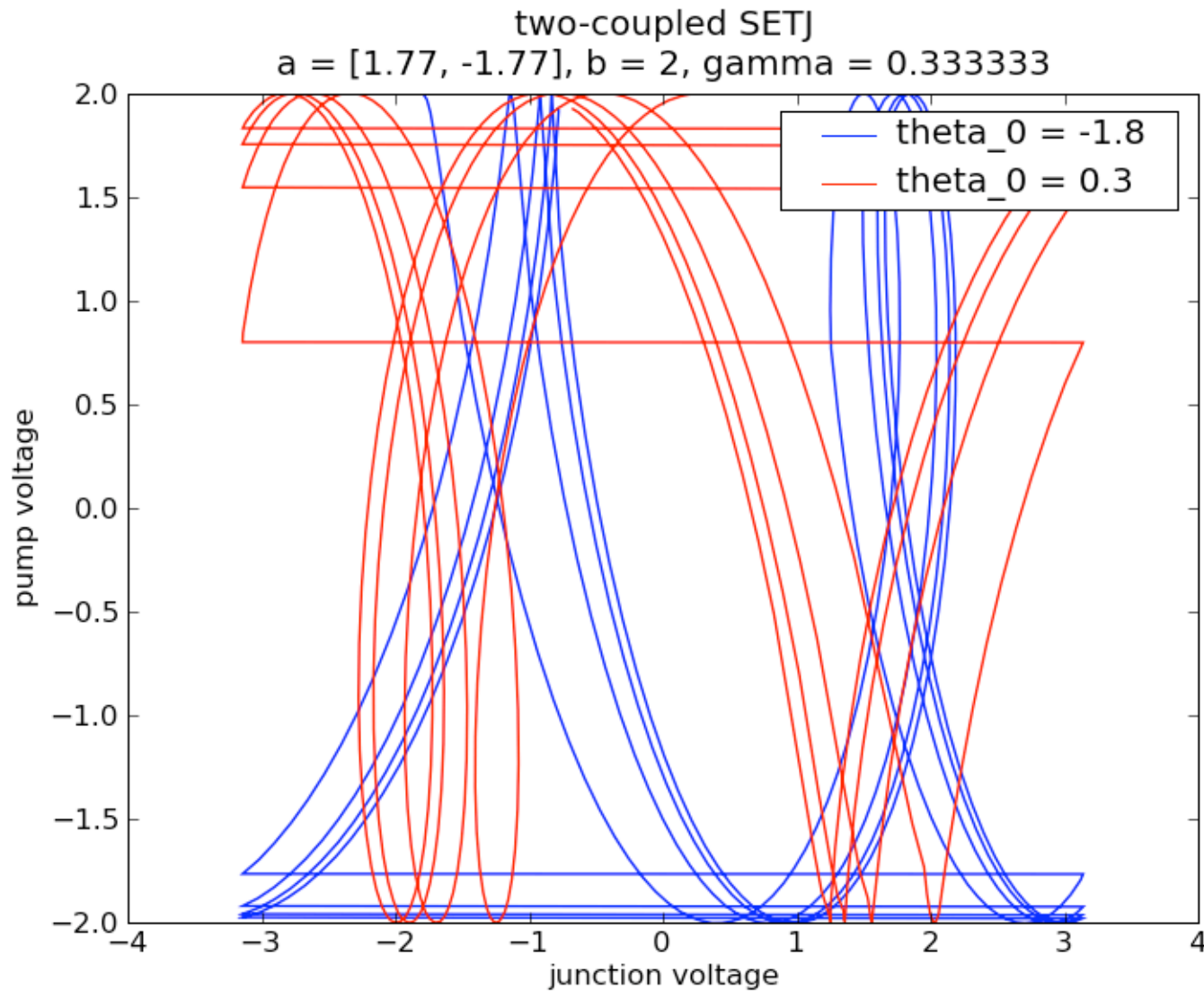# And now, for a two-coupled SETJ circuit

And now, for a two-coupled SETJ circuit

time series for two-coupled SETJ voltages, using fourth-order Runge-Kutta integration and impulsive difference eq
a = [1.77, -1.77], b = 2, gamma = 0.333333

And now, for a two-coupled SETJ circuit

two-coupled SETJ
a = [1.77, -1.77], b = 2, gamma = 0.333333

And now, for a two-coupled SETJ circuit

phase portrait for two-coupled SETJ
a = [1.77, -1.77], b = 2, gamma = 0.333333

r_0 = theta1_0 - theta2_0 = -2.1